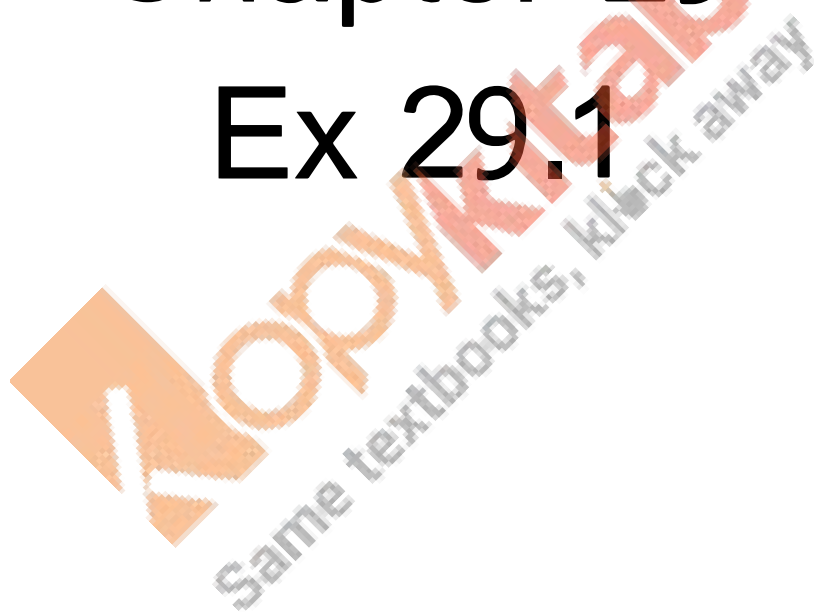


RD Sharma
Solutions
Class 11 Maths
Chapter 29
Ex 29.1



Limits Ex 29.1 Q1

$$\lim_{x \rightarrow 0} \frac{x}{|x|}$$

We know that $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

$$\therefore \lim_{x \rightarrow 0^+} \frac{x}{|x|} = \lim_{x \rightarrow 0^+} \frac{x}{x} = \lim_{x \rightarrow 0^+} 1 = 1$$

$$\text{Also, } \lim_{x \rightarrow 0^-} \frac{x}{|x|} = \lim_{x \rightarrow 0^-} \frac{x}{-x} = \lim_{x \rightarrow 0^-} -1 = -1$$

$$\Rightarrow \text{LHL of } f(x) \neq \text{RHL of } f(x)$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{x}{|x|} \text{ does not exist}$$

Limits Ex 29.1 Q2

$$\begin{aligned} \lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^-} (2x + 3) \\ &= 2(2) + 3 \\ &= 7 \end{aligned}$$

$$\therefore \lim_{x \rightarrow 2^-} f(x) = 7$$

Also,

$$\begin{aligned} \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} (x + k) \\ &= (2 + k) \end{aligned}$$

Since, $\lim_{x \rightarrow 2} f(x)$ exists (given)

$$\therefore \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$$

$$\Rightarrow 7 = 2 + k$$

$$\Rightarrow k = 5$$

Limits Ex 29.1 Q3

Let $f(x) = \frac{1}{x}$, this function is defined for every value of x except at $x = 0$

$$\text{As } x \rightarrow 0^+, \frac{1}{x} \rightarrow \infty.$$

$$\text{As } x \rightarrow 0^-, \frac{1}{x} \rightarrow -\infty$$

$$\therefore \lim_{x \rightarrow 0} \frac{1}{x} \text{ does not exist.}$$

Limits Ex 29.1 Q4

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{3x}{-x + 2x} = \lim_{x \rightarrow 0^-} \frac{3x}{x} = 3$$

$$[\because \text{ as } x \rightarrow 0^-, |x| = -x]$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{3x}{x + 2x} = 1$$

$$[\because \text{ as } x \rightarrow 0^+, |x| = x]$$

$$\text{thus, } \lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

$$\therefore \lim_{x \rightarrow 0} f(x) \text{ does not exist.}$$

Limits Ex 29.1 Q5

$$\begin{aligned} \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} x + 1 \\ &= \lim_{h \rightarrow 0} (0 + h) + 1 = 1 \end{aligned}$$

$$\begin{aligned} \text{Also, } \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} x - 1 \\ &= \lim_{h \rightarrow 0} (0 - h) - 1 = -1 \end{aligned}$$

$$\Rightarrow \lim_{x \rightarrow 0^+} f(x) \neq \lim_{x \rightarrow 0^-} f(x)$$

Hence, limit does not exist.

Limits Ex 29.1 Q6

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 0^-} f(x) \\ &= \lim_{h \rightarrow 0} f(0 - h) \\ &= \lim_{h \rightarrow 0} -h - 4 \\ &= 0 - 4 \\ &= -4 \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 0^+} f(x) \\ &= \lim_{h \rightarrow 0} f(0 + h) \\ &= \lim_{h \rightarrow 0} h + 5 \\ &= 0 + 5 \\ &= 5 \end{aligned}$$

$$\therefore \lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

Hence $\lim_{x \rightarrow 0} f(x)$ does not exist.

Limits Ex 29.1 Q7

$$\lim_{x \rightarrow 3^+} f(x) = 4$$

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (x + 1) = \lim_{h \rightarrow 0} (3 - h + 1) = 3 + 1 = 4$$

$$\text{Since, } \lim_{x \rightarrow 3^+} f(x) = 4 = \lim_{x \rightarrow 3^-} f(x)$$

$$\therefore \lim_{x \rightarrow 3} f(x) \text{ is } 4$$

Limits Ex 29.1 Q8

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 3(x+1) = 3$$

and

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} 2(x) + 3 = 3$$

$$\therefore \lim_{x \rightarrow 0} f(x) = 3$$

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} 3(x+1) = 3+3 = 6$$

$$\therefore \lim_{x \rightarrow 1} f(x) = 6$$

Limits Ex 29.1 Q9

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 - 1 = \lim_{h \rightarrow 0} (-1-h)^2 - 1 = 1 - 1 = 0$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} -x^2 - 1 = \lim_{h \rightarrow 0} -(1+h)^2 - 1 = -2$$

$$\text{Since, } \lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$$

$$\therefore \lim_{x \rightarrow 1} f(x) \text{ does not exist.}$$

Limits Ex 29.1 Q10

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x)$$

$$\Rightarrow \lim_{x \rightarrow 0^-} \frac{|x|}{x}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{|0-h|}{0-h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{+h}{-h} = -1 \quad \text{---(i)}$$

and,

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x)$$

$$\Rightarrow \lim_{x \rightarrow 0^+} \frac{|x|}{x}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{|0+h|}{0+h} = \lim_{h \rightarrow 0} \frac{h}{h} = 1 \quad \text{---(ii)}$$

So, LHL $\neq$ RHL

$$\therefore \lim_{x \rightarrow 0} f(x) \text{ does not exist.}$$

Limits Ex 29.1 Q11

$$\begin{aligned}
 & \lim_{x \rightarrow a_1} f(x) \\
 \Rightarrow & \lim_{x \rightarrow a_1} (x - a_1)(x - a_2) \dots (x - a_n) && [\text{Putting limit } x \rightarrow a_1] \\
 \Rightarrow & (a_1 - a_1)(a_1 - a_2) \dots (a_1 - a_n) \\
 \Rightarrow & 0
 \end{aligned}$$

$$\begin{aligned}
 \text{And, } & \lim_{x \rightarrow a} f(x) \\
 \Rightarrow & \lim_{x \rightarrow a} (x - a_1)(x - a_2) \dots (x - a_n) && [\text{Putting limit } x \rightarrow a] \\
 \Rightarrow & (a - a_1)(a - a_2) \dots (a - a_n).
 \end{aligned}$$

Limits Ex 29.1 Q12

$$\lim_{x \rightarrow 1^+} \frac{1}{(x-1)} = \lim_{h \rightarrow 0} \frac{1}{(1+h-1)} = \lim_{h \rightarrow 0} \frac{1}{h} = \infty$$

Limits Ex 29.1 Q13(i)

$$\begin{aligned}
 & \lim_{x \rightarrow 2^+} \frac{x-3}{x^2-4} \\
 &= \lim_{h \rightarrow 0} \frac{(2+h)-3}{(2+h)^2-2^2} && \left[\because \lim_{x \rightarrow 2^+} f(x) = \lim_{h \rightarrow 0} f(2+h) \right] \\
 &= \lim_{h \rightarrow 0} \frac{(2-3+h)}{(2+h-2)(2+h+2)} \\
 &= \lim_{h \rightarrow 0} \frac{(h-1)}{(h)(4+h)} \\
 &= \lim_{h \rightarrow 0} \frac{1-\frac{1}{h}}{4+h} \\
 &= \frac{1-\frac{1}{0}}{4} = -\infty
 \end{aligned}$$

Limits Ex 29.1 Q13(ii)

$$\begin{aligned}
 \lim_{x \rightarrow 2^-} \frac{x-3}{x^2-4} &= \lim_{h \rightarrow 0} \frac{(2-h)-3}{(2-h)^2-4} \\
 &= \lim_{h \rightarrow 0} \frac{(2-h-3)}{(2-h+2)(2-h-2)} && \left[\because \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2-h) \right] \\
 &= \lim_{h \rightarrow 0} \frac{-1-h}{(4-h)(-h)} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{1}{h}+1}{(4-h)} \\
 &= \frac{\frac{1}{0}+1}{4} = \infty
 \end{aligned}$$

Limits Ex 29.1 Q13(iii)

$$\begin{aligned}
 & \lim_{x \rightarrow 0^+} \frac{1}{3x} \\
 &= \lim_{h \rightarrow 0} \frac{1}{3(0+h)} \\
 &= \lim_{h \rightarrow 0} \frac{1}{0+3h} \\
 &= \frac{1}{0} = \infty
 \end{aligned}$$

Limits Ex 29.1 Q13(iv)

$$\begin{aligned}
 & \lim_{x \rightarrow -8^+} \frac{2x}{x+8} \\
 &= \lim_{h \rightarrow 0} \frac{2(-8+h)}{(-8+h)+8} \\
 &= \lim_{h \rightarrow 0} \frac{-16+2h}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-16}{h} + 2 \\
 \Rightarrow & \frac{-16}{0} + 2 = \infty
 \end{aligned}$$

Limits Ex 29.1 Q13(v)

$$\begin{aligned}
 & \lim_{x \rightarrow 0^+} \frac{2}{x^5} \\
 &= \lim_{h \rightarrow 0} \frac{2}{(0+h)^5} \\
 \Rightarrow & \frac{2}{0} = \infty
 \end{aligned}$$

Limits Ex 29.1 Q13(vi)

$$\begin{aligned}
 & \lim_{x \rightarrow \frac{\pi}{2}^-} \tan x \\
 &= \lim_{h \rightarrow 0} \tan\left(\frac{\pi}{2} - h\right) \\
 &= \tan\left(\frac{\pi}{2} - 0\right) \\
 \Rightarrow & \tan \frac{\pi}{2} = \infty
 \end{aligned}$$

Limits Ex 29.1 Q13(vii)

$$\begin{aligned}
 \lim_{x \rightarrow -\frac{\pi}{2}^+} \sec x &= \lim_{h \rightarrow 0} \sec \left(-\frac{\pi}{2} + h \right) \\
 &= \sec \left(-\frac{\pi}{2} + 0 \right) \\
 &= \sec \left(-\frac{\pi}{2} \right) \\
 &= \frac{1}{\cos \left(-\frac{\pi}{2} \right)} \\
 &= \frac{-1}{\left(\cos \frac{\pi}{2} \right)} \\
 &= \frac{-1}{0} = -\infty
 \end{aligned}$$

Limits Ex 29.1 Q13(viii)

$$\begin{aligned}
 \lim_{x \rightarrow 0^+} \frac{x^2 - 3x + 2}{x^3 - 2x^2} &= \lim_{x \rightarrow 0^+} \frac{x^2 - x - 2x + 2}{x^2(x - 2)} \\
 &= \lim_{x \rightarrow 0^+} \frac{x(x - 1) - 2(x - 1)}{x^2(x - 2)} \\
 &= \lim_{x \rightarrow 0^+} \frac{(x - 1)(x - 2)}{x^2(x - 2)} \\
 &= \lim_{x \rightarrow 0^+} \frac{(x - 1)}{x^2} \\
 &= \lim_{h \rightarrow 0} \frac{(0 - h - 1)}{(0 - h)^2} \\
 &= \frac{-h}{h^2} = \frac{-1}{h} = \frac{-1}{0} = -\infty
 \end{aligned}$$

Limits Ex 29.1 Q13(ix)

$$\begin{aligned}
 \lim_{x \rightarrow -2^+} \frac{x^2 - 1}{2x + 4} &= \lim_{x \rightarrow -2^+} \frac{(x - 1)(x + 1)}{2(x + 2)} \\
 &= \lim_{h \rightarrow 0} \frac{(-2 + h - 1)(-2 + h + 1)}{2(-2 + h + 2)} \\
 &= \lim_{h \rightarrow 0} \frac{(-3 + h)(h - 1)}{2h} \\
 \Rightarrow \frac{-3 \times -1}{2 \times 0} &= \frac{1}{0} = \infty
 \end{aligned}$$

Limits Ex 29.1 Q13(x)

$$\begin{aligned}
 \lim_{x \rightarrow 0^-} 2 - \cot x &= \lim_{h \rightarrow 0} 2 - \cot(0 - h) \\
 &= \lim_{h \rightarrow 0} 2 - (-1) \coth \\
 &= \lim_{h \rightarrow 0} 2 + \coth \\
 &= \lim_{h \rightarrow 0} 2 + \frac{1}{\tanh} \\
 \Rightarrow 2 + \frac{1}{0} &\leftarrow \infty
 \end{aligned}$$

Limits Ex 29.1 Q13(xi)

$$\begin{aligned}
 \lim_{x \rightarrow 0^-} 1 + \operatorname{cosec} x &= \lim_{x \rightarrow 0^-} 1 + \operatorname{cosec}(0 - h) \\
 &= \lim_{h \rightarrow 0} 1 - \operatorname{cosec} h \\
 &= \lim_{h \rightarrow 0} 1 - \frac{1}{\sinh} \\
 \Rightarrow 1 - \frac{1}{0} &= -\infty
 \end{aligned}$$

Limits Ex 29.1 Q14

$$\begin{aligned}
 \lim_{x \rightarrow 0} e^{\frac{-1}{x}} &= \lim_{x \rightarrow 0^+} e^{\frac{-1}{x}} = \lim_{x \rightarrow 0^+} \frac{1}{e^{\frac{1}{x}}} = \lim_{h \rightarrow 0} \frac{1}{e^{\frac{1}{0+h}}} = \frac{1}{e^{\frac{1}{0}}} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0
 \end{aligned}$$

$$\text{And, } \lim_{x \rightarrow 0^-} e^{\frac{-1}{x}} = \lim_{x \rightarrow 0^-} \frac{1}{e^{\frac{1}{x}}} = \lim_{h \rightarrow 0} \frac{1}{e^{\frac{1}{0-h}}} = \lim_{h \rightarrow 0} \frac{1}{e^{-\frac{1}{h}}} = \frac{1}{e^{-\frac{1}{0}}} = \frac{1}{e^{-\infty}} = e^{\infty} = \infty$$

$$\Rightarrow \lim_{x \rightarrow 0^+} e^{\frac{-1}{x}} \neq \lim_{x \rightarrow 0^-} e^{\frac{-1}{x}}$$

$$\therefore \lim_{x \rightarrow 0} e^{\frac{-1}{x}} \text{ does not exist.}$$

Limits Ex 29.1 Q15

$$(i) \lim_{x \rightarrow 2} [x]$$

$$\lim_{x \rightarrow 2^-} [x] = 1$$

$$\lim_{x \rightarrow 2^+} [x] = 2$$

Thus, $\lim_{x \rightarrow 2} [x]$ does not exist.

$$(ii) \lim_{x \rightarrow \frac{5}{2}} [x]$$

$$\lim_{x \rightarrow \frac{5}{2}^+} [x] = 2$$

$$\lim_{x \rightarrow \frac{5}{2}^-} [x] = 2$$

$$\Rightarrow \lim_{x \rightarrow \frac{5}{2}} [x] = 2$$

$$(iii) \lim_{x \rightarrow 1} [x]$$

$$\lim_{x \rightarrow 1^-} [x] = 0$$

$$\lim_{x \rightarrow 1^+} [x] = 1$$

$$\Rightarrow \lim_{x \rightarrow 1^-} [x] \neq \lim_{x \rightarrow 1^+} [x]$$

Thus, $\lim_{x \rightarrow 1} [x]$ does not exist

Limits Ex 29.1 Q16

$$\lim_{x \rightarrow a^+} [x]$$

$$\Rightarrow \lim_{h \rightarrow 0^+} [a + h] = [a]$$

$$\Rightarrow \lim_{h \rightarrow 0^+} [x] = [a] \forall a \in \mathbb{R}$$

Also, $\lim_{x \rightarrow 1^-} [x]$

$$= \lim_{h \rightarrow 0^+} [1 - h]$$

$$= 0$$

$$\Rightarrow \lim_{x \rightarrow 1^-} [x] = 0$$

Limits Ex 29.1 Q17

$$\lim_{x \rightarrow 2^-} \frac{x}{[x]} = \lim_{x \rightarrow 2^-} \frac{x}{1} = \frac{2}{1} = 2$$

$$\left[\because \lim_{x \rightarrow k^-} [x] = k - 1 \right]$$

$$\text{Also, } \lim_{x \rightarrow 2^+} \frac{x}{[x]} = \lim_{x \rightarrow 2^+} \frac{x}{3} = \frac{2}{3}$$

$$\left[\lim_{x \rightarrow k^+} [x] = k + 1 \right]$$

$$\Rightarrow \lim_{x \rightarrow 2^-} \frac{x}{[x]} \neq \lim_{x \rightarrow 2^+} \frac{x}{[x]}$$

$$\lim_{x \rightarrow 3^+} \frac{x}{[x]} = \lim_{x \rightarrow 3^+} \frac{x}{3} = \frac{3}{3} = 1$$

$$\lim_{x \rightarrow 3^-} \frac{x}{[x]} = \lim_{x \rightarrow 3^-} \frac{x}{2} = \frac{3}{2} = 1.5$$

$$\text{Therefore, } \lim_{x \rightarrow 3^+} \frac{x}{[x]} \neq \lim_{x \rightarrow 3^-} \frac{x}{[x]}$$

Limits Ex 29.1 Q19

$$\lim_{x \rightarrow \frac{5}{2}} [x]$$

$$\lim_{x \rightarrow \frac{5}{2}} [x] = \left[\frac{5}{2} \right],$$

$$= [2.5] = 2$$

[By definition of greatest integer function]

$$\Rightarrow \lim_{x \rightarrow \frac{5}{2}} [x] = 2$$

Limits Ex 29.1 Q20

$$\begin{aligned} \lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^-} (x - [x]) \\ &= \lim_{x \rightarrow 2^-} x - \lim_{x \rightarrow 2^-} [x] \\ &= 2 - 1 = 1 \end{aligned}$$

$$[\because \lim_{x \rightarrow k^-} [x] = k - 1]$$

$$\begin{aligned} \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} (3x - 5) \\ &= 3(2) - 5 \\ &= 6 - 5 \\ &= 1 \end{aligned}$$

$$[\because x > 2]$$

$$\text{Thus, } \lim_{x \rightarrow 2^-} f(x) = 1 = \lim_{x \rightarrow 2^+} f(x)$$

$$\Rightarrow \lim_{x \rightarrow 2} f(x) = 1$$

Limits Ex 29.1 Q21

$$\begin{aligned} \lim_{x \rightarrow 0^-} \sin \frac{1}{x} &= \lim_{h \rightarrow 0} \sin \frac{1}{0-h} = - \lim_{h \rightarrow 0} \sin \frac{1}{h} \\ &= - (\text{An oscillating number which oscillates between } -1 \text{ and } 1). \end{aligned}$$

$$\text{So, } \lim_{x \rightarrow 0^-} \sin \frac{1}{x} \text{ does not exist.}$$

$$\text{Similarly, } \lim_{x \rightarrow 0^+} \sin \frac{1}{x} \text{ does not exist.}$$

$$\lim_{x \rightarrow 0} \sin \frac{1}{x} \text{ does not exist.}$$

Let $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{where } x \neq \frac{\pi}{2} \\ 3, & \text{where } x = \frac{\pi}{2} \end{cases}$ and if $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$, find the value of k .

$$\text{LHL} = \lim_{x \rightarrow \frac{\pi}{2}^-} f(x)$$

$$= \lim_{h \rightarrow \frac{\pi}{2}} f\left(\frac{\pi}{2} - h\right)$$

$$= \lim_{h \rightarrow \frac{\pi}{2}} \frac{k \cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)}$$

$$= \lim_{h \rightarrow \frac{\pi}{2}} \frac{k \cos\left(\frac{\pi}{2} - h\right)}{2h}$$

$$= \frac{k \cos\left(\frac{\pi}{2} - \frac{\pi}{2}\right)}{\pi}$$

$$= \frac{k}{\pi}$$

$$\text{RHL} = \lim_{x \rightarrow \frac{\pi}{2}^+} f(x)$$

$$= \lim_{h \rightarrow \frac{\pi}{2}} f\left(\frac{\pi}{2} + h\right)$$

$$= \lim_{h \rightarrow \frac{\pi}{2}} \frac{k \cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)}$$

$$= \lim_{h \rightarrow \frac{\pi}{2}} \frac{k \cos\left(\frac{\pi}{2} + h\right)}{-2h}$$

$$= \frac{k \cos\left(\frac{\pi}{2} + \frac{\pi}{2}\right)}{-\pi}$$

$$= \frac{k}{\pi}$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^+} f(x)$$

Hence $\lim_{x \rightarrow \frac{\pi}{2}} f(x)$ exists.

$$\lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\frac{k}{\pi} = 3$$

$$k = 3\pi$$