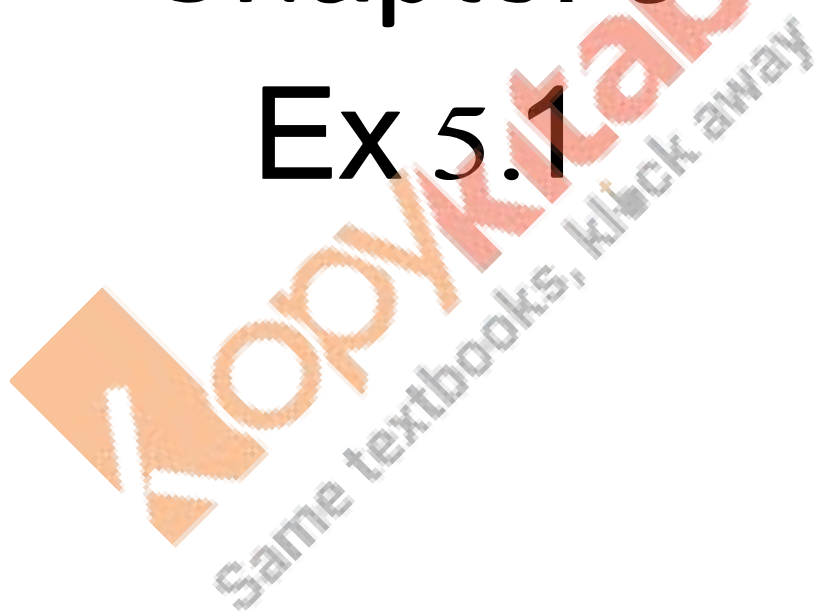


RD Sharma
Solutions
Class 11 Maths
Chapter 5
Ex 5.1



Trigonometric Functions Ex 5.1 Q1

$$\begin{aligned}
\text{LHS} &= \sec^4 \theta - \sec^2 \theta \\
&= \sec^2 \theta (\sec^2 \theta - 1) \\
&= (1 + \tan^2 \theta) \tan^2 \theta & [\because \sec^2 \theta = 1 + \tan^2 \theta] \\
&= \tan^2 \theta + \tan^4 \theta \\
&= \tan^4 \theta + \tan^2 \theta \\
&= \text{RHS} \\
\text{LHS} &= \text{RHS} \\
&\text{Proved}
\end{aligned}$$

Trigonometric Functions Ex 5.1 Q2

$$\begin{aligned}
\text{LHS} &= \sin^6 \theta + \cos^6 \theta \\
&= (\sin^2 \theta)^3 + (\cos^2 \theta)^3 \\
&= (\sin^2 \theta + \cos^2 \theta) \left[(\sin^2 \theta)^2 - \sin^2 \theta \cos^2 \theta + (\cos^2 \theta)^2 \right] & (\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)) \\
&= (\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 2 \sin^2 \theta \cos^2 \theta - \sin^2 \theta \cos^2 \theta \\
&\quad \left[\text{adding and subtracting } 2 \sin^2 \theta \cos^2 \theta \text{ and using identity } \sin^2 \theta + \cos^2 \theta = 1 \right] \\
&= (\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta \\
&= (\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta \\
&= 1^2 - 3 \sin^2 \theta \cos^2 \theta & (\because \sin^2 \theta + \cos^2 \theta = 1) \\
&= 1 - 3 \sin^2 \theta \cos^2 \theta \\
&= \text{RHS}
\end{aligned}$$

$$\begin{aligned}
\therefore \text{LHS} &= \text{RHS} \\
&\text{Proved}
\end{aligned}$$

Trigonometric Functions Ex 5.1 Q3

$$\begin{aligned}
\text{LHS} &= (\cos \sec \theta - \sin \theta) (\sec \theta - \cos \theta) (\tan \theta + \cot \theta) \\
&= \left(\frac{1}{\sin \theta} - \sin \theta \right) \left(\frac{1}{\cos \theta} - \cos \theta \right) \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) & \left[\because \cos \sec \theta = \frac{1}{\sin \theta}, \sec \theta = \frac{1}{\cos \theta}, \right. \\
&\quad \left. \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta} \right] \\
&= \left(\frac{1 - \sin^2 \theta}{\sin \theta} \right) \left(\frac{1 - \cos^2 \theta}{\cos \theta} \right) \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right) \\
&= \frac{\cos^2 \theta \cdot \sin^2 \theta \cdot 1}{\sin^2 \theta \cdot \cos^2 \theta} & \left(\because \sin^2 \theta + \cos^2 \theta = 1 \right. \\
&\quad \left. \Rightarrow 1 - \sin^2 \theta = \cos^2 \theta, \text{ and } 1 - \cos^2 \theta = \sin^2 \theta \right) \\
&= 1 \\
&= \text{RHS} \\
&\text{Proved}
\end{aligned}$$

Trigonometric Functions Ex 5.1 Q4

$$\begin{aligned}
\text{LHS} &= \cos \sec \theta (\sec \theta - 1) - \cot \theta (1 - \cos \theta) \\
&= \frac{1}{\sin \theta} \left(\frac{1}{\cos \theta} - 1 \right) - \frac{\cos \theta}{\sin \theta} (1 - \cos \theta) & \left[\because \cos \sec \theta = \frac{1}{\sin \theta}, \sec \theta = \frac{1}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta} \right] \\
&= \frac{(1 - \cos \theta)}{\sin \theta \cos \theta} - \frac{\cos \theta (1 - \cos \theta)}{\sin \theta} \\
&= \frac{(1 - \cos \theta) - \cos^2 \theta (1 - \cos \theta)}{\sin \theta \cos \theta} \\
&= \frac{(1 - \cos \theta)(1 - \cos^2 \theta)}{\sin \theta \cos \theta} \\
&= \frac{(1 - \cos \theta) \sin^2 \theta}{\sin \theta \cos \theta} & (\because 1 - \cos^2 \theta = \sin^2 \theta)
\end{aligned}$$

$$= (1 - \cos \theta) \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\sin \theta}{\cos \theta} - \sin \theta$$

$$= \tan \theta - \sin \theta \quad (\because \tan \theta = \sin \theta - \cos \theta)$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q5

$$\text{LHS} = \frac{1 - \sin A \cos A}{\cos A (\sec A - \cos \sec A)} \cdot \frac{\sin^2 A - \cos^2 A}{\sin^3 A + \cos^3 A}$$

$$= \frac{1 - \sin A \cos A}{\cos A \left(\frac{1}{\cos A} - \frac{1}{\sin A} \right)} \cdot \frac{(\sin A + \cos A)(\sin A - \cos A)}{(\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A \cos A)}$$

$$\left[\begin{array}{l} \text{Using } a^2 - b^2 = (a - b)(a + b) \\ \text{and } a^3 + b^3 = (a + b)(a^2 + b^2 - ab) \end{array} \right]$$

$$= \frac{(1 - \sin A \cos A)}{\cos A \left(\frac{\sin A - \cos A}{\cos A \sin A} \right)} \cdot \frac{(\sin A - \cos A)}{(1 - \sin A \cos A)} \quad (\because \sin^2 A + \cos^2 A = 1)$$

$$= \frac{\cos A \sin A}{\cos A}$$

= sin A

= RHS

Proved

Trigonometric Functions Ex 5.1 Q6

$$\text{LHS} = \frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$$

$$= \frac{(\sin A / \cos A)}{\left(1 - \frac{\cos A}{\sin A}\right)} + \frac{(\cos A / \sin A)}{1 - \frac{\sin A}{\cos A}}$$

$$= \frac{\sin A}{\cos A \frac{(\sin A - \cos A)}{\sin A}} + \frac{\cos A}{\sin A \frac{(\cos A - \sin A)}{\cos A}}$$

$$= \frac{\sin^2 A}{\cos A (\sin A - \cos A)} + \frac{\cos^2 A}{\sin A (\cos A - \sin A)}$$

$$= \frac{\sin^3 A - \cos^3 A}{\cos A \sin A (\sin A - \cos A)}$$

$$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{\cos A \sin A (\sin A - \cos A)}$$

$$\left[\text{Using } a^3 - b^3 = (a - b)(a^2 + b^2 + ab) \right]$$

$$= \frac{1 + \sin A \cos A}{\sin A \cos A} \quad \left[\because \sin^2 A + \cos^2 A = 1 \right]$$

$$= \frac{1}{\sin A \cos A} + \frac{\sin A \cos A}{\sin A \cos A}$$

$$= \sec A \csc A + 1$$

$$\left[\because \frac{1}{\cos A} = \sec A, \frac{1}{\sin A} = \csc A \right]$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q7

$$\text{LHS} = \frac{\sin^3 A + \cos^3 A}{\sin A \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A \cos A}$$

$$\sin A + \cos A \quad \sin A - \cos A$$

$$= \frac{(\sin A + \cos A) \{ \sin^2 A + \cos^2 A - \sin A \cos A \}}{(\sin A + \cos A)} + \frac{(\sin A - \cos A) \{ \sin^2 A + \cos^2 A \}}{\sin A - \cos A}$$

$$\left(\text{Using } a^3 + b^3 = (a+b)(a^2 + b^2 - ab) \text{ and } a^3 - b^3 = (a-b)(a^2 + b^2 + ab) \right)$$

$$= (1 - \sin A \cos A) + (1 + \sin A \cos A) \{ \because \sin^2 A + \cos^2 A = 1 \}$$

$$= 2$$

$$= \text{RHS}$$

Trigonometric Functions Ex 5.1 Q8

$$\text{LHS} = (\sec A \sec B + \tan A \tan B)^2 - (\sec A \tan B + \tan A \sec B)^2$$

$$= (\sec A \sec B)^2 + (\tan A \tan B)^2 + 2 \sec A \sec B \tan A \tan B$$

$$- \left\{ (\sec A \tan B)^2 + (\tan A \sec B)^2 + 2 \sec A \tan B \tan A \sec B \right\} \quad \left[\text{Using } (a+b)^2 = a^2 + b^2 + 2ab \right]$$

$$= \sec^2 A \sec^2 B + \tan^2 A \tan^2 B + 2 \sec A \sec B \tan A \tan B$$

$$- \sec^2 A \tan^2 B - \tan^2 A \sec^2 B - 2 \sec A \sec B \tan A \tan B \quad \left[\text{Using } (ab)^2 = a^2 b^2 \right]$$

$$= \sec^2 A \sec^2 B - \sec^2 A \tan^2 B + \tan^2 A \tan^2 B - \tan^2 A \sec^2 B$$

$$= \sec^2 A \{ \sec^2 B - \tan^2 B \} + \tan^2 A \{ \tan^2 B - \sec^2 B \}$$

$$= \sec^2 A \cdot 1 - \tan^2 A \cdot 1 \quad \left[\because \sec^2 \theta = 1 + \tan^2 \theta \right. \\ \left. \Rightarrow \sec^2 \theta - \tan^2 \theta = 1 \right]$$

$$= 1 + \tan^2 A - \tan^2 A$$

$$= 1$$

$$= \text{RHS}$$

Proved

Trigonometric Functions Ex 5.1 Q9

$$\text{RHS} = \frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta - \sin \theta}$$

$$= \frac{\{(1 + \cos \theta) - \sin \theta\} \{(1 + \cos \theta) + \sin \theta\}}{\{(1 + \cos \theta) - \sin \theta\} \{(1 + \cos \theta) + \sin \theta\}}$$

$$= \frac{\{(1 + \cos \theta) + \sin \theta\}^2}{(1 + \cos \theta)^2 - \sin^2 \theta}$$

$$\left(\text{Using } (a+b)(a+b) = (a+b)^2 \right. \\ \left. \& (a+b)(a-b) = a^2 - b^2 \right)$$

$$= \frac{(1 + \cos \theta)^2 + \sin^2 \theta + 2 \sin \theta (1 + \cos \theta)}{1 + \cos^2 \theta + 2 \cos \theta - \sin^2 \theta} \quad \left(\text{Using } (a+b)^2 = a^2 + b^2 + 2ab \right)$$

$$= \frac{1 + \cos^2 \theta + 2 \cdot 1 \cos \theta + \sin^2 \theta + 2 \sin \theta (1 + \cos \theta)}{1 + \cos^2 \theta + 2 \cos \theta - (1 - \cos^2 \theta)} \quad \left(\text{Using } \sin^2 \theta = 1 - \cos^2 \theta \right)$$

$$= \frac{1 + 1 + 2 \cos \theta + 2 \sin \theta (1 + \cos \theta)}{1 - 1 + \cos^2 \theta + \cos^2 \theta + 2 \cos \theta} \quad \left(\text{Using } \sin^2 \theta + \cos^2 \theta = 1 \right)$$

$$= \frac{2 + 2 \cos \theta + 2 \sin \theta (1 + \cos \theta)}{2 \cos^2 \theta + 2 \cos \theta}$$

$$= \frac{2(1 + \cos \theta) + 2 \sin \theta (1 + \cos \theta)}{2 \cos \theta (\cos \theta + 1)}$$

$$= \frac{(1 + \cos \theta) (2 + 2 \sin \theta)}{2 \cos \theta (1 + \cos \theta)}$$

$$= \frac{1 + \sin \theta}{\cos \theta}$$

$$= \frac{1 + \sin \theta}{\cos \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta}$$

$$= \frac{\cos^2 \theta}{\cos \theta (1 - \sin \theta)}$$

$$\cos \theta$$

$$= 1 - \sin \theta$$

Trigonometric Functions Ex 5.1 Q10

$$\text{LHS} = \frac{\tan^3 \theta}{1 + \tan^2 \theta} + \frac{\cot^3 \theta}{1 + \cot^2 \theta}$$

$$= \frac{\sin^3 \theta}{\cos^3 \theta \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta}\right)} + \frac{\cos^3 \theta}{\sin^3 \theta \left(1 + \frac{\cos^2 \theta}{\sin^2 \theta}\right)}$$

$$\left(\begin{array}{l} \because \tan \theta = \frac{\sin \theta}{\cos \theta} \\ \text{and } \cot \theta = \frac{\cos \theta}{\sin \theta} \end{array} \right)$$

$$= \frac{\sin^3 \theta \cos^2 \theta}{\cos^3 \theta (\cos^2 \theta + \sin^2 \theta)} + \frac{\cos^3 \theta \sin^2 \theta}{\sin^3 \theta (\sin^2 \theta + \cos^2 \theta)}$$

$$= \frac{\sin^3 \theta}{\cos \theta} + \frac{\cos^3 \theta}{\sin \theta}$$

$$\left(\because \cos^2 \theta + \sin^2 \theta = 1 \right)$$

$$= \frac{\sin^4 \theta + \cos^4 \theta}{\sin \theta \cos \theta}$$

$$= \frac{(\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 2 \sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta} \quad \left(\text{adding and subtracting } 2 \sin^2 \theta \cos^2 \theta \right)$$

$$= \frac{(\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{1^2 - 2 \sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta}$$

$$\left(\because \sin^2 \theta + \cos^2 \theta = 1 \right)$$

$$= \frac{1 - 2 \sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta}$$

$$= \text{RHS}$$

Proved

Trigonometric Functions Ex 5.1 Q11

$$\text{LHS} = 1 - \frac{\sin^2 \theta}{1 + \cot \theta} - \frac{\cos^2 \theta}{1 + \tan \theta}$$

$$= 1 - \frac{\sin^2 \theta}{1 + \frac{\cos \theta}{\sin \theta}} - \frac{\cos^2 \theta}{1 + \frac{\sin \theta}{\cos \theta}} \quad \left(\because \cot \theta = \frac{\cos \theta}{\sin \theta}, \tan \theta = \frac{\sin \theta}{\cos \theta} \right)$$

$$= 1 - \frac{\sin^2 \theta}{\frac{\sin \theta + \cos \theta}{\sin \theta}} - \frac{\cos^2 \theta}{\frac{\cos \theta + \sin \theta}{\cos \theta}}$$

$$= 1 - \frac{\sin^3 \theta}{\sin \theta + \cos \theta} - \frac{\cos^3 \theta}{\cos \theta + \sin \theta}$$

$$= \frac{\sin \theta + \cos \theta - (\sin^3 \theta + \cos^3 \theta)}{\sin \theta + \cos \theta}$$

$$= \frac{\sin \theta + \cos \theta - (\sin \theta + \cos \theta)(\sin^2 \theta + \cos^2 \theta - \sin \theta \cos \theta)}{\sin \theta + \cos \theta}$$

$$\left(\text{Using } a^3 + b^3 = (a + b)(a^2 + b^2 - ab) \right)$$

$$\frac{(\sin \theta + \cos \theta)(1 - (\sin^2 \theta + \cos^2 \theta - \sin \theta \cos \theta))}{\sin \theta + \cos \theta}$$

$$\left(\text{Using } \sin^2 \theta + \cos^2 \theta = 1 \right)$$

$$= \sin \theta \cos \theta$$

$$= \text{RHS}$$

$$\text{Proved}$$

Trigonometric Functions Ex 5.1 Q12

$$\begin{aligned}
 \text{LHS} &= \left(\frac{1}{\sec^2 \theta - \cos^2 \theta} + \frac{1}{\operatorname{cosec}^2 \theta - \sin^2 \theta} \right) \sin^2 \theta \cos^2 \theta \\
 &= \left(\frac{1}{\frac{1}{\cos^2 \theta} - \cos^2 \theta} + \frac{1}{\left(\frac{1}{\sin^2 \theta} - \sin^2 \theta \right)} \right) \sin^2 \theta \cos^2 \theta \\
 &= \left(\frac{1}{\frac{1 - \cos^4 \theta}{\cos^2 \theta}} + \frac{1}{\frac{1 - \sin^4 \theta}{\sin^2 \theta}} \right) \sin^2 \theta \cos^2 \theta \\
 &= \left(\frac{\cos^2 \theta}{(1 - \cos^2 \theta)(1 + \cos^2 \theta)} + \frac{\sin^2 \theta}{(1 - \sin^2 \theta)(1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta \\
 &= \left(\frac{\cos^2 \theta}{\sin^2 \theta (1 + \cos^2 \theta)} + \frac{\sin^2 \theta}{\cos^2 \theta (1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta \quad \left(\begin{array}{l} \text{Using } 1 - a^4 = 1 - (a^2)^2 \\ = (1 - a^2)(1 + a^2) \end{array} \right) \\
 &= \left(\frac{\cos^4 \theta (1 + \sin^2 \theta) + \sin^4 \theta (1 + \cos^2 \theta)}{\sin^2 \theta \cos^2 \theta (1 + \cos^2 \theta)(1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta \quad \left(\begin{array}{l} \text{Using } 1 - \cos^2 \theta = \sin^2 \theta \\ \& \quad 1 - \sin^2 \theta = \cos^2 \theta \end{array} \right) \\
 &= \frac{\cos^4 \theta + \sin^2 \theta \cos^4 \theta + \sin^4 \theta + \cos^2 \theta \sin^4 \theta}{(1 + \cos^2 \theta)(1 + \sin^2 \theta)} \\
 &= \frac{(\cos^2 \theta)^2 + (\sin^2 \theta)^2 + 2 \cos^2 \theta \sin^2 \theta - 2 \cos^2 \theta \sin^2 \theta + \sin^2 \theta \cos^4 \theta + \cos^2 \theta \sin^4 \theta}{(1 + \cos^2 \theta)(1 + \sin^2 \theta)} \\
 &= \frac{(\cos^2 \theta + \sin^2 \theta)^2 - 2 \cos^2 \theta \sin^2 \theta + \sin^2 \theta \cos^2 \theta (\cos^2 \theta + \sin^2 \theta)}{1 + \sin^2 \theta + \cos^2 \theta + \sin^2 \theta \cos^2 \theta} \quad (\text{adding and subtracting } 2 \cos^2 \theta \sin^2 \theta) \\
 &= \frac{1^2 - 2 \cos^2 \theta \sin^2 \theta + \sin^2 \theta \cos^2 \theta \cdot 1}{1 + 1 + \sin^2 \theta \cos^2 \theta} \\
 &= \frac{1 - \sin^2 \theta \cos^2 \theta}{2 + \sin^2 \theta \cos^2 \theta} \\
 &= \text{RHS} \\
 &\quad \text{Proved}
 \end{aligned}$$

Trigonometric Functions Ex 5.1 Q13

$$\begin{aligned}
 \text{LHS} &= (1 + \tan \alpha \tan \beta)^2 + (\tan \alpha - \tan \beta)^2 \\
 &= 1 + (\tan \alpha + \tan \beta)^2 + 2 \tan \alpha \tan \beta + (\tan \alpha)^2 + (\tan \beta)^2 - 2 \tan \alpha \tan \beta \\
 &\quad \left(\text{Using } (a + b)^2 = a^2 + b^2 + 2ab \text{ and } (a - b)^2 = a^2 + b^2 - 2ab \right) \\
 &= 1 + \tan^2 \alpha + \tan^2 \beta + 2 \tan \alpha \tan \beta + \tan^2 \alpha + \tan^2 \beta - 2 \tan \alpha \tan \beta \\
 &= 1 + \tan^2 \alpha + \tan^2 \alpha + \tan^2 \beta + \tan^2 \beta \\
 &= \sec^2 \alpha + \tan^2 \beta (1 + \tan^2 \alpha) \quad (\because 1 + \tan^2 \alpha = \sec^2 \alpha) \\
 &= \sec^2 \alpha + \tan^2 \beta \cdot \sec^2 \alpha \\
 &= \sec^2 \alpha (1 + \tan^2 \beta)
 \end{aligned}$$

$$\begin{aligned}
 &= \sec^{-1} \alpha \cdot \sec^{-1} \mu \quad (\because 1 + \tan^{-1} \mu = \sec^{-1} \mu) \\
 &= \text{RHS} \\
 &\quad \text{Proved}
 \end{aligned}$$

Trigonometric Functions Ex 5.1 Q14

$$\begin{aligned}
 \text{LHS} &= \frac{(1 + \cot \theta + \tan \theta)(\sin \theta - \cos \theta)}{\sec^3 \theta - \cos \sec^3 \theta} \\
 &= \frac{\left(1 + \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}\right)}{\left(\frac{1}{\cos^3 \theta} - \frac{1}{\sin^3 \theta}\right)} \quad \left(\because \cot \theta = \frac{\cos \theta}{\sin \theta}, \tan \theta = \frac{\sin \theta}{\cos \theta}\right) \\
 &\quad \left(\sec \theta = \frac{1}{\cos \theta}, \cos \sec \theta = \frac{1}{\sin \theta}\right) \\
 &= \frac{\left(1 + \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta}\right)}{\left(\frac{\sin^3 \theta - \cos^3 \theta}{\cos^3 \theta \sin^3 \theta}\right)} (\sin \theta - \cos \theta) \\
 &= \frac{(\sin \theta \cos \theta + 1) \sin^3 \theta \cos^3 \theta}{\sin \theta \cos \theta (\sin^3 \theta - \cos^3 \theta)} (\sin \theta - \cos \theta) \quad (\because \sin^2 \theta + \cos^2 \theta = 1) \\
 &= \frac{(1 + \sin \theta \cos \theta) \sin^2 \theta \cos^2 \theta (\sin \theta - \cos \theta)}{(\sin \theta - \cos \theta) (\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)} \quad (\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)) \\
 &= \frac{(1 + \sin \theta \cos \theta) \cdot \sin^2 \theta \cos^2 \theta}{(1 + \sin \theta \cos \theta)} \\
 &= \sin^2 \theta \cos^2 \theta \\
 &= \text{RHS} \\
 &\quad \text{Proved}
 \end{aligned}$$

Trigonometric Functions Ex 5.1 Q15

$$\begin{aligned}
 \text{LHS} &= \frac{2 \sin \theta \cos \theta - \cos \theta}{1 - \sin \theta + \sin^2 \theta - \cos^2 \theta} \\
 &= \frac{\cos \theta (2 \sin \theta - 1)}{1 - \cos^2 \theta + \sin^2 \theta - \sin \theta} \\
 &= \frac{\cos \theta (2 \sin \theta - 1)}{\sin^2 \theta + \sin^2 \theta - \sin \theta} \quad (\because 1 - \cos^2 \theta = \sin^2 \theta) \\
 &= \frac{\cos \theta (2 \sin \theta - 1)}{2 \sin^2 \theta - \sin \theta} \\
 &= \frac{\cos \theta (2 \sin \theta - 1)}{\sin \theta (2 \sin \theta - 1)} \\
 &= \frac{\cos \theta}{\sin \theta} \\
 &= \cot \theta \\
 &= \text{RHS} \\
 &\quad \text{Proved}
 \end{aligned}$$

Trigonometric Functions Ex 5.1 Q16

$$\begin{aligned}
 \text{LHS} &= \cos \theta (\tan \theta + 2) (2 \tan \theta + 1) \\
 &= \cos \theta \left(\frac{\sin \theta}{\cos \theta} + 2\right) \left(\frac{2 \sin \theta}{\cos \theta} + 1\right) \left(\because \tan \theta = \frac{\sin \theta}{\cos \theta}\right) \\
 &= \cos \frac{(\sin \theta + 2 \cos \theta) (2 \sin \theta + \cos \theta)}{\cos \theta \cdot \cos \theta} \\
 &\quad (2 \sin^2 \theta + \sin \theta \cos \theta + 4 \sin \theta \cos \theta + 2 \cos^2 \theta)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{2(\sin^2 \theta + \cos^2 \theta) + 5 \sin \theta \cos \theta}{\cos \theta} \\
 &= \frac{2 + 5 \sin \theta \cos \theta}{\cos \theta} \quad (\because \sin^2 \theta + \cos^2 \theta = 1) \\
 &= \frac{2}{\cos \theta} + \frac{5 \sin \theta \cos \theta}{\cos \theta} \\
 &= 2 \sec \theta + 5 \sin \theta \\
 &= \text{RHS} \\
 &\text{Proved}
 \end{aligned}$$

Trigonometric Functions Ex 5.1 Q17

$$\begin{aligned}
 \frac{2 \sin \theta}{1 + \cos \theta + \sin \theta} &= x \\
 \Rightarrow \frac{2 \sin \theta (1 - \cos \theta + \sin \theta)}{(1 + \cos \theta + \sin \theta)(1 - \cos \theta + \sin \theta)} &= x \quad [\text{Rationalizing the denominator}] \\
 \Rightarrow \frac{2 \sin \theta (1 - \cos \theta + \sin \theta)}{(1 + \sin \theta)^2 - \cos^2 \theta} &= x \\
 \Rightarrow \frac{2 \sin \theta - 2 \sin \theta \cos \theta + 2 \sin^2 \theta}{1 + \sin^2 \theta + 2 \sin \theta - \cos^2 \theta} &= x \\
 \Rightarrow \frac{2 \sin \theta (1 + \cos \theta - \sin \theta)}{2 \sin^2 \theta + 2 \sin \theta} &= x \\
 \Rightarrow \frac{2 \sin \theta (1 + \cos \theta - \sin \theta)}{2 \sin \theta (1 + \sin \theta)} &= x \\
 \Rightarrow \frac{1 + \cos \theta - \sin \theta}{1 + \sin \theta} &= x \quad [\text{Cancelling the } 2 \sin \theta \text{ in both Numerator and Denominator}]
 \end{aligned}$$

Hence Proved

Trigonometric Functions Ex 5.1 Q18

$$\text{Now, } \cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$\begin{aligned}
 &= \sqrt{\frac{1 - (a^2 - b^2)^2}{(a^2 + b^2)^2}} \quad \left[\because \sin \theta = \frac{a^2 - b^2}{a^2 + b^2} \right] \\
 &= \sqrt{\frac{(a^2 + b^2)^2 - (a^2 - b^2)^2}{(a^2 + b^2)^2}} \\
 &= \sqrt{\frac{(a^2 + b^2 + a^2 - b^2)(a^2 + b^2 - a^2 + b^2)}{a^2 + b^2}} \quad (\text{Using } x^2 - y^2 = (x - y)(x + y)) \\
 &= \sqrt{\frac{2a^2 \times 2b^2}{a^2 + b^2}} \\
 &= \frac{2ab}{a^2 + b^2} \dots\dots\dots (ii)
 \end{aligned}$$

$$\text{Now } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\begin{aligned}
 &= \frac{\frac{a^2 - b^2}{a^2 + b^2}}{\frac{2ab}{a^2 + b^2}} \\
 &= \frac{a^2 - b^2}{2ab}
 \end{aligned}$$

$$= \frac{1}{a^2 + b^2}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{a^2 + b^2}{2ab} \quad (\text{from (i)})$$

$$\text{and } \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{a^2 + b^2}{a^2 - b^2} \quad (\text{from (i)})$$

Trigonometric Functions Ex 5.1 Q19

$$\begin{aligned} & \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} \\ &= \sqrt{\frac{\frac{a}{b} + 1}{\frac{a}{b} - 1}} + \sqrt{\frac{\frac{a}{b} - 1}{\frac{a}{b} + 1}} \quad [\text{Dividing both Numerator and denominator by } b] \\ &= \sqrt{\frac{\tan \theta + 1}{\tan \theta - 1}} + \sqrt{\frac{\tan \theta - 1}{\tan \theta + 1}} \\ &= \sqrt{\frac{\frac{\sin \theta}{\cos \theta} + 1}{\frac{\sin \theta}{\cos \theta} - 1}} + \sqrt{\frac{\frac{\sin \theta}{\cos \theta} - 1}{\frac{\sin \theta}{\cos \theta} + 1}} \\ &= \sqrt{\frac{\frac{\sin \theta + \cos \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\cos \theta}}} + \sqrt{\frac{\frac{\sin \theta - \cos \theta}{\cos \theta}}{\frac{\sin \theta + \cos \theta}{\cos \theta}}} \\ &= \sqrt{\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}} + \sqrt{\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta}} \\ &= \frac{\sin \theta + \cos \theta + \sin \theta - \cos \theta}{\sqrt{\sin^2 \theta - \cos^2 \theta}} \\ &= \frac{2 \sin \theta}{\sqrt{\sin^2 \theta - \cos^2 \theta}} \end{aligned}$$

Trigonometric Functions Ex 5.1 Q20

$$\text{Given } \tan \theta = \frac{a}{b}$$

$$\text{To show: } \frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} = \frac{a^2 - b^2}{a^2 + b^2}$$

$$\text{Since, } \tan \theta = \frac{a}{b}$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{a}{b}$$

$$\Rightarrow b \sin \theta = a \cos \theta = \lambda \text{ (say)}$$

$$\Rightarrow \sin \theta = \frac{\lambda}{b} \text{ and } \cos \theta = \frac{\lambda}{a}$$

$$\text{how } \frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} = \frac{\frac{a \cdot \lambda}{b} - \frac{b \cdot \lambda}{a}}{\frac{a \cdot \lambda}{b} + \frac{b \cdot \lambda}{a}}$$

$$= \frac{\lambda \left(\frac{a}{b} - \frac{b}{a} \right)}{\lambda \left(\frac{a}{b} + \frac{b}{a} \right)}$$

$$= \frac{\frac{a}{b} - \frac{b}{a}}{\frac{a}{b} + \frac{b}{a}}$$

$$= \frac{\frac{a^2 - b^2}{ab}}{\frac{a^2 + b^2}{ab}}$$

$$= \frac{a^2 - b^2}{a^2 + b^2}$$

$$a^2 + b^2$$

Proved

Trigonometric Functions Ex 5.1 Q21

Given, $\sec \theta - \sin \theta = a^3$, $\sec \theta - \cos \theta = b^3$

To show: $a^2 b^2 (a^2 + b^2) = 1$

Since, $\sec \theta - \sin \theta = a^3$

$$\Rightarrow \frac{1}{\sin \theta} - \sin \theta = a^3 \quad \left(\because \sec \theta = \frac{1}{\sin \theta} \right)$$

$$\Rightarrow \frac{1 - \sin^2 \theta}{\sin \theta} = a^3$$

$$\Rightarrow \frac{\cos^2 \theta}{\sin \theta} = a^3 \quad \left(\because 1 - \sin^2 \theta = \cos^2 \theta \right)$$

$$\Rightarrow a = \frac{\cos^{2/3} \theta}{\sin^{1/3} \theta}$$

$$\text{Since, } \frac{1}{\cos \theta} - \cos \theta = b^3 \quad \left(\because \sec \theta = \frac{1}{\cos \theta} \right)$$

$$\Rightarrow \frac{1 - \cos^2 \theta}{\cos \theta} = b^3$$

$$\Rightarrow \frac{\sin^2 \theta}{\cos \theta} = b^3 \quad \left(\because 1 - \cos^2 \theta = \sin^2 \theta \right)$$

$$\Rightarrow b = \frac{\sin^{2/3} \theta}{\cos^{1/3} \theta}$$

$$\text{Now, } a^2 b^2 (a^2 + b^2) = \frac{\cos^{4/3} \theta}{\sin^{2/3} \theta} \times \frac{\sin^{4/3} \theta}{\cos^{2/3} \theta} \left(\frac{\cos^{4/3} \theta}{\sin^{2/3} \theta} + \frac{\sin^{4/3} \theta}{\cos^{2/3} \theta} \right)$$

$$= \cos^{2/3} \theta \times \sin^{2/3} \theta \times \frac{(\cos^{6/3} \theta + \sin^{6/3} \theta)}{\sin^{2/3} \theta \cdot \cos^{2/3} \theta}$$

$$= \cos^2 \theta + \sin^2 \theta$$

$$= 1$$

Proved

Trigonometric Functions Ex 5.1 Q22

Let,

$$\cot \theta (1 + \sin \theta) = 4m \quad \text{---(i)}$$

$$\text{and, } \cot \theta (1 - \sin \theta) = 4n \quad \text{---(ii)}$$

To show: $(m^2 - n^2)^2 = mn$

From (i) and (ii), we get

$$m = \frac{\cot \theta (1 + \sin \theta)}{4} \quad \& \quad n = \frac{\cot \theta (1 - \sin \theta)}{4}$$

$$\begin{aligned} \text{LHS} &= (m^2 - n^2)^2 \\ &= ((m+n)(m-n))^2 \\ &= (m+n)^2 (m-n)^2 \\ &= \left(\frac{\cot \theta (1 + \sin \theta) + \cot \theta (1 - \sin \theta)}{4} \right)^2 \left(\frac{\cot \theta (1 + \sin \theta) - \cot \theta (1 - \sin \theta)}{4} \right)^2 \\ &= \left(\frac{\cot \theta (1 + \sin \theta + 1 - \sin \theta)}{4} \right)^2 \times \left(\frac{\cot \theta (1 + \sin \theta - 1 + \sin \theta)}{4} \right)^2 \\ &= \left(\frac{2 \cot \theta}{4} \right)^2 \times \left(\frac{2 \cot \theta}{4} \right)^2 \\ &= \cot^2 \theta \times \cot^2 \theta \\ &= \cot^4 \theta \end{aligned}$$

$$\begin{aligned}
 &= \frac{\cot \theta}{16} \times 4 \times \frac{\cot \theta}{16} \times 4 \sin^2 \theta \\
 &= \frac{\cot^2 \theta}{16} \times \frac{\cos^2 \theta}{\sin^2 \theta} \sin^2 \theta \\
 &= \frac{\cot \theta}{4} \times \frac{\cot \theta}{4} \times (1 - \sin^2 \theta) \\
 &= \frac{\cot \theta (1 + \sin \theta)}{4} \times \frac{\cot \theta (1 - \sin \theta)}{4} \\
 &= mn
 \end{aligned}$$

$$\begin{aligned}
 &\left[\because \cot \theta = \frac{\cos \theta}{\sin \theta} \right] \\
 &\left[\because \cos^2 \theta = 1 - \sin^2 \theta \right]
 \end{aligned}$$

Trigonometric Functions Ex 5.1 Q23

To show: $\sin^6 \theta + \cos^6 \theta = \frac{4 - 3(m^2 - 1)^2}{4}$, where $m^2 \leq 2$

Since, $\sin \theta + \cos \theta = m \quad \dots (i)$

$$\Rightarrow (\sin \theta + \cos \theta)^2 = m^2$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = m^2$$

$$\Rightarrow 1 + 2 \sin \theta \cos \theta = m^2 \quad \left(\because \sin^2 \theta + \cos^2 \theta = 1 \right)$$

$$\Rightarrow 2 \sin \theta \cos \theta = m^2 - 1$$

$$\Rightarrow \sin \theta \cos \theta = \frac{m^2 - 1}{2} \quad \dots (ii)$$

$$\therefore \text{LHS} = \sin^6 \theta + \cos^6 \theta$$

$$= (\sin^2 \theta)^3 + (\cos^2 \theta)^3$$

$$= (\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta)^2 + (\cos^2 \theta)^2 - \sin^2 \theta \cos^2 \theta$$

$$= 1 \cdot \left((\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 2 \sin^2 \theta \cos^2 \theta - \sin^2 \theta \cos^2 \theta \right)$$

$$\left(\text{adding and subtracting } 2 \sin^2 \theta \cos^2 \theta \right)$$

$$= (\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta$$

$$= 1 - 3 \sin^2 \theta \cos^2 \theta$$

$$= 1 - 3(\sin \theta \cos \theta)^2$$

$$= 1 - 3 \frac{(m^2 - 1)^2}{4} \quad (\text{from (ii)})$$

$$= \frac{4 - 3(m^2 - 1)^2}{4}, \text{ where } m^2 \leq 2$$

$$= \text{RHS}$$

Proved

Trigonometric Functions Ex 5.1 Q24

$$\text{LHS} = ab + a - b + 1$$

$$= (\sec \theta - \tan \theta)(\operatorname{cosec} \theta + \cot \theta) + \sec \theta - \tan \theta - \operatorname{cosec} \theta - \cot \theta + 1$$

$$= \left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \right) \left(\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \right) + \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} - \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} + 1$$

$$= \frac{1}{\sin \theta \cos \theta} + \frac{1}{\cos \theta} \times \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} \times \frac{1}{\sin \theta} - \tan \theta \times \cot \theta + \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} - \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} + 1$$

$$= \frac{1}{\sin \theta \cos \theta} + \frac{1}{\sin \theta} - \frac{1}{\cos \theta} - 1 + \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} - \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} + 1$$

$$= \frac{1}{\sin \theta \cos \theta} - \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}$$

$$= \frac{1}{\sin \theta \cos \theta} - \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}$$

$$= \frac{1 - \sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{1 - (\cos^2 \theta + \sin^2 \theta)}{\sin \theta \cos \theta}$$

$$= \frac{1 - 1}{\sin \theta \cos \theta} = 0 = \text{RHS. Hence Proved}$$

Trigonometric Functions Ex 5.1 Q25

$$\dots \left| \sqrt{1 - \sin \theta} \right| \left| \sqrt{1 + \sin \theta} \right|$$

$$\text{LHS} = \left| \sqrt{1 + \sin \theta} + \sqrt{1 - \sin \theta} \right|$$

$$= \left| \frac{\left(\sqrt{1 - \sin \theta} \right)^2 + \left(\sqrt{1 + \sin \theta} \right)^2}{\sqrt{(1 + \sin \theta)(1 - \sin \theta)}} \right|$$

$$= \left| \frac{1 - \sin \theta + 1 + \sin \theta}{\sqrt{1 - \sin^2 \theta}} \right|$$

$$= \left| \frac{2}{\cos \theta} \right| \quad \left(\because 1 - \sin^2 \theta = \cos^2 \theta \Rightarrow \sqrt{1 - \sin^2 \theta} = \cos \theta \right)$$

$$= \frac{-2}{\cos \theta} \quad \left(\because \frac{\pi}{2} < \theta < \pi \Rightarrow \cos \theta < 0 \right)$$

$$= \text{RHS}$$

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