RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.6

Limits Ex 29.6 Q1

$$\lim_{x\to\infty}\frac{(3x-1)(4x-2)}{(x+8)(x-1)} \qquad \left[\text{Expression is } \frac{\infty}{\infty}\right]$$

$$= \lim_{x\to\infty}\frac{\left(12x^2-10x+2\right)}{\left(x^2+9x-8\right)}$$

$$= \lim_{x\to\infty}\left(\frac{12-\frac{10}{x}+\frac{2}{x^2}}{1+\frac{9}{x}-\frac{8}{x^2}}\right)$$

$$= \frac{12-0+0}{1+0-1}$$

$$= 12$$

Limits Ex 29.6 Q2

$$\lim_{x\to\infty}\frac{3x^3-4x^2+6x-1}{2x^3+x^2-5x+7}$$

$$= \lim_{x\to\infty}\frac{3-\frac{4}{x}+\frac{6}{x^2}-\frac{1}{x^3}}{2+\frac{1}{x}-\frac{5}{x^2}+\frac{7}{x^3}}$$

$$= \frac{3-0+0-0}{2+0-0+0}$$

$$= \frac{3}{2}$$

Limits Ex 29.6 Q3

$$\lim_{x\to\infty}\frac{5x^3-6}{\sqrt{9+4x^6}}$$

$$= \lim_{x\to\infty}\frac{5-\frac{6}{x^3}}{\sqrt{\frac{9}{x^6}+\frac{4x^6}{x^6}}}$$

$$= \lim_{x\to\infty} \frac{\left(5 - \frac{6}{x^3}\right)}{\sqrt{\frac{9}{x^6} + 4}}$$

$$= \frac{5}{\sqrt{4}} = \frac{5}{2}$$

Limits Ex 29.6 Q4

$$\lim_{x\to\infty} \sqrt{x^2 + cx} - x$$

$$= \lim_{x\to\infty} \left(\left(\sqrt{x^2 + cx} - x\right) \frac{\left(\sqrt{x^2 + cx} + x\right)}{\sqrt{x^2 + cx} + x} \right)$$

$$= \lim_{x\to\infty} \frac{\left(x^2 + cx - x^2\right)}{\sqrt{x^2 + cx} + x}$$

$$= \lim_{x\to\infty} \frac{cx}{\sqrt{x^2 + cx} + x} \quad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \lim_{x\to\infty} \frac{c}{\sqrt{1 + \frac{c}{x}} + 1}$$

$$= \frac{c}{1+1} = \frac{c}{2}$$

Limits Ex 29.6 Q5

$$\lim_{x\to\infty} \sqrt{x+1} - \sqrt{x}$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{x+1} - \sqrt{x}\right)\left(\sqrt{x+1} + \sqrt{x}\right)}{\left(\sqrt{x+1} + \sqrt{x}\right)}$$

$$= \lim_{x\to\infty} \frac{(x + 1 - x)}{\sqrt{x+1} + \sqrt{x}}$$

$$= \lim_{x\to\infty} \left(\frac{1}{\sqrt{x+1} + \sqrt{x}}\right)$$

$$= \frac{1}{\infty}$$
$$= 0$$

Limits Ex 29.6 Q6

$$\lim_{x \to \infty} \sqrt{x^2 + 7x} - x$$

$$= \lim_{x \to \infty} \left(\frac{\left(\sqrt{x^2 + 7x} - x\right)\left(\sqrt{x^2 + 7x} + x\right)}{\sqrt{x^2 + 7x} + x} \right)$$

$$= \lim_{x \to \infty} \left(\frac{\left(x^2 + 7x\right) - x^2}{\sqrt{x^2 + 7x} + x} \right)$$

$$= \lim_{x \to \infty} \frac{7x}{\sqrt{x^2 + 7x} + x}$$

$$= \lim_{x \to \infty} \frac{7}{\sqrt{\frac{x^2}{x^2} + \frac{7x}{x^2}} + 1}$$

$$= \lim_{x \to \infty} \frac{7}{\sqrt{1 + \frac{7}{x}} + 1}$$

$$= \frac{7}{2}$$

Limits Ex 29.6 Q7

$$\lim_{x \to \infty} \frac{x}{\sqrt{4x^2 + 1} - 1}$$

$$= \lim_{x \to \infty} \frac{1}{\sqrt{4 + \frac{1}{x^2}} - \frac{1}{x}}$$

$$= \frac{1}{\sqrt{4} - 0}$$

$$= \frac{1}{2}$$

Limits Ex 29.6 Q8

$$\lim_{n\to\infty}\frac{n^2}{1+2+3+\ldots+n}$$

$$=\lim_{n\to\infty}\frac{n^2}{\frac{1}{2}n(n+1)} \qquad \left[\because 1+2+3+\ldots+n=\frac{n(n+1)}{2}\right]$$

$$=\lim_{n\to\infty}\frac{2n^2}{n^2+n}$$

$$=2\lim_{n\to\infty}\frac{n^2}{n^2+n}$$

$$=2\lim_{n\to\infty}\frac{n^2}{n^2\left(1+\frac{1}{n}\right)}$$

$$=2\times\frac{1}{1+0}$$

$$=2$$

Limits Ex 29.6 Q9

$$\lim_{n\to\infty}\frac{\frac{3}{x}+\frac{4}{x^2}}{\frac{5}{x}+\frac{6}{x^2}} \qquad \left[\frac{0}{0}\text{ form}\right]$$

$$=\lim_{x\to\infty}\frac{\frac{1}{x}\left(3+\frac{4}{x}\right)}{\frac{1}{x}\left(5+\frac{6}{x}\right)}$$

$$=\lim_{x\to\infty}\frac{(3+0)}{(5+0)}=\frac{3}{5}$$

Limits Ex 29.6 Q10

$$\lim_{x\to\infty}\frac{\sqrt{x^2+a^2}-\sqrt{x^2+b^2}}{\sqrt{x^2+c^2}-\sqrt{x^2+d^2}} \qquad \left[\frac{\infty}{\infty}\text{ form}\right]$$

$$\lim_{x\to\infty}\frac{\left(\sqrt{x^2+a^2}-\sqrt{x^2+b^2}\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)}\times\frac{\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}{\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(\left(x^2+a^2\right)-\left(x^2-b^2\right)\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)}{\left(x^2+c^2-x^2-d^2\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)}{\left(c^2-d^2\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(x\sqrt{1+\frac{c^2}{x^2}}+x\sqrt{1+\frac{d^2}{x^2}}\right)}{\left(c^2-d^2\right)\left(x\sqrt{1+\frac{a^2}{x^2}}+x\sqrt{1+\frac{b^2}{x^2}}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(\sqrt{1+\frac{c^2}{x^2}}+\sqrt{1+\frac{d^2}{x^2}}\right)}{\left(c^2-d^2\right)\left(\sqrt{1+\frac{a^2}{x^2}}+\sqrt{1+\frac{b^2}{x^2}}\right)}$$

$$=\frac{\left(a^2-b^2\right)\left(\sqrt{1+0}+\sqrt{1+0}\right)}{\left(c^2-d^2\right)\left(\sqrt{1+0}+\sqrt{1+0}\right)}$$

$$=\frac{\left(a^2-b^2\right)(1+1)}{\left(c^2-d^2\right)(1+1)}$$

$$=\frac{\left(a^2-b^2\right)(2)}{\left(c^2-d^2\right)(2)}=\frac{a^2-b^2}{c^2-d^2}$$

Limits Ex 29.6 Q11

$$\lim_{n\to\infty} \frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$$

We know that $(n+2) = (n+2)(n+1)!$

$$\Rightarrow \quad \lim_{n\to\infty} \frac{(n+2)(n+1)!+(n+1)!}{(n+2)(n+1)!-(n+1)!}$$

$$= \lim_{n\to\infty} \frac{(n+1)!\left[(n+2)+1\right]}{(n+1)\left[(n+2)-1\right]}$$

$$= \lim_{n\to\infty} \frac{n+3}{n+1} \qquad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \lim_{n\to\infty} \frac{1+\frac{3}{n}}{1+\frac{1}{n}}$$

$$= \frac{1+0}{1+0} = 1$$

$$= 1$$

Limits Ex 29.6 Q12

$$\lim_{x\to\infty} x\left[\sqrt{x^2+1}-\sqrt{x^2-1}\right]$$

$$= \lim_{x\to\infty} x\left[\sqrt{x^2+1}-\sqrt{x^2-1}\right] \times \frac{\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)}{\left(\sqrt{x^2+1}+\sqrt{x^2+1}\right)}$$

$$= \lim_{x\to\infty} x \frac{x\left(x^2+1-x^2+1\right)}{\sqrt{x^2+1}+\sqrt{x^2-1}}$$

$$= \lim_{x\to\infty} \frac{x(2)}{x\left(\sqrt{1+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x^2}}\right)}$$

$$= \lim_{x\to\infty} \frac{2}{\sqrt{1+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x^2}}}$$

$$= \frac{2}{2} = 1$$

Limits Ex 29.6 Q13

$$\lim_{x\to\infty}\left[\sqrt{x+1}-\sqrt{x}\right]\sqrt{x+2}$$

$$= \lim_{x\to\infty}\left[\sqrt{x+1}-\sqrt{x}\right]\frac{\left[\sqrt{x+1}+\sqrt{x}\right]}{\left[\sqrt{x+1}+\sqrt{x}\right]}\times\frac{\sqrt{x+2}\times\sqrt{x+2}}{\sqrt{x+2}}$$

$$= \lim_{x\to\infty}\frac{(x+1-x)}{\sqrt{x+1}+\sqrt{x}}\times\frac{(x+2)}{\sqrt{x+2}}$$

$$= \lim_{x\to\infty}\frac{1(x+2)}{\left(\sqrt{x+1}+\sqrt{x}\right)\left(\sqrt{x+2}\right)}$$

$$= \lim_{x\to\infty}\frac{x\left(1+\frac{2}{x}\right)}{\sqrt{x}\left(\sqrt{1+\frac{1}{x}}+\sqrt{1}\right)\left(\sqrt{1+\frac{2}{x}}\right)\sqrt{x}}$$

$$= \lim_{x\to\infty}\frac{\left(1+\frac{2}{x}\right)}{\left(\sqrt{1+\frac{1}{x}}+\sqrt{1}\right)\left(\sqrt{1+\frac{2}{x}}\right)}$$

$$= \lim_{x\to\infty}\frac{(1+0)}{(1+1)\times 1} = \frac{1}{2}$$