RD Sharma Solutions Class 11 Maths Chapter 9 Ex 9.2

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 1

L.H.S,

$$\sin 5\theta = \sin(3\theta + 2\theta)$$

$= \sin 3\theta \cos 2\theta + \cos 3\theta . \sin 2\theta$

$= (3\sin\theta - 4\sin^3\theta)(1 - 2\sin^2\theta) + (4\cos^3\theta - 3\cos\theta) 2\sin\theta\cos\theta.$

$= 3\sin\theta - 4\sin^3\theta - 6\sin^3\theta + 8\sin^5\theta + (8\cos^4\theta - 6\cos^2\theta)\sin\theta$

$= 3\sin\theta - 10\sin^3\theta + 8\sin^5\theta + 8\sin\theta\left((1-\sin^2\theta)^2 - 6\sin\theta(1-\sin^2\theta)\right)$

$= 3\sin\theta - 10\sin^3\theta + 8\sin^5\theta + 8\sin\theta - 16\sin^3\theta + 8\sin^5\theta - 6\sin\theta + 6\sin^3\theta$

$= 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta = \text{RHS}$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 2

Consider the L.H.S of the given equation

$4(\cos^3 10^\circ + \sin^3 20^\circ) = 3(\cos 10^\circ + \sin 20^\circ)$

Since $\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$

and $\sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin 3.20^\circ = \cos 3.10^\circ$

$\Rightarrow \quad 3\sin 20^\circ - 4\sin^3 20^\circ = 4\cos^3 10^\circ - 3\cos 10^\circ$

$\Rightarrow \quad 4(\cos^3 10^\circ + \sin^3 20^\circ) = 3(\cos 10^\circ + \sin 20^\circ)$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 3

$\cos^3\theta \sin 3\theta + \sin^3\theta \cos 3\theta = \frac{3}{4}\sin 4\theta$

LHS $= \cos^3\theta\sin 3\theta + \sin^3\theta\cos 3\theta$

$= \left(\frac{\cos 3\theta + 3\cos\theta}{4}\right)\sin 3\theta + \left(\frac{3\sin\theta - \sin 3\theta}{4}\right)\cos 3\theta \quad \left\{\because \begin{array}{l}\sin 3\theta = 3\sin\theta - 4\sin^3\theta \\ \cos 3\theta = 4\cos^3\theta - 3\cos\theta\end{array}\right\}$

$= \frac{1}{4}[3(\sin 3\theta\cos\theta + \sin\theta\cos 3\theta) + \cos 3\theta\sin 3\theta - \sin 3\theta\cos 3\theta]$

$= \frac{1}{4}[3\sin(3\theta + \theta) + 0]$

$= \frac{3}{4}\sin 4\theta$

So,

$\cos^3\theta\sin 3\theta + \sin^3\theta\cos 3\theta = \frac{3}{4}\sin 4\theta$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 4

We have to prove that

$\sin 5A = 5\cos^4 A\sin A - 10\cos^2 A\sin^3 A + \sin^5 A$

L.H.S= $\sin 5A = \sin(3A + 2A)$

$= \sin 3A\cos 2A + \cos 3A.\sin 2A$

$= (3\sin A - 4\sin^3 A)(2\cos^2 A - 1) + (4\cos^3 A - 3\cos A)2\sin A\cos A.$

$= -3\sin A + 4\sin^3 A + 6\sin A\cos^2 A - 8\sin^3 A\cos^2 A + 8\cos^4 A\sin A - 6\cos^2 A\sin A$

$= 8\cos^4 A\sin A - 8\sin^3 A\cos^2 A - 3\sin A + 4\sin^3 A$

$= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - 3\sin A + 3\cos^4 A\sin A + 4\sin^3 A + 2\sin^3 A\cos^2 A$

$= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - 3\sin A(1 - \cos^4 A) + 2\sin^3 A(2 + \cos^2 A)$

$= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - 3\sin A(1 - \cos^2 A)(1 + \cos^2 A) + 2\sin^3 A(2 + \cos^2 A)$

$= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - 3\sin^3 A(1 + \cos^2 A) + 2\sin^3 A(2 + \cos^2 A)$

$= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - \sin^3 A[3(1 + \cos^2 A) - 2(2 + \cos^2 A)]$

$= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - \sin^3 A[3 + 3\cos^2 A - 4 - 2\cos^2 A]$

$= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - \sin^3 A[\cos^2 A - 1]$

$= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A + \sin^5 A$

= RHS

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 5

$\tan A \times \tan(A+60°) + \tan A \times \tan(A-60°) + \tan(A+60°)\tan(A-60°)$

$= \tan(A)\dfrac{[\tan(A) - \tan(60°)]}{[1+\tan(A)\tan(60°)]}$

$+ \tan(A)\dfrac{[\tan(A) + \tan(60°)]}{[1 - \tan(A)\tan(60°)]}$

$+ \{\dfrac{[\tan(A) - \tan(60°)]}{[1+\tan(A)\tan(60°)]}\}\{\dfrac{[\tan(A)+\tan(60°)]}{[1-\tan(A)\tan(60°)]}\}$

$= \tan(A)\dfrac{[\tan(A) - \tan(60°)][1 - \tan(A)\tan(60°)]}{[1 - \tan^2(A)\tan^2(60°)]}$

$+ \tan(A)\dfrac{[\tan(A) + \tan(60°)][1 + \tan(A)\tan(60°)]}{[1 - \tan^2(A)\tan^2(60°)]}$

$+ \dfrac{[\tan(A) - \tan(60°)][\tan(A) + \tan(60°)]}{[1 - \tan^2(A)\tan^2(60°)]}$

$= \tan(A)\dfrac{[\tan(A) - \sqrt{3}][1 - \sqrt{3}\tan(A)]}{[1 - 3\tan^2(A)]}$

$+ \tan(A)\dfrac{[\tan(A) + \sqrt{3}][1 + \sqrt{3}\tan(A)]}{[1 - 3\tan^2(A)]}$

$+ \dfrac{[\tan(A) - \sqrt{3}][\tan(A) + \sqrt{3}]}{[1 - 3\tan^2(A)]}$

$= \tan(A)\dfrac{[4\tan(A) - \sqrt{3} - \sqrt{3}\tan^2(A)]}{[1 - 3\tan^2(A)]}$

$+ \tan(A)\dfrac{[4\tan(A) + \sqrt{3} + \sqrt{3}\tan^2(A)]}{[1 - 3\tan^2(A)]}$

$+ \dfrac{[\tan^2(A) - 3]}{[1 - 3\tan^2(A)]}$

$= \dfrac{[9\tan^2(A) - 3]}{[1 - 3\tan^2(A)]}$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 6

$\tan A + \tan(60° + A) - \tan(60° - A) = 3\tan 3A$

$\text{LHS} = \tan A + \tan(60° + A) - \tan(60° - A)$

$= \tan A + \dfrac{\tan 60° + \tan A}{1 - \tan 60° \tan A} - \dfrac{\tan 60° - \tan A}{1 + \tan 60° \tan A}$

$= \tan A + \dfrac{\sqrt{3} + \tan A}{1 - \sqrt{3}\tan A} - \dfrac{\sqrt{3} - \tan A}{1 + \sqrt{3}\tan A}$

$= \tan A + \left[\dfrac{\sqrt{3} + 3\tan A + \tan A + \sqrt{3}\tan^2 A - \sqrt{3} + 3\tan A + \tan A - \sqrt{3}\tan^2 A}{(1 - \sqrt{3}\tan A)(1 + \sqrt{3}\tan A)}\right]$

$= \tan A + \dfrac{8\tan A}{1 - 3\tan^2 A}$

$= \dfrac{\tan A - 3\tan^3 A + 8\tan A}{1 - 3\tan^2 A}$

$= \dfrac{9\tan A - 3\tan^3 A}{1 - 3\tan^2 A}$

$= 3\left(\dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}\right)$

$= 3\tan 3A$

so,

$\tan A + \tan(60° + A) - \tan(60° - A) = 3\tan 3A$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 7

$\text{LHS} = \cot A + \cot(60° + A) - \cot(60° - A)$

$= \dfrac{1}{\ldots} + \dfrac{1}{\ldots} - \dfrac{1}{\ldots}$

$$\tan A \quad \tan(60^\circ + A) \quad \tan(60^\circ - A)$$

$$= \frac{1}{\tan A} + \frac{1 - \sqrt{3}\tan A}{\sqrt{3} + \tan A} - \frac{1 + \sqrt{3}\tan A}{\sqrt{3} - \tan A}$$

$$= \frac{1}{\tan A} - \frac{8\tan A}{3 - \tan^2 A}$$

$$= \frac{3 - \tan^2 A - 8\tan^2 A}{3\tan A - \tan^3 A}$$

$$= \frac{3 - 9\tan^2 A}{3\tan A - \tan^3 A}$$

$$= 3\left(\frac{1 - 3\tan^2 A}{3\tan A - \tan^3 A}\right)$$

$$= \frac{3}{\tan 3A}$$

$$= 3\cot 3A$$

= RHS

LHS = RHS

Hence proved.

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 8

$$\text{LHS} = \cot A + \cot(60^\circ + A) + \cot(120^\circ + A)$$

$$= \cot A + \cot(60^\circ + A) - \cot\left[180^\circ - (120^\circ + A)\right]$$

$$\{\text{since } -\cot\theta = \cot(180^\circ - \theta)\}$$

$$= \cot A + \cot(60^\circ + A) - \cot(60^\circ - A)$$

$$= \frac{1}{\tan A} + \frac{1}{\tan(60^\circ + A)} - \frac{1}{\tan(60^\circ - A)}$$

$$= \frac{1}{\tan A} + \frac{1 - \sqrt{3}\tan A}{\sqrt{3} + \tan A} - \frac{1 + \sqrt{3}\tan A}{\sqrt{3} - \tan A}$$

$$= \frac{1}{\tan A} - \frac{8\tan A}{3 - \tan^2 A}$$

$$= \frac{3 - \tan^2 A - 8\tan^2 A}{3\tan A - \tan^3 A}$$

$$= \frac{3 - 9\tan^2 A}{3\tan A - \tan^3 A}$$

$$= \frac{3(1 - 3\tan^2 A)}{3\tan A - \tan^3 A}$$

$$= \frac{3}{\tan 3A}$$

$$= 3\cot 3A$$

LHS = RHS

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 9

$$\text{LHS} = \sin^3 A + \sin^3\left(\frac{2\pi}{3} + A\right) + \sin^3\left(\frac{4\pi}{3} + A\right)$$

$$\left\{\text{we know that } \sin^3 A = \frac{3\sin A - \sin 3A}{4}\right\}$$

$$= \left(\frac{3\sin A - \sin 3A}{4}\right) + \left\{\frac{3\sin\left(\frac{2\pi}{3} + A\right) - \sin 3\left(\frac{2\pi}{3} + A\right)}{4}\right\} + \left\{\frac{3\sin\left(\frac{4\pi}{3} + A\right) - \sin 3\left(\frac{4\pi}{3} + A\right)}{4}\right\}$$

$$\left[\frac{3\sin A - \sin 3A}{\cdots}\right] \quad \left[3\sin\left[\pi\left(\frac{2\pi}{3} + A\right)\right] - \sin(2\pi + 3A)\right] \quad \left[3\sin\left(\pi + \left(\frac{\pi}{3} + A\right)\right) - \sin(4\pi + 3A)\right]$$

$= \left[\frac{\ldots}{4}\right] + \left[\frac{\ldots}{4}\right] + \left[\frac{\ldots}{4}\right]$

$= \frac{1}{4}\left\{\left[3\sin A - \sin 3A\right] + \left[3\sin\left(\frac{\pi}{3} - A\right) - \sin 3A\right] - \left[3\sin\left(\frac{\pi}{3} + A\right) + \sin 3A\right]\right\}$

$= \frac{1}{4}\left[3\sin A - \sin 3A + 3\sin\left(\frac{\pi}{3} - A\right) - 3\sin\left(\frac{\pi}{3} + A\right) - \sin 3A - \sin 3A\right]$

$= \frac{1}{4}\left[3\sin A - 3\sin 3A + 3\left(\sin\left(\frac{\pi}{3} - A\right) - \sin\left(\frac{\pi}{3} + A\right)\right)\right]$

$= \frac{1}{4}\left[3\sin A - 3\sin 3A + 3\left\{2\cos\frac{\frac{\pi}{3} - A + \frac{\pi}{3} + A}{2}\sin\frac{\frac{\pi}{3} - A - \frac{\pi}{3} - A}{2}\right\}\right]$

$= \frac{1}{4}\left[3\sin A - 3\sin 3A + 6\cos\frac{\pi}{3}\sin(-A)\right]$

$= \frac{1}{4}\left[3\sin A - 3\sin 3A - 3\sin A\right]$

$= -\frac{3}{4}\sin 3A$

= RHS

LHS = RHS

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 10

$\left|\sin\theta\sin(60 - \theta)\sin(60 + \theta)\right|$

$= \left|\sin\theta\left(\sin^2 60 - \sin^2\theta\right)\right|$

$\left\{\text{since } \sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B\right\}$

$= \left|\sin\theta\left(\frac{3}{4} - \sin^2\theta\right)\right|$

$= \left|\frac{1}{4}\sin\theta\left(3 - 4\sin^2\theta\right)\right|$

$= \left|\frac{1}{4}\sin 3\theta\right|$

$= \frac{1}{4}\left|\sin 3\theta\right|$

$\le \frac{1}{4}$ $\qquad \left\{\sin\theta\left|\sin 3\theta\right|\right|\le\left|\right\}$

So,

$\left|\sin\theta\sin\left(60^\circ - \theta\right)\sin\left(60^\circ + \theta\right)\right| \le \frac{1}{4}$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 11

$\left|\cos\theta\cos\left(60^\circ - \theta\right)\cos\left(60^\circ + \theta\right)\right|$

$= \left|\cos\theta\left(\cos^2 60^\circ - \sin^2\theta\right)\right|$

$\left\{\text{since } \cos(A - B)\cos(A + B) = \cos^2 A - \sin^2 B\right\}$

$= \left|\cos\theta\left(\frac{1}{4} - \sin^2\theta\right)\right|$

$= \left|\cos\theta\frac{1}{4}\left(1 - 4\sin^2\theta\right)\right|$

$= \left|\frac{1}{4}\cos\theta\left(1 - 4\left(1 - \cos^2\theta\right)\right)\right|$

$= \left|\frac{1}{4} \ldots\right|$

$$= \left|\frac{1}{4}\cos\theta\left(-3+4\cos^2\theta\right)\right|$$

$$= \left|\frac{1}{4}\left(4\cos 3\theta - 3\cos\theta\right)\right|$$

$$= \left|\frac{1}{4}\cos 3\theta\right|$$

$$\le \frac{1}{4} \qquad \left\{\text{since } \left|\cos 3\theta\right| \le 1\right\}$$

So,

$$\left|\cos\theta\cos\left(60^\circ-\theta\right)\cos\left(60^\circ+\theta\right)\right| \le \frac{1}{4}$$