RD Sharma Solutions
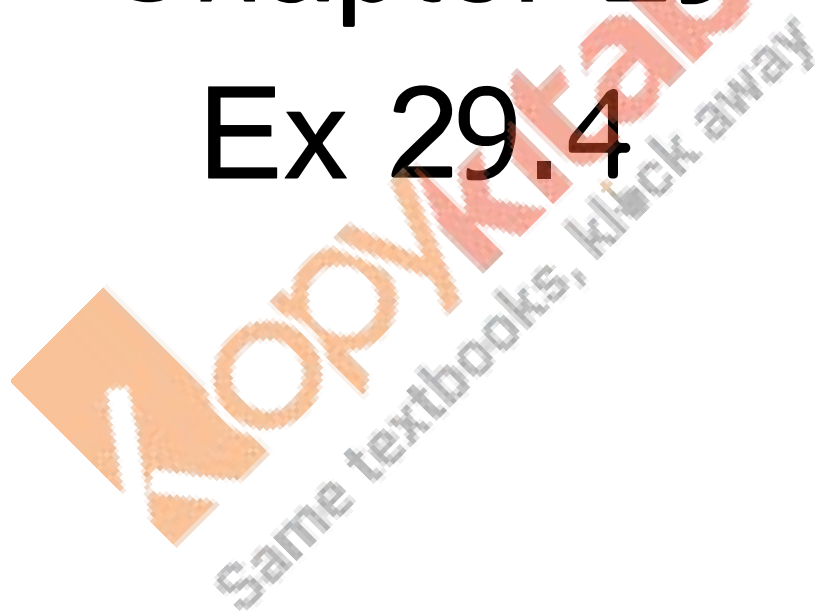
Class 11 Maths

Chapter 29

Ex 29.4

Limits Ex 29.4 Q1

$$\lim_{x \to 0} \frac{\sqrt{1+x+x^2} - 1}{x}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1+x+x^2} - 1\right)\left(\sqrt{1+x+x^2} + 1\right)}{x\left(\sqrt{1+x+x^2} + 1\right)}$$

$$= \lim_{x \to 0} \frac{\left(\left(1+x+x^2\right) - 1\right)}{x\left(\sqrt{1+x+x^2} + 1\right)}$$

$$= \lim_{x \to 0} \frac{x(1+x)}{x\left(\sqrt{1+x+x^2} + 1\right)}$$

$$= \lim_{x \to 0} \frac{1+x}{\sqrt{1+x+x^2} + 1}$$

$$= \frac{1+0}{\sqrt{1+0+0} + 1}$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2}$$

Limits Ex 29.4 Q2

$$\lim_{x \to 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}}$$

$$= \lim_{x \to 0} \frac{2x}{\left(\sqrt{a+x} - \sqrt{a-x}\right)} \times \frac{\sqrt{a+x} + \sqrt{a-x}}{\left(\sqrt{a+x} + \sqrt{a-x}\right)}$$

$$= \lim_{x \to 0} \frac{2x \left(\sqrt{a+x} + \sqrt{a-x}\right)}{\left((a+x) - (a-x)\right)}$$

$$= \lim_{x \to 0} \frac{2x \left(\sqrt{a+x} + \sqrt{a-x}\right)}{2x}$$

$$= \lim_{x \to 0} \left(\sqrt{a+x} + \sqrt{a-x}\right)$$

$$= \sqrt{a} + \sqrt{a}$$

$$= 2\sqrt{a}$$

Limits Ex 29.4 Q3

$$\lim_{x \to 0} \frac{\sqrt{a^2 + x^2} - a}{x^2}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{a^2 + x^2} - a\right)}{x^2} \times \frac{\left(\sqrt{a^2 + x^2} + a\right)}{\left(\sqrt{a^2 + x^2} + a\right)}$$

$$= \lim_{x \to 0} \frac{\left(a^2 + x^2 - a^2\right)}{x^2 \sqrt{a^2 + x^2} + a}$$

$$= \lim_{x \to 0} \frac{x^2}{x^2 \left(\sqrt{a^2 + x^2} + a\right)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{a^2 + x^2} + a}$$

$$= \frac{1}{a + a}$$

$$= \frac{1}{2a}$$

Limits Ex 29.4 Q4

$$\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1+x} - \sqrt{1-x}\right)}{2x} \times \frac{\left(\sqrt{1+x} + \sqrt{1-x}\right)}{\left(\sqrt{1+x} + \sqrt{1-x}\right)}$$

$$= \lim_{x \to 0} \frac{(1+x) - (1-x)}{2x\left(\sqrt{1+x} + \sqrt{1-x}\right)}$$

$$= \lim_{x \to 0} \frac{2x}{2x\left(\sqrt{1+x} + \sqrt{1-x}\right)}$$

$$= \lim_{x \to 0} \frac{1}{\left(\sqrt{1+x} + \sqrt{1-x}\right)}$$

$$= \frac{1}{\sqrt{1} + \sqrt{1}}$$

$$= \frac{1}{2}$$

Limits Ex 29.4 Q5

$$\lim_{x \to 2} \frac{\sqrt{3-x} - 1}{2-x}$$

$$= \lim_{x \to 2} \frac{\left(\sqrt{3-x} - 1\right)}{2-x} \times \frac{\left(\sqrt{3-x} + 1\right)}{\left(\sqrt{3-x} + 1\right)}$$

$$= \lim_{x \to 2} \frac{(3-x) - 1}{(2-x)\left(\sqrt{3-x} + 1\right)}$$

$$= \lim_{x \to 2} \frac{(2-x)}{(2-x)\left(\sqrt{3-x} + 1\right)}$$

$$= \lim_{x \to 2} \frac{1}{\left(\sqrt{3-x} + 1\right)}$$

$$= \frac{1}{\sqrt{3-2} + 1} = \frac{1}{1+1}$$

$$= \frac{1}{2}$$

Limits Ex 29.4 Q6

$$\lim_{x \to 3} \frac{x - 3}{\sqrt{x - 2} - \sqrt{4 - x}}$$

$$= \lim_{x \to 3} \frac{x - 3}{\left(\sqrt{x - 2} - \sqrt{4 - x}\right)} \times \frac{\left(\sqrt{x - 2} + \sqrt{4 - x}\right)}{\left(\sqrt{x - 2} + \sqrt{4 - x}\right)}$$

$$= \lim_{x \to 3} \frac{(x - 3)\left(\sqrt{x - 2} + \sqrt{4 - x}\right)}{(x - 2) - (4 - x)}$$

$$= \lim_{x \to 3} \frac{(x - 3)\left(\sqrt{x - 2} + \sqrt{4 - x}\right)}{x - 2 - 4 + x}$$

$$= \lim_{x \to 3} \frac{(x - 3)\left(\sqrt{x - 2} + \sqrt{4 - x}\right)}{2(x - 3)}$$

$$= \frac{1}{2} \lim_{x \to 3} \left(\sqrt{x - 2} + \sqrt{4 - x}\right)$$

$$= \frac{1}{2}\left(\sqrt{3 - 2} + \sqrt{4 - 3}\right)$$

$$= \frac{1}{2}\left(\sqrt{1} + \sqrt{1}\right)$$

$$= \frac{1}{2}(1 + 1) = \frac{2}{2}$$

$$= 1$$

Limits Ex 29.4 Q7

$$\lim_{x \to 0} \frac{x}{\left(\sqrt{1 + x} - \sqrt{1 - x}\right)}$$

$$= \lim_{x \to 0} \frac{x}{\left(\sqrt{1 + x} - \sqrt{1 - x}\right)} \times \frac{\left(\sqrt{1 + x} + \sqrt{1 - x}\right)}{\left(\sqrt{1 + x} + \sqrt{1 - x}\right)}$$

$$= \lim_{x \to 0} \frac{x\left(\sqrt{1 + x} + \sqrt{1 - x}\right)}{\left(\sqrt{1 + x}\right)^2 - \left(\sqrt{1 - x}\right)^2}$$

$$= \lim_{x \to 0} \frac{x\left(\sqrt{1 + x} + \sqrt{1 - x}\right)}{1 + x - 1 + x}$$

$$= \lim_{x \to 0} \frac{x\left(\sqrt{1 + x} + \sqrt{1 - x}\right)}{2x}$$

$$= \frac{1}{2} \lim_{x \to 0} \left(\frac{\sqrt{1 + x} + \sqrt{1 - x}}{x}\right) x$$

$$= \frac{1}{2} \lim_{x \to 0} \left(\sqrt{1 + x} + \sqrt{1 - x}\right)$$

$$= \frac{1}{2}\left(\sqrt{1} + \sqrt{1}\right)$$

$$= \frac{2}{2} = 1$$

Limits Ex 29.4 Q8

$$\lim_{x \to 1} \frac{\left(\sqrt{5x-4} - \sqrt{x}\right)}{x-1}$$

$$= \lim_{x \to 1} \frac{\left(\sqrt{5x-4} - \sqrt{x}\right)}{x-1} \times \frac{\left(\sqrt{5x-4} + \sqrt{x}\right)}{\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{\left((5x-4) - x\right)}{(x-1)\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= 4 \lim_{x \to 1} \frac{(x-1)}{(x-1)\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= 4 \lim_{x \to 1} \frac{1}{\sqrt{5x-4} + \sqrt{x}}$$

$$= 4 \times \frac{1}{\sqrt{5-4} + \sqrt{1}}$$

$$= 4 \times \frac{1}{\sqrt{1} + \sqrt{1}}$$

$$= \frac{4}{2} = 2$$

Limits Ex 29.4 Q9

$$\lim_{x \to 1} \frac{(x-1)}{\left(\sqrt{x^2+3} - 2\right)}$$

$$= \lim_{x \to 1} \frac{(x-1) \times \left(\sqrt{x^2+3} + 2\right)}{\left(\sqrt{x^2+3} - 2\right)\left(\sqrt{x^2+3} + 2\right)}$$

$$= \lim_{x \to 1} \frac{(x-1)\left(\sqrt{x^2+3} + 2\right)}{\left(x^2+3-4\right)}$$

$$= \lim_{x \to 1} \frac{(x-1)\left(\sqrt{x^2+3} + 2\right)}{\left(x^2-1\right)}$$

$$= \lim_{x \to 1} \frac{\sqrt{x^2+3} + 2}{x+1}$$

Putting the value $x = 1$

$$\Rightarrow \quad \frac{\sqrt{1+3} + 2}{1+1}$$

$$= \frac{2+2}{2}$$

$$= \frac{4}{2} = 2$$

Limits Ex 29.4 Q10

$$\lim_{x \to 3} \frac{\left(\sqrt{x+3} - \sqrt{6}\right)}{\left(x^2 - 9\right)}$$

$$= \lim_{x \to 3} \frac{\left(\sqrt{x+3} - \sqrt{6}\right)}{(x-3)(x+3)} \frac{\left(\sqrt{x+3} - \sqrt{6}\right)}{\left(\sqrt{x+3} + \sqrt{6}\right)}$$

$$= \lim_{x \to 3} \frac{\left((x+3) - 6\right)}{(x-3)(x+3)\left(\sqrt{x+3} + \sqrt{6}\right)}$$

$$= \lim_{x \to 3} \frac{(x-3)}{(x-3)(x+3)\left(\sqrt{x+3} + \sqrt{6}\right)}$$

$$= \lim_{x \to 3} \frac{1}{(x+3)\left(\sqrt{x+3} + \sqrt{6}\right)}$$

$$= \frac{1}{(3+3)\sqrt{3+3} + \sqrt{6}}$$

$$= \frac{1}{6\left(\sqrt{6} + \sqrt{6}\right)} = \frac{1}{6 \times 2\sqrt{6}}$$

$$= \frac{1}{12\sqrt{6}}$$

Limits Ex 29.4 Q11

$$\lim_{x \to 1} \frac{\left(\sqrt{5x-4} - \sqrt{x}\right)}{\left(x^2 - 1\right)}$$

$$= \lim_{x \to 1} \frac{\left(\sqrt{5x-4} - \sqrt{x}\right)}{(x-1)(x+1)} \times \frac{\left(\sqrt{5x-4} + \sqrt{x}\right)}{\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{\left((5x-4) - x\right)}{(x-1)(x+1)\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{4(x-1)}{(x-1)(x+1)\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{4}{(x+1)\left(\sqrt{5x-4} + \sqrt{x}\right)}$$

$$= \frac{4}{(1+1)\left(\sqrt{5-4} + \sqrt{1}\right)}$$

$$= \frac{4}{2(1+1)}$$

$$= \frac{4}{4} = 1$$

Limits Ex 29.4 Q12

$$\lim_{x \to 0} \frac{\left(\sqrt{1+x} - 1\right)}{x}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1+x} - 1\right)}{x} \times \frac{\left(\sqrt{1+x} + 1\right)}{\left(\sqrt{1+x} + 1\right)}$$

$$= \lim_{x \to 0} \frac{(1 + x - 1)}{x \left(\sqrt{1+x} + 1\right)}$$

$$= \lim_{x \to 0} \frac{x}{x \left(\sqrt{1+x} + 1\right)}$$

$$= \lim_{x \to 0} \frac{1}{\left(\sqrt{1+x} + 1\right)}$$

$$= \frac{1}{\sqrt{1} + 1} = \frac{1}{2}$$

Limits Ex 29.4 Q13

$$\lim_{x \to 2} \frac{\sqrt{x^2 + 1} - \sqrt{5}}{(x - 2)}$$

$$= \lim_{x \to 2} \frac{\sqrt{x^2 + 1} - \sqrt{5}}{(x - 2)} \times \frac{\left(\sqrt{x^2 + 1} + \sqrt{5}\right)}{\left(\sqrt{x^2 + 1} + \sqrt{5}\right)}$$

$$= \lim_{x \to 2} \frac{\left(x^2 + 1 - 5\right)}{(x - 2)\left(\sqrt{x^2 + 1} + \sqrt{5}\right)}$$

$$= \lim_{x \to 2} \frac{(x + 2)(x - 2)}{(x - 2)\left(\sqrt{x^2 + 1} + \sqrt{5}\right)}$$

$$= \lim_{x \to 2} \frac{(x + 2)}{\left(\sqrt{x^2 + 1} + \sqrt{5}\right)}$$

$$= \frac{(2 + 2)}{\sqrt{4 + 1} + \sqrt{5}}$$

$$= \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$$