RD Sharma Solutions Class 11 Maths Chapter 8 Ex 8.2

Transformation Formulae Ex 8.2 Q1

(i) $\sin 12\theta + \sin 4\theta$ $\left[\because \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$$= 2\sin\left(\frac{12\theta + 4\theta}{2}\right)\cos\left(\frac{12\theta - 4\theta}{2}\right)$$

$$= 2\sin 8\theta \cos 4\theta$$

(ii) $\sin 5\theta - \sin\theta$ $\left[\because \sin C - \sin D = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$

$$= 2\cos\left(\frac{5\theta + \theta}{2}\right)\sin\left(\frac{5\theta - \theta}{2}\right)$$

$$= 2\sin 2\theta \cos 3\theta$$

(iii) $\cos 12\theta + \cos 8\theta$ $\left[\because \cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$$= 2\cos 10\theta \cos 2\theta$$

(iv) $\cos 12\theta - \cos 4\theta$

$$= -2\sin\left(\frac{12\theta + 4\theta}{2}\right)\sin\left(\frac{12\theta - 4\theta}{2}\right)$$ $\left[\because \cos D - \cos C = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$

$$= -2\sin 8\theta \sin 4\theta$$

(v) $\sin 2\theta + \cos 4\theta$

$$= \sin 2\theta + \sin(90 - 4\theta)$$

$$= 2\sin\frac{(2\theta + 90 - 4\theta)}{2}\cos\frac{(2\theta - 90 + 4\theta)}{2}$$

$$= 2\sin\left(\frac{\pi}{4} + \theta\right)\cos\left(\frac{\pi}{4} - 3\theta\right)$$

Transformation Formulae Ex 8.2 Q2

$\sin 38° + \sin 22° = \sin 82°$

LHS $= \sin 38° + \sin 22°$

$\because \quad \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\Rightarrow \quad \sin 38° + \sin 22° = 2\sin\frac{60°}{2}\cos\frac{16°}{2}$

$= 2\sin 30° \cos 8°$

$= 2 \times \frac{1}{2}\cos 8°$

$= \cos(90-82)°$

$= \sin 82° = \text{RHS}$ $\quad [\because \cos\theta = \sin(90-\theta)]$

Transformation Formulae Ex 8.2 Q2(i)

$\cos 100° + \cos 20° = \cos 40°$

LHS $= \cos 100° + \cos 20°$ $\quad \left[\because \cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$\Rightarrow \quad 2\cos\frac{(100°+20°)}{2}\cos\frac{(100°-20°)}{2}$

$= 2\cos 60° \cos 40°$

$= 2 \times \frac{1}{2}\cos 40°$ $\quad \left[\because \cos 60° = \frac{1}{2}\right]$

$= \cos 40° = \quad \text{RHS}$

Transformation Formulae Ex 8.2 Q2(ii)

$\sin 50° + \sin 10° = \cos 20°$

LHS $= \sin 50° + \sin 10°$ $\quad \left[\because \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$\sin 50° + \sin 10° = 2\sin\frac{60°}{2}\cos 20°$

$= 2\sin 30° \cos 20°$

$= 2 \times \frac{1}{2}\cos 20°$

$= \cos 20° = \quad \text{RHS}$ $\quad \left[\because \sin 30° = \frac{1}{2}\right]$

Transformation Formulae Ex 8.2 Q2(iii)

$\sin 30° + \sin 37° = \cos 7°$

LHS $= \sin 23° + \sin 37°$

$= 2\sin\left(\frac{23°+37°}{2}\right)\cos\left(\frac{23°-37°}{2}\right)$ $\quad \left[\because \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$= 2\sin(30°)\cos(-7°)$

$= 2 \times \frac{1}{2}\cos 7$ $\quad \left[\because \cos(-\theta) = \cos\theta, \sin 30° = \frac{1}{2}\right]$

$= \cos 7° = \quad \text{RHS}$

Transformation Formulae Ex 8.2 Q2(iv)

$\text{LHS} = \sin 105° + \cos 105°$

$= \sin 105° + \cos(90° + 15°)$

$= \sin 105° - \sin 15°$

$= 2\sin\left(\frac{105° - 15°}{2}\right)\cos\left(\frac{105° + 15°}{2}\right)$

$= 2\sin 45° \cos 60°$

$= 2\frac{1}{\sqrt{2}}\frac{1}{2}$

$= \frac{1}{\sqrt{2}}$

$= \cos 45°$

Transformation Formulae Ex 8.2 Q2(v)

$\sin 40° + \sin 20° = \cos 10°$

LHS $= \sin 40° + \sin 20°$

$= 2\sin\left(\frac{40°+20°}{2}\right)\cos\left(\frac{40°-20°}{2}\right)$ $\quad \left[\because \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

$= 2\sin 30° \cos 10°$

$= 2 \times \frac{1}{2}\cos 10°$

2

$= \cos 10°$

$= RHS \qquad \left[\because \sin 30° = \frac{1}{2}\right]$

Transformation Formulae Ex 8.2 Q3(i)

$\cos 55 + \cos 65 + \cos 175 = 0$

$\cos 175 = -\cos 5$

substitute above value in the equation we get

$\cos 55 + \cos 65 = \cos 5$

applying rule $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\cos 55 + \cos 65 = 2\cos\left(\frac{65+55}{2}\right)\cos\left(\frac{65-55}{2}\right) = 2\cos 60 \cos 5 = 2\times\frac{1}{2}\times\cos 5 = \cos 5$

Hence Proved

Transformation Formulae Ex 8.2 Q3(ii)

$\sin 50° - \sin 70° + \sin 10° = 0$

$(\sin 50° - \sin 70°) + \sin 10°$

$\Rightarrow \left(2\sin\left(\frac{50° - 70°}{2}\right)\cos\left(\frac{50° + 70°}{2}\right)\right) + \sin 10° \qquad \left[\because \sin C - \sin D = 2\sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right)\right]$

$= 2\sin(-10°)\cos 60° + \sin 10°$

$= -2\sin 10° \times \frac{1}{2} + \sin 10° \qquad \left[\because \cos 60° = \frac{1}{2}\right]$

$= 0$

$= RHS$

Transformation Formulae Ex 8.2 Q3(iii)

$\cos 80° + \cos 40° - \cos 20° = 0$

$(\cos 80° + \cos 40°) - \cos 20°$

$= 2\cos\left(\frac{80° + 40°}{2}\right)\cos\left(\frac{80° - 40°}{2}\right) - \cos 20° \qquad \left[\because \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$

$= 2\cos 60° \cos 20° - \cos 20°$

$= 2\times\frac{1}{2}\cos 20° - \cos 20°$

$= \cos 20° - \cos 20°$

$= 0$

$= RHS$

Transformation Formulae Ex 8.2 Q3(iv)

$\cos 20° + \cos 100° + \cos 140° = 0$

$\Rightarrow (\cos 20° + \cos 100°) + \cos 140°$

$= 2\cos\left(\frac{20° + 100°}{2}\right)\cos\left(\frac{20° - 100°}{2}\right) + \cos 140° \qquad \left[\because \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$

$= 2\cos 60° \cos(-40°) + \cos 140°$

$= 2\times\frac{1}{2}\cos 40° + \cos 140° \qquad \left[\because \cos 60° = \frac{1}{2}\right]$

$= \cos 40° + \cos(180° - 40°)$

$= \cos 40° - \cos 40°$

$= 0$

$= RHS$

Transformation Formulae Ex 8.2 Q3(v)

$\sin\frac{5\pi}{18} - \cos\frac{4\pi}{9} = \sqrt{3}\sin\frac{\pi}{9}$

LHS $= \sin\frac{5\pi}{18} - \cos\frac{4\pi}{9}$

$= \sin 50° - \cos 80°$

$= \sin 50° - \sin 10°$

$= 2\sin\left(\frac{50° - 10°}{2}\right)\cos\left(\frac{50° + 10°}{2}\right)$

$= 2\sin 20° \cos 30°$

$= 2\sin 20° \times \frac{\sqrt{3}}{2}$

$= \sqrt{3}\sin\frac{\pi}{9}$

Transformation Formulae Ex 8.2 Q3(vi)

$\cos\frac{\pi}{12} - \sin\frac{\pi}{12} = \frac{1}{\sqrt{2}}$

Multiplying and dividing by $\sqrt{2}$ on LHS

$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos\frac{\pi}{12} - \frac{1}{\sqrt{2}}\sin\frac{\pi}{12}\right)$

$$= \sqrt{2}\left(\sin\frac{\pi}{4}\cos\frac{\pi}{12} - \cos\frac{\pi}{4}\sin\frac{\pi}{12}\right) \qquad \left[\because \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4} = \sin\frac{\pi}{4}\right]$$

$$= \sqrt{2}\left(\sin\left(\frac{\pi}{4} - \frac{\pi}{12}\right)\right) \qquad \left[\because \sin(A - B) = \sin A\cos B - \cos A\sin B\right]$$

$$= \sqrt{2}\left(\sin\frac{\pi}{6}\right)$$

$$= \sqrt{2} \times \frac{1}{2}$$

$$= \frac{1}{\sqrt{2}}$$

= RHS

Transformation Formulae Ex 8.2 Q3(vii)

$\sin 80° - \cos 70° = \cos 50°$

LHS $= \sin 80° = \cos 50° + \cos 70°$

Now,

$$\cos C + \cos D = 2\cos\frac{C + D}{2}\cos\frac{C - D}{2}$$

RHS $= \cos 50° + \cos 70°$

$$= 2\cos\left(\frac{50° + 70°}{2}\right)\cos\left(\frac{50° - 70°}{2}\right)$$

$$= 2\cos 60°\cos(-10°)$$

$$= 2 \times \frac{1}{2}\cos 10° \qquad \left[\cos(-\theta) = \cos\theta\right]$$

$$= \cos 10°$$

$$= \sin 80°$$

$$= \text{LHS} \qquad \left[\because \cos\theta = \sin(90 - \theta)\right]$$

Transformation Formulae Ex 8.2 Q3(viii)

$\sin 51° + \cos 81° = \cos 21°$

$\sin 51° = \cos 21° - \cos 81°$

RHS $= \cos 21° - \cos 81°$

$$= -2\sin(51°)\sin(-30°) \qquad \left[\because \cos C - \cos D = -2\sin\frac{C + D}{2}\sin\frac{C - D}{2}\right]$$

$$= +2\sin 51°\sin 30°$$

$$= 2\sin 51° \times \frac{1}{2}$$

$$= \sin 51°$$

= LHS

Transformation Formulae Ex 8.2 Q4

We have,

LHS $= \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$

$$= -\left[\cos\left(\frac{3\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} + x\right)\right]$$

$$= -\left[2\sin\frac{3\pi}{4}\sin x\right] \qquad \left[\because \cos(A - B) - \cos(A + B) = 2\sin A\sin B\right]$$

$$= -2\sin\frac{3\pi}{4}\sin x$$

$$= -2\sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right)\sin x$$

$$= -2\cos\frac{\pi}{4}\sin x$$

$$= -2 \times \frac{1}{\sqrt{2}} \times \sin x$$

$$= -\frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}}\sin x$$

$$= -\sqrt{2}\sin x$$

= RHS

$\therefore \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2}\sin x$ Hence proved.

Transformation Formulae Ex 8.2 Q4(i)

We have,

LHS $= \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right)$

$$= 2\cos\frac{\pi}{4}\cos x \qquad \left[\because \cos(A + B) + \cos(A - B) = 2\cos A\cos B\right]$$

$$= 2 \times \frac{1}{\sqrt{2}} \times \cos x$$

$$= \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}}\cos x$$

$= \sqrt{2}\cos x$
$= \text{RHS}$

$\therefore \quad \cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right) = \sqrt{2}\cos x.$

Transformation Formulae Ex 8.2 Q5(i)

We have,

$$\begin{aligned}
\text{LHS} &= \sin 65° + \cos 65° \\
&= \sin(45°+20°) + \cos(90°-25°) \\
&= \sin(45°+20°) + \sin 25° \\
&= \sin(45°+20°) + \sin(45°-20°) \\
&= 2\sin 45° \cos 20° \\
&= 2 \times \frac{1}{\sqrt{2}} \cos 20° \\
&= \frac{\sqrt{2}\times\sqrt{2}}{\sqrt{2}} \times \cos 20° \\
&= \sqrt{2}\cos 20° \\
&= \text{RHS}
\end{aligned}$$

$\therefore \quad \sin 65° + \cos 65° = \sqrt{2}\cos 20°$ Hence proved.

Transformation Formulae Ex 8.2 Q5(ii)

We have,

$$\begin{aligned}
\text{LHS} &= \sin 47° + \cos 77° \\
&= \sin(90°-43°) + \cos 77° \\
&= \cos 43° + \cos 77° \\
&= \cos(60°-17°) + \cos(60°+17°) \\
&= 2\cos 60° \cos 17° \\
&= 2 \times \frac{1}{2} \times \cos 17° \\
&= \cos 17° \\
&= \text{RHS}
\end{aligned}$$

$\therefore \quad \sin 47° + \cos 77° = \cos 17°$ Hence proved.

Transformation Formulae Ex 8.2 Q 6(i)

We have,

$$\begin{aligned}
\text{LHS} &= \cos 3A + \cos 5A + \cos 7A + \cos 15A \\
&= [\cos 5A + \cos 3A] + [\cos 15A + \cos 7A] \\
&= \left[2\cos\frac{(5A+3A)}{2}\cos\frac{(5A-3A)}{2}\right] + \left[2\cos\frac{(15A+7A)}{2}\cos\frac{(15A-7A)}{2}\right] \\
&= 2\cos 4A\cos A + 2\cos 11A\cos 4A \\
&= 2\cos 4A[\cos A + \cos 11A] \\
&= 2\cos 4A[\cos 11A + \cos A] \\
&= 2\cos 4A\left[2\cos\frac{(11A+A)}{2}\cos\frac{(11A-A)}{2}\right] \\
&= 4\cos A[\cos 6A\cos 5A] \\
&= 4\cos 4A\cos 5A\cos 6A \\
&= \text{RHS}
\end{aligned}$$

$\therefore \quad \cos 3A + \cos 5A + \cos 7A + \cos 15A = 4\cos 4A\cos 5A\cos 6A$ Hence proved.

Transformation Formulae Ex 8.2 Q 6(ii)

We have,

$$\begin{aligned}
\text{LHS} &= \cos A + \cos 3A + \cos 5A + \cos 7A \\
&= (\cos 3A + \cos A) + (\cos 7A + \cos 5A) \\
&= \left[2\cos\left(\frac{3A+A}{2}\right)\cos\left(\frac{3A-A}{2}\right)\right] + \left[2\cos\left(\frac{7A+5A}{2}\right)\cos\left(\frac{7A-5A}{2}\right)\right] \\
&= 2\cos 2A\cos A + 2\cos 6A\cos A \\
&= 2\cos A[\cos 2A + \cos 6A] \\
&= 2\cos A[\cos 6A + \cos 2A] \\
&= 2\cos A\left[2\cos\left(\frac{6A+2A}{2}\right)\cos\left(\frac{6A-2A}{2}\right)\right] \\
&= 4\cos A[\cos 4A\cos 2A] \\
&= \text{RHS}
\end{aligned}$$

$\therefore \quad \cos A + \cos 3A + \cos 5A + \cos 7A = 4\cos A\cos 2A\cos 4A.$ Hence proved.

Transformation Formulae Ex 8.2 Q 6(iii)

We have,

$$\begin{aligned}
\text{LHS} &= \sin A + \sin 2A + \sin 4A + \sin 5A \\
&= (\sin 2A + \sin A) + (\sin 5A + \sin 4a) \\
&= \left[2\sin\left(\frac{2A+A}{2}\right)\cos\left(\frac{2A-A}{2}\right)\right] + \left[2\sin\left(\frac{5A+4A}{2}\right)\cos\left(\frac{5A-4A}{2}\right)\right] \\
&= 2\sin\frac{3A}{2}\cos\frac{A}{2} + 2\sin\frac{9A}{2}\cos\frac{A}{2} \\
&= 2\cos\frac{A}{2}\left[\sin\frac{3A}{2} + \sin\frac{9A}{2}\right] \\
&= 2\cos\frac{A}{2}\left[\sin\frac{9A}{2} + \sin\frac{3A}{2}\right] \\
&= 2\cos\frac{A}{2}\left[2\sin\left\{\frac{1}{2}\left(\frac{9A}{2}+\frac{3A}{2}\right)\right\}\cos\left\{\frac{1}{2}\left(\frac{9A}{2}-\frac{3A}{2}\right)\right\}\right] \\
&= 4\cos\frac{A}{2}\left[\sin\frac{12A}{4}\cos\frac{6A}{4}\right] \\
&= 4\cos\frac{A}{2}\sin 3A\cos\frac{3A}{2} \\
&= 4\cos\frac{A}{2}\cos\frac{3A}{2}\sin 3A \\
&= \text{RHS}
\end{aligned}$$

$\therefore \sin A + \sin 2A + \sin 4A + \sin 5A = 4\cos\frac{A}{2}\cos\frac{3A}{2}\sin 3A.$ Hence proved.

Transformation Formulae Ex 8.2 Q 6(iv)

We have,

$$\begin{aligned}
\text{LHS} &= \sin 3A + \sin 2A - \sin A \\
&= \sin 3A - \sin A + \sin 2A \\
&= 2\sin\left(\frac{3A-A}{2}\right)\cos\left(\frac{3A+A}{2}\right) + \sin 2A \\
&= 2\sin A\cos 2A + \sin 2A \\
&= 2\sin A\cos 2A + 2\sin A\cos A \\
&= 2\sin A[\cos 2A + \cos A] \\
&= 2\sin A\left[2\cos\left(\frac{2A+A}{2}\right)\cos\left(\frac{2A-A}{2}\right)\right] \\
&= 4\sin A\cos\frac{3A}{2}\cos\frac{A}{2} \\
&= 4\sin A\cos\frac{A}{2}\cos\frac{3A}{2} \\
&= \text{RHS}
\end{aligned}$$

$\therefore \sin 3A + \sin 2A - \sin A = 4\sin A\cos\frac{A}{2}\cos\frac{3A}{2}.$ Hence proved.

Transformation Formulae Ex 8.2 Q 6(v)

We have,

$$\begin{aligned}
\text{LHS} &= \cos 20^\circ\cos 100^\circ + \cos 100^\circ\cos 140^\circ - \cos 140^\circ\cos 200^\circ \\
&= \frac{1}{2}[2\cos 100^\circ\cos 20^\circ + 2\cos 140^\circ\cos 100^\circ - 2\cos 200^\circ\cos 140^\circ] \\
&= \frac{1}{2}\left[\begin{array}{r}\cos(100^\circ+20^\circ) + \cos(100^\circ-20^\circ) + \cos(140^\circ+100^\circ) + \cos(140^\circ-100^\circ) \\ -\{\cos(200^\circ+140^\circ) + \cos(200^\circ-140^\circ)\}\end{array}\right] \\
&= \frac{1}{2}[\cos 120^\circ + \cos 80^\circ + \cos 240^\circ + \cos 40^\circ - \cos 340^\circ - \cos 60^\circ] \\
&= \frac{1}{2}\left[\cos(90^\circ+30^\circ) + \cos 80^\circ + \cos 40^\circ - \cos(180^\circ+60^\circ) - \cos(360^\circ-20^\circ) - \frac{1}{2}\right] \\
&= \frac{1}{2}\left[-\sin 30^\circ + 2\cos\left(\frac{80^\circ+40^\circ}{2}\right)\cos\left(\frac{80^\circ-40^\circ}{2}\right) - \cos 60^\circ - \cos 20^\circ - \frac{1}{2}\right] \\
&= \frac{1}{2}\left[-\frac{1}{2} + 2\cos 60^\circ\cos 20^\circ - \frac{1}{2} - \cos 20^\circ - \frac{1}{2}\right] \\
&= \frac{1}{2}\left[-\frac{3}{2} + 2\times\frac{1}{2}\times\cos 20^\circ - \cos 20^\circ\right] \\
&= \frac{1}{2}\left[-\frac{3}{2} + \cos 20^\circ - \cos 20^\circ\right] \\
&= \frac{1}{2}\left[-\frac{3}{2} + 0\right] \\
&= -\frac{3}{4} \\
&= \text{RHS}
\end{aligned}$$

$\therefore \quad \cos 20° \cos 100° + \cos 100° \cos 140° - \cos 140° \cos 200° = -\frac{3}{4}.$ Hence proved.

Transformation Formulae Ex 8.2 Q 6(vi)

We have,

$$\text{LHS} = \sin\frac{\theta}{2}\sin\frac{7\theta}{2} + \sin\frac{3\theta}{2}\sin\frac{11\theta}{2}$$

$$= \frac{1}{2}\left[2\sin\frac{7\theta}{2}\sin\frac{\theta}{2} + 2\sin\frac{11\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$= \frac{1}{2}\left[\cos\left(\frac{7\theta}{2}-\frac{\theta}{2}\right) - \cos\left(\frac{7\theta}{2}+\frac{\theta}{2}\right) + \cos\left(\frac{11\theta}{2}-\frac{3\theta}{2}\right) - \cos\left(\frac{11\theta}{2}+\frac{3\theta}{2}\right)\right]$$

$$= \frac{1}{2}\left[\cos\frac{6\theta}{2} - \cos\frac{8\theta}{2} + \cos\frac{8\theta}{2} - \cos\frac{14\theta}{2}\right]$$

$$= \frac{1}{2}\left[\cos 3\theta - \cos 4\theta + \cos 4\theta - \cos 7\theta\right]$$

$$= \frac{1}{2}\left[\cos 3\theta - \cos 7\theta\right]$$

$$= \frac{-1}{2}\left[\cos 7\theta - \cos 3\theta\right]$$

$$= \frac{-1}{2}\left[-2\sin\left(\frac{7\theta+3\theta}{2}\right)\sin\left(\frac{7\theta-3\theta}{2}\right)\right]$$

$$= \sin\frac{10\theta}{2}\sin\frac{4\theta}{2}$$

$$= \sin 5\theta \sin 2\theta$$

$$= \sin 2\theta \sin 5\theta$$

$$= \text{RHS}$$

$\therefore \quad \sin\frac{\theta}{2}\sin\frac{7\theta}{2} + \sin\frac{3\theta}{2}\sin\frac{11\theta}{2} = \sin 2\theta \sin 5\theta.$ Hence proved.

Transformation Formulae Ex 8.2 Q 7(i)

We have,

$$\text{LHS} = \frac{\sin A + \sin 3A}{\cos A - \cos 3A}$$

$$= \frac{2\sin\left(\frac{A+3A}{2}\right)\cos\left(\frac{A-3A}{2}\right)}{-2\sin\left(\frac{A+3A}{2}\right)\sin\left(\frac{A-3A}{2}\right)}$$

$$= \frac{-\sin 2A \times \cos(-A)}{\sin 2A \sin(-A)}$$

$$= \frac{-\cos(-A)}{\sin(-A)}$$

$$= \frac{-\cos A}{-\sin A} \qquad [\because \cos(-\theta) = \cos\theta \text{ and } \sin(-\theta) = -\sin\theta]$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A$$

$$= \text{RHS}$$

$\therefore \quad \frac{\sin A + \sin 3A}{\cos A - \cos 3A} = \cot A.$ Hence proved.

Transformation Formulae Ex 8.2 Q 7(ii)

We have,

$$\text{LHS} = \frac{\sin 9A - \sin 7A}{\cos 7A - \cos 9A}$$

$$= \frac{2\sin\left(\frac{9A-7A}{2}\right)\cos\left(\frac{9A+7A}{2}\right)}{-2\sin\left(\frac{7A+9A}{2}\right)\sin\left(\frac{7A-9A}{2}\right)}$$

$$= \frac{-\sin A \cos 8A}{\sin 8A \sin(-A)}$$

$$= \frac{-\sin A \cos 8A}{-\sin A \times \sin 8A} \qquad [\because \sin(-\theta) = -\sin\theta]$$

$$= \frac{\cos 8A}{\sin 8A}$$

$$= \cot 8A$$

$$= \text{RHS}$$

$\therefore \quad \frac{\sin 9A - \sin 7A}{\cos 7A - \cos 9A} = \cot 8A.$ Hence proved.

Transformation Formulae Ex 8.2 Q 7(iii)

We have,

$$\text{LHS} = \frac{\sin A - \sin B}{\cos A + \cos B}$$

$$= \frac{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$$

$$= \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A-B}{2}\right)}$$

$$= \tan\left(\frac{A-B}{2}\right)$$

$$= \text{RHS}$$

$$\therefore \frac{\sin A - \sin B}{\cos A + \cos B} = \tan\left(\frac{A-B}{2}\right).$$ Hence proved.

Transformation Formulae Ex 8.2 Q7(iv)

We have,

$$\text{LHS} = \frac{\sin A + \sin B}{\sin A - \sin B}$$

$$= \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}$$

$$= \frac{\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}$$

$$= \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$$

$$= \text{RHS}$$

$$\therefore \frac{\sin A + \sin B}{\sin A - \sin B} = \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right).$$ Hence proved.

Transformation Formulae Ex 8.2 Q 7(v)

We have,

$$\text{LHS} = \frac{\cos A + \cos B}{\cos B - \cos A}$$

$$= \frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{-2\sin\left(\frac{B+A}{2}\right)\sin\left(\frac{B-A}{2}\right)}$$

$$= \frac{-\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)} \qquad [\because \sin(-\theta) = -\sin\theta]$$

$$= \frac{-\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{-\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}$$

$$= \cot\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right)$$

$$= \text{RHS}$$

$$\therefore \frac{\cos A + \cos B}{\cos B - \cos A} = \cot\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right).$$ Hence proved.

Transformation Formulae Ex 8.2 Q 8(i)

We have,

$$\text{LHS} = \frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A}$$

$$= \frac{(\sin 5A + \sin A) + \sin 3A}{(\cos 5A + \cos A) + \cos 3A}$$

$$= \frac{2\sin\left(\frac{5A+A}{2}\right)\cos\left(\frac{5A-A}{2}\right) + \sin 3A}{2\cos\left(\frac{5A+A}{2}\right)\cos\left(\frac{5A-A}{2}\right) + \cos 3A}$$

$2\sin 3A\cos 2A + \sin 3A$

$$= \frac{}{2\cos 3A \cos 2A + \cos 3A}$$
$$= \frac{\sin 3A\,(2\cos 2A + 1)}{\cos 3A\,(2\cos 2A + 1)}$$
$$= \frac{\sin 3A}{\cos 3A}$$
$$= \tan 3A$$
$$= \text{RHS}$$

$$\therefore \quad \frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A \qquad \text{Hence proved.}$$

Transformation Formulae Ex 8.2 Q 8(ii)

We have,

$$\text{LHS} = \frac{\cos 3A + 2\cos 5A + \cos 7A}{\cos A + 2\cos 3A + \cos 5A}$$
$$= \frac{(\cos 7A + \cos 3A) + 2\cos 5A}{(\cos 5A + \cos A) + 2\cos 3A}$$
$$= \frac{2\cos\left(\frac{7A+3A}{2}\right)\cos\left(\frac{7A-3A}{2}\right) + 2\cos 5A}{2\cos\left(\frac{5A+A}{2}\right)\cos\left(\frac{5A-A}{2}\right) + \cos 3A}$$
$$= \frac{2\cos 5A\cos 2A + 2\cos 5A}{2\cos 3A\cos 2A + 2\cos 3A}$$
$$= \frac{2\cos 5A\,(\cos 2A + 1)}{2\cos 3A\,(\cos 2A + 1)}$$
$$= \frac{\cos 5A}{\cos 3A}$$
$$= \text{RHS}$$

$$\therefore \quad \frac{\cos 3A + 2\cos 5A + \cos 7A}{\cos A + 2\cos 3A + \cos 5A} = \frac{\cos 5A}{\cos 3A} \qquad \text{Hence proved.}$$

Transformation Formulae Ex 8.2 Q 8(iii)

We have,

$$\text{LHS} = \frac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A + \sin 2A}$$
$$= \frac{(\cos 4A + \cos 2A) + \cos 3A}{(\sin 4A + \sin 2A) + \sin 3A}$$
$$= \frac{2\cos\left(\frac{4A+2A}{2}\right)\cos\left(\frac{4A-2A}{2}\right) + \cos 3A}{2\sin\left(\frac{4A+2A}{2}\right)\cos\left(\frac{4A-2A}{2}\right) + \sin 3A}$$
$$= \frac{2\cos 3A\cos A + \cos 3A}{2\sin 3A\cos A + \sin 3A}$$
$$= \frac{\cos 3A\,(2\cos A + 1)}{\sin 3A\,(2\cos A + 1)}$$
$$= \frac{\cos 3A}{\sin A}$$
$$= \cot 3A$$
$$= \text{RHS}$$

$$\therefore \quad \frac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A + \sin 2A} = \cot 3A \text{ Hence proved.}$$

Transformation Formulae Ex 8.2 Q 8(iv)

We have,

$$\text{LHS} = \frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A}$$
$$= \frac{(\sin 9A + \sin 3A) + (\sin 7A + \sin 5A)}{(\cos 9A + \cos 3A) + (\cos 7A + \cos 5A)}$$
$$= \frac{2\sin\left(\frac{9A+3A}{2}\right)\cos\left(\frac{9A-3A}{2}\right) + 2\sin\left(\frac{7A+5A}{2}\right)\cos\left(\frac{7A-5A}{2}\right)}{2\cos\left(\frac{9A+3A}{2}\right)\cos\left(\frac{9A-3A}{2}\right) + 2\cos\left(\frac{7A+5A}{2}\right)\cos\left(\frac{7A-5A}{2}\right)}$$
$$= \frac{2\sin 6A\cos 3A + 2\sin 6A\cos A}{2\cos 6A\cos 3A + 2\cos 6A\cos A}$$
$$= \frac{2\sin 6A\,(\cos 3A + \cos A)}{2\cos 6A\,(\cos 3A + \cos A)}$$
$$= \frac{\sin 6A}{\cos 6A}$$
$$= \tan 6A$$
$$= \text{RHS}$$

$$\therefore \quad \frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A} = \tan 6A \qquad \text{Hence proved.}$$

Transformation Formulae Ex 8.2 Q 8(v)

We have,

$$\text{LHS} = \frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A}$$

$$= \frac{-(\sin 7A - \sin 5A) + (\sin 8A - \sin 4A)}{-(\cos 7A - \cos 5A) - (\cos 8A - \cos 4A)}$$

$$= \frac{-\left[2\sin\left(\frac{7A-5A}{2}\right)\cos\left(\frac{7A+5A}{2}\right)\right] + \left[2\sin\left(\frac{8A-4A}{2}\right)\cos\left(\frac{8A+4A}{2}\right)\right]}{-2\sin\left(\frac{7A+5A}{2}\right)\sin\left(\frac{7A-5A}{2}\right) - \left[-2\sin\left(\frac{8A+4A}{2}\right)\sin\left(\frac{8A-4A}{2}\right)\right]}$$

$$= \frac{-2\sin A\cos 6A + 2\sin 2A\cos 6A}{-2\sin 6A\sin A + 2\sin 6A\sin 2A}$$

$$= \frac{2\cos 6A[-\sin A + \sin 2A]}{2\sin 6A[-\sin A + \sin 2A]}$$

$$= \frac{\cos 6A}{\sin 6A}$$

$$= \cot 6A$$

$$= \text{RHS}$$

$$\therefore \quad \frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A} = \cot 6A \qquad \text{Hence proved.}$$

Transformation Formulae Ex 8.2 Q 8(vi)

We have,

$$\text{LHS} = \frac{\sin 5A\cos 2A - \sin 6A\cos A}{\sin A\sin 2A - \cos 2A\cos 3A}$$

$$= \frac{2(\sin 5A\cos 2A - \sin 6A\cos A)}{2(\sin A\sin 2A - \cos 2A\cos 3A)}$$

$$= \frac{2\sin 5A\cos 2A - 2\sin 6A\cos A}{2\sin A\sin 2A - 2\cos 2A\cos 3A}$$

$$= \frac{\sin(5A+2A) + \sin(5A-2A) - [\sin(6A+A) + \sin(6A-A)]}{\cos(2A-A) - \cos(2A+A) - [\cos(3A+2A) + \cos(3A-2A)]}$$

$$= \frac{\sin 7A + \sin 3A - \sin 7A - \sin 5A}{\cos A - \cos 3A - \cos 5A - \cos A}$$

$$= \frac{\sin 3a - \sin 5A}{-\cos 3A - \cos 5A}$$

$$= \frac{-(\sin 5A - \sin 3A)}{-(\cos 5A + \cos 3A)}$$

$$= \frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A}$$

$$= \frac{2\sin\left(\frac{5A-3A}{2}\right)\cos\left(\frac{5A+3A}{2}\right)}{2\cos\left(\frac{5A+3A}{2}\right)\cos\left(\frac{5A-3A}{2}\right)}$$

$$= \frac{\sin A\cos 4A}{\cos 4A\cos A}$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

$$= \text{RHS}$$

$$\therefore \quad \frac{\sin 5A\cos 2A - \sin 6A\cos A}{\sin A\sin 2A - \cos 2A\cos 3A} = \tan A \qquad \text{Hence proved.}$$

Transformation Formulae Ex 8.2 Q 8(vii)

We have,

$$\text{LHS} = \frac{\sin 11A\sin A + \sin 7A\sin 3A}{\cos 11A\sin A + \cos 7A\sin 3A}$$

$$= \frac{2(\sin 11A\sin A + \sin 7A\sin 3A)}{2(\cos 11A\sin A + \cos 7A\sin 3A)}$$

$$= \frac{2\sin 11A\sin A + 2\sin 7A\sin 3A}{2\cos 11A\sin A + 2\cos 7A\sin 3A}$$

$$= \frac{\cos(11A-A) - \cos(11A+A) + \cos(7A-3A) - \cos(7A+3A)}{\sin(11A+A) - \sin(11A-A) + \sin(7A+3A) - \sin(7A-3A)}$$

$$= \frac{\cos 10A - \cos 12A + \cos 4A - \cos 10A}{\sin 12A - \sin 10A + \sin 10A - \sin 4A}$$

$$= \frac{-(\cos 12A - \cos 4A)}{\sin 12A - \sin 4A}$$

$$\left[\quad \left(\frac{12A+4A}{\quad}\right) \quad \left(\frac{12A-4A}{\quad}\right)\right]$$

$$= \frac{-\left[-2\sin\left(\frac{12A+4A}{2}\right)\sin\left(\frac{12A-4A}{2}\right)\right]}{2\sin\left(\frac{12A-4A}{2}\right)\cos\left(\frac{12A+4A}{2}\right)}$$

$$= \frac{2\sin 8A \sin 4A}{2\sin 4A \cos 8A}$$

$$= \frac{\sin 8A}{\cos 8A}$$

$$= \tan 8A$$

$$= \text{RHS}$$

$$\therefore \frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A} = \tan 8A$$ Hence proved.

Transformation Formulae Ex 8.2 Q 8(viii)

$$\text{LHS} = \frac{\sin 3A \cos 4A - \sin A \cos 2A}{\sin 4A \sin A + \cos 6A \cos A}$$

$$= \frac{2(\sin 3A \cos 4A - \sin A \cos 2A)}{2(\sin 4A \sin A + \cos 6A \cos A)}$$

$$= \frac{2\sin 3A \cos 4A - 2\sin A \cos 2A}{2\sin 4A \sin A + 2\cos 6A \cos A}$$

$$= \frac{\sin(4A+3A) - \sin(4A-3A) - [\sin(2A+A) - \sin(2A-A)]}{\cos(4A-A) - \cos(4A+A) + \cos(6A+A) + \cos(6A-A)}$$

$$= \frac{\sin(7A) - \sin(A) - \sin(3A) + \sin(A)}{\cos(3A) - \cos(5A) + \cos(7A) + \cos(5A)}$$

$$= \frac{\sin(7A) - \sin(3A)}{\cos(3A) + \cos(7A)}$$

$$= \frac{2\sin\left(\frac{7A-3A}{2}\right)\cos\left(\frac{7A+3A}{2}\right)}{2\cos\left(\frac{7A+3A}{2}\right)\cos\left(\frac{7A-3A}{2}\right)}$$

$$= \frac{\sin 2A}{\cos 2A}$$

$$= \tan 2A$$

$$= RHS$$

Transformation Formulae Ex 8.2 Q 8(ix)

We have,

$$\text{LHS} = \frac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A}$$

$$= \frac{2[\sin A \sin 2A + \sin 3A \sin 6A]}{2[\sin A \cos 2A + \sin 3A \cos 6A]}$$

$$= \frac{2\sin 2A \sin A + 2\sin 6A \sin 3A}{2\cos 2A \sin A + 2\cos 6A \sin 3A}$$

$$= \frac{\cos(2A-A) - \cos(2A+A) + \cos(6A-3A) - \cos(6A+3A)}{\sin(2A+A) - \sin(2A-A) + \sin(6A+3A) - \sin(6A-3A)}$$

$$= \frac{\cos A - \cos 3A + \cos 3A - \cos 9A}{\sin 3A - \sin A + \sin 9A - \sin 3A}$$

$$= \frac{\cos A - \cos 9A}{\sin 9A - \sin A}$$

$$= \frac{-[\cos 9A - \cos A]}{\sin 9A - \sin A}$$

$$= \frac{-\left(-2\sin\left(\frac{9A+A}{2}\right) \times \sin\left(\frac{9A-A}{2}\right)\right)}{2\sin\left(\frac{9A-A}{2}\right) \times \cos\left(\frac{9A+A}{2}\right)}$$

$$= \frac{\sin 5A \sin 4A}{\sin 4A \cos 5A}$$

$$= \tan 5A$$

$$= \text{RHS}$$

$$\therefore \frac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A} = \tan 5A$$ Hence proved.

Transformation Formulae Ex 8.2 Q 8(x)

We have,

$$\text{LHS} = \frac{\sin A + 2\sin 3A + \sin 5A}{\sin 3A + 2\sin 5A + \sin 7A}$$

$$= \frac{\sin 5A + \sin A + 2\sin 3A}{\sin 7A + \sin 3A + 2\sin 5A}$$

$\sin 7A + \sin 3A + 2\sin 5A$

$$= \frac{2\sin\left(\frac{5A+A}{2}\right)\cos\left(\frac{5A-A}{2}\right) + 2\sin 3A}{2\sin\left(\frac{7A+3A}{2}\right)\cos\left(\frac{7A-3A}{2}\right) + 2\sin 5A}$$

$$= \frac{2\sin 3A\cos 2A + 2\sin 3A}{2\sin 5A\cos 2A + 2\sin 5A}$$

$$= \frac{2\sin 3A(\cos 2A + 1)}{2\sin 5A(\cos 2A + 1)}$$

$$= \frac{\sin 3A}{\sin 5A}$$

$= \text{RHS}$

$\therefore \frac{\sin A + 2\sin 3A + \sin 5A}{\sin 3A + 2\sin 5A + \sin 7A} = \frac{\sin 3A}{\sin 5A}$ Hence proved.

Transformation Formulae Ex 8.2 Q 8(xi)

We have,

$$\text{LHS} = \frac{\sin(\theta+\phi) - 2\sin\theta + \sin(\theta-\phi)}{\cos(\theta+\phi) - 2\cos\theta + \cos(\theta-\phi)}$$

$$= \frac{\sin(\theta+\phi) + \sin(\theta-\phi) - 2\sin\theta}{\cos(\theta+\phi) + \cos(\theta-\phi) - 2\cos\theta}$$

$$= \frac{2\sin\left[\frac{(\theta+\phi)+(\theta-\phi)}{2}\right]\cos\left[\frac{(\theta+\phi)-(\theta-\phi)}{2}\right] - 2\sin\theta}{2\cos\left[\frac{(\theta+\phi)+(\theta-\phi)}{2}\right]\cos\left[\frac{(\theta+\phi)-(\theta-\phi)}{2}\right] - 2\cos\theta}$$

$$= \frac{2\sin(\theta)\cos(\phi) - 2\sin\theta}{2\cos(\theta)\cos(\phi) - 2\cos\theta}$$

$$= \frac{2\sin\theta(\cos\phi - 1)}{2\cos\theta(\cos\phi - 1)}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$= \text{RHS}$

$\therefore \frac{\sin(\theta+\phi) - 2\sin\theta + \sin(\theta-\phi)}{\cos(\theta+\phi) - 2\cos\theta + \cos(\theta-\phi)} = \tan\theta$ Hence proved.

Transformation Formulae Ex 8.2 Q9(i)

We have,

$\text{LHS} = \sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma)$

$= (\sin\alpha + \sin\beta) + (\sin\gamma - \sin(\alpha+\beta+\gamma))$

$$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + 2\sin\left(\frac{\gamma-(\alpha+\beta+\gamma)}{2}\right)\cos\left(\frac{\gamma+\alpha+\beta+\gamma}{2}\right)$$

$$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + 2\sin\left(\frac{-\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)$$

$$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) - 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)$$

$$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[\cos\left(\frac{\alpha-\beta}{2}\right) - \cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right]$$

$$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[-2\sin\frac{\left[\frac{\alpha-\beta}{2} + \frac{\alpha+\beta+2\gamma}{2}\right]}{2}\sin\frac{\left[\frac{\alpha-\beta}{2} - \frac{\alpha+\beta+2\gamma}{2}\right]}{2}\right]$$

$$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[-2\sin\left[\frac{2\alpha+2\gamma}{2\times 2}\right]\sin\left[\frac{-2\beta-2\gamma}{2\times 2}\right]\right]$$

$$= -4\sin\left(\frac{\alpha+\beta}{2}\right)\left[\sin\left(\frac{\alpha+\gamma}{2}\right)\sin\left[\frac{-(\beta+\gamma)}{2}\right]\right]$$

$$= 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)$$

$$= 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)$$

$= \text{RHS}$

$\therefore \sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma) = 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)$ Hence proved.

Transformation Formulae Ex 8.2 Q9(ii)

We have,

$\text{LHS} = \cos(A+B+C) + \cos(A-B+C) + \cos(A+B-C) + \cos(-A+B+C)$

LHS $= \cos(A+B+C)+\cos(A-B+C)+\cos(A+B-C)+\cos(-A+B+C)$

$= [\cos(A+B+C)+\cos(A-B+C)]+[\cos(A+B-C)+\cos(-A+B+C)]$

$= 2\cos\left\{\frac{A+B+C+A-B+C}{2}\right\}\cos\left\{\frac{A+B+C-A+B-C}{2}\right\}+2\left\{\begin{matrix}\cos\left\{\frac{A+B-C-A+B+C}{2}\right\} \\ \cos\left\{\frac{A+B-C+A-B-C}{2}\right\}\end{matrix}\right\}$

$= 2\cos\left\{\frac{2A+2C}{2}\right\}\cos\left\{\frac{2B}{2}\right\}+2\cos\left\{\frac{2B}{2}\right\}\cos\left\{\frac{2A-2C}{2}\right\}$

$= 2\cos(A+C)\cos(B)+2\cos(B)\cos(A-C)$

$= 2\cos(B)[\cos(A+C)+\cos(A-C)]$

$= 2\cos(B)\left[2\cos\left(\frac{A+C+A-C}{2}\right)\cos\left(\frac{A+C-A+C}{2}\right)\right]$

$= 2\cos(B)[2\cos A\cos C]$

$= 4\cos A\cos B\cos C.$

Transformation Formulae Ex 8.2 Q10

We have,

$\cos A+\cos B = \frac{1}{2}$

and, $\sin A+\sin B = \frac{1}{4}$

Now,

$\frac{\sin A+\sin B}{\cos A+\cos B} = \frac{\frac{1}{4}}{\frac{1}{2}}$ [On dividing]

$\Rightarrow \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)} = \frac{1}{2}$

$\Rightarrow \frac{\sin\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)} = \frac{1}{2}$

$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \frac{1}{2}$ Hence proved.

Transformation Formulae Ex 8.2 Q 11.

We have,

$\operatorname{cosec}A+\sec A = \operatorname{cosec}B+\sec B$

$\Rightarrow \sec A-\sec B = \operatorname{cosec}B-\operatorname{cosec}A$

$\Rightarrow \frac{1}{\cos A}-\frac{1}{\cos B} = \frac{1}{\sin B}-\frac{1}{\sin A}$

$\Rightarrow \frac{\cos B-\cos A}{\cos A\cos B} = \frac{\sin A-\sin B}{\sin A\sin B}$

$\Rightarrow \frac{\sin A\sin B}{\cos A\cos B} = \frac{\sin A-\sin B}{\cos B-\cos A}$

$\Rightarrow \tan A\tan B = \frac{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{-2\sin\left(\frac{B-A}{2}\right)\sin\left(\frac{B+A}{2}\right)}$

$\Rightarrow \tan A\tan B = \frac{-\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{-\sin\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right)}$ $[\because \sin(-\theta) = -\sin\theta]$

$\Rightarrow \tan A\tan B = \cot\left(\frac{A+B}{2}\right)$ Hence proved.

Transformation Formulae Ex 8.2 Q 12.

We have,

$\sin 2A = \lambda\sin 2B$

$\Rightarrow \lambda = \frac{\sin 2A}{\sin 2B}$

Now,

$\frac{\lambda+1}{\lambda-1} = \frac{\frac{\sin 2A}{\sin 2B}+1}{\frac{\sin 2A}{\sin 2B}-1}$

$\frac{\sin 2A+\sin 2B}{}$

$$= \frac{\frac{\sin 2B}{\sin 2A - \sin 2B}}{\sin 2B}$$

$$= \frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B}$$

$$= \frac{2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right)}{2\sin\left(\frac{2A-2B}{2}\right)\cos\left(\frac{2A+2B}{2}\right)}$$

$$= \frac{\sin(A+B)\cos(A-B)}{\sin(A-B)\cos(A+B)}$$

$$= \frac{\sin(A+B)\cos(A-B)}{\cos(A+B)\sin(A-B)}$$

$$= \frac{\tan(A+B)}{\tan(A-B)}$$

$$\therefore \quad \frac{\lambda+1}{\lambda-1} = \frac{\tan(A+B)}{\tan(A-B)}$$

$$\Rightarrow \quad \frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}$$ Hence proved.

Transformation Formulae Ex 8.2 Q13(i)

We have,

$$\text{LHS} = \frac{\cos(A+B+C)+\cos(-A+B+C)+\cos(A-B+C)+\cos(A+B-C)}{\sin(A+B+C)+\sin(-A+B+C)+\sin(A-B+C)-\sin(A+B-C)}$$

$$= \frac{2\cos\left\{\frac{A+B+C-A+B+C}{2}\right\}\cos\left\{\frac{A+B+C+A-B-C}{2}\right\}+2\cos\left\{\frac{A-B+C+A+B-C}{2}\right\}\cos\left\{\frac{A-B+C-A-B+C}{2}\right\}}{2\sin\left\{\frac{A+B+C-A+B+C}{2}\right\}\cos\left\{\frac{A+B+C+A-B-C}{2}\right\}+2\sin\left\{\frac{A-B+C-A-B+C}{2}\right\}\cos\left\{\frac{A-B+C+A+B-C}{2}\right\}}$$

$$= \frac{2\cos(B+C)\cos A + 2\cos A\cos(C-B)}{2\sin(B+C)\cos A + 2\sin(C-B)\cos A}$$

$$= \frac{2\cos A\left[\cos(B+C)+\cos(C-B)\right]}{2\cos A\left[\sin(B+C)+\sin(C-B)\right]}$$

$$= \frac{\cos(B+C)+\cos(C-B)}{\sin(B+C)+\sin(C-B)}$$

$$= \frac{2\cos\left\{\frac{B+C+C-B}{2}\right\}\cos\left\{\frac{B+C-C+B}{2}\right\}}{2\sin\left\{\frac{B+C+C-B}{2}\right\}\cos\left\{\frac{B+C-C+B}{2}\right\}}$$

$$= \frac{2\cos C\cos B}{2\sin C\cos B}$$

$$= \frac{\cos C}{\sin C}$$

$$= \cot C$$

$$= \text{RHS}$$

$$\therefore \quad \frac{\cos(A+B+C)+\cos(-A+B+C)+\cos(A-B+C)+\cos(A+B-C)}{\sin(A+B+C)+\sin(-A+B+C)+\sin(A-B+C)-\sin(A+B-C)} = \cot C.$$ Hence proved.

Transformation Formulae Ex 8.2 Q13(ii)

We have,

$$\text{LHS} = \sin(B-C)\cos(A-D)+\sin(C-A)\cos(B-D)+\sin(A-B)\cos(C-D)$$

$$= \frac{1}{2}\left[2\sin(B-C)\cos(A-D)+2\sin(C-A)\cos(B-D)+2\sin(A-B)\cos(C-D)\right]$$

$$= \frac{1}{2}\left[\begin{array}{r}\sin(B-C+A-D)+\sin(B-C-A+D)+\sin(C-A+B-D)+\sin(C-A-B+D)\\ +\sin(A-B+C-D)+\sin(A-B-C+D)\end{array}\right]$$

$$= \frac{1}{2}\left[\begin{array}{r}\sin(A+B-C-D)+\sin(B+D-C-A)+\sin(B+C-A-D)+\sin(C+D-A-B)\\ +\sin(A+C-B-D)+\sin(A+D-B-C)\end{array}\right]$$

$$= \frac{1}{2}\left[\begin{array}{r}\sin(A+B-C-D)-\sin(A+C-B-D)-\sin(A+D-B-C)-\sin(A+B-C-D)\\ +\sin(A+C-B-D)+\sin(A+D-B-C)\end{array}\right]$$

$$= \frac{1}{2}[0]$$

$$= 0$$

$$= \text{RHS}$$

$$\therefore \quad \sin(B-C)\cos(A-D)+\sin(C-A)\cos(B-D)+\sin(A-B)\cos(C-D) = 0$$ Hence proved.

Transformation Formulae Ex 8.2 Q 14.

We have,

$$\frac{\cos(A-B)}{\cos(A+B)}+\frac{\cos(C+D)}{\cos(C-D)}=0$$

$$\Rightarrow \quad \frac{\cos(A-B)}{\cos(A+B)}=-\frac{\cos(C+D)}{\cos(C-D)} \qquad \text{---(i)}$$

Now,

$$\frac{\cos(A-B)}{\cos(A+B)}=-\frac{\cos(C+D)}{\cos(C-D)}$$

$$\Rightarrow \quad \frac{\cos(A-B)}{\cos(A+B)}+1=\frac{-\cos(C+D)}{\cos(C-D)}+1$$

$$\Rightarrow \quad \frac{\cos(A-B)+\cos(A+B)}{\cos(A+B)}=\frac{-\cos(C+D)+\cos(C-D)}{\cos(C-D)}$$

$$\Rightarrow \quad \frac{\cos(A+B)+\cos(A-B)}{\cos(A+B)}=\frac{-\left[\cos(C+D)-\cos(C-D)\right]}{\cos(C-D)} \qquad \text{---(ii)}$$

Again,

$$\frac{\cos(A-B)}{\cos(A+B)}=\frac{-\cos(C+D)}{\cos(C-D)} \qquad \text{[By equation (i)]}$$

$$\Rightarrow \quad \frac{\cos(A-B)}{\cos(A+B)}-1=\frac{-\cos(C+D)}{\cos(C-D)}-1$$

$$\Rightarrow \quad \frac{\cos(A-B)-\cos(A+B)}{\cos(A+B)}=\frac{-\cos(C+D)-\cos(C-D)}{\cos(C-D)}$$

$$\Rightarrow \quad \frac{-\{\cos(A+B)-\cos(A-B)\}}{\cos(A+B)}=\frac{-\left[\cos(C+D)+\cos(C-D)\right]}{\cos(C-D)}$$

$$\Rightarrow \quad \frac{\cos(A+B)-\cos(A-B)}{\cos(A+B)}=\frac{\cos(C+D)+\cos(C-D)}{\cos(C-D)} \qquad \text{---(iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\frac{\cos(A+B)+\cos(A-B)}{\cos(A+B)-\cos(A-B)}=\frac{-\left[\cos(C+D)-\cos(C-D)\right]}{\cos(C+D)+\cos(C-D)}$$

$$\Rightarrow \quad \frac{2\cos\left\{\frac{A+B+A-B}{2}\right\}\cos\left\{\frac{A+B-A+B}{2}\right\}}{-2\sin\left\{\frac{A+B+A-B}{2}\right\}\sin\left\{\frac{A+B-A+B}{2}\right\}}=\frac{-\left[2\sin\left\{\frac{C+D+C-D}{2}\right\}\sin\left\{\frac{C+D-C+D}{2}\right\}\right]}{2\cos\left\{\frac{C+D+C-D}{2}\right\}\cos\left\{\frac{C+D-C+D}{2}\right\}}$$

$$\Rightarrow \quad \frac{\cos A\cos B}{-\sin A\sin B}=\frac{\sin C\sin D}{\cos C\cos D}$$

$$\Rightarrow \quad \frac{1}{-\tan A\tan B}=\tan C\tan D$$

$$\Rightarrow \quad -1=\tan A\tan B\tan C\tan D$$

$\therefore \quad \tan A\tan B\tan C\tan D=-1$ Hence proved.

Transformation Formulae Ex 8.2 Q 15.

We have,

$$\cos(\alpha+\beta)\sin(\gamma+\delta)=\cos(\alpha-\beta)\sin(\gamma-\delta)$$

$$\Rightarrow \quad \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)} \qquad \text{---(i)}$$

Now,

$$\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}$$

$$\Rightarrow \quad \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}+1=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}+1$$

$$\Rightarrow \quad \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)+\sin(\gamma+\delta)}{\sin(\gamma+\delta)} \qquad \text{---(ii)}$$

Again,

$$\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)} \qquad \text{[By equation (i)]}$$

$$\Rightarrow \quad \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}-1=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}-1$$

$$\Rightarrow \quad \frac{\cos(\alpha+\beta)-\cos(\alpha-\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)-\sin(\gamma+\delta)}{\sin(\gamma+\delta)} \qquad \text{---(iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)+\sin(\gamma+\delta)}{\sin(\gamma-\delta)-\sin(\gamma+\delta)}$$

$$\Rightarrow \quad \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)}=-\left[\frac{\sin(\gamma+\delta)+\sin(\gamma-\delta)}{\sin(\gamma+\delta)-\sin(\gamma-\delta)}\right]$$

$$\Rightarrow \quad \frac{2\cos\left\{\frac{\alpha+\beta+\alpha-\beta}{2}\right\}\cos\left\{\frac{\alpha+\beta-\alpha+\beta}{2}\right\}}{-2\sin\left\{\frac{\alpha+\beta+\alpha-\beta}{2}\right\}\sin\left\{\frac{\alpha+\beta-\alpha+\beta}{2}\right\}}=-\left[\frac{2\sin\left\{\frac{\gamma+\delta+\gamma-\delta}{2}\right\}\cos\left\{\frac{\gamma+\delta-\gamma+\delta}{2}\right\}}{2\sin\left\{\frac{\gamma+\delta-\gamma+\delta}{2}\right\}\cos\left\{\frac{\gamma+\delta+\gamma-\delta}{2}\right\}}\right]$$

$$\Rightarrow \quad \frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}=\frac{\sin\gamma\cos\delta}{\sin\delta\cos\gamma}$$

$$\Rightarrow \quad \cot\alpha\cot\beta=\frac{\sin\gamma\cos\delta}{\cos\gamma\sin\delta}$$

$$\Rightarrow \quad \cot\alpha\cot\beta=\frac{\cot\delta}{\cot\gamma}$$

$$\Rightarrow \quad \cot\alpha\cot\beta\cot\gamma=\cot\delta$$

$\therefore \quad \cot\alpha\cot\beta\cot\gamma=\cot\delta$ Hence proved.

Transformation Formulae Ex 8.2 Q 16.

We have,

$$y\sin\phi=x\sin(2\theta+\phi)$$

$$\Rightarrow \quad \frac{\sin\phi}{\sin(2\theta+\phi)}=\frac{x}{y} \qquad \text{---(i)}$$

Now,

$$\frac{\sin\phi}{\sin(2\theta+\phi)} = \frac{x}{y}$$

$$\Rightarrow \quad \frac{\sin\phi}{\sin(2\theta+\phi)}+1 = \frac{x}{y}+1$$

$$\Rightarrow \quad \frac{\sin\phi+\sin(2\theta+\phi)}{\sin(2\theta+\phi)} = \frac{x+y}{y} \qquad ---\text{(ii)}$$

Again,

$$\frac{\sin\phi}{\sin(2\theta+\phi)} = \frac{x}{y} \qquad [\text{By equation (i)}]$$

$$\Rightarrow \quad \frac{\sin\phi}{\sin(2\theta+\phi)}-1 = \frac{x}{y}-1$$

$$\Rightarrow \quad \frac{\sin\phi-\sin(2\theta+\phi)}{\sin(2\theta+\phi)} = \frac{x-y}{y} \qquad ---\text{(iii)}$$

Dividing equation (ii) by equation (iii),we get

$$\frac{\sin\phi+\sin(2\theta+\phi)}{\sin\phi-\sin(2\theta+\phi)} = \frac{x+y}{x-y}$$

$$\Rightarrow \quad \frac{2\sin\left(\frac{\phi+2\theta+\phi}{2}\right)\cos\left(\frac{\phi-2\theta-\phi}{2}\right)}{2\sin\left(\frac{\phi-2\theta-\phi}{2}\right)\cos\left(\frac{\phi+2\theta+\phi}{2}\right)} = \frac{x+y}{x-y}$$

$$\Rightarrow \quad \frac{\sin(\theta+\phi)\cos(\theta-\phi)}{\sin(-\theta)\cos(\theta+\phi)} = \frac{x+y}{x-y}$$

$$\Rightarrow \quad \frac{\sin(\theta+\phi)\cos(\theta)}{\cos(\theta+\phi)[-\sin(\theta)]} = \frac{x+y}{x-y}$$

$$\Rightarrow \quad \frac{-\cot(\theta)}{\cot(\theta+\phi)} = \frac{x+y}{x-y}$$

$$\Rightarrow \quad -(x-y)\cot\theta = (x+y)\cot(\theta+\phi)$$

$$\Rightarrow \quad (y-x)\cot\theta = (x+y)\cot(\theta+\phi)$$

$$\Rightarrow \quad (x+y)\cot(\theta+\phi) = (y-x)\cot\theta \qquad \text{Hence proved.}$$

Transformation Formulae Ex 8.2 Q 17.

We have,

$$\cos(A+B)\sin(C-D) = \cos(A-B)\sin(C+D)$$

$$\Rightarrow \quad \frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C+D)}{\sin(C-D)} \qquad ---\text{(i)}$$

Now,

$$\frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C+D)}{\sin(C-D)}$$

$$\Rightarrow \quad \frac{\cos(A+B)}{\cos(A-B)}+1 = \frac{\sin(C+D)}{\sin(C-D)}+1$$

$$\Rightarrow \quad \frac{\cos(A+B)+\cos(A-B)}{\cos(A-B)} = \frac{\sin(C+D)+\sin(C-D)}{\sin(C-D)} \qquad ---\text{(ii)}$$

Again,

$$\frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C+D)}{\sin(C-D)} \qquad [\text{By equation (i)}]$$

$$\Rightarrow \quad \frac{\cos(A+B)}{\cos(A-B)}-1 = \frac{\sin(C+D)}{\sin(C-D)}-1$$

$$\Rightarrow \quad \frac{\cos(A+B)-\cos(A-B)}{\cos(A-B)} = \frac{\sin(C+D)-\sin(C-D)}{\sin(C-D)} \qquad ---\text{(iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\frac{\cos(A+B)+\cos(A-B)}{\cos(A+B)-\cos(A-B)} = \frac{\sin(C+D)+\sin(C-D)}{\sin(C+D)-\sin(C-D)}$$

$$\Rightarrow \quad \frac{2\cos\left\{\frac{A+B+A-B}{2}\right\}\cos\left\{\frac{A+B-A+B}{2}\right\}}{-2\sin\left\{\frac{A+B+A-B}{2}\right\}\sin\left\{\frac{A+B-A+B}{2}\right\}} = \frac{2\sin\left\{\frac{C+D+C-D}{2}\right\}\cos\left\{\frac{C+D-C+D}{2}\right\}}{2\sin\left\{\frac{C+D-C+D}{2}\right\}\cos\left\{\frac{C+D+C-D}{2}\right\}}$$

$$\Rightarrow \quad \frac{\cos A\cos B}{-\sin A\sin B} = \frac{\sin C\cos D}{\sin D\cos C}$$

$$\Rightarrow \quad \frac{1}{-\tan A\tan B} = \frac{\sin C\cos D}{\cos C\sin D}$$

$$\Rightarrow \quad \frac{-1}{\tan A\tan B} = \frac{\tan C}{\tan D}$$

$$\Rightarrow \quad -\tan D = \tan A\tan B\tan C$$

$$\Rightarrow \quad \tan A\tan B\tan C = -\tan D$$

$$\Rightarrow \quad \tan A\tan B\tan C+\tan D = 0 \qquad \text{Hence proved.}$$

Transformation Formulae Ex 8.2 Q 18.

$$\text{Given } x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right) = k(\text{say})$$

$$x = \frac{k}{\cos\theta}$$

$$y = \frac{k}{\cos\left(\theta+\frac{2\pi}{3}\right)}$$

$$z = \frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}$$

$$xy + yz + zx = k^2\left[\frac{1}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)} + \frac{1}{\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)} + \frac{1}{\cos\left(\theta + \frac{4\pi}{3}\right)\cos\theta}\right]$$

$$= k^2\left[\frac{\cos\left(\theta + \frac{4\pi}{3}\right) + \cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right)}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right]$$

$$= k^2\left[\frac{\cos\theta\cos\frac{4\pi}{3} - \sin\theta\sin\frac{4\pi}{3} + \cos\theta + \cos\theta\cos\frac{2\pi}{3} - \sin\theta\sin\frac{2\pi}{3}}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right]$$

$$= k^2\left[\frac{\cos\theta\left(\frac{-1}{2}\right) - \sin\theta\left(\frac{-\sqrt{3}}{2}\right) + \cos\theta + \cos\theta\left(\frac{-1}{2}\right) - \sin\theta\left(\frac{\sqrt{3}}{2}\right)}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right]$$

$$= k^2\left[\frac{-\cos\theta + \sin\theta\left(\frac{\sqrt{3}}{2}\right) + \cos\theta + -\sin\theta\left(\frac{\sqrt{3}}{2}\right)}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right]$$

$$= 0$$

Hence Proved