RD Sharma Solutions Class 11 Maths Chapter 3 Ex 3.4

Functions Ex 3.4 Q1

We have,

$$f(x) = x^3 + 1 \text{ and } g(x) = x + 1$$

Now,

$f + g : R \to R$ given by $(f+g)(x) = x^3 + x + 2$

$f - g : R \to R$ given by $(f-g)(x) = x^3 + 1 - (x+1)$

$= x^3 - x$

$cf : R \to R$ given by $(cf)(x) = c(x^3 + 1)$

$fg : R \to R$ given by $(fg)(x) = (x^3+1)(x+1)$

$= x^4 + x^3 + x + 1$

$\frac{1}{f} : R - \{-1\} \to R$ given by $\left(\frac{1}{f}\right)(x) = \frac{1}{x^3+1}$

$\frac{f}{g} : R - \{-1\} \to R$ given by $\left(\frac{f}{g}\right)(x) = \frac{(x+1)(x^2-x+1)}{x+1}$

$= x^2 - x + 1$

We have,

$$f(x) = \sqrt{x-1} \text{ and } g(x) = \sqrt{x+1}$$

Now,

$f + g : (1, \infty) \to R$ defined by $(f+g)(x) = \sqrt{x-1} + \sqrt{x+1}$,

$f - g : (1, \infty) \to R$ defined by $(f-g)(x) = \sqrt{x-1} - \sqrt{x+1}$,

$cf : (1, \infty) \to R$ defined by $(cf)(x) = c\sqrt{x-1}$,

$fg : (1, \infty) \to R$ defined by $(fg)(x) = (\sqrt{x-1})(\sqrt{x+1})$

$= \sqrt{x^2 - 1}$

$\frac{1}{f} : (1, \infty) \to R$ defined by $\left(\frac{1}{f}\right)(x) = \frac{1}{\sqrt{x-1}}$

$\frac{f}{g} : (1, \infty) \to R$ defined by $\left(\frac{f}{g}\right)(x) = \sqrt{\frac{x-1}{x+1}}$

Functions Ex 3.4 Q2

We have,

$$f(x) = 2x + 5 \text{ and } g(x) = x^2 + x$$

We observe that $f(x) = 2x + 5$ is defined for all $x \in R$.

So, domain$(f) = R$

Clearly $g(x) = x^2 + x$ is defined for all $x \in R$

So, domain $(g) = R$

$\therefore$ Domain$(f) \cap$ Domain$(g) = R$

(i) Clearly, $(f+g) : R \to R$ is given by

$(f+g)(x) = f(x) + g(x)$

$= 2x + 5 + x^2 + x$

$= x^2 + 3x + 5$

$$\text{Domain}(f+g) = R$$

(ii) We find that $f - g : R \to R$ is defined as

$$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= 2x + 5 - \left(x^2 + x\right)\\ &= 2x + 5 - x^2 - x\\ &= -x^2 + x + 5\end{aligned}$$

$$\text{Domain}(f-g) = R$$

(iii) We find that $fg : R \to R$ is given by

$$\begin{aligned}(fg)(x) &= f(x) \times g(x)\\ &= (2x+5) \times \left(x^2 + x\right)\\ &= 2x^3 + 2x^2 + 5x^2 + 5x\\ &= 2x^3 + 7x^2 + 5x\end{aligned}$$

$$\text{Domain}(fg) = R$$

(iv) We have,

$$g(x) = x^2 + x$$

$\therefore \quad f(x) = 0 \Rightarrow x^2 + x = 0$

$\Rightarrow \quad x(x+1) = 0$

$\Rightarrow \quad x = 0 \quad \text{or,} \quad x = -1$

So, $\text{domain}\left(\dfrac{f}{g}\right) = \text{domain}(f) \cap \text{domain}(g) - \{x : g(x) = 0\}$

$$= R - \{-\phi, 0\}$$

We find that, $\dfrac{f}{g} : R - \{-1, 0\} \to R$ is given by $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2x+5}{x^2+x}$

$\text{Domain}\left(\dfrac{f}{g}\right) = R - \{-1, 0\}$

Functions Ex 3.4 Q3

We have,

$$f(x) = \begin{cases} -1, & -2 \le x \le 0 \\ x - 1, & 0 < x \le 2 \end{cases}$$

Now,

$$f(|x|) = |x| - 1, \text{ where } -2 \le x \le 2$$

and $$|f(x)| = \begin{cases} 1, & -2 \le x \le 0 \\ -(x-1), & 0 \le x \le 1 \\ (x-1), & 1 \le x \le 2 \end{cases}$$

$$\therefore \quad g(x) = f(|x|) + |f(x)|$$

$$= \begin{cases} -x & -2 \le x \le 0 \\ 0, & 0 < x < 1 \\ 2(x-1), & 1 \le x \le 2 \end{cases}$$

Functions Ex 3.4 Q4

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9 - x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \ge -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9 - x^2}$ is defined for

$$9 - x^2 \ge 0 \Rightarrow x^2 - 9 \le 0$$

$\Rightarrow \quad x^2 - 3^2 \le 0$

$\Rightarrow \quad (x-3)(x+3) \le 0$

$\Rightarrow \quad x \in [-3, 3]$

$\therefore \quad \text{domain}(g) = [-3, 3]$

Now

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3]$$
$$= [-1, 3]$$

$f + g : [-1, 3] \to R$ is given by $(f + g)(x) = f(x) + g(x) = \sqrt{x+1} + \sqrt{9 - x^2}$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9 - x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9 - x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$
$\Rightarrow \quad x^2 - 3^2 \leq 0$

$\Rightarrow \quad (x - 3)(x + 3) \leq 0$

$\Rightarrow \quad x \in [-3, 3]$

$\therefore \quad \text{domain}(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3]$$
$$= [-1, 3]$$

$g - f : [-, 3] \to R$ is given by $(g - f)(x) = g(x) - f(x) = \sqrt{9 - x^2} - \sqrt{x+1}$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9 - x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty)$

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$\Rightarrow \quad (x - 3)(x + 3) \leq 0$

$\Rightarrow \quad x \in [-3, 3]$

$\therefore \quad \text{domain}(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty) \cap [-3, 3]$$
$$= [-1, 3]$$

$fg : [-, 3] \to R$ is given by $(fg)(x) = f(x) \times g(x) = \sqrt{x+1} \times \sqrt{9 - x^2}$

$$= \sqrt{9 + 9x - x^2 - x^3}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9 - x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9 - x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$
$\Rightarrow \quad x^2 - 3^2 \leq 0$

$\Rightarrow \quad (x - 3)(x + 3) \leq 0$

$\Rightarrow \quad x \in [-3, 3]$

$\therefore \quad \text{domain}(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3]$$
$$= [-1, 3]$$

We have, $g(x) = \sqrt{9 - x^2}$

$\therefore \quad 9 - x^2 = 0 \Rightarrow x^2 - 9 = 0$

$\Rightarrow \quad (x - 3)(x + 3) = 0$

$\Rightarrow \quad x = \pm 3$

So, $\text{domain}\left(\frac{f}{g}\right) = [-1, 3] - [-3, 3] = [-1, 3]$

$\therefore \quad \frac{f}{g} : [-1, 3] \to R$ is given by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+1}}{\sqrt{9-x^2}}$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$
$$\Rightarrow \quad x^2 - 3^2 \leq 0$$
$$\Rightarrow \quad (x-3)(x+3) \leq 0$$
$$\Rightarrow \quad x \in [-3, 3]$$

$\therefore \quad \text{domain}(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3] = [-1, 3]$$

We have,

$$f(x) = \sqrt{x+1}$$
$$\therefore \quad \sqrt{x+1} = 0$$
$$\Rightarrow \quad x + 1 = 0$$
$$\Rightarrow \quad x = -1$$

So, $\text{domain}\left(\frac{g}{f}\right) = [-1, 3] - \{-1\} = [-1, 3]$

$\therefore \quad \frac{g}{f} : [-1, 3] \to R$ is given by $\frac{g}{f}(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{9-x^2}}{\sqrt{x+1}}$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$
$$\Rightarrow \quad x^2 - 3^2 \leq 0$$
$$\Rightarrow \quad (x-3)(x+3) \leq 0$$
$$\Rightarrow \quad x \in [-3, 3]$$

$\therefore \quad \text{domain}(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3] = [-1, 3]$$

$2f - \sqrt{5}g : [-, 3] \to R$ defined by $(2f - \sqrt{5}g)(x) = 2\sqrt{x+1} - \sqrt{5}\sqrt{9-x^2}$
$= 2\sqrt{x+1} - \sqrt{45 - 5x^2}.$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$
$$\Rightarrow \quad x^2 - 3^2 \leq 0$$
$$\Rightarrow \quad (x-3)(x+3) \leq 0$$
$$\Rightarrow \quad x \in [-3, 3]$$

$\therefore \quad \text{domain}(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3] = [-1, 3]$$

$f^2 + 7f : [-1, \infty] \to R$ defined by $(f^2 + 7f)(x) = f^2(x) + 7f(x) \qquad [\because D(f) = [-1, \infty]]$

$$= \left(\sqrt{x+1}\right)^2 + 7\sqrt{x+1}$$
$$= x + 1 + 7\sqrt{x+1}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$
$$\Rightarrow \quad x^2 - 3^2 \leq 0$$
$$\Rightarrow \quad (x-3)(x+3) \leq 0$$
$$\Rightarrow \quad x \in [-3, 3]$$
$$\therefore \quad \text{domain}(g) = [-3, 3]$$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3]$$
$$= [-1, 3]$$

We have,

$$g(x) = \sqrt{9-x^2}$$
$$\therefore \quad 9 - x^2 = 0 \Rightarrow x^2 - 9 = 0$$
$$\Rightarrow \quad (x-3)(x+3) = 0$$
$$\Rightarrow \quad x = \pm 3$$

So, $\text{domain}\left(\frac{1}{g}\right) = [-3, 3] - \{-3, 3\}$

$$= (-3, 3)$$

$$\therefore \quad \frac{5}{g} = (-3, 3) \to R \text{ defined by } \left(\frac{5}{g}\right)(x) = \frac{5}{\sqrt{9-x^2}}$$

Functions Ex 3.4 Q5

We have,

$$f(x) = \log_e(1-x)$$
$$\text{and} \quad g(x) = [x]$$

$f(x) = \log_e(1-x)$ is defined, if $1 - x > 0$

$$\Rightarrow \quad 1 > x$$
$$\Rightarrow \quad x < 1$$
$$\Rightarrow \quad x \in (-\infty, 1)$$

$\therefore$ Domain$(f) = (-\infty, 1)$

$g(x) = [x]$ is defined for all $x \in R$

$\therefore$ Domain$(g) = R$

$$\therefore \quad \text{Domain}(f) \cap R \ \text{Domain}(g) = (-\infty, 1) \cap R$$
$$= (-\infty, 1)$$

(i) $f + g : (-\infty, 1) \to R$ defined by $(f+g)(x) = f(x) + g(x)$

$$= \log_e(1-x) + [x]$$

(ii) $fg : (-\infty, 1) \to R$ defined by $(fg)(x) = f(x) \times g(x)$

$$= \log_e(1-x) \times [x]$$
$$= [x] \log_e(1-x)$$

(iii) $g(x) = [x]$

$$\therefore \quad [x] = 0$$
$$\Rightarrow \quad x \in (0, 1)$$

So, $\text{domain}\left(\frac{f}{g}\right) = \text{domain}(f) \cap \text{domain}(g) - \{x : g(x) = 0\}$

$$= (-\infty, 0)$$

$$\therefore \quad \frac{f}{g} : (-\infty, 0) \to R \text{ defined by } \left(\frac{f}{g}\right)(x) = \frac{\log_e(1-x)}{[x]}$$

(iv) We have,

(iv) We have,

$$f(x) = \log_e(1-x)$$

$\Rightarrow$ $$\frac{1}{f(x)} = \frac{1}{\log_e(1-x)}$$

$\therefore$ $\frac{1}{f(x)}$ is defined if $\log_e(1-x)$ is defined and $\log_e(1-x) \neq 0$

$\Rightarrow$ $1-x > 0$ and $1-x \neq 0$

$\Rightarrow$ $x < 1$ and $x \neq 0$

$\Rightarrow$ $x \in (-\infty, 0) \cup (0,1)$

$\therefore$ $\text{domain}\left(\frac{g}{f}\right) = (-\infty,0) \cup (0,1)$

$\frac{g}{f} : (-\infty,0) \cup (0,1) \to R$ defined by $\left(\frac{g}{f}\right)(x) = \frac{[x]}{\log_e(1-x)}$

Now,

$$(f+g)(-1) = f(-1) + g(-1)$$
$$= \log_e(1-(-1)) + [-1]$$
$$= \log_e 2 - 1$$

$\Rightarrow$ $(f+g)(-1) = \log_e 2 - 1$

(v) $fg(0) = \log_e(1-0) \times [0]$
$= 0$

(vi) $\left(\frac{f}{g}\right)\left(\frac{1}{2}\right)$ = does not exist

(vii) $$\left(\frac{g}{f}\right)\left(\frac{1}{2}\right) = \frac{\left[\frac{1}{2}\right]}{\log_e\left(1-\frac{1}{2}\right)} = 0$$

Functions Ex 3.4 Q6

We have,

$$f(x) = \sqrt{x+1},\ g(x) = \frac{1}{x}$$

and $h(x) = 2x^2 - 3$

Clearly, $f(x)$ is defined for $x + 1 \geq 0$

$\Rightarrow$ $x \geq -1$

$\Rightarrow$ $x \in [-1, \infty]$

$\therefore$ $\text{Domain}(f) = [-1, \infty]$

$g(x)$ is defined for $x \neq 0$

$\Rightarrow$ $x \in R - \{0\}$

and, $h(x)$ is defined for all $x \in R$

$\therefore$ $\text{Domain}(f) \cap \text{Domain}(g) \cap \text{Domain}(h) = [-1,\infty] - \{0\}$

Clearly,

$2f + g - h : [-1,\infty] - \{0\} \to R$ is given by

$$(2f+g-h)(x) = 2f(x) + g(x) - h(x)$$
$$= 2\sqrt{x+1} + \frac{1}{x} - 2x^2 + 3$$

$\therefore$ $$(2f+g-h)(1) = 2\sqrt{1+1} + \frac{1}{1} - 2\times(1)^2 + 3$$
$$= 2\sqrt{2} + 1 - 2 + 3$$
$$= 2\sqrt{2} + 4 - 2$$
$$= 2\sqrt{2} + 2$$

and, $(2f+g-h)(0)$ does not exist, it is not lies in the domain $x \in [-1,\infty] - \{0\}$.

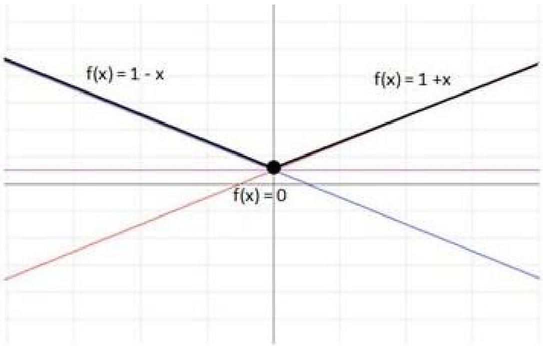
Functions Ex 3.4 Q7

Let,

$$y = f(x) = \begin{cases} 1-x, & x < 0 \\ 1, & x = 0 \\ x+1, & x > 0 \end{cases}$$

The graph of f(x) for x <0 is the part of the line y = 1-x that lies to the left of origin.
The graph of f(x) for x >0 is the part of the line y = 1+x that lies to the right of origin.
For x = 0, the graph of f(x) represents the point (0,1)

The graph of f(x) is shown below.



Functions Ex 3.4 Q8

$f: R \to R$ defined by $(f+g)(x) = 3x - 2$

$f: R \to R$ defined by $(f-g)(x) = -x + 4$

$f: R - \left\{\frac{3}{2}\right\} \to R$ defined by $\frac{f}{g}(x) = \frac{x+1}{2x-3}$

Functions Ex 3.4 Q9

$f + g: [0, \infty) \to R$ defined by $(f+g)(x) = \sqrt{x} + x;$

$f - g: [0, \infty) \to R$ defined by $(f-g)(x) = \sqrt{x} - x;$

$fg: [0, \infty) \to R$ defined by $(fg)(x) = x^{3/2};$

$\frac{f}{g}: [0, \infty) \to R$ defined by $\left(\frac{f}{g}\right)(x) = \frac{1}{\sqrt{x}};$

Functions Ex 3.4 Q10

$(f+g): R \to [0, \infty)$ defined by $(f+g)(x) = x^2 + 2x + 1 = (x+1)^2$

$(f-g): R \to R$ defined by $(f-g)(x) = x^2 - 2x - 1$

$(fg): R \to R$ defined by $(fg)(x) = 2x^3 + x^2$

$\left(\frac{f}{g}\right): R \to R$ defined by $\left(\frac{f}{g}\right)(x) = \frac{x^2}{2x+1}$