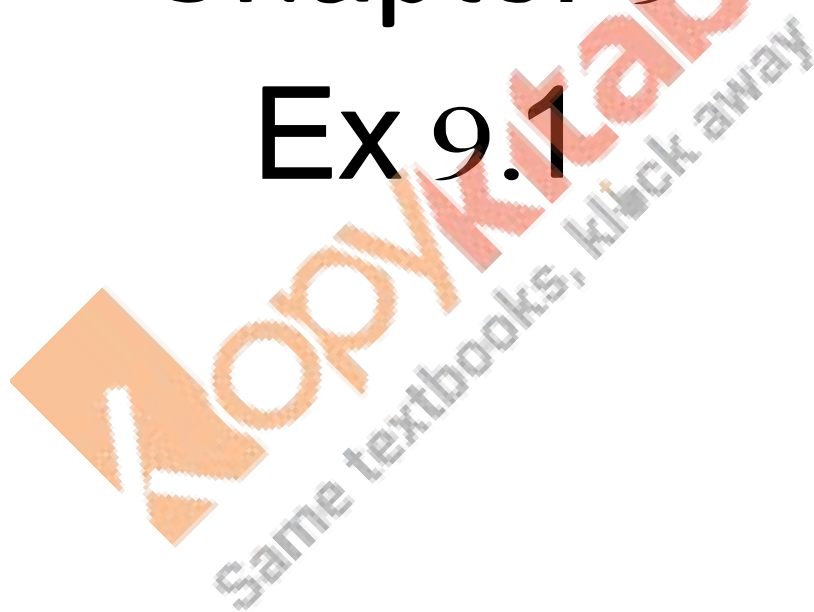


RD Sharma
Solutions
Class 11 Maths
Chapter 9
Ex 9.1



Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 1

We have,

$$\begin{aligned}\frac{\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}} &= \frac{\sqrt{2\sin^2 \theta}}{\sqrt{2\cos^2 \theta}} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta = \text{RHS}\end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 2

LHS,

$$\begin{aligned}\frac{\sin 2\theta}{1-\cos 2\theta} &= \frac{2\sin \theta \cos \theta}{2\sin^2 \theta} \\ &= \frac{\cos \theta}{\sin \theta} \\ &= \cot \theta = \text{RHS}\end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 3

LHS,

$$\begin{aligned}\frac{\sin 2\theta}{1+\cos 2\theta} &= \frac{2\sin \theta \cdot \cos \theta}{2\cos^2 \theta} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta = \text{RHS}\end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 4

LHS,

$$\begin{aligned}&\sqrt{2+\sqrt{2+2\cos 4\theta}} \\ &= \sqrt{2+\sqrt{2(1+\cos 4\theta)}} \\ &= \sqrt{2+\sqrt{2 \cdot 2\cos^2 2\theta}} \\ &= \sqrt{2+2\cos 2\theta} \\ &= \sqrt{2(1+\cos 2\theta)} \\ &= \sqrt{2 \cdot 2\cos^2 \theta} \\ &= 2\cos \theta = \text{RHS}\end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 5

LHS,

$$\begin{aligned}&\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta} \\ &= \frac{2\sin^2 \theta+2\sin \theta \cdot \cos \theta}{2\cos^2 \theta+2\sin \theta \cdot \cos \theta} \\ &= \frac{2\sin \theta(\sin \theta+\cos \theta)}{2\cos \theta(\cos \theta+\sin \theta)} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta = \text{RHS}\end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 6

LHS,

$$\begin{aligned}
 &= \frac{\sin \theta + \sin 2\theta}{1 + \cos \theta + \cos 2\theta} \\
 &= \frac{\sin \theta + 2 \sin \theta \cdot \cos \theta}{\cos \theta + (1 + \cos 2\theta)} \\
 &= \frac{\sin \theta (1 + 2 \cos \theta)}{\cos \theta + 2 \cos^2 \theta} \\
 &= \frac{\sin \theta (1 + 2 \cos \theta)}{\cos \theta (1 + 2 \cos \theta)} \\
 &= \frac{\sin \theta}{\cos \theta} \\
 &= \tan \theta \\
 &= \text{RHS}
 \end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 7

LHS,

$$\begin{aligned}
 &\frac{\cos 2\theta}{1 + \sin 2\theta} \\
 &= \frac{\cos^2 \theta - \sin^2 \theta}{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta} \quad \left[\begin{array}{l} \because \cos 2\theta = \cos^2 \theta - \sin^2 \theta \\ \& \sin^2 \theta + \cos^2 \theta = 1 \end{array} \right] \\
 &= \frac{(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)}{(\cos \theta + \sin \theta)^2} \quad [\because a^2 - b^2 = (a + b)(a - b)] \\
 &= \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}
 \end{aligned}$$

Dividing numerator and denominator by $\cos \theta$

$$\begin{aligned}
 &= \frac{1 - \tan \theta}{1 + \tan \theta} \\
 &= \tan \left(\frac{\pi}{4} - \theta \right) = \text{RHS}
 \end{aligned}$$

Note: $\tan \left(\frac{\pi}{4} - \theta \right) = \frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \tan \theta}$

$$= \frac{1 - \tan \theta}{1 + \tan \theta}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 8

$$\begin{aligned}
 &\frac{\cos \theta}{1 - \sin \theta} \\
 &= \frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{\sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} - 2 \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}} \quad \left[\begin{array}{l} \because \cos 2A = \cos^2 A - \sin^2 A \& \\ \sin^2 A + \cos^2 A = 1 \end{array} \right] \\
 &= \frac{(\cos \frac{\theta}{2} - \sin \frac{\theta}{2})(\cos \frac{\theta}{2} + \sin \frac{\theta}{2})}{(\cos \frac{\theta}{2} - \sin \frac{\theta}{2})^2} \\
 &= \frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}
 \end{aligned}$$

Dividing numerator and denominator by $\cos \frac{\theta}{2}$

$$= \frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}}$$

$$= \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \text{RHS}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 9

LHS,

$$\begin{aligned} & \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{5\pi}{8} + \cos^2 \frac{7\pi}{8} \\ &= \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \left(\pi - \frac{3\pi}{8}\right) + \cos^2 \left(\pi - \frac{\pi}{8}\right) \\ &= \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{\pi}{8} \\ &= 2 \left(\cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} \right) \\ &= 2 \left(\cos^2 \frac{\pi}{8} + \cos^2 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) \right) \\ &= 2 \left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \right) \\ &= 2 \\ &= \text{RHS} \end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 10

LHS,

$$\begin{aligned} & \sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{5\pi}{8} + \sin^2 \frac{7\pi}{8} \\ &= \sin^2 \frac{\pi}{8} + \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) + \sin^2 \frac{5\pi}{8} + \sin^2 \left(\pi - \frac{\pi}{8} \right) \\ &= \sin^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} + \sin^2 \left(\pi - \frac{3\pi}{8} \right) + \sin^2 \frac{\pi}{8} \\ &= 1 + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{\pi}{8} \\ &= 1 + \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) + \sin^2 \frac{\pi}{8} \\ &= 1 + \cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \\ &= 1 + 1 \\ &= 2 \\ &= \text{RHS} \end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 11

LHS,

$$\begin{aligned} & (\cos \lambda + \cos \beta)^2 + (\sin \lambda + \sin \beta)^2 \\ &= \cos^2 \lambda + \cos^2 \beta + 2 \cos \lambda \cos \beta + \sin^2 \lambda + \sin^2 \beta + 2 \sin \lambda + \sin \beta \\ &= (\cos^2 \lambda + \sin^2 \lambda) + (\cos^2 \beta + \sin^2 \beta) + 2 (\cos \lambda \cos \beta + \sin \lambda \sin \beta) \\ &= 1 + 1 + 2 \cos (\lambda - \beta) \\ &= 2 + 2 \cos (\lambda - \beta) \\ &= 2 (1 + \cos (\lambda - \beta)) \\ &= 2 \cdot 2 \cos^2 \left(\frac{\lambda - \beta}{2} \right) \\ &= 4 \cos^2 \left(\frac{\lambda - \beta}{2} \right) \\ &= \text{RHS} \end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 12

LHS,

$$\begin{aligned}
 & \sin^2 \left(\frac{\pi}{8} + \frac{A}{2} \right) - \sin^2 \left(\frac{\pi}{8} - \frac{A}{2} \right) \\
 &= \left[\sin \left(\frac{\pi}{8} + \frac{A}{2} \right) + \sin \left(\frac{\pi}{8} - \frac{A}{2} \right) \right] \left[\sin \left(\frac{\pi}{8} + \frac{A}{2} \right) - \sin \left(\frac{\pi}{8} - \frac{A}{2} \right) \right] \\
 &= \left[\sin \frac{\pi}{8} \cdot \cos \frac{A}{2} + \cos \frac{\pi}{8} \cdot \sin \frac{A}{2} + \sin \frac{\pi}{8} \cdot \cos \frac{A}{2} - \cos \frac{\pi}{8} \cdot \sin \frac{A}{2} \right] \\
 &= \left[2 \sin \frac{\pi}{8} \cdot \cos \frac{A}{2} \right] \\
 &= 2 \sin \frac{\pi}{8} \cdot \cos \frac{A}{2} \\
 &= \sin 2 \cdot \frac{\pi}{8} \cdot \sin 2 \cdot \frac{A}{2} \\
 &= \sin \frac{\pi}{4} \cdot \sin A \\
 &= \frac{1}{\sqrt{2}} \sin A \\
 &= \text{RHS}
 \end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 13

LHS,

$$\begin{aligned}
 & 1 + \cos^2 2\theta \\
 &= 1 + \{\cos^2 \theta - \sin^2 \theta\}^2 \quad \left[\because \cos 2\theta = \cos^2 \theta - \sin^2 \theta \right] \\
 &= 1 + \cos^4 \theta + \sin^4 \theta - 2 \sin^2 \theta \cdot \cos^2 \theta \\
 &= \{\sin^2 \theta + \cos^2 \theta\}^2 + \cos^4 \theta + \sin^4 \theta - 2 \sin^2 \theta \cdot \cos^2 \theta \quad \left[\because \sin^2 \theta + \cos^2 \theta = 1 \right] \\
 &= \sin^4 \theta + \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \cos^4 \theta + \sin^4 \theta - 2 \sin^2 \theta \cos^2 \theta \\
 &= 2(\cos^4 \theta + \sin^4 \theta) \\
 &= \text{RHS}
 \end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 14

$$\cos^3 2\theta + 3 \cos 2\theta = 4(\cos^6 \theta - \sin^6 \theta)$$

$$\begin{aligned}
 \text{RHS} &= 4 \left[(\cos^2 \theta)^3 - (\sin^2 \theta)^3 \right] \\
 &= 4 (\cos^2 \theta - \sin^2 \theta) [\cos^4 \theta + \sin^4 \theta + \sin^2 \theta \cos^2 \theta] \\
 &= 4 \cos^2 \theta \left[(\cos^2 \theta - \sin^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta + \sin^2 \theta \cos^2 \theta \right] \\
 &= 4 \cos^2 \theta [\cos^2 2\theta + 3 \sin^2 \theta \cos^2 \theta] \\
 &= 4 \cos^2 \theta \left[\cos^2 2\theta + 3 \left(\frac{1 - \cos^2 \theta}{2} \right) \left(\frac{1 + \cos^2 \theta}{2} \right) \right] \\
 &= 4 \cos^2 \theta \left[\cos^2 2\theta + \frac{3}{4} (1 - \cos^2 2\theta) \right] \\
 &= \cos^2 \theta [4 \cos^2 2\theta + 3 - 3 \cos^2 2\theta] \\
 &= \cos^2 \theta [\cos^2 2\theta + 3] \\
 &= \cos^3 2\theta + 3 \cos^2 \theta \\
 &= \text{LHS}
 \end{aligned}$$

LHS = RHS

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 15

$$\text{LHS} = (\sin 3A + \sin A) \sin A (\cos 3A - \cos A) \cos A$$

$$\left[\because \sin C + \sin D = 2 \sin \frac{C+D}{2} \cdot \cos \frac{C-D}{2} \right]$$

$$\Rightarrow 2 \sin 2A \cdot \cos A \cdot \sin A + \{-2 \sin 2A \cdot \sin A \cos A\}$$

$$\left[\cos C - \cos D = -2 \sin \frac{C+D}{2} \cdot \sin \frac{C-D}{2} \right]$$

$$\Rightarrow 2 \sin 2A \cos A \cdot \sin A - 2 \sin 2A \cos A \cdot \sin A$$

$$\Rightarrow 0 = \text{RHS}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q16

$$\text{LHS} = \cos^2 \left(\frac{\pi}{4} - \theta \right) - \sin^2 \left(\frac{\pi}{4} - \theta \right)$$

$$= \cos 2 \left(\frac{\pi}{4} - \theta \right) \quad \left[\because \cos 2\theta = \cos^2 \theta - \sin^2 \theta \right]$$

$$= \cos \left(\frac{\pi}{2} - 2\theta \right) \quad \left[\because \cos \left(\frac{\pi}{2} - \theta \right) = \sin \theta \right]$$

$$= \sin 2\theta$$

$$= \text{RHS}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q17

$$\text{LHS} = \cos 4A$$

$$= \cos 2 \cdot 2A$$

$$= 2 \cos^2 2A - 1 \quad \left[\because \cos 2\theta = 2 \cos^2 \theta - 1 \right]$$

$$= 2 \left(2 \cos^2 A - 1 \right)^2 - 1$$

$$= 2 \left(4 \cos^4 A - 4 \cos^2 A + 1 \right) - 1$$

$$= 8 \cos^4 A - 8 \cos^2 A + 1$$

$$= 1 - 8 \cos^2 A + 8 \cos^4 A$$

$$= \text{RHS}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q18

$$\text{LHS} = \sin 4A$$

$$= \sin 2 \cdot 2A$$

$$= 2 \sin 2A \cos 2A$$

$$= 2 \left(2 \sin A \cos A \right) \cdot \left(\cos^2 A - \sin^2 A \right)$$

$$= 4 \sin A \cos^3 A - 4 \sin^3 A \cos A$$

$$= \text{RHS}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q19

$$\text{LHS} = 3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$$

$$= 3[\sin^4 x - 4 \sin^3 x \cos x + 6 \sin^2 x \cos^2 x - 4 \sin x \cos^3 x + \cos^4 x]$$

$$+ 6[\sin^2 x + 2 \sin x \cos x + \cos^2 x] + 4[\sin^6 x + \cos^6 x]$$

$$\left[\because (a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4 \text{ by binomial expansion} \right]$$

$$= 3[\sin^4 x + \cos^4 x - 4 \sin x \cos x (\sin^2 x + \cos^2 x) + 6 \sin^2 x \cos^2 x]$$

$$+ 6[1 + 2 \sin x \cos x] + 4[(\cos^2 x + \sin^2 x)(\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x)]$$

$$\left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2) \right]$$

$$= 7[\sin^4 x + \cos^4 x] + 18 \sin^2 x \cos^2 x - 4 \sin^2 x \cos^2 x + 6$$

$$= 7[\sin^4 x + \cos^4 x + 2 \sin^2 x \cos^2 x] + 6$$

$$= 7[\sin^2 x + \cos^2 x]^2 + 6$$

$$= 7 + 6$$

$$= 13$$

$$= \text{RHS}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q20

$$\text{L.H.S} = 2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1$$

$$\begin{aligned}
&= 2 \left[(\sin^2 x)^3 + (\cos^2 x)^3 \right] - 3 (\sin^4 x + \cos^4 x) + 1 \quad \left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2) \right] \\
&= 2 \left[(\sin^2 x + \cos^2 x) (\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x) \right] - 3 (\sin^4 x + \cos^4 x) + 1 \\
&= - [\sin^4 x + \cos^4 x + 2 \sin^2 x \cos^2 x] + 1 \\
&= - [\sin^2 x + \cos^2 x] + 1 \\
&= -1 + 1 \\
&= 0 \\
&= \text{RHS}
\end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q21

$$\begin{aligned}
\text{L.H.S} &= \cos^6 A - \sin^6 A \\
&= (\cos^2 A)^3 - (\sin^2 A)^3 \\
&= (\cos^2 A - \sin^2 A) (\cos^4 A + \sin^2 A \cos^2 A + \sin^4 A) \quad \left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2) \right] \\
&= \cos 2A (\cos^4 A + 2 \sin^2 A \cos^2 A + \sin^4 A - \sin^2 A \cos^2 A) \\
&\quad \left[\because \cos^2 A - \sin^2 A = \cos^2 A \text{ \& \text{ Adding and subtracting } } \sin^2 A \cos^2 A \right] \\
&= \cos 2A \left[(\sin^2 A + \cos^2 A)^2 - \frac{4}{4} \sin^2 A \cos^2 A \right] \\
&= \cos 2A \left[1 - \frac{1}{4} (2 \sin A \cos A)^2 \right] \\
&= \cos 2A \left[1 - \frac{1}{4} \sin^2 2A \right] \\
&= \text{RHS}
\end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q22

$$\begin{aligned}
\text{L.H.S} &= \tan \left(\frac{\pi}{4} + \theta \right) + \tan \left(\frac{\pi}{4} - \theta \right) \\
&= \frac{\tan \frac{\pi}{4} + \tan \theta}{1 - \tan \frac{\pi}{4} \tan \theta} + \frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \tan \theta} \\
&= \frac{1 + \tan \theta}{1 - \tan \theta} + \frac{1 - \tan \theta}{1 + \tan \theta} \quad \left[\because \tan \frac{\pi}{4} = 1 \right] \\
&= \frac{(1 + \tan^2 \theta + 2 \tan \theta) + (1 + \tan^2 \theta - 2 \tan \theta)}{(1 - \tan \theta)(1 + \tan \theta)} \\
&= \frac{2(1 + \tan^2 \theta)}{1 - \tan^2 \theta} \\
&= \frac{2 \sec^2 \theta}{1 - \frac{\sin^2 \theta}{\cos^2 \theta}} \quad \left[\because \sec^2 \theta = 1 + \tan^2 \theta \right] \\
&= \frac{2 \sec^2 \theta \cdot \cos^2 \theta}{\cos^2 \theta - \sin^2 \theta} \quad \left[\because \sec = \frac{1}{\cos \theta} \right] \\
&= \frac{2}{\cos 2\theta} \\
&= 2 \sec 2\theta \\
&= \text{RHS}
\end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q23

$$\begin{aligned}
\text{L.H.S} &= \cot^2 A - \tan^2 A \\
&= \frac{\cos^2 A}{\sin^2 A} - \frac{\sin^2 A}{\cos^2 A} \\
&= \frac{(\cos^2 A)^2 - (\sin^2 A)^2}{\sin^2 A \cos^2 A} \\
&= \frac{(\cos^2 A + \sin^2 A) (\cos^2 A - \sin^2 A)}{(\sin A \cos A)^2} \quad \left[\because a^2 - b^2 = (a+b)(a-b) \right] \\
&= \frac{\cos 2A}{\frac{1}{4} (2 \sin A \cos A)^2} \quad \left[\because \cos 2A = \cos^2 A - \sin^2 A \right] \\
&= \frac{4 \cos 2A}{\sin^2 2A}
\end{aligned}$$

$$= \frac{\cos \theta}{\sin 2A} \cdot \frac{1}{\sin 2A} \quad \left[\because \operatorname{cosec} \theta = \frac{1}{\sin \theta} \right]$$

$$= 4 \cot 2A \cdot \operatorname{cosec} 2A$$

$$= \text{RHS}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q24

$$\cos 4\theta - \cos 4\alpha = 2\cos^2 2\theta - 2\cos^2 2\alpha$$

$$= 2(\cos 2\theta + \cos 2\alpha)(\cos 2\theta - \cos 2\alpha)$$

$$= 2(2\cos^2 \theta - 1 + 1 - 2\sin^2 \alpha)(2\cos^2 \theta - 1 - 2\cos^2 \alpha + 1)$$

$$= 8(\cos^2 \theta - \sin^2 \alpha)(\cos^2 \theta - \cos^2 \alpha)$$

$$= 8(\cos \theta - \sin \alpha)(\cos \theta + \sin \alpha)(\cos \theta - \cos \alpha)(\cos \theta + \cos \alpha)$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q25

$$\sin 3x + \sin 2x - \sin x$$

$$= (\sin 3x - \sin x) + \sin 2x$$

$$= 2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right) + 2\sin x \cos x \dots \dots \dots \left[\begin{array}{l} \because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\ \sin 2x = 2\sin x \cos x \end{array} \right]$$

$$= 2\cos(2x)\sin(x) + 2\sin x \cos x$$

$$= 2\sin x [\cos(2x) + \cos x]$$

$$= 2\sin x \left[2\cos\left(\frac{2x+x}{2}\right) + \cos\left(\frac{2x-x}{2}\right) \right] \dots \dots \dots \left[\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \right]$$

$$= 4\sin x \cos \frac{3x}{2} \cos \frac{x}{2}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 26

$$\tan 82\frac{1^\circ}{2} = \tan\left(90 - 7\frac{1^\circ}{2}\right)$$

$$= \cot 7\frac{1^\circ}{2}$$

$$= \cot A \quad \text{If } A = 7\frac{1^\circ}{2}$$

Now

$$\begin{aligned} \cot A &= \frac{\cos A}{\sin A} \\ &= \frac{2\cos^2 A}{2\sin A \cos A} \\ &= \frac{1+\cos^2 A}{\sin^2 A} \\ \cot A &= \frac{1+\cos 15}{\sin 15} \\ &= \frac{1+\cos(45-30)}{\sin 15} \\ &= \frac{1+\left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}\right)}{\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}} \\ &= \frac{2\sqrt{2} + \{\sqrt{3}+1\}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \\ &= \frac{2\sqrt{2}(\sqrt{3}+1) + \{\sqrt{3}+1\}^2}{3-1} \\ &= \frac{2\sqrt{2} + 2\sqrt{2} + 4 + 2\sqrt{3}}{2} \end{aligned}$$

$$\cot A = \sqrt{2} + \sqrt{2} + 2 + \sqrt{3} \dots \dots \dots (1)$$

$$= \sqrt{2} + 2 + \sqrt{2} + \sqrt{3}$$

$$= \sqrt{2}(1+\sqrt{2}) + \sqrt{3}(\sqrt{2}+1)$$

$$\cot A = (\sqrt{2}+1)(\sqrt{2}+\sqrt{3}) \dots \dots \dots (2)$$

From equation (1) and (2)

$$\begin{aligned} \tan 82\frac{1^\circ}{2} = \cot 7\frac{1^\circ}{2} &= \sqrt{2} + \sqrt{2} + 2 + \sqrt{3} \\ &= (\sqrt{2}+1)(\sqrt{2}+\sqrt{3}) \end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 27

We know that,

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1-\cos A}{2}}$$

Put $A = 45^\circ$,

$$\sin 22 \frac{1^\circ}{2} = \sqrt{\frac{1 - \cos 45^\circ}{2}} \quad \left\{ \text{since } \sin 22 \frac{1^\circ}{2}, \text{ is positive} \right\}$$

$$= \sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}}$$

$$\sin 22 \frac{1^\circ}{2} = \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}}$$

And

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

Put $A = 45^\circ$

$$\cos 22 \frac{1^\circ}{2} = \sqrt{\frac{1 + \cos 45^\circ}{2}}$$

$$= \sqrt{\frac{1 + \frac{1}{\sqrt{2}}}{2}}$$

$$\cos 22 \frac{1^\circ}{2} = \sqrt{\frac{\sqrt{2} + 1}{2\sqrt{2}}}$$

Now,

$$\cot 22 \frac{1^\circ}{2} = \frac{\cos 22 \frac{1^\circ}{2}}{\sin 22 \frac{1^\circ}{2}}$$

$$= \frac{\sqrt{\frac{\sqrt{2} + 1}{2\sqrt{2}}}}{\sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}}}$$

$$= \sqrt{\frac{\sqrt{2} + 1}{\sqrt{2} - 1}}$$

Rationalizing denominator,

$$= \sqrt{\frac{\sqrt{2} + 1 \times \sqrt{2} + 1}{\sqrt{2} - 1 \times \sqrt{2} + 1}}$$

$$= \sqrt{\frac{(\sqrt{2} + 1)^2}{2 - 1}}$$

$$\cot 22 \frac{1^\circ}{2} = \sqrt{2} + 1$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 28(i)

$$\text{Since } \cos x = -\frac{3}{5} = \frac{b}{h}$$

$$\Rightarrow b = 3, h = 5$$

$$\Rightarrow p = 4$$

Now, x lies on third quad.

$$\therefore \sin 2x = 2 \sin x \cdot \cos x$$

$$= 2 \cdot \left(\frac{-4}{5} \right) \cdot \left(\frac{-3}{5} \right) = \frac{24}{25}$$

$$\therefore \pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

Which means $\frac{x}{2}$ lies in second quadrant

$$\text{so, } \cos \frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}}$$

$$[\because 1 + \cos 2\theta = 2 \cos^2 \theta]$$

$$= \sqrt{\frac{1 - \frac{3}{5}}{2}} = \frac{-1}{\sqrt{5}}$$

(-ve sign because of second quad.)
where $\cos D$ is -ve

Also,

$$\sin \frac{x}{2} = \frac{\sin x}{2 \cos \frac{x}{2}}$$

$$[\because \sin 2A = 2 \sin A \cos A]$$

$$= \left(\frac{-4}{5} \div (-1) \right)$$

$$\left(\sqrt[2]{\frac{2}{\sqrt{5}}} \right)$$

$$= \frac{2}{\sqrt{5}}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 28(ii)

$\because x$ lies in IInd quadrant,

$$\Rightarrow \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \pi < 2x < 2\pi \Rightarrow 2x \text{ lies in 1st quad.}$$

$$\text{Also, } \cos x = \frac{-3}{5} = \frac{b}{h} \Rightarrow b = 3$$

$$h = 5$$

$$\Rightarrow p = 4.$$

$$\text{so, } \sin x = \frac{p}{h} = \frac{4}{5}$$

$$\therefore \sin 2x = 2 \sin x \cos x$$

$$= 2 \cdot \frac{4}{5} \cdot \left(\frac{-3}{5} \right) = \frac{-24}{25}$$

$$\sin \frac{x}{2} = \frac{\sin x}{2 \cos \frac{x}{2}} \text{ or } \sqrt{\frac{1 - \cos x}{2}}$$

$$= \sqrt{\frac{1 - \left(1 - \frac{3}{5}\right)}{2}}$$

$$= \frac{2}{\sqrt{5}}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 29

$\because x$ lies in IInd quad,

$$\Rightarrow \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Which means $\frac{x}{2}$ lies in first quad.

$$\text{Now, } \sin x = \frac{\sqrt{5}}{3} = \frac{p}{h} \Rightarrow \begin{matrix} p = \sqrt{5} \\ h = 3 \end{matrix} \Rightarrow b = 2$$

$$\text{so, } \cos x = \frac{b}{h} = \frac{-2}{3} \quad (\text{-ve due to IInd quad})$$

Thus,

$$\cos \frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}} = \sqrt{\frac{1 - \frac{2}{3}}{2}} = \frac{1}{\sqrt{6}}$$

$$\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}} = \sqrt{\frac{1 + \frac{2}{3}}{2}} = \sqrt{\frac{5}{6}}$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\sqrt{\frac{5}{6}}}{\frac{1}{\sqrt{6}}} = \sqrt{5}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q30(i)

Since x lies in IInd quadrant

$$\Rightarrow \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}, \text{ which means } \frac{x}{2} \text{ lies in 1st quad.}$$

Now,

$$\sin x = \frac{1}{4} = \frac{p}{h} \Rightarrow \begin{matrix} p = 1 \\ h = 4 \end{matrix} \Rightarrow b = \sqrt{15}$$

$$\text{so, } \cos x = \frac{b}{h} = \frac{-\sqrt{15}}{4} \quad (\text{-ve due to IInd quad})$$

Thus,

$$\cos \frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}} = \sqrt{\frac{1 - \frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{4 - \sqrt{15}}{8}}$$

$$\cos \frac{\gamma}{2} = \sqrt{\frac{1}{2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}} = \sqrt{\frac{1 + \frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{4 + \sqrt{15}}{8}}$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\sqrt{\frac{4 + \sqrt{15}}{8}}}{\sqrt{\frac{4 - \sqrt{15}}{8}}} = \sqrt{\frac{4 + \sqrt{15}}{4 - \sqrt{15}}}$$

$$= \sqrt{\frac{(4 + \sqrt{15})(4 + \sqrt{15})}{(4 - \sqrt{15})(4 + \sqrt{15})}}$$

$$= 4 + \sqrt{15}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 30(ii)

Since θ is acute, so $0 < 2\theta < \pi$

$$\text{Now, } \cos \theta = \frac{4}{5} = \frac{b}{h} \Rightarrow b = 4 \quad \Rightarrow p = 3$$

$$h = 5$$

$$\therefore \sin \theta = \frac{p}{h} = \frac{3}{5}$$

$$\tan \theta = \frac{p}{b} = \frac{3}{4}$$

$$\text{so, } \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \cdot \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2}$$

$$= \frac{\frac{6}{4}}{\frac{7}{16}} = \frac{24}{7}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 30(iii)

$$\sin \theta = \frac{4}{5} = \frac{p}{h} \Rightarrow p = 4 \quad \Rightarrow b = 3$$

$$h = 5$$

$$\therefore \cos \theta = \frac{b}{h} = \frac{3}{5}$$

$$\text{Now, } \sin 2\theta = 2 \sin \theta \cdot \cos \theta = 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{-7}{25}$$

$$\text{so, } \sin 4\theta = \sin 2 \cdot 2\theta = 2 \sin 2\theta \cdot \cos 2\theta$$

$$= 2 \cdot \frac{24}{25} \cdot \left(\frac{-7}{25}\right)$$

$$= \frac{-336}{625}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q31

$$\frac{\sqrt{a+b}}{\sqrt{a-b}} + \frac{\sqrt{a-b}}{\sqrt{a+b}} = \frac{(a+b) + (a-b)}{\sqrt{(a-b)(a+b)}}$$

$$= \frac{2a}{\sqrt{a^2 - b^2}}$$

$$= \frac{2}{\sqrt{1 - \left(\frac{b}{a}\right)^2}}$$

$$= \frac{2}{\sqrt{1 - \tan^2 x}} \dots \dots \dots \left[\because \tan x = \frac{b}{a} \right]$$

$$= \frac{2 \cos x}{\cos 2x - \sin 2x}$$

$$\frac{\sqrt{\cos x - \sin x}}{2 \cos x} = \frac{1}{\sqrt{\cos 2x}}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 32

We have,

$$\tan A = \frac{1}{7} \quad \& \quad \tan B = \frac{1}{3}$$

$$\begin{aligned} \therefore \cos 2A &= \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{1}{7}\right)^2}{1 + \left(\frac{1}{7}\right)^2} = \frac{48}{49} \\ &= \frac{48}{50} = \frac{24}{25} \dots\dots\dots (A) \end{aligned}$$

Also,

$$\sin 4B = \sin 2.2B$$

$$= 2 \sin 2B \cdot \cos 2B$$

$$= 2 \cdot \left(\frac{2 \tan B}{1 + \tan^2 B} \right) \cdot \left(\frac{1 - \tan^2 B}{1 + \tan^2 B} \right)$$

$$= 4 \cdot \left(\frac{\frac{1}{3}}{1 + \frac{1}{9}} \right) \cdot \left(\frac{1 - \frac{1}{9}}{1 + \frac{1}{9}} \right)$$

$$= \frac{4 \cdot \frac{1}{3} \cdot \frac{8}{9}}{\frac{10}{9} \times \frac{10}{9}}$$

$$= \frac{32 \times 3}{100}$$

$$= \frac{8 \times 3}{25} = \frac{24}{25} \dots\dots\dots (B)$$

from (A) & (B)

$$\cos 2A = \sin 4B$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 33

LHS,

$$\cos 7^\circ \cos 14^\circ \cos 28^\circ \cos 56^\circ$$

Divide and multiply by $2 \sin 7^\circ$, we get

$$\frac{1}{2 \sin 7^\circ} \cdot 2 \sin 7^\circ \cdot \cos 7^\circ \cdot \cos 14^\circ \cdot \cos 28^\circ \cdot \cos 56^\circ$$

$$= \frac{2 \sin 14^\circ}{2 \cdot 2 \sin 7^\circ} \cdot \cos 14^\circ \cdot \cos 28^\circ \cdot \cos 56^\circ \quad [\because 2 \sin A \cos A = \sin 2A]$$

$$= \frac{2 \sin 28^\circ}{2 \cdot 4 \sin 7^\circ} \cdot \cos 28^\circ \cdot \cos 56^\circ$$

$$= \frac{2 \sin 56^\circ}{2 \cdot 8 \sin 7^\circ} \cdot \cos 56^\circ$$

$$= \frac{\sin 112^\circ}{16 \sin 7^\circ}$$

$$= \frac{\sin (180^\circ - 68^\circ)}{16 \sin (90^\circ - 83^\circ)}$$

$$= \frac{\sin 68^\circ}{16 \cos 83^\circ}$$

$$= \text{RHS}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q34

$$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a) \quad (1)$$

$$\text{for } a=b, \sin(2a) = 2 \sin(a) \cos(a) \quad (2)$$

$$\begin{aligned} \text{let } a &= 16\pi/15 & (3) \\ (\text{so } 2a &= 32\pi/15) \end{aligned}$$

then using (3) in (2), we have

$$\begin{aligned} \sin(2a) &= 2 \sin(a) \cos(a) \\ &= 2 (2 \sin(a/2) \cos(a/2)) \cos(a) \\ &= 2 (2 (2 \sin(a/4) \cos(a/4)) \cos(a/2)) \cos(a) \\ &= 2 (2 (2 (2 \sin(a/8) \cos(a/8)) \cos(a/4)) \cos(a/2)) \cos(a) \\ &= 16 \sin(a/8) (\cos(a/8) \cos(a/4) \cos(a/2) \cos(a)) \end{aligned}$$

now note $\sin(2a) = \sin(2\pi/15)$ and $\sin(a/8) = \sin(2\pi/15)$

so,

$$\cos(a/8) \cos(a/4) \cos(a/2) \cos(a) = 1/16$$

or, replacing a with $16\pi/15$,

$$\cos(2\pi/15) \cdot \cos(4\pi/15) \cdot \cos(8\pi/15) \cdot \cos(16\pi/15) = 1/16$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q35

$$\begin{aligned} \cos \frac{\pi}{5} \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5} &= \frac{\sin \frac{2^4\pi}{5}}{2^4 \sin \frac{\pi}{5}} \\ &= \frac{\sin \frac{16\pi}{5}}{16 \sin \frac{\pi}{5}} \\ &= \frac{\sin \left(3\pi + \frac{\pi}{5} \right)}{16 \sin \frac{\pi}{5}} \\ &= \frac{1 \left\{ -\sin \left(\frac{\pi}{5} \right) \right\}}{16 \sin \frac{\pi}{5}} \\ &= \frac{-1}{16} \end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q36

$$\text{LHS} = \cos \frac{\pi}{65} \cdot \cos \frac{2\pi}{65} \cdot \cos \frac{4\pi}{65} \cdot \cos \frac{8\pi}{15} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}$$

Divide and Multiply by $2 \sin \frac{\pi}{65}$, we get

$$\begin{aligned} &= \frac{2 \sin \frac{\pi}{65} \cdot \cos \frac{\pi}{65} \cdot \cos \frac{2\pi}{65} \cdot \cos \frac{4\pi}{65} \cdot \cos \frac{8\pi}{65} \cdot \cos \frac{16\pi}{65} \cdot \cos \frac{32\pi}{65}}{2 \sin \frac{\pi}{65}} \\ &= \frac{2 \sin \frac{2\pi}{65} \cdot \cos \frac{2\pi}{65} \cdot \cos \frac{4\pi}{65} \cdot \cos \frac{8\pi}{65} \cdot \cos \frac{16\pi}{65} \cdot \cos \frac{32\pi}{65}}{2 \cdot 2 \sin \frac{\pi}{65}} \\ &= \frac{2 \sin \frac{4\pi}{65} \cdot \cos \frac{4\pi}{65} \cdot \cos \frac{8\pi}{65} \cdot \cos \frac{16\pi}{65} \cdot \cos \frac{32\pi}{65}}{2 \cdot 4 \sin \frac{\pi}{65}} \\ &= \frac{2 \sin \frac{8\pi}{65} \cdot \cos \frac{8\pi}{65} \cdot \cos \frac{16\pi}{65} \cdot \cos \frac{32\pi}{65}}{2 \cdot 8 \sin \frac{\pi}{65}} \end{aligned}$$

$$\begin{aligned}
 &= \frac{2 \cdot \sin \frac{16\pi}{65}}{2 \cdot 16 \sin \frac{\pi}{65}} \cos \frac{16\pi}{65} \cdot \cos \frac{32\pi}{65} \\
 &\sim \sim \\
 &= \frac{2 \cdot \sin \frac{32\pi}{65}}{2 \cdot 32 \sin \frac{\pi}{65}} \cos \frac{32\pi}{65} \\
 &= \frac{\sin \frac{64\pi}{65}}{64 \cdot \sin \frac{\pi}{65}} \\
 &= \frac{1}{64} \cdot \frac{\sin \left(\pi - \frac{\pi}{65} \right)}{\sin \frac{\pi}{65}} \\
 &= \frac{1}{64} \frac{\sin \frac{\pi}{65}}{\sin \frac{\pi}{65}} \\
 &= \frac{1}{64} \\
 &= \text{RHS}
 \end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 37

We have, $2 \tan \alpha = 3 \tan \beta$

$$\Rightarrow \frac{\tan \alpha}{\tan \beta} = \frac{3}{2}$$

Let $\tan \alpha = 3K$ and $\tan \beta = 2K$

Now, $\tan (\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \cdot \tan \beta} = \frac{3K - 2K}{1 + 3K \cdot 2K} = \frac{K}{1 + 6K^2} \quad \dots (A)$

Also,

$$\begin{aligned}
 \frac{\sin 2\beta}{5 - \cos 2\beta} &= \frac{\frac{2 \tan \beta}{1 + \tan^2 \beta}}{5 - \left(\frac{1 - \tan^2 \beta}{1 + \tan^2 \beta} \right)} \\
 &= \frac{\frac{2 \cdot 2K}{1 + 4K^2}}{5 - \left(\frac{1 - 4K^2}{1 + 4K^2} \right)} \\
 &= \frac{4K}{5 + 20K^2 - 1 + 4K^2} \\
 &= \frac{4K}{4 + 24K^2} = \frac{K}{1 + 6K^2} \quad \dots (B)
 \end{aligned}$$

form (A) & (B)

$$\tan (\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 38(i)

We have,

$$\sin \alpha + \sin \beta = a \text{ \& \; } \cos \alpha + \cos \beta = b \text{ (A)}$$

Squaring and adding, we get

$$\begin{aligned}
 \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta + \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta &= a^2 + b^2 \\
 \Rightarrow 1 + 1 + 2 (\sin \alpha \sin \beta + \cos \alpha \cos \beta) &= a^2 + b^2 \\
 \Rightarrow 2 (\sin \alpha \sin \beta + \cos \alpha \cos \beta) &= a^2 + b^2 - 2 \\
 \therefore 2 \cos (\alpha - \beta) &= a^2 + b^2 - 2
 \end{aligned}$$

$$\text{Thus, } \cos (\alpha - \beta) = \frac{a^2 + b^2 - 2}{2} \text{ (ii)}$$

Again,

$$\sin \alpha + \sin \beta = a \quad \Rightarrow \quad 2 \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2} = a$$

$$\cos \alpha + \cos \beta = b \quad \Rightarrow \quad 2 \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2} = b$$

$$\Rightarrow \quad \tan \frac{\alpha + \beta}{2} = \frac{a}{b} \dots\dots\dots (B)$$

Now,

$$\sin (\alpha + \beta) = \frac{2 \tan \frac{\alpha + \beta}{2}}{1 + \tan^2 \left(\frac{\alpha + \beta}{2} \right)}$$

$$= \frac{2 \frac{a}{b}}{1 + \frac{a^2}{b^2}} = \frac{2ab}{a^2 + b^2}$$

Thus,

$$\sin (\alpha + \beta) = \frac{2ab}{a^2 + b^2}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 38(ii)

We have,

$$\sin \alpha + \sin \beta = a \quad \& \quad \cos \alpha + \cos \beta = b$$

Squaring and adding, we get

$$\sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta + \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta = a^2 + b^2$$

$$\Rightarrow \quad 1 + 1 + 2 \{ \sin \alpha \sin \beta + \cos \alpha \cos \beta \} = a^2 + b^2$$

$$\Rightarrow \quad 2 \{ \sin \alpha \sin \beta + \cos \alpha \cos \beta \} = a^2 + b^2 - 2$$

$$\therefore \quad 2 \cos (\alpha - \beta) = a^2 + b^2 - 2$$

$$\text{Thus, } \cos (\alpha - \beta) = \frac{a^2 + b^2 - 2}{2}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 39

We have,

$$2 \tan \frac{\alpha}{2} = \tan \frac{\beta}{2}$$

$$\Rightarrow \quad \frac{\tan \frac{\alpha}{2}}{\tan \frac{\beta}{2}} = \frac{1}{2}$$

$$\text{Let } \tan \frac{\alpha}{2} = K \quad \text{and} \quad \tan \frac{\beta}{2} = 2K$$

Then,

$$\cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} = \frac{1 - K^2}{1 + K^2} \dots\dots\dots (A)$$

Also,

$$\frac{3 + 5 \cos \beta}{5 + 3 \cos \beta} = \frac{3 + 5 \left(\frac{1 - \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\beta}{2}} \right)}{5 + 3 \left(\frac{1 - \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\beta}{2}} \right)}$$

$$= \frac{3 + 5 \left(\frac{1 - 4K^2}{1 + 4K^2} \right)}{5 + 3 \left(\frac{1 - 4K^2}{1 + 4K^2} \right)}$$

$$= \frac{8 - 8K^2}{8 + 8K^2} = \frac{1 - K^2}{1 + K^2} \dots\dots\dots (B)$$

from (A) & (B)

$$\cos \alpha = \frac{3 + 5 \cos \beta}{5 + 3 \cos \beta}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q40

We have,

$$\cos \theta = \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cdot \cos \beta}$$

Now,

$$\cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

$$\Rightarrow \frac{1 - \tan^2 \theta/2}{1 + \tan^2 \theta/2} = \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta}$$

by componende and dividendo, we get

$$\frac{(1 - \tan^2 \theta/2) + (1 + \tan^2 \theta/2)}{(1 - \tan^2 \theta/2) - (1 + \tan^2 \theta/2)} = \frac{1 + \cos \alpha \cos \beta + \cos \alpha + \cos \beta}{-(1 + \cos \alpha \cos \beta - \cos \alpha - \cos \beta)}$$

$$\Rightarrow \frac{2}{2 \tan^2 \theta/2} = \frac{(1 + \cos \alpha)(1 + \cos \beta)}{(1 - \cos \alpha)(1 - \cos \beta)}$$

$$\Rightarrow \tan^2 \theta/2 = \frac{(1 - \cos \alpha)(1 - \cos \beta)}{(1 + \cos \alpha)(1 + \cos \beta)}$$

$$= \frac{2 \sin^2 \alpha/2 \cdot 2 \sin^2 \beta/2}{2 \cos^2 \alpha/2 \cdot 2 \cos^2 \beta/2}$$

$$\Rightarrow \tan \theta/2 = \pm \tan \alpha/2 \cdot \tan \beta/2$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q41

We have,

$$\sec(\theta + \alpha) + \sec(\theta - \alpha) = 2 \sec \theta.$$

$$\Rightarrow \frac{1}{\cos \theta \cdot \cos \alpha - \sin \theta \sin \alpha} + \frac{1}{\cos \theta \cdot \cos \alpha + \sin \theta \sin \alpha} = \frac{2}{\cos \theta}$$

$$\Rightarrow \frac{2 \cos \theta \cos \alpha}{\cos^2 \theta \cos^2 \alpha - \sin^2 \theta \sin^2 \alpha} = \frac{2}{\cos \theta}$$

$$\Rightarrow \frac{\cos \theta \cos \alpha}{\cos^2 \theta \cos^2 \alpha - (1 - \cos^2 \theta) \sin^2 \alpha} = \frac{1}{\cos \theta}$$

$$\Rightarrow \cos^2 \theta \cos \alpha = \cos^2 \theta (\cos^2 \alpha + \sin^2 \alpha) - \sin^2 \alpha$$

$$\Rightarrow \cos^2 \theta (1 - \cos \alpha) = \sin^2 \alpha$$

$$\Rightarrow \cos^2 \theta = \frac{\sin^2 \alpha}{2 \sin^2 \alpha/2}$$

$$= \frac{4 \sin^2 \alpha/2 \cdot \cos^2 \alpha/2}{2 \sin^2 \alpha/2}$$

$$\Rightarrow \cos \theta = \pm \sqrt{2} \cos \alpha/2$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q42

We have,

$$\cos \alpha + \cos \beta = \frac{1}{3} \text{ and } \sin \alpha + \sin \beta = \frac{1}{4}$$

Squaring and adding, we get

$$(\cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta) + (\sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta) = \frac{1}{9} + \frac{1}{16}$$

$$\Rightarrow 1 + 1 + 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) = \frac{25}{144}$$

$$\Rightarrow 2 \cos(\alpha - \beta) = \frac{25}{144} - 2 = \frac{-263}{144}$$

$$\Rightarrow \cos(\alpha - \beta) = \frac{-263}{288}$$

Now,

$$\cos\left(\frac{\alpha - \beta}{2}\right) = \sqrt{\frac{1 + \cos(\alpha - \beta)}{2}}$$

$$\sqrt{1 - \frac{263}{288}}$$

$$= \sqrt[3]{-\frac{288}{2}} = \sqrt[3]{-144}$$

$$= \pm \frac{5}{24}$$

$$\therefore \cos\left(\frac{\alpha - \beta}{2}\right) = \pm \frac{5}{24}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 43

We have,

$$\sin \alpha = \frac{4}{5} \quad \& \quad \cos \beta = \frac{5}{13} \quad \Rightarrow \cos \alpha = \frac{3}{5} \quad \& \quad \sin \beta = \frac{12}{13}$$

$$\therefore \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$= \frac{3}{5} \cdot \frac{5}{13} + \frac{4}{5} \cdot \frac{12}{13}$$

$$= \frac{15}{65} + \frac{48}{65} = \frac{63}{65}$$

Now,

$$\cos\left(\frac{\alpha - \beta}{2}\right) = \sqrt{\frac{1 + \cos(\alpha - \beta)}{2}}$$

$$= \sqrt{\frac{1 + \frac{63}{65}}{2}}$$

$$= \sqrt{\frac{128}{65 \times 2}} = \sqrt{\frac{64}{65}}$$

$$= \pm \frac{8}{\sqrt{65}}$$

$$\therefore \cos\left(\frac{\alpha - \beta}{2}\right) = \frac{8}{\sqrt{65}}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q44

$$\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

substitute these values in the given equation, it reduces to

$$a(1 - \tan^2 \theta) + b(2 \tan \theta) = c(1 + \tan^2 \theta)$$

$$(c+a) \tan^2 \theta + 2b \tan \theta + c - a = 0$$

As α and β are roots

$$\text{sum of the roots, } \tan \alpha + \tan \beta = \frac{2b}{c+a}$$

$$\text{Product of roots, } \tan \alpha \tan \beta = \frac{c-a}{c+a}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{2b}{c+a-c+a} = \frac{b}{a}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 45

$$\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$$

squaring on both sides gives

$$\cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta = \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta$$

Bring square terms on one side, we get

$$\cos 2\alpha + \cos 2\beta = -2(-\sin \alpha \sin \beta + \cos \alpha \cos \beta) = -2 \cos(\alpha + \beta)$$