RD Sharma Solutions Class 11 Maths Chapter 9 Ex 9.3

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 1

We have,

$\sin^2 72^\circ - \sin^2 60^\circ.$

$= \sin^2(90^\circ - 18^\circ) - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \cos^2 18^\circ - \frac{3}{4}$

$= \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2 - \frac{3}{4} \quad \left[\because \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}\right]$

$= \frac{10+2\sqrt{5}}{16} - \frac{3}{4}$

$= \frac{10+2\sqrt{5}-12}{16}$

$= \frac{2\sqrt{5}-2}{16}$

$= \frac{\sqrt{5}-1}{8}$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 2

L.H.S= $\sin^2 24^\circ - \sin^2 6^\circ$

$= \sin(24+6)\sin(24-6) \quad [\because \sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B]$

$= \sin 30^\circ \ \sin 18^\circ$

$= \frac{1}{2} \cdot \frac{\sqrt{5}-1}{4} \quad \left[\because \sin 18^\circ = \frac{\sqrt{5}-1}{4}\right]$

$= \frac{\sqrt{5}-1}{8}$

= RHS

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 3

L.H.S= $\sin^2 42^\circ - \cos^2 78^\circ$

$= \sin^2(90-48) - \cos^2(90-12)$

$= \cos^2 48^\circ - \sin^2 12^\circ$

$= \cos(48+12).\cos(48-12)$

$[\because \cos(A+B).\cos(A-B) = \cos^2 A - \sin^2 B]$

$= \cos 60^\circ.\ \cos 36^\circ$

$= \frac{1}{2} \cdot \frac{\sqrt{5}+1}{4} \quad \left[\because \cos 36^\circ = \frac{\sqrt{5}+1}{4}\right]$

$= \frac{\sqrt{5}+1}{8}$

= RHS

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 4

L.H.S= $\cos 78^\circ.\ \cos 42^\circ.\ \cos 36^\circ$

$= \frac{(2\cos 78^\circ.\cos 42^\circ)}{2}.\cos 36^\circ$

$$= \frac{1}{2}\left(\cos 120^\circ + \cos 36^\circ\right).\cos 36^\circ$$

$$= \frac{1}{2}\left(\frac{-1}{2} + \frac{\sqrt{5}+1}{4}\right)\frac{\sqrt{5}+1}{4}$$

$$= \frac{1}{8}\frac{\left[-2\left(\sqrt{5}+1\right)+5+1+2\sqrt{5}\right]}{4}$$

$$= \frac{1}{8}\left[\frac{4}{4}\right]$$

$$= \frac{1}{8}$$

= RHS

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 5

$$\text{L.H.S} = \cos\frac{\pi}{15}.\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{7\pi}{15}$$

$$= \frac{2\sin\frac{\pi}{15}.\cos\frac{\pi}{15}}{2\sin\frac{\pi}{15}}.\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{7\pi}{15} \quad \left[\text{Divide and multiply by } 2\sin\frac{\pi}{15}\right]$$

$$= \frac{2.\sin\frac{2\pi}{15}}{2.2\sin\frac{\pi}{15}}.\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{7\pi}{15}$$

$$= \frac{2.\sin\frac{4\pi}{15}}{2.4\sin\frac{\pi}{15}}.\cos\frac{4\pi}{15}.\cos\frac{7\pi}{15}$$

$$= \frac{2\sin\frac{8\pi}{15}}{2.8\sin\frac{\pi}{15}}.\cos\left(\frac{7\pi}{15}\right)$$

$$= \frac{\sin\left(\frac{8\pi}{15}+\frac{7\pi}{15}\right)+\sin\left(\frac{8\pi}{15}-\frac{7\pi}{15}\right)}{16\sin\frac{\pi}{15}}$$

$$= \frac{\sin\pi + \sin\pi/15}{16\sin\pi/15}$$

$$= \frac{\sin\pi/15}{16\sin\pi/15} \qquad [\because \sin\pi = 0]$$

$$= \frac{1}{16}$$

= RHS

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 6

$$\cos\frac{5\pi}{15} = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\cos\frac{7\pi}{15} = \cos\left(\pi - \frac{8\pi}{15}\right)$$

$$\cos\frac{7\pi}{15} = -\cos\frac{8\pi}{15}$$

Now $\text{LHS} = \cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{3\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$

$$= \left[\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\left(-\cos\frac{8\pi}{15}\right)\right]\left(\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right)\frac{1}{2}$$

$$= -\frac{2^3}{2^4\sin\frac{\pi}{15}}\left[2\sin\frac{\pi}{15}\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right]$$

$$\times\frac{2}{8\sin\frac{3\pi}{15}}\left(2\sin\frac{3\pi}{15}\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right)$$

$$= -\frac{2^3}{16\sin\frac{\pi}{15}}\left[\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right]\times\frac{2}{8\sin\frac{3\pi}{15}}\left(\sin\frac{6\pi}{15}\cos\frac{6\pi}{15}\right)$$

$$= -\frac{2^{2}}{16\sin\frac{\pi}{15}}\left[2\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right]\times\frac{1}{8\sin\frac{3\pi}{15}}\left(2\sin\frac{6\pi}{15}\cos\frac{6\pi}{15}\right)$$

$$= -\frac{2}{16\sin\frac{\pi}{15}}\left[\sin\frac{8\pi}{15}\cos\frac{8\pi}{15}\right]\frac{\sin\frac{12\pi}{15}}{8\sin\frac{3\pi}{15}}$$

$$= -\frac{1}{16\sin\frac{\pi}{15}}\left(\sin\frac{16\pi}{15}\right)\frac{\sin\frac{12\pi}{15}}{8\sin\frac{3\pi}{15}}$$

$$= -\frac{\sin\left(\pi+\frac{\pi}{15}\right)}{128\sin\frac{\pi}{15}}\times\frac{\sin\left(\pi-\frac{3\pi}{15}\right)}{\sin\frac{3\pi}{15}}$$

$$= -\frac{-\sin\frac{\pi}{15}}{128\sin\frac{\pi}{15}}\times\frac{\sin\frac{3\pi}{15}}{\sin\frac{3\pi}{15}}$$

$$= \frac{1}{128}$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 7

L.H.S $= \cos 6^\circ . \cos 42^\circ . \cos 66^\circ . \cos 78^\circ$

$$= \frac{1}{4}\left(2\cos 6^\circ . \cos 66^\circ\right)\left(2\cos 42^\circ . \cos 78^\circ\right)$$

$$= \frac{1}{4}\left(\cos 72^\circ + \cos 60^\circ\right)\left(\cos 120^\circ + \cos 36^\circ\right)$$

$$= \frac{1}{4}\left(\sin 18^\circ + \frac{1}{2}\right)\left(-\frac{2}{2} + \frac{\sqrt{5}+1}{4}\right)$$

$$= \frac{1}{4}\left(\frac{\sqrt{5}-1}{4} + \frac{1}{2}\right)\left(\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right)$$

$$= \frac{1}{4}\left(\frac{\sqrt{5}-1+2}{4}\right)\left(\frac{\sqrt{5}+1-2}{4}\right)$$

$$= \frac{1}{64}\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)$$

$$= \frac{1}{64}\left(\left(\sqrt{5}\right)^2 - 1^2\right)$$

$$= \frac{1}{64}(5-1)$$

$$= \frac{1}{16}$$

= RHS

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 8

L.H.S= $\sin 6^\circ . \sin 42^\circ . \sin 66^\circ . \sin 78^\circ$

$$= \frac{1}{4}\left(2\sin 6^\circ . \sin 66^\circ\right)\left(2\sin 42^\circ . \sin 78^\circ\right)$$

$$= \frac{1}{4}\left(\cos 60 - \cos 72^\circ\right)\left(\cos 36^\circ - \cos 120^\circ\right)$$

$$= \frac{1}{4}\left(\frac{1}{2} - \sin 18^\circ\right)\left(\frac{\sqrt{5}+1}{4} + \frac{1}{2}\right)$$

$$= \frac{1}{4}\left(\frac{1}{2} - \frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4} + \frac{1}{2}\right)$$

$$= \frac{1}{4}\left(\frac{2-\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}+1+2}{4}\right)$$

$$= \frac{1}{64}\left(3^2 - \sqrt{5}^2\right)$$

$$= \frac{1}{64}(9-5)$$

$$= \frac{1}{16}$$

= RHS

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 9

L.H.S= $\cos 36^\circ.\cos 42^\circ.\cos 60^\circ.\cos 78^\circ$

$$= \frac{1}{2}\cos 36^\circ.\ \cos 60^\circ.\ \left(2\cos 42^\circ.\cos 78^\circ\right)$$
$$= \frac{1}{2}\left(\frac{\sqrt{5}+1}{4}\right).\frac{1}{2}\left(\cos 120^\circ + \cos 36^\circ\right)$$
$$= \frac{\left(\sqrt{5}+1\right)}{16}\left(\frac{-1}{2}+\frac{\sqrt{5}+1}{4}\right)$$
$$= \frac{\left(\sqrt{5}+1\right)}{16}\left(\frac{-2+\sqrt{5}+1}{4}\right)$$
$$= \frac{\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)}{64}$$
$$= \frac{5-1}{64}$$
$$= \frac{1}{16}$$
= RHS

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 10

L.H.S,

$\sin 36^\circ.\sin 72^\circ.\sin 108^\circ.\sin 144^\circ$ $\left[\because\ \sin 144^\circ = \sin\left(180^\circ - 36^\circ\right) = \sin 36^\circ \text{ and } \sin 108^\circ = \sin\left(180^\circ - 78^\circ\right) = \sin 72^\circ\right]$

$$= \sin 36^\circ.\sin 72^\circ.\sin 72^\circ.\sin 36^\circ$$
$$= \frac{1}{4}\left(2\sin 36^\circ.\sin 72^\circ\right)^2$$
$$= \frac{1}{4}\left(2\sin 36^\circ\cos 18^\circ\right)^2 \quad \left[\because\ \sin 72^\circ = \cos 18^\circ\right]$$
$$= \frac{4}{4}\left(\frac{\sqrt{10-2\sqrt{5}}}{4}.\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2$$
$$= \frac{1}{64}\left(10-2\sqrt{5}\right)\left(10+2\sqrt{5}\right)$$
$$= \frac{100-20}{64\times 4}$$
$$= \frac{80}{256}$$
$$= \frac{5}{16}$$
= RHS