RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.9

Limits Ex 29.9 Q1

$$= \lim_{x \to \pi} \frac{1+\cos x}{\tan^2 x}$$

As $x \to \pi, x - \pi \to 0$, let $x - \pi = y$

$$= \lim_{y \to 0} \frac{1+\cos(\pi + y)}{\tan^2(\pi + y)}$$

$$= \lim_{y \to 0} \frac{1-\cos y}{\tan^2 y}$$

$$= \lim_{y \to 0} \frac{2\sin^2 \frac{y}{2}}{\tan^2 y}$$

$$= \frac{\lim_{y \to 0} 2\sin^2 \frac{y}{2}}{\lim_{y \to 0} \tan^2 y}$$

$$= \frac{2\left(\lim_{y \to 0} \frac{\sin \frac{y}{2}}{\frac{y}{2}}\right)^2 \times \frac{y^2}{4}}{\left(\lim_{y \to 0} \frac{\tan y}{y}\right) \times y^2}$$

$$= \frac{2 \times 1 \times \frac{y^2}{4}}{1 \times y^2}$$

$$\left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \right.$$
$$\left. \lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1 \right]$$

$$= 2 \times 1 \times \frac{1}{4}$$

$$= \frac{1}{2}$$

Limits Ex 29.9 Q2

$$\lim_{x\to\frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cot^2 x + 1 - 2}{\cot x - 1}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cot^2 x - 1}{\cot x - 1}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{(\cot x - 1)(\cot x + 1)}{(\cot x - 1)}$$

$$= \lim_{x\to\frac{\pi}{4}} (\cot x + 1)$$

$$= \cot\frac{\pi}{4} + 1$$

$$= 1 + 1$$

$$= 2$$

Limits Ex 29.9 Q3

$$\lim_{x\to\frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 1 - 3}{\operatorname{cosec} x - 2}$$

$$= \lim_{x\to\frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 4}{\operatorname{cosec} x - 2}$$

$$= \lim_{x\to\frac{\pi}{6}} \frac{(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 2)}{(\operatorname{cosec} x - 2)}$$

$$= \lim_{x\to\frac{\pi}{6}} (\operatorname{cosec} x + 2)$$

$$= \operatorname{cosec}\frac{\pi}{6} + 2$$

$$= 2 + 2$$

$$= 4$$

Limits Ex 29.9 Q4

$$\lim_{x\to\frac{\pi}{4}}\frac{2-\operatorname{cosec}^2 x}{1-\cot x}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{2-\left(1+\cot^2 x\right)}{1-\cot x}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{2-1-\cot^2 x}{1-\cot x}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{1-\cot^2 x}{1-\cot x}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{(1-\cot x)(1+\cot x)}{(1-\cot x)}$$

$$=\lim_{x\to\frac{\pi}{4}}(1+\cot x)$$

$$=1+\cot\frac{\pi}{4}$$

$$=1+1$$

$$=2$$

Limits Ex 29.9 Q5

$$\lim_{x\to\pi}\frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}$$

$$=\lim_{x\to\pi}\frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}\times\frac{\sqrt{2+\cos x}+1}{\sqrt{2+\cos x}+1}$$

$$=\lim_{x\to\pi}\frac{(2+\cos x)-1}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)}$$

$$=\lim_{x\to\pi}\frac{1+\cos x}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)}$$

Let $\pi-x=y, x\to\pi, y\to 0$

$$\Rightarrow\quad\lim_{x\to\pi}\frac{1+\cos x}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)}=\lim_{y\to 0}\frac{1+\cos(\pi-y)}{y^2\left(\sqrt{2+\cos(\pi-y)}+1\right)}$$

$$= \lim_{y\to 0} \frac{1-\cos y}{y^2\sqrt{2-\cos y}+1}$$

$$= \lim_{y\to 0} \frac{\frac{2\sin^2 y}{2}}{y^2\sqrt{2-\cos y}+1}$$

$$= 2\lim_{y\to 0}\left(\frac{\frac{\sin y}{2}}{\frac{y}{2}}\right)^2 \times \frac{1}{4}\frac{1}{\sqrt{2-\cos y}+1}$$

$$= 2\times\left(\lim_{y\to 0}\frac{\sin y}{2}\right)^2 \times \frac{1}{4}\frac{1}{\lim_{y\to 0}\sqrt{2-\cos y}+1}$$

$$= 2\times 1\times\frac{1}{4}\times\frac{1}{\sqrt{2-\cos 0}+1}$$

$$= 2\times 1\times\frac{1}{4}\times\frac{1}{\sqrt{2-1}+1}$$

$$= 2\times 1\times\frac{1}{4}\times\frac{1}{1+1}$$

$$= 2\times 1\times\frac{1}{4}\times\frac{1}{2}$$

$$= \frac{1}{4}$$

Limits Ex 29.9 Q6

$$\lim_{x\to\frac{3\pi}{2}} \frac{1+\operatorname{cosec}^3 x}{\cot^2 x}$$

$$= \lim_{x\to\frac{3\pi}{2}} \frac{(1+\operatorname{cosec} x)\left(1+\operatorname{cosec}^2 x-\operatorname{cosec} x\right)}{\left(\operatorname{cosec}^2 x-1\right)}$$

$$= \lim_{x\to\frac{3\pi}{2}} \frac{(\operatorname{cosec} x+1)\left(1+\operatorname{cosec}^2 x-\operatorname{cosec} x\right)}{(\operatorname{cosec} x-1)(\operatorname{cosec} x+1)}$$

$$= \lim_{x\to\frac{3\pi}{2}} \frac{\left(1+\operatorname{cosec}^2 x-\operatorname{cosec} x\right)}{\operatorname{cosec} x-1}$$

$$= \frac{1+\operatorname{cosec}^2\frac{3\pi}{2}-\operatorname{cosec}\frac{3\pi}{2}}{\operatorname{cosec}\frac{3\pi}{2}-1}$$

$$= \frac{1+(-1)^2-(-1)}{(-1)-1} \qquad \left[\because \operatorname{cosec}\frac{3\pi}{2}=-1\right]$$

$$= \frac{1+1+1}{-2}$$

$$= \frac{-3}{2}$$