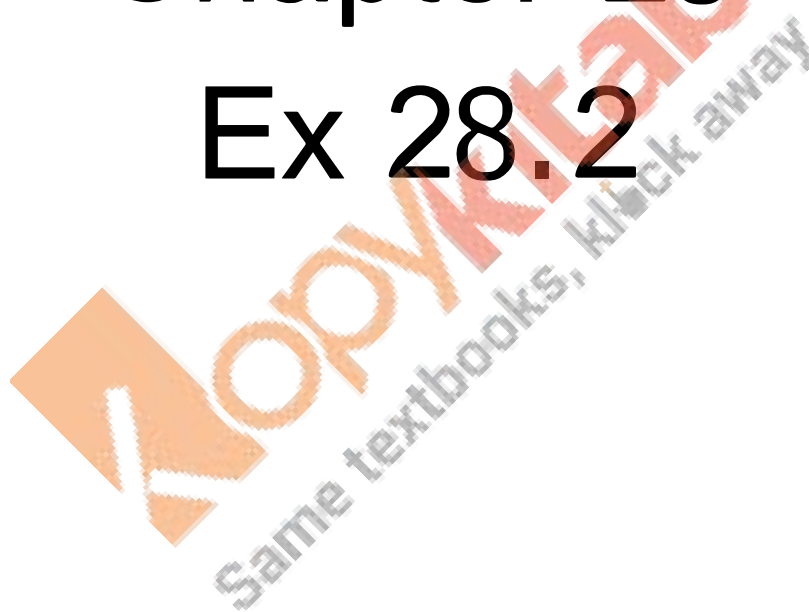


RD Sharma
Solutions
Class 11 Maths
Chapter 28
Ex 28.2



Introduction to 3D Coordinate Geometry Ex 28.2 Q1

(i) Distance between points P and Q

$$\begin{aligned}PQ &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \\&= \sqrt{(1 - 2)^2 + (-1 - 1)^2 + (0 - 2)^2} \\&= \sqrt{(-1)^2 + (-2)^2 + (-2)^2} \\&= \sqrt{1 + 4 + 4}\end{aligned}$$

$$PQ = 3 \text{ units}$$

(ii) Distance between points A and B

$$\begin{aligned}AB &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \\&= \sqrt{(3 + 1)^2 + (2 + 1)^2 + (-1 + 1)^2} \\&= \sqrt{(4)^2 + (3)^2 + (0)^2} \\&= \sqrt{16 + 9 + 0} \\&= \sqrt{25}\end{aligned}$$

$$AB = 5 \text{ units}$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q2

Distance between points P and Q

$$\begin{aligned}PQ &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \\&= \sqrt{(-2 - 2)^2 + (3 - 1)^2 + (1 - 2)^2} \\&= \sqrt{(-4)^2 + (2)^2 + (-1)^2} \\&= \sqrt{16 + 4 + 1}\end{aligned}$$

$$PQ = \sqrt{21} \text{ units}$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q3(i)

$A(4, -3, -1)$, $B(5, -7, 6)$ and $C(3, 1, -8)$

$$\begin{aligned}AB &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \\&= \sqrt{(4 - 5)^2 + (-3 + 7)^2 + (-1 - 6)^2} \\&= \sqrt{(-1)^2 + (4)^2 + (-7)^2} \\&= \sqrt{1 + 16 + 49} \\&= \sqrt{66} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(5 - 3)^2 + (-7 - 1)^2 + (6 + 8)^2} \\&= \sqrt{(2)^2 + (-8)^2 + (14)^2} \\&= \sqrt{4 + 64 + 196} \\&= \sqrt{264} \\&= 2\sqrt{66} \text{ units}\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(4 - 3)^2 + (-3 - 1)^2 + (-1 + 8)^2} \\&= \sqrt{(1)^2 + (-4)^2 + (7)^2} \\&= \sqrt{1 + 16 + 49} \\&= \sqrt{66} \text{ units}\end{aligned}$$

Since $AC + AB = BC$
so, A, B, C are collinear.

$$P(0, 7, -7), Q(1, 4, -5), R(-1, 10, -9)$$

$$PQ = \sqrt{(0-1)^2 + (7-4)^2 + (-7+5)^2}$$

$$= \sqrt{1^2 + 3^2 + (-2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14} \text{ units}$$

$$QR = \sqrt{(1+1)^2 + (4-10)^2 + (-5+9)^2}$$

$$= \sqrt{2^2 + (-6)^2 + 4^2}$$

$$= \sqrt{4+36+16}$$

$$= 2\sqrt{14} \text{ units}$$

$$PR = \sqrt{(0+1)^2 + (7-10)^2 + (-7+9)^2}$$

$$= \sqrt{1^2 + (-3)^2 + 2^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14} \text{ units}$$

Since $PQ + PR = QR$

so, P, Q, R are collinear

$A(3, -5, 1)$, $B(-1, 0, 8)$, and $C(7, -10, -6)$

$$\begin{aligned}AB &= \sqrt{\{3+1\}^2 + \{-5-0\}^2 + \{1-8\}^2} \\&= \sqrt{\{4\}^2 + \{-5\}^2 + \{-7\}^2} \\&= \sqrt{16+25+49} \\&= \sqrt{90} \\&= 3\sqrt{10} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{\{-1-7\}^2 + \{0+10\}^2 + \{8+6\}^2} \\&= \sqrt{\{-8\}^2 + \{10\}^2 + \{14\}^2} \\&= \sqrt{64+100+196} \\&= \sqrt{360} \\&= 6\sqrt{10} \text{ units}\end{aligned}$$

$$\begin{aligned}CA &= \sqrt{\{3-7\}^2 + \{-5+10\}^2 + \{1+6\}^2} \\&= \sqrt{\{-4\}^2 + \{5\}^2 + \{7\}^2} \\&= \sqrt{16+25+49} \\&= \sqrt{90} \\&= 3\sqrt{10} \text{ units}\end{aligned}$$

Since $AB + AC = BC$
so, A , B , and C are collinear

Let the point on xy - plane be $P(x, y, 0)$.

Now P is equidistance from $A(1, -1, 0)$, $B(2, 1, 2)$ and $C(3, 2, -1)$.

So, $AP = BP = CP$

Now,

$$(AP)^2 = (x - 1)^2 + (y + 1)^2 + (0 - 0)^2$$

$$(BP)^2 = (x - 2)^2 + (y - 1)^2 + (0 - 2)^2$$

$$(CP)^2 = (x - 3)^2 + (y - 2)^2 + (0 + 1)^2$$

$$(AP)^2 = (BP)^2 \Rightarrow (x - 1)^2 + (y + 1)^2 = (x - 2)^2 + (y - 1)^2 + 4$$

$$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y + 1 = x^2 + 4 - 4x + y^2 + 1 - 2y + 4$$

$$\Rightarrow 2x + 4y = 7 \quad \dots (1)$$

$$(BP)^2 = (CP)^2 \Rightarrow (x - 2)^2 + (y - 1)^2 + 4 = (x - 3)^2 + (y - 2)^2 + 1$$

$$\Rightarrow x^2 + 4 - 4x + y^2 + 1 - 2y + 4 = x^2 + 9 - 6x + y^2 + 4 - 4y + 1$$

$$\Rightarrow 2x + 2y = 5 \quad \dots (2)$$

$$(AP)^2 = (CP)^2 \Rightarrow (x - 1)^2 + (y + 1)^2 = (x - 3)^2 + (y - 2)^2 + 1$$

$$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y = x^2 + 9 - 6x + y^2 + 4 - 4y + 1$$

$$\Rightarrow 4x + 6y = 12 \quad \dots (3)$$

Solving equation (1) and (2) we get

$$y = 1, \quad x = 3/2$$

Put x and y in equation (3)

$$4(3/2) + 6(1) = 12$$

$$12 = 12$$

So, the required point is $(3/2, 1, 0)$

Let $Q(0, y, z)$ be the required point.

So

$$\begin{aligned}(AQ)^2 &= (BQ)^2 \Rightarrow (0-1)^2 + (y+1)^2 + (z-0)^2 = (0-2)^2 + (y-1)^2 + (z-2)^2 \\ \Rightarrow 1 + y^2 + 1 + 2y + z^2 &= 4 + y^2 + 1 - 2y + z^2 + 4 - 4z \\ \Rightarrow 4y + 4z &= 7 \dots (1)\end{aligned}$$

$$\begin{aligned}(BQ)^2 &= (CQ)^2 \Rightarrow (0-2)^2 + (y-1)^2 + (z-2)^2 = (0-3)^2 + (y-2)^2 + (z+1)^2 \\ \Rightarrow 4 + y^2 + 1 - 2y + z^2 + 4 - 4z &= 9 + y^2 + 4 - 4y + z^2 + 1 + 2z \\ \Rightarrow 2y - 6z &= 5 \dots (2)\end{aligned}$$

$$\begin{aligned}(AQ)^2 &= (CQ)^2 \Rightarrow (0-1)^2 + (y+1)^2 + (z-0)^2 = (0-3)^2 + (y-2)^2 + (z+1)^2 \\ \Rightarrow 1 + y^2 + 2y + 1 + z^2 &= 9 + y^2 + 4 - 4y + 4 + z^2 + 1 + 2z \\ \Rightarrow 6y - 2z &= 12 \dots (3)\end{aligned}$$

Solving equation (1) and (2), we get

$$z = \frac{-3}{16} \text{ and } y = \frac{31}{16}$$

Put the value of y and z in equation (3)

$$6y - 2z = 12 = 12$$

$$6\left(\frac{31}{16}\right) - 2\left(\frac{-3}{16}\right) = 12$$

$$\frac{186}{16} + \frac{6}{16} = 12$$

$$\frac{192}{16} = 12$$

$$12 = 12$$

LHS = RHS.

so,

$$y = \frac{31}{16}, z = \frac{-3}{16}$$

$$\text{Required point} = \left(0, \frac{31}{16}, \frac{-3}{16}\right)$$

Let $R(x, 0, z)$ be the required point.

So

$$\begin{aligned}(AR)^2 &= (BR)^2 \Rightarrow (1-x)^2 + (-1-0)^2 + (0-z)^2 = (2-x)^2 + (1-0)^2 + (2-z)^2 \\ \Rightarrow 1+x^2-2x+1+z^2 &= 4+x^2-4x+1+4+z^2-4z \\ \Rightarrow 2x+4z &= 7 \dots (1)\end{aligned}$$

$$\begin{aligned}(BR)^2 &= (OR)^2 \Rightarrow (z-z)^2 + (1-0)^2 + (2-z)^2 = (3-x)^2 + (2-0)^2 + (-1-z)^2 \\ \Rightarrow 4+x^2-4x+4+z^2-4z &= 9+x^2-6x+4+1+z^2+2z \\ \Rightarrow 2x-6z &= 5 \dots (2)\end{aligned}$$

$$\begin{aligned}(AR)^2 &= (OR)^2 \Rightarrow (1-x)^2 + (1-0)^2 + (0-z)^2 = (3-x)^2 + (2-0)^2 + (-1-z)^2 \\ \Rightarrow 1+x^2-2x+1+z^2 &= 9+6x+4+1+z^2+2z \\ \Rightarrow 4x-2z &= 12 \dots (3)\end{aligned}$$

Solving equation (1) and (2), we get

$$z = \frac{1}{5}, x = \frac{31}{10}$$

Put the value of x and z in equation (3)

$$4x - 2z = 12$$

$$4\left(\frac{31}{10}\right) - 2\left(\frac{1}{5}\right) = 12$$

$$\frac{124}{10} - \frac{2}{10} = 12$$

$$\frac{120}{10} = 12$$

$$12 = 12$$

$$\text{LHS} = \text{RHS.}$$

so,

$$x = \frac{31}{10}, z = \frac{1}{5}$$

$$\text{Required point} = \left(\frac{31}{10}, 0, \frac{1}{5}\right)$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q5

Let $P(0, 0, z)$ be the point equidistant from $Q(1, 5, 7)$ and $R(5, 1, -4)$.

So,

$$\begin{aligned}(PQ)^2 &= (PR)^2 \Rightarrow (0-1)^2 + (0-5)^2 + (z-7)^2 = (0-5)^2 + (0-1)^2 + (z+4)^2 \\ \Rightarrow 1+25+(z-7)^2 &= 25+1+(z+4)^2 \\ \Rightarrow 26+z^2+49-14z &= 26+z^2+8z+16 \\ \Rightarrow -14z-8z &= 16-49 \\ \Rightarrow -22z &= -33 \\ \Rightarrow z &= \frac{-33}{-22} \\ \Rightarrow z &= \frac{3}{2}\end{aligned}$$

$$\text{Required point} = (0, 0, 3/2)$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q6

Let $P(0, y, 0)$ be a point on y -axis which is equidistant from $Q(3, 1, 2)$ and $R(5, 5, 2)$.

So,

$$\begin{aligned} (PR)^2 &= (PQ)^2 \Rightarrow (0-5)^2 + (y-5)^2 + (0-2)^2 = (0-3)^2 + (y-1)^2 + (0-2)^2 \\ \Rightarrow 25 + y^2 + 25 - 10y + 4 &= 9 + y^2 + 1 - 2y + 4 \\ \Rightarrow -10y + 2y &= 14 - 54 \\ \Rightarrow -8y &= -40 \\ \Rightarrow y &= 5 \end{aligned}$$

So, the required point is $(0, 5, 0)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q7

Let $P(0, 0, z)$ be at a distance of $\sqrt{21}$ from $Q(1, 2, 3)$.

So

$$\begin{aligned} PQ &= \sqrt{(0-1)^2 + (0-2)^2 + (z-3)^2} \\ \sqrt{21} &= \sqrt{(1)^2 + (2)^2 + (z-3)^2} \\ 21 - 5 &= (z-3)^2 \\ 16 &= (z-3)^2 \\ z-3 &= \pm 4 \\ z &= 7 \text{ and } z = -1 \end{aligned}$$

So, the required points are $(0, 0, 7)$ and $(0, 0, -1)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q8

Let the triangle formed be $\triangle ABC$

$$\begin{aligned} AB &= \sqrt{(1-2)^2 + (2-3)^2 + (3-1)^2} \\ &= \sqrt{(-1)^2 + (-1)^2 + (2)^2} \\ &= \sqrt{6} \text{ units} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(2-3)^2 + (3-1)^2 + (1-2)^2} \\ &= \sqrt{(-1)^2 + (2)^2 + (-1)^2} \\ &= \sqrt{6} \text{ units} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(1-3)^2 + (2-1)^2 + (3-2)^2} \\ &= \sqrt{(-2)^2 + (1)^2 + (1)^2} \\ &= \sqrt{6} \text{ units} \end{aligned}$$

since, $AB = BC = CA$

So, $\triangle ABC$ is an equilateral $\triangle$

Introduction to 3D Coordinate Geometry Ex 28.2 Q9

Let $A = (0, 7, 10)$, $B = (-1, 6, 6)$ and $C = (-4, 9, 6)$

$$\begin{aligned} AB &= \sqrt{(0+1)^2 + (7-6)^2 + (10-6)^2} \\ &= \sqrt{(1)^2 + (1)^2 + (4)^2} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \text{ units} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(-1+4)^2 + (6-9)^2 + (6-6)^2} \\ &= \sqrt{(3)^2 + (3)^2 + 0} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \text{ units} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2} \\ &= \sqrt{(4)^2 + (-2)^2 + (4)^2} \\ &= \sqrt{36} \\ &= 6 \text{ units} \end{aligned}$$

$$\begin{aligned} (AB)^2 + (BC)^2 &= (3\sqrt{2})^2 + (3\sqrt{2})^2 \\ &= 18 + 18 \\ &= 36 \\ &= (AC)^2 \end{aligned}$$

Also $I(AB) = I(BC)$

Hence $(0, 7, 10)$, $(-1, 6, 6)$ and $(-4, 9, 6)$ are the vertices of an isosceles right-angled triangle.

Here points are $A(3, 3, 3)$, $B(0, 6, 3)$, $C(1, 7, 7)$ and $D(4, 4, 7)$.

$$\begin{aligned}AB &= \sqrt{(3-0)^2 + (3-6)^2 + (3-3)^2} \\&= \sqrt{9+9} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(0-1)^2 + (6-7)^2 + (3-7)^2} \\&= \sqrt{1+1+16} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(3-1)^2 + (3-7)^2 + (3-7)^2} \\&= \sqrt{4+16+16} \\&= 6 \text{ units}\end{aligned}$$

$$\begin{aligned}BD &= \sqrt{(0-4)^2 + (6-4)^2 + (3-7)^2} \\&= \sqrt{16+4+16} \\&= 6 \text{ units}\end{aligned}$$

$$\begin{aligned}CD &= \sqrt{(1-4)^2 + (7-4)^2 + (7-7)^2} \\&= \sqrt{9+9} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}AD &= \sqrt{(3-4)^2 + (3-4)^2 + (3-7)^2} \\&= \sqrt{1+1+16} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

Since,

$$AB = BC = CD = DA$$

And $AC = BD$

So,

A, B, C, D are vertices of a square.

Here,

$$\begin{aligned}AB &= \sqrt{(1+5)^2 + (3-5)^2 + (0-2)^2} \\&= \sqrt{36 + 4 + 4} \\&= \sqrt{44} \\&= 2\sqrt{11} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(-5+9)^2 + (5+1)^2 + (2-2)^2} \\&= \sqrt{16 + 36} \\&= \sqrt{52} \\&= 2\sqrt{13} \text{ units}\end{aligned}$$

$$\begin{aligned}CD &= \sqrt{(-9+3)^2 + (-1+3)^2 + (2-0)^2} \\&= \sqrt{36 + 4 + 4} \\&= 2\sqrt{11} \text{ units}\end{aligned}$$

$$\begin{aligned}DA &= \sqrt{(-3-4)^2 + (-3-3)^2 + 0} \\&= \sqrt{16 + 36} \\&= \sqrt{52} \\&= 2\sqrt{13} \text{ units}\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(1+9)^2 + (3+1)^2 + (0-2)^2} \\&= \sqrt{150 + 16 + 4} \\&= \sqrt{120} \\&= 4\sqrt{3} \text{ units}\end{aligned}$$

$$\begin{aligned}BD &= \sqrt{(-3+5)^2 + (-3-5)^2 + (0-2)^2} \\&= \sqrt{4 + 64 + 4} \\&= \sqrt{72} \\&= 6\sqrt{2} \text{ units}\end{aligned}$$

Since,

$$AB = CD \text{ and } BC = DA$$

$\Rightarrow$ $ABCD$ is a parallelogram $\Rightarrow BD$

but, $AC \neq BD$

$\Rightarrow$ $ABCD$ is not a rectangle.

Here,

$$\begin{aligned}AB &= \sqrt{(1+1)^2 + (3-6)^2 + (4-10)^2} \\&= \sqrt{4+9+36} \\&= 7 \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(-1+7)^2 + (6-4)^2 + (0-7)^2} \\&= \sqrt{36+4+9} \\&= 7 \text{ units}\end{aligned}$$

$$\begin{aligned}CD &= \sqrt{(-7+5)^2 + (4-1)^2 + (7-1)^2} \\&= \sqrt{4+9+36} \\&= 7 \text{ units}\end{aligned}$$

$$\begin{aligned}DA &= \sqrt{(-5-1)^2 + (1-3)^2 + (1-4)^2} \\&= \sqrt{36+4+9} \\&= \sqrt{52} \\&= 7 \text{ units}\end{aligned}$$

Since, $AB = BC = CD = DA$

So, $ABCD$ is a rhombus.

Here,

$$\begin{aligned}AB &= \sqrt{(0-1)^2 + (1-0)^2 + (1-1)^2} \\&= \sqrt{1+1} \\&= \sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(1-1)^2 + (0-1)^2 + (1-0)^2} \\&= \sqrt{1+1} \\&= \sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}CA &= \sqrt{(1-0)^2 + (1-1)^2 + (0-1)^2} \\&= \sqrt{1+0+1} \\&= \sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}DA &= \sqrt{(0-0)^2 + (0-1)^2 + (0-1)^2} \\&= \sqrt{1+1} \\&= \sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}OB &= \sqrt{(0-1)^2 + (0-0)^2 + (0-1)^2} \\&= \sqrt{1+1} \\&= \sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}OC &= \sqrt{(0-1)^2 + (0-1)^2 + (0-0)^2} \\&= \sqrt{1+1} \\&= \sqrt{2} \text{ units}\end{aligned}$$

Since, $OA = OB = OC = AB = BC = CA$

So, O, A, B, C represent a regular tetrahedron

Here,

$$\begin{aligned}OA &= \sqrt{(1-3)^2 + (3-2)^2 + (4-2)^2} \\&= \sqrt{4+1+4} \\&= 3 \text{ units}\end{aligned}$$

$$\begin{aligned}OB &= \sqrt{(1+1)^2 + (3-1)^2 + (4-3)^2} \\&= \sqrt{4+4+1} \\&= 3 \text{ units}\end{aligned}$$

$$\begin{aligned}OC &= \sqrt{(1-0)^2 + (3-5)^2 + (4-6)^2} \\&= \sqrt{1+4+4} \\&= 3 \text{ units}\end{aligned}$$

$$\begin{aligned}OD &= \sqrt{(1-2)^2 + (3-1)^2 + (4-2)^2} \\&= \sqrt{1+4+4} \\&= 3 \text{ units}\end{aligned}$$

Since, $OA = OC = OD = OB$, points A, B, C, D lie on a sphere with centre O.

Radius = 3 units

Introduction to 3D Coordinate Geometry Ex 28.2 Q15

Let the required point be $P(x_1, y_1, z_1)$

Here, $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 0)$, $C(0, 0, 8)$

Since, $(OP)^2 = (PA)^2$

$$\begin{aligned}(x-0)^2 + (y-0)^2 + (z-0)^2 &= (x-2)^2 + (y-0)^2 + (z-0)^2 \\x^2 + y^2 + z^2 &= x^2 - 4x + 4 + y^2 + z^2 \\4x &= 4 \\x &= 1\end{aligned}$$

$$(OP)^2 = (PB)^2$$

$$\begin{aligned}(x-0)^2 + (y-0)^2 + (z-0)^2 &= (x-0)^2 + (y-3)^2 + (z-0)^2 \\x^2 + y^2 + z^2 &= x^2 + y^2 - 6y + 9 + z^2 \\6y &= 9 \\y &= \frac{3}{2}\end{aligned}$$

$$(OP)^2 = (PC)^2$$

$$\begin{aligned}(x-0)^2 + (y-0)^2 + (z-0)^2 &= (x-0)^2 + (y-0)^2 + (z-8)^2 \\x^2 + y^2 + z^2 &= x^2 + y^2 + z^2 - 16z + 64 \\16z &= 64 \\z &= 4\end{aligned}$$

The required point = $\left(1, \frac{3}{2}, 4\right)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q16

Let P be (x, y, z) , here, $A(-2, 2, 3)$ and $B(13, -3, 13)$
and $3PA = 2PB$

$$\Rightarrow 3\sqrt{(x+2)^2 + (y-2)^2 + (z-3)^2} = 2\sqrt{(x-13)^2 + (y+3)^2 + (z-13)^2}$$

squaring both the sides,

$$\begin{aligned}\Rightarrow & 9[x^2 + 4x + 4 + y^2 + 4 - 4y + z^2 + 9 - 6z] \\ & = 4[x^2 + 169 - 26x + y^2 + 9 + 6y + z^2 + 169 - 26z] \\ \Rightarrow & 9x^2 - 4x^2 + 36x + 104x + 36 - 676 + 9y^2 - 4y^2 \\ & + 36 - 36 - 36y - 24y + 9z^2 - 4z^2 + 81 - 676 - 54z + 6yz = 0 \\ \Rightarrow & 5x^2 + 5y^2 + 5z^2 + 140x - 60y + 50z - 1235 = 0 \\ \Rightarrow & 5(x^2 + y^2 + z^2) + 140x - 60y + 50z - 1235 = 0\end{aligned}$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q17

Let $P(x, y, z)$, here, $A(3, 4, 5)$, $B(-1, 3, -7)$
 $PA^2 + PB^2 = 2k^2$

$$\begin{aligned}\Rightarrow & (x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (z+7)^2 = 2k^2 \\ \Rightarrow & x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z + x^2 + 1 + 2x \\ & + y^2 + 9 - 6y + z^2 + 49 + 14z = 2k^2 \\ \Rightarrow & 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = 2k^2 \\ \Rightarrow & 2(x^2 + y^2 + z^2) - 4x - 14y + 4z + 109 - 2k^2 = 0\end{aligned}$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q18

Here, $A(a, b, c)$, $B(b, c, a)$, $C(c, a, b)$

$$\begin{aligned}AB &= \sqrt{(a-b)^2 + (b-c)^2 + (c-a)^2} \\ &= \sqrt{a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ac}\end{aligned}$$

$$AB = \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac}$$

$$\begin{aligned}BC &= \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2} \\ &= \sqrt{b^2 + c^2 - 2bc + c^2 + a^2 - 2ca + a^2 + b^2 - 2ab}\end{aligned}$$

$$BC = \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca}$$

$$\begin{aligned}CA &= \sqrt{(a-c)^2 + (b-a)^2 + (c-b)^2} \\ &= \sqrt{a^2 + c^2 - 2ac + b^2 + a^2 - 2ab + b^2 + c^2 - 2bc}\end{aligned}$$

$$CA = \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca}$$

Since, $AB = BC = CA$, so
 $\triangle ABC$ is an isosceles $\triangle$

Introduction to 3D Coordinate Geometry Ex 28.2 Q19

Here, $A(3, 6, 9)$, $B(10, 20, 30)$, $C(25, 41, 5)$

$$\begin{aligned} (AB)^2 &= (3-10)^2 + (6-20)^2 + (9-30)^2 \\ &= (-7)^2 + (-14)^2 + (-21)^2 \\ &= 49 + 196 + 441 \\ &= 586 \end{aligned}$$

$$\begin{aligned} (BC)^2 &= (10-25)^2 + (20+41)^2 + (30-5)^2 \\ &= (-15)^2 + (61)^2 + (25)^2 \\ &= 225 + 3721 + 625 \\ &= 4571 \end{aligned}$$

$$\begin{aligned} (CA)^2 &= (3-25)^2 + (6+41)^2 + (9-5)^2 \\ &= (-22)^2 + (47)^2 + (4)^2 \\ &= 484 + 2209 + 16 \\ &= 2709 \end{aligned}$$

Since, $AB^2 + BC^2 \neq AC^2$

$$AB^2 + AC^2 \neq BC^2$$

$$BC^2 + AC^2 \neq AB^2$$

So, $\triangle ABC$ is not a right triangle.

Introduction to 3D Coordinate Geometry Ex 28.2 Q20(i)

Here, $A(0, 7, -10)$, $B(1, 6, -6)$, $C(4, 9, -6)$

$$\begin{aligned} AB &= \sqrt{(0-1)^2 + (7-6)^2 + (-10+6)^2} \\ &= \sqrt{1+1+16} \\ &= 3\sqrt{2} \text{ units} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(1-4)^2 + (6-9)^2 + (-6+6)^2} \\ &= \sqrt{9+9} \\ &= 3\sqrt{2} \text{ units} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2} \\ &= \sqrt{16+4+16} \\ &= 6 \text{ units} \end{aligned}$$

Since, $AB = BC$

So, $\triangle ABC$ is an isosceles $\triangle$

Introduction to 3D Coordinate Geometry Ex 28.2 Q20(ii)

Here, $A(0, 7, 10)$, $B(-1, 6, 6)$, $C(-4, 9, 6)$

$$\begin{aligned}AB &= \sqrt{(0+1)^2 + (7-6)^2 + (10-6)^2} \\&= \sqrt{1+1+16} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(-1+4)^2 + (6-9)^2 + (6-6)^2} \\&= \sqrt{9+9+0} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}CA &= \sqrt{(-4-0)^2 + (9-7)^2 + (6+10)^2} \\&= \sqrt{16+4+16} \\&= \sqrt{36} \text{ units}\end{aligned}$$

$$\text{Since, } (AB)^2 + (BC)^2 = (AC)^2$$

So, $\triangle ABC$ is a right triangle.

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Here, $A(-1, 2, 1)$, $B(1, -2, 5)$, $C(4, -7, 8)$, $D(2, -3, 4)$

$$\begin{aligned}AB &= \sqrt{(-1-1)^2 + (2+2)^2 + (1-5)^2} \\&= \sqrt{4+16+16} \\&= 6 \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(1-4)^2 + (-2+7)^2 + (5-8)^2} \\&= \sqrt{9+25+9} \\&= \sqrt{43} \text{ units}\end{aligned}$$

$$\begin{aligned}CD &= \sqrt{(4-2)^2 + (-7+3)^2 + (8-4)^2} \\&= \sqrt{4+16+16} \\&= 6 \text{ units}\end{aligned}$$

$$\begin{aligned}DA &= \sqrt{(2+1)^2 + (-3-2)^2 + (4-1)^2} \\&= \sqrt{9+25+9} \\&= \sqrt{43} \text{ units}\end{aligned}$$

$$\text{Since, } AB = CD \text{ and } BC = DA$$

So, $\triangle ABC$ is a parallelogram

Introduction to 3D Coordinate Geometry Ex 28.2 Q20(iv)

Let $A(5, -1, 1)$, $B(7, -4, 7)$, $C(1, -6, 10)$ and $D(-1, -3, 4)$ be the given points.

$$AB = \sqrt{(7-5)^2 + (-4+1)^2 + (7-1)^2} = \sqrt{4+9+36} = 7$$

$$BC = \sqrt{(1-7)^2 + (-6+4)^2 + (10-7)^2} = \sqrt{36+4+9} = 7$$

$$CD = \sqrt{(-1-1)^2 + (-3+6)^2 + (4-10)^2} = \sqrt{4+9+36} = 7$$

$$AD = \sqrt{(-1-5)^2 + (-3+1)^2 + (4-1)^2} = \sqrt{36+4+9} = 7$$

So $AB = BC = CD = AD$

Hence ABCD is a rhombus.

Introduction to 3D Coordinate Geometry Ex 28.2 Q21

Let the point $P(x_1, y_1, z_1)$ which is equidistance from $A(1, 2, 3)$ and $B(3, 2, -1)$, so

$$AP = BP$$

$$(AP)^2 = (BP)^2$$

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

$$x^2 + 1 - 2x + y^2 + 4 - 4y + z^2 + 9 - 6z = x^2 + 9 - 6x + y^2 + 4 - 4y + z^2 + 1 + 2z$$

$$4x - 8z = 14 - 14$$

$$4x - 8z = 0$$

$$x - 2z = 0$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q22

Let locus of $P(x_1, y_1, z_1)$ is the required locus, so

$$PA + PB = 10$$

$$\Rightarrow \sqrt{(x-4)^2 + (y-0)^2 + (z-0)^2} + \sqrt{(x+4)^2 + (y-0)^2 + (z-0)^2} = 10$$

$$\Rightarrow \sqrt{x^2 + 16 - 8x + y^2 + z^2} = 10 - \sqrt{x^2 + 8x + 16 + y^2 + z^2}$$

$$\Rightarrow x^2 + y^2 + z^2 - 8x + 16 = (10)^2 + (x^2 + y^2 + z^2 + 8x + 16) - 20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow -8x + 16 - 100 - 8x - 16 = -20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow -16x - 100 = -20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow -4(4x + 25) = -20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow (4x + 25) = 5\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

squaring both the sides,

$$(4x + 25)^2 = 25(x^2 + y^2 + z^2 + 8x + 16)$$

$$\Rightarrow 16x^2 + 625 + 200x = 25(x^2 + y^2 + z^2 + 8x + 16)$$

$$\Rightarrow 16x^2 + 625 + 200x = 25x^2 + 25y^2 + 25z^2 + 200x + 400$$

$$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q23

$$\begin{aligned}
 AB &= \sqrt{(1+1)^2 + (2+2)^2 + (3+1)^2} \\
 &= \sqrt{4+16+16} \\
 &= 6 \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 BC &= \sqrt{(-1-2)^2 + (-2-3)^2 + (-1-2)^2} \\
 &= \sqrt{9+25+9} \\
 &= \sqrt{43} \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 CD &= \sqrt{(2-4)^2 + (3-7)^2 + (2-6)^2} \\
 &= \sqrt{4+16+16} \\
 &= 6 \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 DA &= \sqrt{(4-1)^2 + (7-2)^2 + (6-3)^2} \\
 &= \sqrt{9+25+9} \\
 &= \sqrt{43} \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 AC &= \sqrt{(1-2)^2 + (2-3)^2 + (3-2)^2} \\
 &= \sqrt{1+1+1} \\
 &= \sqrt{3} \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 BD &= \sqrt{(-1-4)^2 + (-2-7)^2 + (-1-6)^2} \\
 &= \sqrt{25+81+49} \\
 &= \sqrt{155} \text{ units}
 \end{aligned}$$

Since,

$$AB = DC \text{ and } BC = DA$$

so,

$ABCD$ is a parallelogram

$$AB^2 + BC^2 = 36 + 43 = 97$$

$$AC = \sqrt{3}$$

$$\therefore AB^2 + BC^2 \neq AC^2$$

i.e. $\angle B$ is not a right angle.

$ABCD$ is not a rectangle.

Let the point be P (x, y, z)

Given

$$A=(3, 4, -5)$$

$$B=(-2, 1, 4)$$

$$PA=PB \Rightarrow PA^2=PB^2$$

$$PA^2 = (x-3)^2 + (y-4)^2 + (z+5)^2$$

$$PB^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$$

$$PA^2=PB^2 \Rightarrow$$

$$(x-3)^2 + (y-4)^2 + (z+5)^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$$

All square terms will be cancelled on both sides, we get

$$-6x+9-8y+16+10z+25=4x+4-2y+1-8z+16$$

$$10x+6y-18z-29=0 \text{ is the required equation}$$