RD Sharma Solutions Class 11 Maths Chapter 3 Ex 3.3

Functions Ex 3.3 Q1

We have,

$$f(x) = \frac{1}{x}$$

Clearly, $f(x)$ assumes real values for all real values for all x except for the values of $x = 0$

Hence, Domain$(f) = R - \{0\}$

We have,

$f(x) = \frac{1}{x-7}$

Clearly, $f(x)$ assumes real values for all real values for all x except for the values of x satisfying $x - 7 = 0$ i.e., $x = 7$

Hence, Domain$(f) = R - \{7\}$

We have,

$$f(x) = \frac{3x-2}{x+1}$$

We observe that $f(x)$ is a rational function of x as $\frac{3x-2}{x+1}$ is a rational expression.

Clearly, $f(x)$ assumes real values for all x except for the values of x for which $x + 1 = 0$ i.e., $x = -1$

Hence, Domain $= R - \{-1\}$

We have,

$$f(x) = \frac{2x+1}{x^2-9}$$
$$= \frac{2x+1}{(x^2-3^2)}$$
$$= \frac{2x+1}{(x-3)(x+3)} \qquad [\because a^2 - b^2 = (a-b)(a+b)]$$

We observe that $f(x)$ is a rational function of x as $\frac{2x+1}{x^2-9}$ is a rational expression.

Clearly, $f(x)$ assumes real values for all x except for all those values of x for which $x^2 - 9 = 0$ i.e., $x = -3, 3$

Hence, Domain$(f) = R - \{-3, 3\}$.

We have,

$$f(x) = \frac{x^2+2x+1}{x^2-8x+12}$$
$$= \frac{x^2+2x+1}{x^2-6x-2x+12}$$
$$= \frac{x^2+2x+1}{x(x-6)-2(x-6)}$$
$$= \frac{x^2+2x+1}{(x-6)(x-2)}$$

Clearly, $f(x)$ is a rational function of x as $\frac{x^2+2x+1}{x^2-8x+12}$ is a rational expression in x.

We observe that $f(x)$ assumes real values for all x except for all those values of x for which $x^2 - 8x + 12 = 0$ i.e., $x = 2, 6$

$\therefore$ Domain$(f) = R - \{2, 6\}$

Functions Ex 3.3 Q2

(i) We have,

$$f(x) = \sqrt{x-2}$$

Clearly, $f(x)$ assumes real values, if

$$x - 2 \geq 0$$
$$\Rightarrow \quad x \geq 2$$
$$\Rightarrow \quad x \in [2, \infty)$$

Hence, Domain$(f) = [2, \infty]$

(ii) We have,

$$f(x) = \frac{1}{\sqrt{x^2-1}}$$

Clearly, $f(x)$ assumes real values, if

$$x^2 - 1 > 0$$
$$\Rightarrow \quad (x-1)(x+1) > 0 \qquad [\because a^2 - b^2 = (a-b)(a+b)]$$
$$\Rightarrow \quad x < -1 \text{ or } x > 1$$
$$\Rightarrow \quad x \in (-\infty, -1) \cup (1, \infty)$$

Hence, domain$(f) = (-\infty, -1) \cup (1, \infty)$

(iii) We have,

$$f(x) = \sqrt{9 - x^2}$$

Clearly, $f(x)$ assumes real values, if

$9 - x^2 \geq 0$

$\Rightarrow \quad 9 \geq x^2$

$\Rightarrow \quad x^2 \leq 9$

$\Rightarrow \quad -3 \leq x \leq 3$

$\Rightarrow \quad x \in [-3, 3]$

Hence, domain$(f) = [-3, 3]$

(iv) We have,

$$f(x) = \sqrt{\frac{x - 2}{3 - x}}$$

Clearly, $f(x)$ assumes real values, if

$x - 2 \geq 0 \quad$ and $\quad 3 - x > 0$

$\Rightarrow \quad x \geq 2 \quad$ and $\quad 3 > x$

$\Rightarrow \quad x \in [2, 3]$

Hence, domain$(f) = [2, 3)$.

Functions Ex 3.3 Q3

We have,

$$f(x) = \frac{ax + b}{bx - a}$$

We observe that $f(x)$ is a rational function of x as $\frac{ax + b}{bx - a}$ is a rational expression.

Clearly, $f(x)$ assumes real values for all x except for the values of x for which

$bx - a = 0$ i.e., $bx = a$

$\Rightarrow \quad x = \frac{a}{b}$

$\therefore$ Domain$(f) = R - \left\{\frac{a}{b}\right\}$

Range of f: Let $f(x) = y$

$\Rightarrow \quad \frac{ax + b}{bx - a} = y$

$\Rightarrow \quad ax + b = y(bx - a)$

$\Rightarrow \quad ax + b = bxy - ax$

$\Rightarrow \quad b + ay = bxy - ax$

$\Rightarrow \quad b + ay = x(by - a)$

$\Rightarrow \quad \frac{b + ay}{b - ay} = x$

$\Rightarrow \quad x = \frac{b + ay}{by - a}$

Clearly, x will take real value for all $x \in R$ except for

$by - a = 0$

$\Rightarrow \quad by = a$

$\Rightarrow \quad y = \frac{a}{b}$

$\therefore$ Range$(f) = R - \left\{\frac{a}{b}\right\}$.

We have,

$$f(x) = \frac{ax - b}{cx - d}$$

We observe that $f(x)$ is a rational function of x as $\frac{ax - b}{cx - d}$ is a rational expression.

Clearly, $f(x)$ assumes real values for all x except for all those values of x for which

$cx - d = 0$ i.e., $cx = d$

$\Rightarrow \quad x = \frac{d}{c}$

$\therefore$ Domain$(f) = R - \left\{\frac{d}{c}\right\}$

Range: Let $f(x) = y$

$\Rightarrow \quad \dfrac{ax-b}{cx-d} = y$

$\Rightarrow \quad ax - b = y(cx - d)$

$\Rightarrow \quad ax - b = cxy - dy$

$\Rightarrow \quad dy - b = cxy - 9x$

$\Rightarrow \quad dy - b = x(cy - a)$

$\Rightarrow \quad \dfrac{dy-b}{cy-a} = x$

Clearly, x assumes real values for all y except

$cy - a = 0$ i.e., $y = \dfrac{a}{c}$

Hence, range$(f) = R - \left\{\dfrac{a}{c}\right\}$

We have,

$f(x) = \sqrt{x-1}$

Clearly, $f(x)$ assumes real values, if

$x - 1 \ge 0$

$\Rightarrow \quad x \ge 1$

$\Rightarrow \quad x \in [1, \infty)$

Hence, domain$(f) = [1, \infty)$

Range: For $x \ge 1$, we have,

$x - 1 \ge 0$

$\Rightarrow \quad \sqrt{x-1} \ge 0$

$\Rightarrow \quad f(x) \ge 0$

Thus, $f(x)$ takes all real values greater than zero.

Hence, range$(f) = [0, \infty)$

We have,

$f(x) = \sqrt{x-3}$

Clearly, $f(x)$ assumes real values, if

$x - 3 \ge 0$

$\Rightarrow \quad x \ge 3$

$\Rightarrow \quad x \in [3, \infty)$

Hence, domain$(f) = [3, \infty)$

Range: For $x \ge 3$, we have,

$x - 3 \ge 0$

$\Rightarrow \quad \sqrt{x-3} \ge 0$

$\Rightarrow \quad f(x) \ge 3$

Thus, $f(x)$ takes all real values greater than zero.

Hence, range$(f) = [0, \infty)$

We have,

$f(x) = \dfrac{x-2}{2-x}$

Domain of f: Clearly, $f(x)$ is defined for all $x \in R$ except for which

$2 - x \ne 0$ i.e., $x \ne 2$

Hence, domain$(f) = R - \{2\}$

Range of f: Let $f(x) = y$

$\Rightarrow \quad \dfrac{x-2}{2-x} = y$

$\Rightarrow \quad \dfrac{-1(2-x)}{2-x} = y$

$\Rightarrow \quad -1 = y$

$\Rightarrow \quad y = -1$

$\therefore$ Range$(f) = \{-1\}$

We have,

$f(x) = |x - 1|$

Clearly, $f(x)$ is defined for all $x \in R$

Clearly, $f(x)$ is defined for all $x \in R$

$\Rightarrow \quad \text{Domain}(f) = R$

Range: Let $f(x) = y$

$\Rightarrow \quad |x - 1| = y$

$\Rightarrow \quad f(x) \geq 0 \; \forall \; x \in R$

It follows from the above relation that y takes all real values greater or equal to zero.

$\therefore \; \text{Range}(f) = [0, \infty)$

As |x|is defined for all real numbers, its domain is R and range is only negative numbers because, |x| is always positive real number for all real numbers and -|x| is always negative real numbers.

In order to have F(x) has defined value, term inside square root should always be greater than or equal to zero which gives domain as $-3 \leq x \leq 3$

Where as Range of above function is limited to [0 , 3]