RD Sharma Solutions Class 11 Maths Chapter 26 Ex 26.1

Ellipse Ex 26.1 Q1

Let $P(x, y)$ be any point on the ellipse whose focus is $S(1, -2)$ and eccentricity $e = \frac{1}{2}$. Let PM be perpendicular from P on the directrix. Then,

$$SP = ePM$$

$$\Rightarrow \quad SP = \frac{1}{2}(PM)$$

$$\Rightarrow \quad SP^2 = \frac{1}{4}(PM)^2$$

$$\Rightarrow \quad 4SP^2 = (PM)^2$$

$$\Rightarrow \quad 4\left[(x-1)^2 + (y+2)^2\right] = \left[\frac{3x - 2y + 5}{\sqrt{(3)^2 + (-2)^2}}\right]^2$$

$$\Rightarrow \quad 4\left[x^2 + 1 - 2x + y^2 + 4 + 4y\right] = \frac{(3x - 2y + 5)^2}{\left(\sqrt{13}\right)^2}$$

$$\Rightarrow \quad 4\left[x^2 + y^2 - 2x + 4y + 5\right] = \frac{(3x - 2y + 5)^2}{13}$$

$$\Rightarrow \quad 52\left[x^2 + y^2 - 2x + 4y + 5\right] = (3x - 2y + 5)^2$$

$$\Rightarrow \quad 52x^2 + 52y^2 - 104x + 208y + 260 = (3x - 2y + 5)^2$$

$$\Rightarrow \quad 52x^2 + 52y^2 - 104x + 208y + 260 = (3x)^2 + (-2y)^2 + (5)^2 + 2 \times 3x \times (-2y) + 2 \times (-2y) \times 5 + 2 \times 5 \times 3x$$

$$\left[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca\right]$$

$$\Rightarrow \quad 52x^2 + 52y^2 - 104x + 208y + 260 = 9x^2 + 4y^2 + 25 - 12xy - 20y + 30x$$

$$\Rightarrow \quad 52x^2 - 9x^2 + 52y^2 - 4y^2 + 12xy - 104x - 30x + 208y + 20y + 260 - 25 = 0$$

$$\Rightarrow \quad 43x^2 + 48y^2 + 12xy - 134x + 228y + 235 = 0$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q2(i)

Let $P(x, y)$ be a point on the ellips. Then, by definition

$$SP = e\,PM$$

Here $e = \frac{1}{2}$, coordinates of S are $(0,1)$ and the equation of the directrix is

$$x + y = 0.$$

$$\therefore \quad SP = \frac{1}{2} PM$$

$$\Rightarrow \quad SP^2 = \frac{1}{4}(PM)^2$$

$$\Rightarrow \quad 4SP^2 = (PM)^2$$

$$\Rightarrow \quad 4\left[(x-0)^2 + (y-1)^2\right] = \left[\frac{x+y}{\sqrt{1^2+1^2}}\right]$$

$$\Rightarrow \quad 4\left[x^2 + y^2 + 1 - 2y\right] = \frac{(x+y)^2}{2}$$

$$\Rightarrow \quad 4 \times 2\left[x^2 + y^2 - 2y + 1\right] = x^2 + y^2 + 2xy$$

$$\Rightarrow \quad 8x^2 + 8y^2 - 16y + 8 = x^2 + y^2 + 2xy$$

$$\Rightarrow \quad 8x^2 - x^2 + 8y^2 - y^2 - 2xy - 16y + 8 = 0$$

$$\Rightarrow \quad 7x^2 + 7y^2 - 2xy - 16y + 8 = 0$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q2(ii)

Let $P(x,y)$ be a point on the ellipse. Then, by definition

$$SP = e\,PM$$

Here $e = \frac{1}{2}$, coordinates of S are $(-1,1)$ and the equation of directrix is

$$x - y + 3 = 0$$

$$\therefore \quad SP = \frac{1}{2}PM$$

$$\Rightarrow \quad SP^2 = \frac{1}{4}(PM)^2$$

$$\Rightarrow \quad 4SP^2 = PM^2$$

$$\Rightarrow \quad 4\left[(x+1)^2 + (y-1)^2\right] = \left[\frac{x-y+3}{\sqrt{1^2+(-1)^2}}\right]^2$$

$$\Rightarrow \quad 4\left[x^2+1+2x+y^2+1-2y\right] = \frac{(x-y+3)^2}{2}$$

$$\Rightarrow \quad 8\left[x^2+y^2+2x-2y+2\right] = (x-y+3)^2$$

$$\Rightarrow \quad 8x^2+8y^2+16x-16y+16 = x^2+(-y)^2+3^2+2\times(-y)\times 3+2\times(x)\times(-y)+2\times 3\times x$$

$$\left[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca\right]$$

$$\Rightarrow \quad 8x^2+8y^2+16x-16y+16 = x^2+y^2+9-6y-2xy+6x$$

$$\Rightarrow \quad 8x^2-x^2+8y^2-y^2+2xy+16x-6x-16y+6y+16-9 = 0$$

$$\Rightarrow \quad 7x^2+7y^2+2xy+10x-10y+7 = 0$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q2(iii)

Let $P(x, y)$ be a point the ellipse. Then, by definition

$$SP = e\,PM$$

Here $e = \frac{4}{5}$, coordinates of S are $(-2, 3)$ and the equation of directrix is

$$2x + 3y + 4 = 0$$

$$\therefore \quad SP = \frac{4}{5} Pm$$

$$\Rightarrow \quad SP^2 = \frac{16}{25}(PM)^2$$

$$\Rightarrow \quad 25\,SP^2 = 16\,PM^2$$

$$\Rightarrow \quad 25\left[(x+2)^2 + (y-3)^2\right] = 16\left[\frac{2x+3y+4}{\sqrt{2^2+3^2}}\right]^2$$

$$\Rightarrow \quad 25\left[x^2 + 4 + 4x + y^2 + 9 - 6y\right] = \frac{16(2x+3y+4)^2}{13}$$

$$\Rightarrow \quad 325\left[x^2 + y^2 + 4x - 6y + 13\right] = 16(2x+3y+4)^2$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q2(iv)

Let $P(x, y)$ be a point on the ellipse. Then, by definition

$$SP = e\,PM$$

Here $e = \frac{1}{2}$, coordinates of S are $(1, 2)$ and the equation of directrix is

$$3x + 4y - 5 = 0$$

$$\therefore \quad SP = \frac{1}{2}Pm$$

$$\Rightarrow \quad SP^2 = \frac{1}{4}(PM)^2$$

$$\Rightarrow \quad 4SP^2 = PM^2$$

$$\Rightarrow \quad 4\left[(x-1)^2 + (y-2)^2\right] = \left[\frac{3x + 4y - 5}{\sqrt{3^2 + 4^2}}\right]^2$$

$$\Rightarrow \quad 4\left[x^2 + 1 - 2x + y^2 + 4 - 4y\right] = \frac{(3x + 4y - 5)^2}{25}$$

$$\Rightarrow \quad 100\left[x^2 + y^2 - 2x - 4y + 5\right] = (3x + 4y - 5)^2$$

$$\Rightarrow \quad 100x^2 + 100y^2 - 200x - 400y + 500 = (3x + 4y - 5)^2$$

$$\Rightarrow \quad 100x^2 + 100y^2 - 200x - 400y + 500 = (3x)^2 + (4y)^2 + (-5)^2 + 2 \times 3x \times 4y + 2 \times 4y \times (-5) + 2 \times (-5) \times 3x$$

$$\Rightarrow \quad 100x^2 + 100y^2 - 200x - 400y + 500 = 9x^2 + 16y^2 + 25 + 24xy - 40y - 30x$$

$$\Rightarrow \quad 100x^2 - 9x^2 + 100y^2 - 16y^2 - 24xy - 200x + 30x - 400y + 40y + 500 - 25 = 0$$

$$\Rightarrow \quad 91x^2 + 84y^2 - 24xy - 170x - 360y + 475 = 0$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q3(i)

$$4x^2 + 9y^2 = 1$$

$$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{9}} = 1$$

$$eccentricity = \sqrt{\frac{\frac{1}{4} - \frac{1}{9}}{\frac{1}{4}}} = \frac{\sqrt{5}}{3}$$

$$\text{Length of latus rectum} = \frac{2 \times \frac{1}{9}}{\frac{1}{2}} = \frac{4}{9}$$

$$Foci\ are\ \left(\frac{\sqrt{5}}{6}, 0\right); \left(-\frac{\sqrt{5}}{6}, 0\right)$$

Ellipse Ex 26.1 Q3(ii)

$5x^2+4y^2=1$

$$\frac{x^2}{\frac{1}{5}}+\frac{y^2}{\frac{1}{4}}=1$$

$$eccentricity=\sqrt{\frac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}}}=\frac{1}{\sqrt{5}}$$

$$\text{Length of latus rectum}=\frac{2\times\frac{1}{5}}{\frac{1}{2}}=\frac{4}{5}$$

$$Foci\ are\ (0,\frac{1}{2\sqrt{5}});(0,-\frac{1}{2\sqrt{5}})$$

Ellipse Ex 26.1 Q3(iii)

We have,

$$4x^2+3y^2=1$$

$$\Rightarrow \quad \frac{x^2}{\frac{1}{4}}+\frac{y^2}{\frac{1}{3}}=1 \ \ldots\ldots\ldots(i)$$

This is of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a^2=\frac{1}{4}$ and $b^2=\frac{1}{3}$ *i.e.*,

$$a=\frac{1}{2} \text{ and } b=\frac{1}{\sqrt{3}}.$$

Clearly, $b>a$, therefore the major and minor axes of the ellipse (i) are along y and x axes respectively.

Let e be the eccentricity of the ellipse. Then,

$$e=\sqrt{1-\frac{a^2}{b^2}}$$

$$=\sqrt{1-\frac{\frac{1}{4}}{\frac{1}{3}}}$$

$$=\sqrt{1-\frac{3}{4}}$$

$$=\sqrt{\frac{1}{4}}$$

$$\therefore \quad e=\frac{1}{2}$$

The coordinates of the foci are $(0,be)$ and $(0,-be)$ *i.e.*, $\left(0,\frac{1}{2\sqrt{3}}\right)$ and $\left(0,\frac{-1}{2\sqrt{3}}\right)$.

Now,

$$\text{Length of the latus rectum}=\frac{2a^2}{b}$$

$$=2\times\frac{\frac{1}{4}}{\frac{1}{\sqrt{3}}}$$

$$=\frac{\sqrt{3}}{2}$$

Ellipse Ex 26.1 Q3(iv)

We have,

$$25x^2 + 16y^2 = 1600$$

$$\Rightarrow \quad \frac{25x^2}{1600} + \frac{16y^2}{1600} = 1$$

$$\Rightarrow \quad \frac{x^2}{64} + \frac{y^2}{100} = 1 \quad \text{..........(i)}$$

This is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a^2 = 64$ and $b^2 = 100$ i.e.,

$a = 8$ and $b = 10$.

Clearly, $b > a$, therefore the major and minor axes of the ellipse (i) are along y and x axes respectively.
Let e be the eccentricity of the ellipse. Then,

$$e = \sqrt{1 - \frac{a^2}{b^2}}$$

$$= \sqrt{1 - \frac{64}{100}}$$

$$= \sqrt{\frac{36}{100}}$$

$$= \frac{6}{10}$$

$$\therefore \quad = \frac{3}{5}$$

The coordinates of the foci are $(0, be)$ and $(0, -be)$ i.e., $(0, 6)$ and $(0, -6)$.
Now,

$$\text{Length of the latus rectum} = \frac{2a^2}{b}$$

$$= 2 \times \frac{64}{10}$$

$$= \frac{64}{5}$$

Ellipse Ex 26.1 Q4

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ .... (i)}$$

$$\therefore \quad e=\sqrt{1-\frac{b^2}{a^2}}$$

$$\Rightarrow \quad \sqrt{\frac{2}{5}}=\sqrt{1-\frac{b^2}{a^2}} \qquad \left[\because \text{ eccentricity}=\sqrt{\frac{2}{5}}\right]$$

$$\Rightarrow \quad \frac{2}{5}=1-\frac{b^2}{a^2}$$

$$\Rightarrow \quad \frac{b^2}{a^2}=1-\frac{2}{5}$$

$$\Rightarrow \quad \frac{b^2}{a^2}=\frac{3}{5}$$

$$\Rightarrow \quad 5b^2=3a^2$$

$$\Rightarrow \quad b^2=\frac{3a^2}{5} \text{ .... (ii)}$$

Putting the value of $b^2=\dfrac{3a^2}{5}$ in equation (ii), we get

$$\frac{9}{a^2}+\frac{1}{\frac{3a^2}{5}}=1$$

$$\Rightarrow \quad \frac{9}{a^2}+\frac{5}{3a^2}=1$$

$$\Rightarrow \quad \frac{1}{a^2}\left[9+\frac{5}{3}\right]=1$$

$$\Rightarrow \quad \frac{1}{a^2}\left[9+\frac{5}{3}\right]=1$$

$$\Rightarrow \quad 9+\frac{5}{3}=a^2$$

$$\Rightarrow \quad a^2=\frac{32}{3} \text{ ....(iii)}$$

Putting $a^2 = \frac{32}{3}$ in equation (ii), we get

$$b^2 = \frac{3}{5}\times\frac{32}{3}=\frac{32}{5} \text{ ....(iv)}$$

$\therefore$ The required equation of ellipse is

$$\frac{x^2}{\frac{32}{3}}+\frac{y^2}{\frac{32}{5}}=1$$

$$\Rightarrow \quad \frac{3x^2}{32}+\frac{5y^2}{32}=1$$

$$\Rightarrow \quad 3x^2+5y^2=32.$$

This is the required equation of ellipse.

Ellipse Ex 26.1 Q5(i)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ ....(i)}$$

The coordinates of the foci are $(\pm 2,0)$. This means that the major and minor axes of the ellipse are along x and y axes respectively and the coordinates of foci are $(\pm ae, o)$

$\therefore \quad ae = 2$

$\Rightarrow \quad a \times \frac{1}{2} = 2 \qquad \left[\because e = \frac{1}{2}\right]$

$\Rightarrow \quad a = 4$

$\Rightarrow \quad a^2 = 16$

Now, $\quad b^2 = a^2\left(1-e^2\right)$

$\Rightarrow \quad b^2 = (4)^2\left[1-\left(\frac{1}{2}\right)^2\right]$

$\Rightarrow \quad b^2 = 16\left[1-\frac{1}{4}\right]$

$\Rightarrow \quad b^2 = 16 \times \frac{3}{4} = 12$

Substituting the value of a^2 and b^2 in (i), we get

$$\frac{x^2}{16}+\frac{y^2}{12}=1$$

$\Rightarrow \quad 3x^2+4y^2=48$

required equation of ellipse.

Ellipse Ex 26.1 Q5(ii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \;\ldots\ldots\;(i)$$

The length of latus-rectum = 5

$$\therefore \quad \frac{2b^2}{a}=5$$

$$\Rightarrow \quad b^2=\frac{5a}{2} \;\ldots\ldots\;(ii)$$

Now, $b^2=a^2\left(1-e^2\right)$

$$\Rightarrow \quad \frac{5a}{2}=a^2\left[1-\left(\frac{2}{3}\right)^2\right] \qquad \left[\because e=\frac{2}{3}\right]$$

$$\Rightarrow \quad \frac{5a}{2}=a^2\left[1-\frac{4}{9}\right]$$

$$\Rightarrow \quad \frac{5}{2}=a\left(\frac{5}{9}\right)$$

$$\Rightarrow \quad \frac{5}{2}\times\frac{9}{5}=a$$

$$\Rightarrow \quad a=\frac{9}{2}$$

$$\Rightarrow \quad a^2=\frac{81}{4}$$

Putting $a=\frac{9}{2}$ in $b^2=\frac{5a}{2}$, we get

$$b^2=\frac{5}{2}\times\frac{9}{2}$$

$$\Rightarrow \quad b^2=\frac{45}{4}$$

Substituting $a^2=\frac{81}{4}$ and $b^2=\frac{45}{4}$ in equation (i), we get

$$\frac{x^2}{\frac{81}{4}}+\frac{y^2}{\frac{45}{4}}=1$$

$$\Rightarrow \quad \frac{4x^2}{81}+\frac{4y^2}{45}=1$$

$$\Rightarrow \quad \frac{4x^2\times5+4y^2\times9}{405}=1$$

$$\Rightarrow \quad 20x^2+36y^2=405$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(iii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ .... (i)}$$

Then, semi-major axis = a

$\therefore \quad a = 4 \quad\quad [\because \text{ semi-major axis} = 4]$

$\Rightarrow \quad a^2 = 16$

Now,

$$b^2 = a^2\left(1 - e^2\right)$$

$$\Rightarrow \quad b^2 = 16\left[1 - \left(\frac{1}{2}\right)^2\right] \quad\quad \left[\because e = \frac{1}{2}\right]$$

$$\Rightarrow \quad b^2 = 16\left[1 - \frac{1}{4}\right]$$

$$\Rightarrow \quad b^2 = 16 \times \frac{3}{4}$$

$$\Rightarrow \quad b^2 = 12$$

Substituting the value of a^2 and b^2 in (i), we get

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$

$$\Rightarrow \quad 3x^2 + 4y^2 = 48$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q5(iv)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ where major axis} = 2a \ldots\ldots\ldots\ldots \text{(i)}$$

Now,

$$2a = 12 \qquad [\because \text{Major axis} = 12]$$

$$\Rightarrow \quad a = 6$$

$$\Rightarrow \quad a^2 = 36$$

Now,

$$b^2 = a^2\left(1 - e^2\right)$$

$$\Rightarrow \quad b^2 = 36\left(1 - \frac{1}{4}\right) \qquad \left[\because e = \frac{1}{2}\right]$$

$$\Rightarrow \quad b^2 = 36 \times \frac{3}{4}$$

$$\Rightarrow \quad b^2 = 27$$

Substituting the value of a^2 and b^2 in (i), we get

$$\frac{x^2}{36} + \frac{y^2}{27} = 1$$

$$\Rightarrow \quad \frac{1}{9}\left[\frac{x^2}{4} + \frac{y^2}{3}\right] = 1$$

$$\Rightarrow \quad \frac{3x^2 + 4y^2}{12} = 9$$

$$\Rightarrow \quad 3x^2 + 4y^2 = 108$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(v)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ ....(i)}$$

Since the ellipse passes through

$(1,4)$ and $(-6,1)$.

$$\therefore \quad \frac{(1)^2}{a^2}+\frac{(4)^2}{b^2}=1$$

$$\Rightarrow \quad \frac{1}{a^2}+\frac{16}{b^2}=1$$

$$\Rightarrow \quad b^2+16a^2=a^2b^2 \text{ ....(ii)}$$

$$\text{and} \quad \frac{(-6)^2}{a^2}+\frac{(1)^2}{b^2}=1$$

$$\Rightarrow \quad \frac{36}{a^2}+\frac{1}{b^2}=1$$

$$\Rightarrow \quad \frac{36}{a^2}+\frac{1}{b^2}=1$$

$$\Rightarrow \quad 36b^2+a^2=a^2b^2 \text{ ....(iii)}$$

Multiplying equation (iii) by 16, we get

$$576b^2+16a^2=16a^2b^2 \text{ ....(iv)}$$

Multipliying equation (iii) by 16, we get

$$576b^2 + 16a^2 = 16a^2b^2 \text{ ....(iv)}$$

Substituting equation (ii) from equation (iv), we get

$$576b^2 - b^2 = 16a^2b^2 - a^2b^2$$

$$\Rightarrow \quad 575b^2 = 15a^2b^2$$

$$\Rightarrow \quad 575 = 15a^2$$

$$\Rightarrow \quad a^2 = \frac{575}{15} = \frac{115}{3}$$

Putting $a^2 = \frac{115}{3}$ in equation (ii), we get

$$b^2 + 16 \times \frac{115}{3} = \frac{115}{3} \times b^2$$

$$\Rightarrow \quad b^2 - \frac{115}{3}b^2 = -16 \times \frac{115}{3}$$

$$\Rightarrow \quad \frac{3b^2 - 115b^2}{3} = -\frac{16 \times 115}{3}$$

$$\Rightarrow \quad -112b^2 = -16 \times 115$$

$$\Rightarrow \quad b^2 = \frac{16 \times 115}{112}$$

Substituting the value of a^2 and b^2 in (i), we get

$$\frac{x^2}{\frac{115}{3}} + \frac{y^2}{\frac{115}{7}} = 1$$

$$\Rightarrow \quad \frac{3x^2 + 7y^2}{115} = 1$$

$$\Rightarrow \quad 3x^2 + 7y^2 = 115$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q5(vi)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The coordinates of its vertices and foci are $(\pm a, 0)$ and $(\pm ae, 0)$ respectively.

$$\therefore \quad a = 5 \text{ and } ae = 4 \quad \Rightarrow \quad e = \frac{4}{5}$$

Now, $b^2 = a^2(1 - e^2) \Rightarrow b^2 = 25\left(1 - \frac{16}{25}\right) = 9.$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{25} + \frac{y^2}{9} = 1,$$ which is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(vii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \ldots\ldots\ldots (i)$$

The coordinates of its vertices and foci are $(0, \pm b)$ and $(0, \pm be)$ respectively.

$\therefore \quad b = 13 \qquad [\because \text{ vertices: } (0, \pm 13)]$

$\Rightarrow \quad b^2 = 169$

and $\quad be = 5 \qquad [\because \text{ foci: } (0, \pm 5)]$

$\Rightarrow \quad 13 \times e = 5$

$\Rightarrow \quad e = \frac{5}{13}$

Now, $\quad a^2 = b^2\left(1-e^2\right)$

$\Rightarrow \quad a^2 = (13)^2\left[1-\left(\frac{5}{13}\right)^2\right]$

$\Rightarrow \quad a^2 = 169\left[1-\frac{25}{169}\right]$

$\Rightarrow \quad a^2 = 169\left[\frac{144}{169}\right]$

$\Rightarrow \quad a^2 = 144$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{144}+\frac{y^2}{169}=1$$

This is the required equation of ellipse.

Ellipse Ex 26.1 Q5(viii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ ....(i)}$$

The coordinates of its vertices and foci are $(\pm a, 0)$ and $(\pm ae, 0)$ respectively.

$\therefore \quad a = 6 \qquad [\because \text{ vertices: } (\pm 6, 0)]$

$\Rightarrow \quad a^2 = 36$

and $\quad ae = 4 \qquad [\because \text{ foci: } (\pm 4, 0)]$

$\Rightarrow \quad 6 \times e = 4$

$\Rightarrow \quad e = \frac{4}{6} = \frac{2}{3}$

Now, $\quad b^2 = a^2(1-e^2)$

$\Rightarrow \quad b^2 = 36\left[1-\left(\frac{2}{3}\right)^2\right]$

$= 36 \times \left[1-\frac{4}{9}\right]$

$= 36 \times \frac{5}{9}$

$= 4 \times 5$

$= 20$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{36}+\frac{y^2}{20}=1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(ix)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ ....(i)}$$

The coordinates of its ends of major axis and minor axis are $(\pm a, 0)$ and $(0, \pm b)$ respectively.

$\therefore \quad a = 3 \qquad [\because \text{ Ends of major axis } = (\pm 3, 0)]$

$\Rightarrow \quad a^2 = 9$

and $\quad b = 2 \qquad [\because \text{ Ends of major axis} = (0, \pm 2)]$

$\Rightarrow \quad b^2 = 4$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(x)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ .... (i)}$$

The coordinates of its ends of major axis and minor axis are $(0,\pm b)$ and $(\pm a,0)$ respectively.

$\therefore \quad b=\sqrt{5} \qquad \left[\because \text{ ends of major axis} = \left(0,\pm\sqrt{5}\right)\right]$

$\Rightarrow \quad b^2=5$

and $\quad a=1 \qquad \left[\because \text{ ends of major axis} = (\pm 1, 0)\right]$

$\Rightarrow \quad a^2=1$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{1}+\frac{y^2}{5}=1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(xi)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ .... (i)}$$

we have,
Length of major axis = 26

$\Rightarrow \quad 2a=26$

$\Rightarrow \quad a=\frac{26}{2}=13$

$\Rightarrow \quad a^2=169$

The coordinates of foci are $(\pm ae, 0)$.

$\therefore \quad ae=5$

$\Rightarrow \quad 13\times e=5$

$\Rightarrow \quad e=\frac{5}{13}$

Now, $b^2=a^2\left(1-e^2\right)$

$\Rightarrow \quad b^2=169\left[1-\left(\frac{5}{13}\right)^2\right]$

$\Rightarrow \quad b^2=169\left[1-\frac{25}{169}\right]$

$\Rightarrow \quad b^2=169\left[\frac{144}{169}\right]$

$\Rightarrow \quad b^2=144$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{169}+\frac{y^2}{144}=1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(xii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ ....(i)}$$

we have,
Length of major axis = 16

$\Rightarrow \quad 2a = 16$

$\Rightarrow \quad a=\frac{16}{2}=8$

$\Rightarrow \quad a^2 = 64$

The coordinates of foci are $(0, \pm be)$.

$\therefore \quad be = 6 \qquad [\because \text{foci: } (0, \pm 6)]$

$\Rightarrow \quad (be)^2 = 36$

Now, $a^2 = b^2(1-e^2)$

$\Rightarrow \quad a^2 = b^2 - b^2e^2$

$\Rightarrow \quad 64 = b^2 - 36 \qquad \left[\because (be)^2 = 36 \text{ and } a^2 = 64\right]$

$\Rightarrow \quad 64 + 36 = b^2$

$\Rightarrow \quad b^2 = 100$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{64}+\frac{y^2}{100}=1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(xiii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ ....} (i)$$

we have,

$$a = 4$$

$$\Rightarrow \quad a^2 = 16$$

and, the coordinates of foci are $(\pm 3, 0)$

$$\therefore \quad ae = 3$$

$$\Rightarrow \quad 4 \times e = 3$$

$$\Rightarrow \quad e = \frac{3}{4}$$

Now, $b^2 = a^2\left(1-e^2\right)$

$$= 4^2\left[1-\left(\frac{3}{4}\right)^2\right]$$

$$= 16 \times \left(1-\frac{9}{16}\right)$$

$$= 16 \times \frac{7}{16}$$

$$= 7$$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{16}+\frac{y^2}{7}=1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q6

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \ldots\ldots\ldots(i)$$

The coordinates of foci are $(+ae, 0)$ and $(-ae, 0)$.

$\therefore \quad ae = 4 \qquad [\because \text{focai}: (\pm 4, 0)]$

$\Rightarrow \quad a \times \frac{1}{3} = 4 \qquad \left[\because e = \frac{1}{3}\right]$

$\Rightarrow \quad a = 12$

$\Rightarrow \quad a^2 = 144$

Now, $b^2 = a^2(1-e^2)$

$\Rightarrow \quad b^2 = 144\left[1-\left(\frac{1}{3}\right)^2\right]$

$\Rightarrow \quad b^2 = 144\left[1-\frac{1}{9}\right]$

$\Rightarrow \quad b^2 = 144 \times \frac{8}{9}$

$\Rightarrow \quad b^2 = 16 \times 8 = 128$

Substituting $a^2 = 144$ and $b^2 = 128$ in equation (i), we get

$$= \frac{x^2}{144}+\frac{y^2}{128}=1$$

$$\Rightarrow \quad \frac{1}{16}\left[\frac{x^2}{9}+\frac{y^2}{8}\right]=1$$

$$\Rightarrow \quad \frac{x^2}{9}+\frac{y^2}{8}=16$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q7

The coordinates of foci are $(\pm ae, 0)$.

$\therefore \quad 2ae = 2b \quad$ [given]

$\Rightarrow \quad ae = b$

$\Rightarrow \quad (ae)^2 = b^2 \; \ldots\ldots\ldots (i)$

The length of latus-rectum is 10.

$\Rightarrow \quad \frac{2b^2}{a} = 10 \quad \left[\because \text{latus-rectum} = \frac{2b^2}{a}\right]$

$\Rightarrow \quad b^2 = \frac{10a}{2}$

$\Rightarrow \quad b^2 = 5a \; \ldots\ldots\ldots\ldots\ldots (ii)$

Now,

$b^2 = a^2(1 - e^2)$

$\Rightarrow \quad b^2 = a^2 - a^2e^2$

$\Rightarrow \quad b^2 = a^2 - b^2$

$\Rightarrow \quad 2b^2 = a^2$

$\Rightarrow \quad b^2 = \frac{a^2}{2}$

Substituting $b^2 = \frac{a^2}{2}$ in equation (ii), we get

$\frac{a^2}{2} = 5a$

$\Rightarrow \quad a^2 = 10a$

$\Rightarrow \quad a = 10$

$\Rightarrow \quad a^2 = 100$

Putting $a^2 = 100$ in $b^2 = \frac{a^2}{2}$, we get

$b^2 = \frac{100}{2} = 50$

$\therefore$ The required equation of ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \quad \frac{x^2}{100} + \frac{y^2}{50} = 1$

$\Rightarrow \quad \frac{x^2 + 2y^2}{100} = 1$

$\Rightarrow \quad x^2 + 2y^2 = 100$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q8(i)

Let $2a$ and $2b$ the major and minor axes of the ellipse. Then, its equation is

$$\frac{(x+2)^2}{a^2}+\frac{(y-3)^2}{b^2}=1 \qquad [\because \text{centre}:(-2,3)\ldots\ldots(i)]$$

We have,

semi-major axis = a = 3

$\Rightarrow \quad a^2 = 9$

and semi-major axis = b = 2

$\Rightarrow \quad b^2 = 4$

Putting $a^2 = 9$ and $b^2 = 4$ in equation (i), we get

$$\frac{(x+2)^2}{9}+\frac{(y-3)^2}{4}=1$$

$$\Rightarrow \quad \frac{4(x+2)^2+9(y-3)^2}{36}=1$$

$$\Rightarrow \quad 4(x+2)^2+9(y-3)^2=36$$

$$\Rightarrow \quad 4\left[x^2+4+4x\right]+9\left[y^2+9-6y\right]=36$$

$$\Rightarrow \quad 4x^2+16+16x+9y^2+81-54y=36$$

$$\Rightarrow \quad 4x^2+9y^2+16x-54y+16+81-36=0$$

$$\Rightarrow \quad 4x^2+9y^2+16x-54y+61=0$$

Ellipse Ex 26.1 Q8(ii)

Let $2a$ and $2b$ the minor and major axes of the ellipse. Then, its equation is

$$\frac{(x+2)^2}{a^2} + \frac{(y-3)^2}{b^2} = 1 \qquad [\because \text{centre}: (-2, 3) \ldots\ldots (i)]$$

We have,

semi-major axis $= a = 2$

$\Rightarrow \quad a^2 = 4$

and semi-major axis $= b = 3$

$\Rightarrow \quad b^2 = 9$

Putting $a^2 = 4$ and $b^2 = 9$ in equation (i), we get

$$\frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1$$

$$\Rightarrow \quad \frac{9(x+2)^2 + 4(y-3)^2}{36} = 1$$

$$\Rightarrow \quad 9(x+2)^2 + 4(y-3)^2 = 36$$

$$\Rightarrow \quad 9\left[x^2 + 4 + 4x\right] + 4\left[y^2 + 9 - 6y\right] = 36$$

$$\Rightarrow \quad 9x^2 + 36 + 36x + 4y^2 + 36 - 24y = 36$$

$$\Rightarrow \quad 9x^2 + 4y^2 + 36x - 24y + 36 + 36 - 36 = 0$$

$$\Rightarrow \quad 9x^2 + 4y^2 + 36x - 24y + 36 = 0$$

Ellipse Ex 26.1 Q9(i)

Let $2a$ and $2b$ be the major and minor axes of the ellipse.
(i) when latus-rectum is half of minor axis.

$$\frac{2b^2}{a} = \frac{1}{2} \times 2b$$

$$\Rightarrow \quad 2b^2 = ab$$

$$\Rightarrow \quad \frac{b^2}{b} = \frac{a}{2}$$

$$\Rightarrow \quad b = \frac{a}{2}$$

$$\Rightarrow \quad b^2 = \frac{a^2}{4}$$

Now, $b^2 = a^2\left(1 - e^2\right)$

$$\Rightarrow \quad \frac{a^2}{4} = a^2\left(1 - e^2\right) \qquad \left[\because b^2 = \frac{a^2}{4}\right]$$

$$\Rightarrow \quad \frac{1}{4} = 1 - e^2$$

$$\Rightarrow \quad e^2 = 1 - \frac{1}{4}$$

$$\Rightarrow \quad e^2 = \frac{3}{4}$$

$$\Rightarrow \quad e = \frac{\sqrt{3}}{2}$$

Ellipse Ex 26.1 Q9(ii)

When latus-rectum is half of major-axis.

$$\frac{2b^2}{a} = \frac{1}{2} \times 2a$$

$$\Rightarrow \quad 2b^2 = a^2$$

$$\Rightarrow \quad a^2 = 2b^2$$

Now,

$$\Rightarrow \quad b^2 = a^2\left(1 - e^2\right)$$

$$\Rightarrow \quad b^2 = 2b^2\left(1 - e^2\right) \qquad \left[\because a^2 = 2b^2\right]$$

$$\Rightarrow \quad 1 = 2\left(1 - e^2\right)$$

$$\Rightarrow \quad 1 = 2 - 2e^2$$

$$\Rightarrow \quad 2e^2 = 2 - 1$$

$$\Rightarrow \quad e^2 = \frac{1}{2}$$

$$\Rightarrow \quad e = \frac{1}{\sqrt{2}}$$

Ellipse Ex 26.1 Q10(i)

We have,

$$x^2+2y^2-2x+12y+10=0$$

$$\Rightarrow \quad x^2-2x+2y^2+12y+10=0$$

$$\Rightarrow \quad \left(x^2-2x+1-1\right)+2\left(y^2+6y\right)+10=0$$

$$\Rightarrow \quad \left[(x-1)^2-1\right]+2\left[\left(y^2+2\times y\times 3+9\right)-9\right]+10=0$$

$$\Rightarrow \quad (x-1)^2-1+2\left[(y+3)^2-9\right]+10=0$$

$$\Rightarrow \quad (x-1)^2+2(y+3)^2-18-1+10=0$$

$$\Rightarrow \quad (x-1)^2+2(y+3)^2-19+10=0$$

$$\Rightarrow \quad (x-1)^2+2(y+3)^2-9=0$$

$$\Rightarrow \quad (x-1)^2+2(y+3)^2=9$$

$$\Rightarrow \quad \frac{(x-1)^2}{9}+2\frac{(y+3)^2}{9}=1$$

$$\Rightarrow \quad \frac{(x-1)^2}{9}+\frac{(y+3)^2}{\frac{9}{2}}=1$$

$$\Rightarrow \quad \frac{(x-1)^2}{(3)^2}+\frac{(y+3)^2}{\left(\frac{3}{\sqrt{2}}\right)^2}=1 \quad \ldots\ldots\ldots\ldots(i)$$

$\therefore$ The coordinates of centre of the ellipse are $(1,-3)$.

Shifting the origin at $(1,-3)$ without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y, we have

$$x=X+1 \quad \text{and} \quad y=Y-3 \ldots\ldots\ldots\ldots(ii)$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{3^2}+\frac{Y^2}{\left(\frac{3}{\sqrt{2}}\right)^2}=1 \quad \ldots\ldots\ldots\ldots(iii)$$

This is of the form

$$\frac{X^2}{a^2}+\frac{Y^2}{b^2}=1, \text{ where } a=3,$$

and $b=\frac{3}{\sqrt{2}}$.

Clearly, a>b. so, the given equation represents an ellipse whose major and minor axes are along X and Y axes respectively.

Length of the axes:

$$\text{major-axis} = 2a = 2\times3 = 6$$

and, $\text{minor-axis} = 2b = \frac{2\times3}{\sqrt{2}} = 3\sqrt{2}$

Eccentricity: The eccentricity e is given by

$$e=\sqrt{1-\frac{b^2}{a^2}}$$

$$=\sqrt{1-\frac{\left(\frac{3}{\sqrt{2}}\right)^2}{3^2}}$$

$$=\sqrt{1-\frac{9}{2\times9}}$$

$$=\sqrt{1-\frac{1}{2}}$$

$$=\sqrt{\frac{1}{2}}$$

$$=\frac{1}{\sqrt{2}}$$

Foci: The coordinates of the foci with respect to the new axes are given by $(X=\pm ae, Y=0)$ *i.e.*,

$$\left(X=\pm\frac{3}{\sqrt{2}}, Y=0\right)$$

Putting $X=\pm\frac{3}{\sqrt{2}}$ and $Y=0$ in equation (ii), we get

$$x=\pm\frac{3}{\sqrt{2}}+1 \quad \text{and} \quad y=0-3$$

$$\Rightarrow \quad x=1\pm\frac{3}{\sqrt{2}} \quad \text{and} \quad y=-3$$

Ellipse Ex 26.1 Q10(ii)

We have,

$$x^2 + 4y^2 - 4x + 24y + 31 = 0$$

$$\Rightarrow \quad x^2 - 4x + 4\{y^2 + 6y\} + 31 = 0$$

$$\Rightarrow \quad \left[x^2 - 2 \times x \times 2 + 2^2 - 2^2\right] + 4\left[y^2 + 2 \times 3 \times y + 3^2 - 3^2\right] + 31 = 0$$

$$\Rightarrow \quad \left[(x-2)^2 - 2^2\right] + 4\left[(y+3)^2 - 9\right] + 31 = 0$$

$$\Rightarrow \quad (x-2)^2 - 4 + 4(y+3)^2 - 36 + 31 = 0$$

$$\Rightarrow \quad (x-2)^2 + 4(y+3)^2 - 5 - 4 = 0$$

$$\Rightarrow \quad (x-2)^2 + 4(y+3)^2 = 9$$

$$\Rightarrow \quad \frac{(x-2)^2}{9} + \frac{4(y+3)^2}{9} = 1$$

$$\Rightarrow \quad \frac{(x-2)^2}{9} + \frac{(y+3)^2}{\frac{9}{4}} = 1$$

$$\Rightarrow \quad \frac{(x-2)^2}{3^2} + \frac{(y+3)^2}{\left(\frac{3}{2}\right)^2} = 1 \text{ ....(i)}$$

$\therefore$ The coordinates of centre of the ellipse are $(2,-3)$.

Shifting the origin at $(2,-3)$ without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y, we have

$$x = X + 2 \quad \text{and} \quad y = Y - 3 \text{ ....(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{3^2} + \frac{Y^2}{\left(\frac{3}{2}\right)^2} = 1 \text{ ....(iii)}$$

This is of the form

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1, \text{ where}$$

$$a = 3 \quad \text{and} \quad b = \frac{3}{2}$$

Clearly, $a > b$. so, the given equation represents on ellipse whose major and minor axes are along X and Y axes respectively.

Length of the axes:

Length of the axes:

Major-axis = $2a = 2 \times 3 = 6$

and, Minor-axis = $2b = 2 \times \frac{3}{2} = 3$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{\frac{9}{4}}{9}}$$

$$= \sqrt{1 - \frac{9}{4 \times 9}}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{1 - \frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}.$$

Foci: The coordinates of the foci with respect to the new axes are given by $\left(X = \pm ae,\ Y = 0\right)$

i.e., $\left(X = \pm \frac{3\sqrt{3}}{2}, Y = 0\right)$

Putting $X = \pm \frac{3\sqrt{3}}{2}$ and $Y = 0$ in equation (ii), we get

$x = +2$ and $y = 0 - 3$

$\Rightarrow \quad x = 2 \pm \frac{3\sqrt{3}}{2}$ and $y = -3$.

so, the coordinates of foci with respect to old axes are given by $\left(2 \pm \frac{3\sqrt{3}}{2}, -3\right)$

Ellipse Ex 26.1 Q10(iii)

We have,

$$4x^2 + y^2 - 8x + 2y + 1 = 0$$

$$\Rightarrow \quad 4(x^2 - 2x) + (y^2 + 2y) + 1 = 0$$

$$\Rightarrow \quad 4[(x^2 - 2x + 1) - 1] + [(y^2 + 2y + 1) - 1] + 1 = 0$$

$$\Rightarrow \quad 4[(x-1)^2 - 1] + [(y+1)^2 - 1] + 1 = 0$$

$$\Rightarrow \quad 4(x-1)^2 - 4 + (y+1)^2 - 1 + 1 = 0$$

$$\Rightarrow \quad 4(x-1)^2 + (y+1)^2 - 4 = 0$$

$$\Rightarrow \quad 4(x-1)^2 + (y+1)^2 = 4$$

$$\Rightarrow \quad \frac{(x-1)^2}{1} + \frac{(y+1)^2}{4} = 1$$

$$\Rightarrow \quad \frac{(x-1)^2}{1^2} + \frac{(y+1)^2}{2^2} = 1 \text{ .... (i)}$$

$\therefore$ The coordinates of centre of the ellipse are $(1, -1)$.

Shifting the origin at $(1, -1)$ without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y, we have

$$x = X + 1 \quad \text{and} \quad y = Y - 1 \text{ .... (ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{1^2} + \frac{Y^2}{2^2} = 1$$

This is of the form

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1, \text{ where}$$

$$a = 1 \quad \text{and} \quad b = 2$$

Clearly, $b > a$. so, the given equation represents on ellipse whose major and minor axes are along Y and X axes respectively.

Length of the axes:

Major-axis = $2b = 2 \times 2 = 4$

Minor-axis = $2a = 2 \times 1 = 2$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 - \frac{a^2}{b^2}}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{\frac{4-1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}.$$

Foci: The coordinates of the foci with respect to the new axes are given by $(X = 0, Y = \pm be)$

i.e., $(X = 0, \ Y = \pm\sqrt{3})$

Putting $X = 0$ and $Y = \pm\sqrt{3}$ in equation (iii), we get

$$x = 0 + 1 \text{ and } y = \pm\sqrt{3} - 1$$

$\Rightarrow \quad x = 1$ and $y = -1 \pm \sqrt{3}$

Ellipse Ex 26.1 Q10(iv)

We have,

$$3x^2 + 4y^2 - 12x - 8y + 4 = 0$$

$$\Rightarrow \quad 3x^2 - 12x + 4y^2 - 8y + 4 = 0$$

$$\Rightarrow \quad 3\left(x^2 - 4x\right) + 4\left(y^2 - 2y\right) + 4 = 0$$

$$\Rightarrow \quad 3\left[x^2 - 2 \times x \times 2 + 2^2 - 2^2\right] + 4\left[y^2 - 2 \times y \times 1^2 - 1^2\right] + 4 = 0$$

$$\Rightarrow \quad 3\left[(x-2)^2 - 4\right] + 4\left[(y+1)^2 - 1\right] + 4 = 0$$

$$\Rightarrow \quad 3(x-2)^2 - 12 + 4(y+1)^2 - 4 + 4 = 0$$

$$\Rightarrow \quad 3(x-2)^2 + 4(y-1)^2 - 12 = 0$$

$$\Rightarrow \quad 3(x-2)^2 + 4(y-1)^2 = 12$$

$$\Rightarrow \quad \frac{3(x-2)^2}{12} + \frac{4(y-1)^2}{12} = 1$$

$$\Rightarrow \quad \frac{(x-2)^2}{4} + \frac{(y-1)^2}{3} = 1$$

$$\Rightarrow \quad \frac{(x-2)^2}{2^2} + \frac{(y-1)^2}{\left(\sqrt{3}\right)^2} = 1 \ldots\ldots\ldots\ldots\text{(i)}$$

$\therefore$ The coordinates of centre of the ellipse are $(2, 1)$.

Shifting the origin at $(2, 1)$ without rotating the coordinate axes and denoting the new coordinates w.r.t the new axes by. X and Y, we have

$$x = X + 2 \qquad \text{and} \quad y = Y - 1 \ldots\ldots\ldots\ldots\text{(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{2^2}+\frac{Y^2}{\left(\sqrt{3}\right)^2}=1, \text{ where}$$

$$a=2 \quad \text{and} \quad b=\sqrt{3}$$

Clearly, $a>b$. so, the given equation represents on ellipse whose major and minor axes are along Y and X axes respectively.

Length of the axes:

Major-axis $=2a=2\times 2=4$

and, Minor-axis $=2b=2\times\sqrt{3}=2\sqrt{3}$

Eccentricity: The eccentricity e is given by

$$e=\sqrt{1-\frac{b^2}{a^2}}$$

$$=\sqrt{1-\frac{3}{4}}$$

$$=\sqrt{\frac{1}{4}}$$

$$=\frac{1}{2}$$

Foci: The coordinates of the foci w.r.t the new axes are given by $\left(X=\pm ae,\ Y=0\right)$
i.e., $\left(X=\pm 1,\ Y=0\right)$

Putting $X=\pm 1$ and $Y=0$ in equation (ii), we get

$$x=\pm 1+2 \text{ and } y=0+1$$

$$\Rightarrow \quad x=2\pm 1 \text{ and } y=1$$

so, the coordinates of foci with respect to old axes are given by $(2\pm 1,1)$ *i.e.*, $(3,1)$ and $(1,1)$.

Ellipse Ex 26.1 Q10(v)

We have,

$$4x^2+16y^2-24x-32y-12=0$$

$$\Rightarrow \quad 4x^2-24x+16y^2-32y-12=0$$

$$\Rightarrow \quad 4\left(x^2-6x\right)+16\left(y^2-2y\right)-12=0$$

$$\Rightarrow \quad 4\left[x^2-2\times x\times 3+3^2-3^2\right]+16\left[y^2-2y+1^2-1^2\right]-12=0$$

$$\Rightarrow \quad 4\left[(x-3)^2-9\right]+16\left[(y-1)^2-1\right]-12=0$$

$$\Rightarrow \quad 4(x-3)^2-36+16(y-1)^2-16-12=0$$

$$\Rightarrow \quad 4(x-3)^2+16(y-1)^2-36-28=0$$

$$\Rightarrow \quad 4(x-3)^2+16(y-1)^2-64=0$$

$$\Rightarrow \quad 4(x-3)^2+16(y-1)^2=64$$

$$\Rightarrow \quad \frac{4(x-3)^2}{64}+\frac{16(y-1)^2}{64}=1$$

$$\Rightarrow \quad \frac{(x-3)^2}{16}+\frac{(y-1)^2}{4}=1$$

$$\Rightarrow \quad \frac{(x-3)^2}{(4)^2}+\frac{(y-1)^2}{(2)^2}=1 \text{ ....(i)}$$

$\therefore$ The coordinates of centre of the ellipse are $(3,1)$.

Shifting the origin at $(3,1)$ without rotating the coordinate axes and denoting the new coordinates w.r.t the new axes by. X and Y, we have

$$x=X+3 \quad \text{and} \quad y=Y+1 \text{ ....(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{4^2}+\frac{Y^2}{2^2}=1, \text{ where}$$

$a = 4$ and $b = 2$

Clearly, $a > b$. so, the given equation represents on ellipse whose major and minor axes are along Y and X axes respectively.

Length of the axes:

Major-axis $= 2a = 2 \times 4 = 8$

and, Minor-axis $= 2b = 2 \times 2 = 4$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1-\frac{b^2}{a^2}}$$

$$= \sqrt{1-\frac{4}{16}}$$

$$= \sqrt{1-\frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

Foci: The coordinates of the foci w.r.t the new axes are given by $(X = \pm ae,\ Y = 0)$

i.e., $(X = \pm 2\sqrt{3},\quad Y = 0)$

Putting $X = \pm 2\sqrt{3}$ and $Y = 0$ in equation (ii), we get

$$x = \pm 2\sqrt{3}+3 \text{ and } y = 0+1$$

$$\Rightarrow \quad x = 3 \pm 2\sqrt{3} \text{ and } y = 1$$

Ellipse Ex 26.1 Q10(vi)

We have,

$$x^2+4y^2-2x=0$$

$$\Rightarrow \quad x^2-2x+4y^2=0$$

$$\Rightarrow \quad \left(x^2-2x+1^2-1^2\right)+4y^2=0$$

$$\Rightarrow \quad (x-1)^2-1+4y^2=0$$

$$\Rightarrow \quad (x-1)^2+4y^2=1$$

$$\Rightarrow \quad \frac{(x-1)^2}{1}+\frac{y^2}{\frac{1}{4}}=1$$

$$\Rightarrow \quad \frac{(x-1)^2}{1^2}+\frac{y^2}{\left(\frac{1}{2}\right)^2}=1 \text{ ....(i)}$$

$\therefore$ The coordinates of centre of the ellipse are $(1,0)$.

Shifting the origin at $(1,0)$ without rotating the coordinate axes and denoting the new coordinates w.r.t the new axes by. X and Y, we have

$$x=X+1 \quad \text{and} \quad y=Y \text{ ....(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{1^2}+\frac{Y^2}{\left(\frac{1}{2}\right)^2}=1, \text{ where}$$

$$a=1 \quad \text{and} \quad b=\frac{1}{2}$$

Clearly, $a>b$. so, the given equation represents on ellipse whose major and minor axes are along X and Y axes respectively.

Length of the axes:

Major-axis $= 2 \times a = 2 \times 1 = 2$

and, Minor-axis $= 2 \times b = 2 \times \frac{1}{2} = 1$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{\frac{1}{4}}{1}}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

Foci: The coordinates of the foci w.r.t the new axes are given by $(X = \pm ae,\ Y = 0)$

i.e., $\left(X = \pm\frac{\sqrt{3}}{2},\quad Y = 0\right)$

Putting $X = \pm\frac{\sqrt{3}}{2}$ and $Y = 0$ in equation (ii), we get

$$x = \pm\frac{\sqrt{3}}{2} + 1 \text{ and } y = 0$$

$$\Rightarrow \quad x = 1 \pm \frac{\sqrt{3}}{2} \text{ and } y = 0.$$

so, the coordinates of foci w.r.t the old axes are given by $\left(1 \pm \frac{\sqrt{3}}{2}, 0\right)$.

Ellipse Ex 26.1 Q11

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ ....(i)}$$

The coordinates of its foci are $(\pm ae, 0)$ *i.e.*, $(\pm 3, 0)$.

$$\therefore \quad ae = 3$$

$$\Rightarrow \quad (ae)^2 = 9 \text{ ....(i)}$$

The required ellipse passes through $(4, 1)$.

$$\therefore \quad \frac{(4)^2}{a^2}+\frac{(1)^2}{b^2}=1$$

$$\Rightarrow \quad \frac{16}{a^2}+\frac{1}{b^2}=1$$

$$\Rightarrow \quad 16b^2+a^2=a^2b^2$$

$$\Rightarrow \quad a^2+16b^2=a^2b^2 \text{ ....(ii)}$$

Now,

$$b^2=a^2\left(1-e^2\right)$$

$$\Rightarrow \quad b^2=a^2-a^2e^2$$

$$\Rightarrow \quad b^2=a^2-9 \qquad \text{[Using equation (i)] ....(iii)}$$

Substituting $b^2=a^2-9$ in equation (ii), we get

$$a^2+16\left(a^2-9\right)=a^2\left(a^2-9\right)$$

$$\Rightarrow \quad a^2+16a^2-144=a^4-9a^2$$

$$\Rightarrow \quad 17a^2-144=a^4-9a^2$$

$$\Rightarrow \quad a^4-9a^2-17a^2+144=0$$

$$\Rightarrow \quad a^4-26a^2+144=0$$

$$\Rightarrow \quad a^4-18a^2-8a^2+144=0$$

$$\Rightarrow \quad a^2\left(a^2-18\right)-8\left(a^2-18\right)=0$$

$$\Rightarrow \quad \left(a^2-18\right)\left(a^2-8\right)=0$$

$$\Rightarrow \quad a^2=18 \qquad \text{or,} \qquad a^2\neq 8$$

$$\Rightarrow \quad a^2=18$$

Putting $a^2=18$ in equation (iii), we get

$$b^2=18-9=9$$

$\therefore$ The required equation of the ellipse is

$$\frac{x^2}{18}+\frac{y^2}{9}=1$$

Ellipse Ex 26.1 Q12

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \ldots\ldots\ldots\ldots (i)$$

The length of latus-rectum = 5

$$\therefore \quad \frac{2b^2}{a} = 5$$

$$\Rightarrow \quad b^2 = \frac{\sqrt{9}}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Now,

$$b^2 = a^2\left(1 - e^2\right)$$

$$\Rightarrow \quad \frac{5a}{2} = a^2\left[1 - \left(\frac{2}{3}\right)^2\right] \qquad \left[\because e = \frac{2}{3}\right]$$

$$\Rightarrow \quad \frac{5a}{2} = a^2\left[1 - \frac{4}{9}\right]$$

$$\Rightarrow \quad \frac{5}{2} = a\left(\frac{5}{9}\right)$$

$$\Rightarrow \quad \frac{5}{2} \times \frac{9}{5} = a$$

$$\Rightarrow \quad a = \frac{9}{2}$$

$$\Rightarrow \quad a^2 = \frac{81}{4}$$

Putting $a = \frac{9}{2}$ in $b^2 = \frac{5a}{2}$, we get

$$b^2 = \frac{5}{2} \times \frac{9}{2}$$

$$\Rightarrow \quad b^2 = \frac{45}{4}$$

Substituting $a^2 = \frac{81}{4}$ and $b^2 = \frac{45}{4}$ in equation(i), we get

$$\frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$$

$$\Rightarrow \quad \frac{4x^2}{81} + \frac{4y^2}{45} = 1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q13

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ \text{where } a<b \ \ldots\ldots\ldots\text{(i)} \qquad [\text{foci on y-axis}]$$

Now,

$$a^2=b^2\left(1-e^2\right)$$

$$\Rightarrow \quad a^2=b^2\left[1-\left(\frac{3}{4}\right)^2\right]$$

$$\Rightarrow \quad a^2=b^2\left[1-\frac{9}{16}\right]$$

$$\Rightarrow \quad a^2=b^2\times\frac{7}{16}$$

$$\Rightarrow \quad a^2=\frac{7}{16}b^2 \ \ldots\ldots\ldots\text{(ii)}$$

The required ellipse through $(6,4)$.

$$\therefore \quad \frac{(6)^2}{a^2}+\frac{(4)^2}{b^2}=1$$

$$\Rightarrow \quad \frac{36}{a^2}+\frac{16}{b^2}=1$$

$$\Rightarrow \quad \frac{36}{\frac{7}{16}b^2}+\frac{16}{b^2}=1 \qquad \left[\because a^2=\frac{7}{16}b^2\right]$$

$$\Rightarrow \quad \frac{36\times16}{7b^2}+\frac{16}{b^2}=1$$

$$\Rightarrow \quad \frac{576}{7b^2}+\frac{16}{b^2}=1$$

$$\Rightarrow \quad \frac{36}{\frac{7}{16}b^2} + \frac{16}{b^2} = 1 \qquad \left[\because a^2 = \frac{7}{16}b^2\right]$$

$$\Rightarrow \quad \frac{36 \times 16}{7b^2} + \frac{16}{b^2} = 1$$

$$\Rightarrow \quad \frac{576}{7b^2} + \frac{16}{b^2} = 1$$

$$\Rightarrow \quad \frac{1}{b^2}\left[\frac{576}{7} + \frac{16}{1}\right] = 1$$

$$\Rightarrow \quad \frac{576}{7} + \frac{16}{1} = b^2$$

$$\Rightarrow \quad \frac{576 + 112}{7} = b^2$$

$$\Rightarrow \quad b^2 = \frac{688}{7}.$$

Putting $b^2 = \frac{688}{7}$ in equation (i), we get

$$a^2 = \frac{7}{16} \times \frac{688}{7}$$

$$\Rightarrow \quad a^2 = \frac{688}{16} = 43$$

Putting $a^2 = 43$ and $b^2 = \frac{688}{7}$ in equation (i), we get

$$\frac{x^2}{43} + \frac{y^2}{\frac{688}{7}} = 1$$

$$\Rightarrow \quad \frac{x^2}{43} + \frac{7y^2}{688} = 1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q14

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ \text{ where } a>b \ \ldots\ldots\ldots\ldots(i)$$

The required ellipse passes through $(4,3)$ and $(-1,4)$.

$$\therefore \quad \frac{(4)^2}{a^2}+\frac{(3)^2}{b^2}=1$$

$$\Rightarrow \quad \frac{16}{a^2}+\frac{9}{b^2}=1$$

$$\Rightarrow \quad 16b^2+9a^2=a^2b^2 \ \ldots\ldots\ldots\ldots\ldots(ii)$$

and $\dfrac{(-1)^2}{a^2}+\dfrac{(4)^2}{b^2}=1$

$$\Rightarrow \quad \frac{1}{a^2}\times\frac{16}{b^2}=1$$

$$\Rightarrow \quad b^2+16a^2=a^2b^2 \ldots\ldots\ldots\ldots(iii)$$

Multiplying equation (iii) by 16, we get

$$16b^2+256a^2=16a^2b^2 \ \ldots\ldots\ldots\ldots(iv)$$

Substracting equation (ii) from equation (iv), we get

$$256a^2-9a^2=16a^2b^2-a^2b^2$$

$$\Rightarrow \quad 247a^2=15a^2b^2$$

$$\Rightarrow \quad \frac{247}{15}=b^2$$

$$\Rightarrow \quad b^2=\frac{247}{15}$$

Putting $b^2=\dfrac{247}{15}$ in equation (iii) we get

$$\frac{247}{15}+16a^2=a^2\times\frac{247}{15}$$

$$\Rightarrow \quad 16a^2-\frac{247a^2}{15}=\frac{-247}{15}$$

$$\Rightarrow \quad \frac{240a^2-247a^2}{15}=\frac{-247}{15}$$

$$\Rightarrow \quad -7a^2=-247$$

$$\Rightarrow \quad a^2=\frac{247}{7}$$

Putting $a^2=\dfrac{247}{7}$ and $b^2=\dfrac{247}{15}$ in equation equation (i), we get

$$\frac{x^2}{\frac{247}{7}}+\frac{y^2}{\frac{247}{15}}=1$$

$$\Rightarrow \quad \frac{7x^2}{247}+\frac{15y^2}{247}=1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q15

Let the equation of the ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ \text{where } a>b \qquad [\because \text{ axes lie along the coordinates axes}]$$

Now,

$$b^2=a^2\left(1-e^2\right)$$

$$\Rightarrow \quad b^2=a^2\left[1-\left(\sqrt{\frac{2}{5}}\right)^2\right] \qquad \left[\because e=\sqrt{\frac{2}{5}}\right]$$

$$\Rightarrow \quad b^2=a^2\left[1-\frac{2}{5}\right]$$

$$b^2=a^2\times\frac{3}{5}$$

$$\Rightarrow \quad b^2=\frac{3a^2}{5} \quad \ldots\ldots\ldots(ii)$$

The required ellipse passes through $(-3,1)$

$$\therefore \quad \frac{(-3)^2}{a^2}+\frac{1^2}{b^2}=1$$

$$\Rightarrow \quad \frac{9}{a^2}+\frac{1}{b^2}=1 \quad \ldots\ldots\ldots\ldots(ii)$$

Putting $b^2=\frac{3a^2}{5}$ in equation(ii), we get

$$\frac{9}{a^2}+\frac{1}{\frac{3a^2}{5}}=1$$

$$\Rightarrow \quad \frac{9}{a^2}+\frac{5}{3a^2}=1$$

$$\Rightarrow \quad \frac{1}{a^2}\left[\frac{9}{1}+\frac{5}{3}\right]=1$$

$$\Rightarrow \quad \frac{27+5}{3}=a^2$$

Putting $a^2=\frac{32}{3}$ in equation(ii), we get

$$b^2=\frac{3}{5}\times\frac{32}{3}=\frac{32}{5}$$

Substituting $a^2=\frac{32}{3}$ and $b^2=\frac{32}{5}$ in equation(i), we get

$$\frac{x^2}{\frac{32}{3}}+\frac{y^2}{\frac{32}{5}}=1$$

$$\Rightarrow \quad \frac{3x^2}{32}+\frac{5y^2}{32}=1$$

$$\Rightarrow \quad 3x^2+5y^2=32$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q16

Let the equation of the ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \ldots\ldots\ldots\ldots (i)$$

We have,

$$2ae = 8 \qquad \text{[given]}$$

$$\Rightarrow \quad e = \frac{8}{2a}$$

$$\Rightarrow \quad e = \frac{4}{a} \ldots\ldots\ldots\ldots (ii)$$

Now,

$$\frac{2a}{e} = 18 \qquad \text{[given]}$$

$$\Rightarrow \quad a = \frac{18e}{2}$$

$$\Rightarrow \quad a = 9e \ldots\ldots\ldots\ldots (iii)$$

Using equation (ii) and equation (iii), we get

$$a = \frac{9 \times 4}{a}$$

$$\Rightarrow \quad a^2 = 36$$

Now,

$$b^2 = a^2\left(1-e^2\right)$$

$$\Rightarrow \quad b^2 = 36-(ae)^2$$

$$\Rightarrow \quad b^2 = 36-16 \qquad \text{[Using equation (iii)]}$$

$$\Rightarrow \quad b^2 = 20$$

Putting $a^2 = 36$ and $b^2 = 20$ in equation (i), we get

$$\frac{x^2}{36}+\frac{y^2}{20}=1$$

This is the equation of the required ellipsis.

Ellipse Ex 26.1 Q17

Let the equation of the ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \ldots\ldots\ldots\ldots (i)$$

The coordinates of vertices are $(0, \pm b)$ *i.e.*, $(0, \pm 10)$.

$\therefore \quad b = 10$

$\Rightarrow \quad b^2 = 100$

Now,

$$a^2 = b^2\left(1 - e^2\right)$$

$$\Rightarrow \quad a^2 = 100\left[1 - \left(\frac{4}{5}\right)^2\right]$$

$$\Rightarrow \quad a^2 = 100\left[1 - \frac{16}{25}\right]$$

$$\Rightarrow \quad a^2 = 100\left[\frac{9}{25}\right]$$

$$\Rightarrow \quad a^2 = 4 \times 9 = 36$$

Putting $a^2 = 36$ and $b^2 = 100$ in equation(i), we get

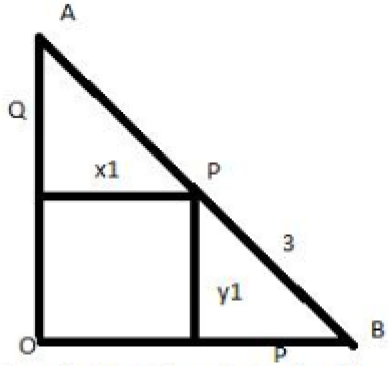
$$\frac{x^2}{36}+\frac{y^2}{100}=1$$

$$\Rightarrow \quad \frac{100x^2 + 36y^2}{3600} = 1$$

$$\Rightarrow \quad 100x^2 + 36y^2 = 3600$$

This is the equation of the required ellipsis.

Ellipse Ex 26.1 Q18



Using similar triangles principle, we can write

$$\frac{Q}{9}=\frac{y_1}{3}$$

$$Q=3y_1$$

$$Similarly,\ p=\frac{x}{3}$$

Point P(x,y)

So OB=$x+\frac{x}{3}$

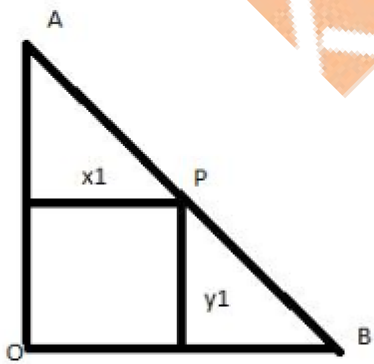
$$OA=y+3y=4y$$

using pythageorus theorem, we get

$$(4y)^2+\left(\frac{4x}{3}\right)^2=12^2$$

$\frac{y^2}{9}+\frac{x^2}{81}=1$ is the equation of ellipse

Ellipse Ex 26.1 Q19



From above figure,

Assume length AB=l

$AP=a, PB=b$

Assume $\widehat{ABO}=\theta$

so $x_1=a\cos\theta,\ y_1=b\sin\theta$

$$\Rightarrow\left(\frac{x_1}{a}\right)^2+\left(\frac{y_1}{b}\right)^2=1$$

Ellipse Ex 26.1 Q20

Let point be (x,y)

Given distances of point from (0, 4) are

2/3 of their distances from the line y = 9

$$\sqrt{(x-0)^2+(y-4)^2}=\frac{2}{3}\left(\sqrt{(y-9)^2}\right)$$

Squaring on both sides, we get

$$9\left[(x-0)^2+(y-4)^2\right]=4\left[(y-9)^2\right]$$

$$9x^2+9y^2+144-72y=4y^2+324-72y$$

$$9x^2+5y^2=180$$