# RD Sharma Solutions Class 11 Maths Chapter 25 Ex 25.1

**Parabola Ex 25.1 Q1(i)**

Let $P(x, y)$ be any point on the parabola whose focus is $S(3,0)$ and the directrix $3x + 4y = 1$. Draw PM perpendicular from $P(x, y)$ on the directrix $3x + 4y = 1$.

Then by definition

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x-3)^2 + (y-0)^2 = \left(\frac{3x + 4y - 1}{\sqrt{(3)^2 + (4)^2}}\right)^2$$

$$\Rightarrow \quad x^2 + 9 - 6x + y^2 = \left(\frac{3x + 4y - 1}{\sqrt{9 + 16}}\right)^2$$

$$\Rightarrow \quad x^2 - 6x + 9 + y^2 = \frac{(3x + 4y - 1)^2}{(\sqrt{25})^2}$$

$$\Rightarrow \quad x^2 - 6x + y^2 + 9 = \frac{(3x + 4y - 1)^2}{25}$$

$$\Rightarrow \quad 25(x^2 - 6x + y^2 + 9) = (3x + 4y - 1)^2$$

$$\Rightarrow \quad 25x^2 - 150x + 25y^2 + 225 = (3x)^2 + (4y)^2 + (-1)^2 + 2 \times 3x \times 4y + 2 \times 4y \times (-1) + 2 \times (-1) \times 3x$$

$$\Rightarrow \quad 25x^2 - 150x + 25y^2 + 225 = 9x^2 + 16y^2 + 1 + 24xy - 8y - 6x$$

$$\Rightarrow \quad 25x^2 - 9x^2 + 25y^2 - 16y^2 - 150x + 6x + 8y - 24xy + 225 - 1 = 0$$

$$\Rightarrow \quad 16x^2 + 9y^2 - 144x + 8y - 24xy + 224 = 0$$

$$\Rightarrow \quad 16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$$

This is the equation of the required parabola.

**Parabola Ex 25.1 Q1(ii)**

Let $P(x, y)$ be any point on the parabola whose focus is $S(1,1)$ and the directrix $x + y + 1 = 0$. Draw PM perpendicular from $P(x, y)$ on the directrix $x + y + 1 = 0$.
Then by definition

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x-1)^2 + (y-1)^2 = \left(\frac{x + y + 1}{\sqrt{1^2 + 1^2}}\right)^2$$

$$\Rightarrow \quad x^2 + 1 - 2x + y^2 + 1 - 2y = \left(\frac{x + y + 1}{\sqrt{2}}\right)^2$$

$$\Rightarrow \quad x^2 + y^2 - 2x - 2y + 2 = \frac{(x + y + 1)^2}{2}$$

$$\Rightarrow \quad 2(x^2 + y^2 - 2x - 2y + 2) = x^2 + y^2 + 1 + 2xy + 2y + 2x$$

$$\Rightarrow \quad 2x^2 + 2y^2 - 4x - 4y + 4 = x^2 + y^2 + 1 + 2xy + 2y + 2x$$

$$\Rightarrow \quad 2x^2 - x^2 + 2y^2 - y^2 - 2xy - 4x - 2x - 4y - 2y + 4 - 1 = 0$$

$$\Rightarrow \quad x^2 + y^2 - 2xy - 6x - 6y + 3 = 0$$

This is the equation of the required parabola.

**Parabola Ex 25.1 Q1(iii)**

Let $P(x, y)$ be any point on the parabola whose focus is $S(0, 0)$ and the directrix $2x - y - 1 = 0$. Draw PM perpendicular from $P(x, y)$ on the directrix $2x - y - 1 = 0$.
Then by definition

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x-0)^2 + (y-0)^2 = \left(\frac{2x - y - 1}{\sqrt{(2)^2 + (-1)^2}}\right)^2$$

$$\Rightarrow \quad x^2 + y^2 = \frac{(2x - y - 1)^2}{(\sqrt{5})^2}$$

$$\Rightarrow \quad 5(x^2 + y^2) = (2x - y - 1)^2$$

$$\Rightarrow \quad 5x^2 + 5y^2 = (2x)^2 + (-y)^2 + (-1)^2 + 2 \times 2x \times (-y) + 2 \times (-y) \times (-1) + 2 \times (-1) \times 2x$$

$$\Rightarrow \quad 5x^2 + 5y^2 = 4x^2 + y^2 + 1 - 4xy + 2y - 4x$$

$$\Rightarrow \quad 5x^2 - 4x^2 + 5y^2 - y^2 + 4xy + 4x - 2y - 1 = 0$$

$$\Rightarrow \quad x^2 + 4y^2 + 4xy + 4x - 2y - 1 = 0$$

This is the equation of the required parabola.

**Parabola Ex 25.1 Q1(iv)**

Let $P(x, y)$ be any point on the parabola whose focus is $S(2, 3)$ and the directrix $x - 4y + 3 = 0$. Draw PM perpendicular from $P(x, y)$ on the directrix $x - 4y + 3 = 0$.

Then by definition

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x-2)^2 + (y-3)^2 = \left(\frac{x - 4y + 3}{\sqrt{1^2 + (-4)^2}}\right)^2$$

$$\Rightarrow \quad x^2 + 4 - 4x + y^2 + 9 - 6y = \frac{(x - 4y + 3)^2}{\left(\sqrt{17}\right)^2}$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 6y + 4 + 9 = \frac{(x - 4y + 3)^2}{17}$$

$$\Rightarrow \quad 17\left(x^2 + y^2 - 4x - 6y + 13\right) = (x - 4y + 3)^2$$

$$\Rightarrow \quad 17x^2 + 17y^2 - 68x - 102y + 221 = x^2 + (-4y)^2 + 3^2 + 2 \times x \times (-4y) + 2 \times (-4y) \times 3 + 2 \times 3 \times x$$

$$\Rightarrow \quad 17x^2 + 17y^2 - 68x - 102y + 221 = x^2 + 16y^2 + 9 - 8xy - 24y + 6x$$

$$\Rightarrow \quad 17x^2 - x^2 + 17y^2 - 16y^2 + 8xy - 68x - 6x - 102y + 24y + 221 - 9 = 0$$

$$\Rightarrow \quad 16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$$

This is the equation of the required parabola.

**Parabola Ex 25.1 Q2**

Let $P(x,y)$ be any point on the parabola whose focus is $S(2,3)$ and the directrix $x-4y+3=0$. Draw PM perpendicular from $P(x,y)$ on the directrix $x-4y+3=0$.

Then by definition

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x-2)^2+(y-3)^2 = \left(\frac{x-4y+3}{\sqrt{1^2+(-4)^2}}\right)^2$$

$$\Rightarrow \quad x^2+4-4x+y^2+9-6y = \frac{(x-4y+3)^2}{\left(\sqrt{17}\right)^2}$$

$$\Rightarrow \quad x^2+y^2-4x-6y+4+9 = \frac{(x-4y+3)^2}{17}$$

$$\Rightarrow \quad 17\left(x^2+y^2-4x-6y+13\right) = (x-4y+3)^2$$

$$\Rightarrow \quad 17x^2+17y^2-68x-102y+221 = x^2+(-4y)^2+3^2+2\times x\times(-4y)+2\times(-4y)\times 3+2\times 3\times x$$

$$\Rightarrow \quad 17x^2+17y^2-68x-102y+221 = x^2+16y^2+9-8xy-24y+6x$$

$$\Rightarrow \quad 17x^2-x^2+17y^2-16y^2+8xy-68x-6x-102y+24y+221-9=0$$

$$\Rightarrow \quad 16x^2+y^2+8xy-74x-78y+212=0$$

This is the equation of the required parabola.

*Latus Rectum = Length of perpendicular from focus* $(2,3)$ *on directrix* $x-4y+3=0$

$$=2\left|\frac{2-12+3}{\sqrt{1+16}}\right|$$

$$=2\left|\frac{-7}{\sqrt{17}}\right|$$

$$=\frac{14}{\sqrt{17}}$$

**Parabola Ex 25.1 Q3(i)**

Given focus (-6, -6)

Vertex(-2,2)

Slope of line connecting vertex and focus is $\frac{2+6}{-2+6} = 2$

Slope of directrix will be $-\frac{1}{2}$, because both lines are perpendicular

Vertex is the midpoint of focus and point on directrix which passes through axis

$-2 = \frac{-6+x}{2}$; $2 = \frac{-6+y}{2}$

$(x, y) = (2, 10)$

Equation of directrix is given by

$y-10 = \frac{-1}{2}(x-2)$

$2y - 20 = -x + 2$

$x + 2y = 22$

Equation of Parabola is $(x+6)^2 + (y+6)^2 = \frac{(x+2y-22)^2}{5}$

$5\left[x^2 + y^2 + 36 + 36 + 12x + 12y\right] = \left[x^2 + 4y^2 + 484 + 4xy - 88y - 44x\right]$

$4x^2 + y^2 - 124 - 4xy + 104x + 148y = 0$

$(2x - y)^2 + 4(26x + 37y - 31) = 0$

**Parabola Ex 25.1 Q3(ii)**

In a parabola, vertex is the mid-point of the focus and the point of the intersection of the axis and directrix. so, let $(x_1, y_1)$ be the coordinate of the point of intersection of the axis and directrix. Then $(0,0)$ is the mid-point of the line segment joining $(0, -3)$ and $(x_1, y_1)$.

$\therefore \quad \frac{x_1 + 0}{2} = 0$ and $\frac{y_1 - 3}{2} = 0$

$\Rightarrow \quad x_1 = 0$ and $y_1 = 3$

Thus, the directrix meets the axis at $(0,3)$

$\therefore$ The equation of the directrix is $y = 3$

Clearly, the required parabola is of the form $x^2 = -4ay$, where $a = 3$

$\therefore$ equation of parabola is $x^2 = -4 \times 3 \times y$

$\Rightarrow \quad x^2 = -12y$

**Parabola Ex 25.1 Q3(iii)**

In a parabola, vertex is the mid-point of the focus and the point of intersection of the axis and directrix. So, let $(x_1, y_1)$ be the coordinate of the point of intersection of the axis and directrix. Then $(-1,-3)$ is the mid-point of the line segment joining $(0,-3)$ and $(x_1, y_1)$.

$$\therefore \quad \frac{x_1+0}{2} = -1 \quad \text{and} \quad \frac{y_1-3}{2} = -3$$

$$\Rightarrow \quad x_1 = -2 \quad \text{and} \quad y_1 = -3$$

Thus, the directrix meets the axis at $(-2,-3)$.
Let A be the vertex and S be the focus of the required parabola.
Then,

$$m_1 = \text{slope of } AS = \frac{-3-(-3)}{0-(-1)} = 0$$

$$\therefore \quad \text{slope of the directrix} = \frac{-1}{0} = \infty \qquad [\because \text{Directrix is perpendicular to the axis}]$$

Thus, the directrix passes through $(-2,-3)$ and has slope $\infty$, so its equation is

$$y-(-3) = \infty(x-(-2))$$

$$\frac{y+3}{\infty} = x+2$$

$$\Rightarrow \quad x+2 = 0$$

Let $P(x, y)$ be a point on the parabola.
Then, $PS$ = Distance of $P$ from the directrix.

$$\sqrt{(x-0)^2+(y+3)^2} = \left|\frac{x+2}{\sqrt{1^2}}\right|$$

$$\Rightarrow \quad x^2+(y+3)^2 = (x+2)^2$$

$$\Rightarrow \quad x^2+y^2+9+6y = x^2+4+4x$$

$$\Rightarrow \quad y^2-4x+6y+9-4 = 0$$

$$\Rightarrow \quad y^2-4x+6y+5 = 0$$

**Parabola Ex 25.1 Q3(iv)**

In a parabola, vertex is the mid-point of the focus and the point of intersection of the axis and directrix. so, let $(x_1, y_1)$ be the coordinates of the point of intersection of the axis and directrix. Then $(a', 0)$ is the mid-point of the line segment joining $(a, 0)$ and $(x_1, y_1)$.

$$\therefore \quad \frac{x_1 + a}{2} = a' \quad \text{and} \quad \frac{y_1 + 0}{2} = 0$$

$$\Rightarrow \quad x_1 = 2a' - a \quad \text{and} \quad y_1 = 0$$

Thus, the directrix meets the axis at $(2a' - a, 0)$.

So the equation of directrix is $x = 2a' - a$

Let $P(x, y)$ be any point on the parabola. Then

$$SP = PM$$

$$\Rightarrow \quad SP^2 = PM^2$$

$$\Rightarrow \quad (x-a)^2 + (y-0)^2 = \left[\frac{x - 2a' + a}{\sqrt{1^2}}\right]^2$$

$$\Rightarrow \quad x^2 + a^2 - 2ax + y^2 = (x - 2a' + a)^2$$

$$\Rightarrow \quad x^2 + a^2 - 2ax + y^2 = x^2 + (-2a')^2 + a^2 + 2 \times x \times (-2a') + 2 \times (-2a') \times a + 2 \times (a) \times (x)$$

$$\Rightarrow \quad x^2 + a^2 - 2ax + y^2 = x^2 + 4(a')^2 + a^2 - 4xa' - 4a'a + 2ax$$

$$\Rightarrow \quad y^2 = x^2 - x^2 + a^2 - a^2 + 2ax + 4(a')^2 - 4xa' - 4a'a + 2ax$$

$$\Rightarrow \quad y^2 = 4ax - 4xa' + 4(a')^2 - 4a'a$$

$$\Rightarrow \quad y^2 = 4ax - 4a'a - 4xa' + 4(a')^2$$
$$= 4a(x - a') - 4a'(x - a')$$
$$= (4a - 4a')(x - a')$$
$$= 4(a - a')(x - a')$$

$$\therefore \quad y^2 = 4(a - a')(x - a')$$

$$\Rightarrow \quad y^2 = -4(a' - a)(x - a')$$

Hence, required equation of parabola is $y^2 = -4(a' - a)(x - a')$

**Parabola Ex 25.1 Q3(v)**

x + y = 1 and x - y = 3

Intersecting point of above lines is

(x,y)=(2,-1)---------vertex

Focus (0,0)

Vertex is the midpoint of focus and point on directrix which passes through

$$2=\frac{0+x}{2};\ -1=\frac{0+y}{2}$$

$$(x,y)=(4,-2)$$

Slope of line passing through focus and vertex is $\frac{-1}{2}$

Slope of directrix is 2, as both are perpendicular lines

y+2=2(x-4)

2x-y=10------ directrix

$$SP^2 = PM^2$$

$$5(x^2+y^2)=(2x-y-10)^2$$

$$x^2+4y^2-100+4xy-20y+40x=0$$

$$(x+2y)^2+20(2x-y-5)=0$$

**Parabola Ex 25.1 Q4(i)**

The given parabola $y^2 = 8x$ is of the form $y^2 = 4ax$, where $4a = 8$

$$\Rightarrow \quad a = \frac{8}{4} = 2.$$

Vertex: The coordinates of the vertex are $(0,0)$.

Focus: The coordinates of the focus are $(2,0)$.

Axes: The equation of the axis is $y = 0$.

Directrix: The equation of the directrix is $x = -2$

Latus-rectum: The length of the latus-rectum $= 4a = 4\times 2 = 8$.

**Parabola Ex 25.1 Q4(ii)**

In the given parabola, $a=\frac{1}{16}$

$$Focus(0,-\frac{1}{16})$$

$$vertex(0,0)$$

$$Directrix,\ y=\frac{1}{16}$$

$$aixs,\ x=0$$

$$LR=\frac{1}{4}$$

The given equation is

$$y^2 - 4y - 3x + 1 = 0$$

$$\Rightarrow \quad y^2 - 4y = 3x - 1$$

$$\Rightarrow \quad y^2 - 4y + 4 = 3x - 1 + 4$$

$$\Rightarrow \quad y^2 - 4y + (2)^2 = 3x + 3$$

$$\Rightarrow \quad (y - 2)^2 = 3(x + 1) \quad \ldots(i)$$

Shifting the origin to the point $(-1, 2)$ without rotating the axes and denoting the new coordinates with respect to these axes by $X$ and $Y$, we have

$$x = X - 1, \qquad y = Y + 2 \ \ldots\ldots.(ii)$$

Using these relations equation $(i)$, reduces to

$$Y^2 = 3X \ \ldots\ldots\ldots.(iii)$$

This is of the form $Y^2 = 4aX$.

On comparing we get,

$$4a = 3$$

$$\Rightarrow \quad a = \frac{3}{4}.$$

Now,

Vertex: The coordinates of the vertex with respect to new axes are $(X = 0, \ \ Y = 0)$.

so, coordinates of the vertex with respect to old axes are, $(-1, 2)$.

Focus: The coordinates of the focus with respect to new axes are $\left(X = \frac{3}{4}, \ \ Y = 0\right)$.

Putting $X = \frac{3}{4}$ and $Y = 0$ in equation $(ii)$, we get

$$x = \frac{3}{4} - 1 \text{ and } y = 0 + 2$$

$$\Rightarrow \quad x = \frac{-1}{4} \text{ and } y = 2.$$

$\therefore$ Coordinates of the focus with respects to old axes are $\left(\frac{-1}{4}, 2\right)$

**Parabola Ex 25.1 Q4(iii)**

Axis: Equation of the axis of the parabola w.r.t new axes is $Y = 0$

$\therefore \quad y = 0 + 2$

$\Rightarrow \quad y = 2$

$\therefore$ equation of axis w.r.t old axes is $y = 2$

Directrix: Equation of the directrix of the parabola w.r.t new axes is $X = \frac{-3}{4}$

$\therefore \quad x = \frac{-3}{4} - 1$

$\Rightarrow \quad x = \frac{-7}{4}$

$\therefore$ Equation of the directrix of the parabola w.r.t old axes is $x = \frac{-7}{4}$

Latus-rectum: The length of the latus-rectum $= 4a$

$= 4 \times \frac{3}{4}$

$= 3.$

**Parabola Ex 25.1 Q4(iv)**

The given equation is

$y^2 - 4y + 4x = 0$

$\Rightarrow \quad y^2 - 4y = -4x$

$\Rightarrow \quad y^2 - 2 \times y \times 2 + (2)^2 = -4x + (2)^2$

$\Rightarrow \quad (y - 2)^2 = -4x + 4$

$\Rightarrow \quad (y - 2)^2 = -4(x - 1) \quad \ldots(i)$

Shifting the origin to the point $(1,2)$ without rotating the axes and denoting the new coordinates with respect to these axes by $X$ and $Y$, we have

$x = X + 1, \quad y = Y + 2 \quad \ldots(ii)$

Using these relations equation $(i)$, reduces to

$Y^2 = -4X \quad \ldots(iii)$

This is of the form $Y^2 = -4aX$.

on comparing, we get, $a = 1$

Now,

Vertex: The coordinates of the vertex w.r.t to new axes are $(X = 0, \quad y = 0)$.

$\therefore \quad x = 0 + 1, \quad y = 0 + 2$ [Using equation ii]

$\Rightarrow \quad x = 1, \quad y = 2$

$\therefore$ Coordinates of the vertex w.r.t old axes are, $(1,2)$

Focus: The coordinates of the focus with respect to new axes are $(X = 1, \quad Y = 0)$.

Putting $X = -1$ and $Y = 0$ in equation $(ii)$, we get

$x = -1 + 1, \quad y = 0 + 2$

$\Rightarrow \quad x = 0, \quad y = 2$

$\therefore$ Coordinates of the focus w.r.t old axes are $(0,2)$.

Axis: Equation of the axis of the parabola w.r.t new axes is $Y = 0$

$\therefore \quad y = 0 + 2$ [Using equation ii]

$\Rightarrow \quad y = 2$

$\therefore$ equation of axis w.r.t old axes is $y = 2$

Directrix: Equation of the directrix of the parabola w.r.t new axes is $X = 1$

$\therefore \quad x = 1 + 1$ $\quad$ [Using equation (ii)]

$\Rightarrow \quad x = 2$

$\therefore$ Equation of the directrix of the parabola w.r.t old axes is $x = 2$.

Latus-rectum: The length of the latus-rectum $= 4a$

$$= 4 \times 1$$
$$= 4$$

## Parabola Ex 25.1 Q4(v)

The given equation is

$$y^2 + 4x + 4y - 3 = 0.$$

$\Rightarrow \quad y^2 + 4y = -4x + 3$

$\Rightarrow \quad y^2 + 2 \times y \times 2 + 2^2 = -4x + 3 + 2^2$

$\Rightarrow \quad (y + 2)^2 = -4x + 3 + 4$

$\Rightarrow \quad (y + 2)^2 = -4x + 7$

$\Rightarrow \quad (y + 2)^2 = -4\left(x - \frac{7}{4}\right) \quad \ldots (i)$

Shifting the origin to the point $\left(\frac{7}{4}, -2\right)$ without rotating the axes and denoting the new coordinates with respect to these axes by $X$ and $Y$, we have

$$x = X + \frac{7}{4}, \quad y = Y - 2 \quad \ldots (ii)$$

Using these relations equation $(i)$, reduces to $Y^2 = -4X \quad \ldots (iii)$

This is of the form $Y^2 = -4aX$ on comparing, we get $a = 1$

Now,

Vertex: The coordinates of the vertex w.r.t new axes are $(X = 0, \quad Y = 0)$

$\therefore \quad x = 0 + \frac{7}{4}, \quad y = 0 - 2$ $\quad$ [Using (ii)]

$\Rightarrow \quad x = \frac{7}{4}, \quad y = -2$

$\therefore$ Coordinates of the vertex w.r.t old axes are $\left(\frac{7}{4}, -2\right)$.

Focus: The coordinates of the focus w.r.t new axes are $(X = -1, \quad = 0)$

$\therefore \quad x = -1 + \frac{7}{4}$ and $y = 0 - 2$ $\quad$ [Using (ii)]

$\Rightarrow \quad x = \frac{3}{4},$ and $y = -2$

$\therefore$ Coordinates oof the focus w.r.t old axes are $\left(\frac{3}{4}, -2\right)$.

Axis: Equation of the axis of the parabola w.r.t new axes is

$$Y = 0$$

$\therefore \quad y = 0 - 2$ $\quad$ [Using equation (ii)]

$\Rightarrow \quad y = -2$

$\therefore$ equation of the w.r.t old axes is $y + 2 = 0$.

## Parabola Ex 25.1 Q4(vi)

The given equation is

$$y^2 = 8x + 8y$$
$$\Rightarrow \quad y^2 - 8y = 8x$$
$$\Rightarrow \quad y^2 - 2\times4\times y + 16 = 8x + 16$$
$$\Rightarrow \quad (y-4)^2 = 8(x+2) \quad \ldots(i)$$

Shifting the origin to the point $(-2, 4)$ without rotating the axes and denoting the new coordinates w.r.t these axes by $X$ and $Y$, we have

$$x = X - 2, \; y = Y + 4 \quad \ldots(ii)$$

Using these relations equation $(i)$, reduces to

$$Y^2 = 8X \quad \ldots(iii)$$

This is of the form $Y^2 = 4aX$, on comparing, we get

$$4a = 8$$
$$\Rightarrow \quad a = 2$$

Now,

Vertex: The coordinates of the vertex w.r.t new axes are $(X = 0, \; Y = 0)$

$$\therefore \quad x = 0 - 2, \; y = 0 + 4$$
$$\Rightarrow \quad x = -2, \; y = 4$$

$\therefore$ Coordinates of the vertex w.r.t old axes are $(-2, 4)$

Focus: The coordinates of the focus w.r.t new axes are $(X = 2, \; Y = 0)$

$$\therefore \quad x = 2 - 2 \text{ and } y = 0 + 4 \quad [\text{Using equation (ii)}]$$
$$\Rightarrow \quad x = 0, \text{ and } y = 4$$

$\therefore$ Coordinates of the focus w.r.t old axes are $(0, 4)$.

Axis: Equation of the axis of the parabola w.r.t new axes is $Y = 0$

$$\therefore \quad y = 0 + 4 \quad [\text{Using equation (ii)}]$$
$$\Rightarrow \quad y = 4$$

$\therefore$ equation of axis w.r.t old axes is $y = 4$

Directrix: Equation of the directrix of the parabola w.r.t new axes is

$$X = -2$$
$$\therefore \quad x = -2 - 2 \quad [\text{using equation (ii)}]$$
$$\Rightarrow \quad x = -4$$
$$\Rightarrow \quad x + 4 = 0$$

$\therefore$ Equation of the directrix of the parabola w.r.t old axes is $x + 4 = 0$

Latus-rectum: THe length of the latus-rectum $= 4a$

$$= 4\times2$$
$$= 8.$$

**Parabola Ex 25.1 Q4(vii)**

The given system of equation is

$$4(y-1)^2 = -7(x-3)$$

$$\Rightarrow \quad (y-1)^2 = \frac{-7}{4}(x-3) \qquad \ldots(i)$$

Shifting the origin to the point $(3,1)$ without rotating the axes and denoting the new coordinates w.r.t these axes by $X$ and $Y$,
we have,

$$x = X+3, \quad y = Y+1 \qquad \ldots(ii)$$

Using these relation $(i)$, redus to

$$Y^2 = \frac{-7}{4}X \qquad \ldots(iii)$$

This is of the form $Y^2 = -4aX$, on comparing, we get

$$4a = \frac{7}{4}$$

$$\Rightarrow \quad a = \frac{7}{16}$$

Now,

Vertex: The coordinates of the vertex w.r.t new axes are $(X=0, \ Y=0)$

$$\therefore \quad x = 0+3, \quad y = 0+1 \qquad [\text{Using equation } (iii)]$$

$$\Rightarrow \quad x = 3, \quad y = 1$$

$\therefore$ Coordinates of the vertex w.r.t old axes are $(3,1)$.

Focus: The coordinates of the focus w.r.t new axes are $\left(x = -\frac{7}{16}, Y = 0\right)$

$$\therefore \quad x = \frac{-7}{16}+3, \quad y = 0+1$$

$$\Rightarrow \quad x = \frac{41}{16}, \quad y = 1$$

$\therefore$ Coordinates of the focus w.r.t old axes are $\left(\frac{41}{16}, 1\right)$.

Axis: Equation of the axis of the parabola w.r.t new axes is

Axis: Equation of the axis of the parabola w.r.t new axes is

$$Y = 0$$

$$\Rightarrow \quad y = 0+1$$

$$\Rightarrow \quad y = 1$$

$\therefore$ equation of axis w.r.t old axes is $y = 1$

Directrix: Equation of the directrix of the parabola w.r.t new axes is

$$Y = \frac{7}{16}$$

$$\therefore \quad x = \frac{7}{16}+3$$

$$\Rightarrow \quad x = \frac{55}{16}$$

$\therefore$ Equation of the directrix of the parabola w.r.t old axes is $x = \frac{55}{16}$.

Latus-rectum: The length of the latus-rectum $= 4a$

$$= 4 \times \frac{7}{16}$$

$$= \frac{7}{4}.$$

**Parabola Ex 25.1 Q4(viii)**

The given system of equation is

$$y^2 = 5x - 4y - 9$$

$$\Rightarrow \quad y^2 + 4y = 5x - 9$$

$$\Rightarrow \quad y^2 + 4y + 4 = 5x - 9 + 4$$

$$\Rightarrow \quad (y + 2)^2 = 5x - 5$$

$$\Rightarrow \quad (y + 2)^2 = 5(x - 1) \qquad \ldots(i)$$

Shifting the origin to the point $(1, -2)$ without rotating the axes and denoting the new coordinates w.r.t these axes by $X$ and $Y$,
we have,

$$x = X + 1, \quad y = Y - 2 \qquad \ldots(ii)$$

using these relations, equation $(i)$ reduces to

$$Y^2 = 5X \qquad \ldots(iii)$$

This is of the from $Y^2 = 4aX$ on comparing we get

$$4a = 5$$

$$\Rightarrow \quad a = \frac{5}{4}$$

Now,

Vertex: The coordinates of the vertex w.r.t new axes are $(X = 0, \ Y = 0)$

$$\therefore \quad x = 0 + 1, \ y = 0 - 2 \qquad [\text{Using equation } (ii)]$$

$$\Rightarrow \quad x = 1, \quad y = -2$$

$\therefore$ Coordinates of the vertex w.r.t old axes are $(1, -2)$.

Focus: The coordinates of the focus w.r.t new are $\left(X = \frac{5}{4}, \ Y = 0\right)$

$$\therefore \quad x = \frac{5}{4} + 1, \quad y = 0 - 2$$

$$\Rightarrow \quad x = \frac{9}{4}, \quad y = -2$$

Axis: Equation of the axis of the parabola w.r.t axes is

$$Y = 0$$

$$\therefore \quad y = 0 - 2$$

$$\Rightarrow \quad y = -2$$

$\therefore$ equation of axis w.r.t old axes is $y = -2$.

Directrix: Equation of the directrix of the parabola w.r.t new axes is

$$X = \frac{-5}{4}$$

$$\therefore \quad x = \frac{-5}{4} + 1$$

$$\Rightarrow \quad x = \frac{-1}{4}$$

$$\Rightarrow \quad 4x + 1 = 0$$

$\therefore$ Equation of the directrix of the parabola w.r.t old axes is $4x + 1 = 0$

Latus-rectum: The length of the latus-rectum $= 4a$

$$= 4 \times \frac{5}{4}$$

$$= 5.$$

**Parabola Ex 25.1 Q4(ix)**

The given of equation is

$$x^2 + y = 6x - 14$$

$\Rightarrow \quad x^2 - 6x = -y - 14$

$\Rightarrow \quad x^2 - 2 \times x \times 3 + 9 = -y - 14 + 9$

$\Rightarrow \quad (x-3)^2 = -y - 5$

$\Rightarrow \quad (x-3)^2 = -1(y+5) \qquad \ldots(i)$

Shifting the origin to the point $(3, -5)$ without rotating the axes and denotinh the new coordinates w.r.t these axes by $X$ and $Y$,
we have,

$$x = X + 3, \quad y = Y - 5 \qquad \ldots(ii)$$

Using these relations, equation (i) reduces to

$$X^2 = -y \qquad \ldots(iii)$$

This is of the form $X^2 = -4aY$, on comparing, we get

$$4a = 1$$

$\Rightarrow \quad a = \frac{1}{4}$

Now,

Vertex: The coordinates of the vertex w.r.t new are $(X = 0, \ Y = 0)$

$\therefore \quad x = 0 + 3, \quad y = 0 - 5$

$\Rightarrow \quad x = 3, \ y = -5$

$\therefore$ Coordinates of the vertex w.r.t old axes are $(3, -5)$.

Focus: The coordinates of the focus w.r.t new axes are $\left(X = 0, \ Y = \frac{-1}{4}\right)$

$\therefore \quad x = 0 + 3, \quad y = \frac{-1}{4} - 5$

$\therefore$ Coordinates of the focus w.r.t old axes are $\left(3, \frac{-21}{4}\right)$.

Axis: Equation of the axis of the parabola w.r.t new axes is

$$X = 0$$

$\therefore$ $x = 0 + 3$

$\Rightarrow$ $x = 3$

$\therefore$ equation of axis w.r.t old axes is $x = 3$.

Directrix: Equation of the directrix of the parabola w.r.t new axes is

$$Y = \frac{1}{4}$$

$\therefore$ $y = \frac{1}{4} - 5$

$\Rightarrow$ $y = \frac{-19}{4}$

$\Rightarrow$ $4y + 19 = 0$

$\therefore$ Equation of the directrix of the parabola w.r.t old axes is $4y + 19 = 0$

Latus-rectum: The length of the latus-rectum $= 4a$

$$= 4 \times \frac{1}{4}$$

$$= 1.$$

**Parabola Ex 25.1 Q5**

Let $PQ$ be the double ordinate of length $8p$ of the parabola $y^2 = 4px$.
Then, $PR = QR = 4p$.

let $AR = x_1$. Then, the coordinates of $P$ and $Q$ are $(x_1, 4p)$ and, $(x_1, -4p)$ respectively.

Since $P$ lies on $y^2 = 4px$

$\therefore \quad (4p)^2 = 4px_1$

$\Rightarrow \quad x_1 = 4p.$

So, coordinates of $P$ and $Q$ are $(4p, 4p)$ and $(4p, -4p)$ respectively.

$\therefore$ The extremities of a double ordinate are $(4p, 4p)$ and $(4p, -4p)$.

Also, the coordinates of the vertex $A$ are $(0,0)$.

$$\therefore \quad m_1 = \text{slope of } AP = \frac{4p - 0}{4p - 0} = 1$$

$$\text{and,} \quad m_2 = \text{slope of } AQ = \frac{-4p - 0}{4p - 0} = -1$$

Clearly, $m_1 m_2 = -1$.

Hence, $AP \perp AQ$

$\therefore$ The lines from the vertex to its extremities are at right angles.

**Parabola Ex 25.1 Q6**

The given equation of the parabola is

$$x^2 = 12y$$

This is of the form $x^2 = 4ay$. on comparing, we get

$$4a = 12$$

$$\Rightarrow \quad a = \frac{12}{4} = 3$$

$\therefore$ Coordinates of the focus $S$ is $(0,3)$.

$P$ and $Q$ lies on the parabola.

$$\therefore \quad x^2 = 12 \times 3$$

$$\Rightarrow \quad x^2 = 36$$

$$\Rightarrow \quad x = \pm 6$$

$\therefore$ $P(-6,3)$ and $Q(6,3)$.

Now, $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(6+6)^2 + (3-3)^2}$$

$$= \sqrt{(12)^2}$$

$$= 12$$

and, $OS = 3$.

$\therefore$ Area of $\triangle POQ = \frac{1}{2} \times PQ \times OS$

$$= \frac{1}{2} \times 12 \times 3$$

$$= 6 \times 3 = 18 \text{ sq. units.}$$

**Parabola Ex 25.1 Q7**

The axis of the parabola is a line ⊥ to the directrix and passing through focus. The equation of a line ⊥ to $3x - 4y - 2 = 0$ is

$$y = \frac{-4}{3} + \lambda$$

$$\left[\because m_1 m_2 = -1 \Rightarrow m_2 = \frac{-1}{m_1} \text{ and } y = m_2 x + \lambda\right]$$

$\Rightarrow \quad 3y + 4x = 3\lambda$

This will pass through focus $(3, 3)$ if,

$3 \times 3 + 4 \times 3 = 3\lambda$

$\Rightarrow \quad 9 + 12 = 3\lambda$

$\Rightarrow \quad 21 = 3\lambda$

$\Rightarrow \quad \lambda = \frac{21}{3} = 7$

so, the equation of axis is $3y + 4x = 3 \times 7 = 21$

$\Rightarrow \quad 3y + 4x = 21 \qquad \ldots(i)$

And the equation of directrix is

$3x - 4y = 2 \qquad \ldots(ii)$

Mutiplying equation (i) by 4 and equation (ii) by 3, we get

$16x + 12y = 84 \qquad \ldots(iii)$

$9x - 12y = 6 \qquad \ldots(iv)$

Adding equation (iii) and (iv), we get

$16x + 9x = 84 + 6$

$\Rightarrow \quad 25x = 90$

$\Rightarrow \quad x = \frac{90}{25} = \frac{18}{5}$

Putting $x = \frac{18}{5}$ in equation (i), we get

Putting $x = \frac{18}{5}$ in equation (i), we get

$3y + 4 \times \frac{18}{5} = 21$

$\Rightarrow \quad 3y + \frac{72}{5} = 21$

$\Rightarrow \quad 3y = 21 - \frac{72}{5}$

$\Rightarrow \quad 3y = \frac{105 - 72}{5}$

$\Rightarrow \quad 3y = \frac{33}{5}$

$\Rightarrow \quad y = \frac{11}{5}$

Hence, the required point of intersection is $\left(\frac{18}{5}, \frac{11}{5}\right)$.

**Parabola Ex 25.1 Q8**

Let the ordinates of the required point is $y$.

$\therefore$ abscissa = $3y$

$\therefore$ The coordinates of the points are $(3y, y)$.

These points lies on the parabola $x^2 = 9y$.

$$\therefore \quad (3y)^2 = 9y$$
$$\Rightarrow \quad 9y^2 = 9y$$
$$\Rightarrow \quad 9y^2 - 9y = 0$$
$$\Rightarrow \quad 9y(y - 1) = 0$$
$$\Rightarrow \quad y - 1 = 0 \qquad [\because y \neq 0]$$
$$\Rightarrow \quad y = 1$$

$\therefore$ abscissa $= 3 \times y = 3$

Hence, the required point is $(3,1)$.

**Parabola Ex 25.1 Q9**

Let the equation of parabola be

$$y^2 = 4ax \qquad \ldots(i) \qquad [\because \text{axis along x-axis}]$$

If passes through $(2,3)$.

$$\therefore \quad (3)^2 = 4 \times a \times 2$$
$$\Rightarrow \quad 9 = 8a$$
$$\Rightarrow \quad a = \frac{9}{8}$$

Putting the value of a in equation $(i)$, we get

$$y^2 = 4 \times \frac{9}{8} \times x$$
$$\Rightarrow \quad y^2 = \frac{9}{2} \times x$$
$$\Rightarrow \quad 2y^2 = 9x$$

Hence, the required equation of parabola is $2y^2 = 9x$.

**Parabola Ex 25.1 Q10**

Let $(x_1, y_1)$ be the coordinates of the point intersection of the axis and the directrix.

$\therefore \quad (x_1, y_1) = (0, 2) \qquad [\because\ y = 2]$

we know that, the vertex is the mid-point of the line segment joining $(0, 2)$ and focus $(x_2, y_2)$

$$\therefore \quad \frac{x_2 + 0}{2} = 0 \quad \text{and} \quad \frac{y_2 + 2}{2} = 0 \qquad [\because \text{ vertex at the origin}]$$

$\therefore \quad x_2 = 0, \quad$ and $y_2 = -2$

$\therefore$ The coordinates of focus is $(0, -2)$.

By the definition of parabola

$$PS = PM$$

$$\Rightarrow \quad PS^2 = PM^2$$

$$\Rightarrow \quad (x - 0)^2 + (y + 2)^2 = \left[\frac{y - 2}{\sqrt{1}}\right]^2$$

$$\Rightarrow \quad x^2 + y^2 + 4 + 4y = (y - 2)^2$$

$$\Rightarrow \quad x^2 + y^2 + 4 + 4y = y^2 + 4 - 4y$$

$$\Rightarrow \quad x^2 = -4y - 4y$$

$$\Rightarrow \quad x^2 = -8y$$

Hence, The required equation of parabola is $x^2 = -8y$.

**Parabola Ex 25.1 Q11**

In a parabola, vertex is the mid point of the focus and the point of intersection of the axis and directrix. So let $(x, y)$ be the coordinates of the point of intersection of the axis and directrix.

Then $(3,2)$ is the mid point of the line segment joining $(5,2)$ and $(x_1, y_1)$

$$\frac{x_1+5}{2}=3 \quad \text{and} \quad \frac{y_1+2}{2}=2$$

$$x_1+5=6 \quad \text{and} \quad y_1+2=4$$

$$x_1=1 \quad \text{and} \quad y_1=2$$

The directrix meets the axis at $(1,2)$

Let A be the vertex and S be the focus of the required parabola

Then

$$m_1 = \text{slope of } AS = \frac{2-2}{5-3} = 0$$

Let $m_2$ be the slope of the directrix

Then

$$m_2 = \infty \qquad [\because \text{Directrix is perpendicular to the axis}]$$

Thus the directrix passes through $(1,2)$ and the slope $\infty$, so its equation is

$$y-2=\infty(x-1)$$

$$\frac{y-2}{\infty}=x-1$$

$$x-1=0$$

Let $P(x,y)$ be a point on the parabola

Then PS=distance of P from the directrix

$$\sqrt{(x-5)^2+(y-2)^2}=\left|\frac{x-1}{\sqrt{1^2}}\right|$$

$$(x-5)^2+(y-2)^2=(x-1)^2$$

$$x^2+25-10x+y^2+4-4y=x^2+1-2x$$

$$y^2-4y-8x+28=0$$

Hence the required equation of the parabola is $y^2-4y-8x+28=0$



## Parabola Ex 25.1 Q12

Let $CAB$ be the bridge and $LOX$ be the road way. Let $A$ be the centre of the bridge. we find that the coordinates of $A$ are $(0,6)$.

Clearly, the bridgeis in the shope of a parabola having its vertex at $A$ $(0,6)$.
Let its equation be $x^2 = 4a(y-6)$ ...(i)

It posses through $B(50,30)$. Therefore, $(50)^2 = 4a(30-6)$

$$\Rightarrow \quad 2500 = 4a \times 24$$

$$\Rightarrow \quad \frac{2500}{4 \times 24} = a$$

$$\Rightarrow \quad a = \frac{625}{24}$$

Putting the value of a in $(i)$, we get

$$x^2 = 4 \times \frac{625}{24}(y-6)$$

$$\Rightarrow \quad x^2 = \frac{625}{6}(y-6) \quad \text{...(ii)}$$

Let $l$ metres be the length of the vertical supporting cable 18 metres from the centre. Then, $P(18,l)$ lies on $(ii)$. Therefore

$$(18)^2 = \frac{625}{6}(l-6)$$

$$\Rightarrow \quad 324 \times 6 = 625(l-6)$$

$$\Rightarrow \quad 324 \times 6 = 625(l-6)$$

$$\Rightarrow \quad \frac{1944}{625} = l - 6$$

$$\Rightarrow \quad \frac{1944}{625} + 6 = l$$

$$\Rightarrow \quad \frac{1944 + 3750}{625} = l$$

$$\Rightarrow \quad l = \frac{5694}{625} = 9.11\,\text{m} \quad (\text{approx})$$

Hence, the required length of a supporting wire is $9.11\,\text{m}$.

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**Parabola Ex 25.1 Q13**

When x=24, then y=±12

So two points are A(24, 12) and B(24, -12)

Equation of lines joining vertex and A is

$y=\frac{1}{2}x$

Equation of lines joining vertex and B is

$y=-\frac{1}{2}x$

**Parabola Ex 25.1 Q14**

In given parabola

a=2

Given focal distance=a+x=4, so x=2

So points are (2, 4) and (2, -4)

**Parabola Ex 25.1 Q15**

$$y = x\tan\theta$$
$$y^2 = 4ax$$

Intersection point of both the curves are $\left(\frac{4a}{\tan^2\theta}, \frac{4a}{\tan\theta}\right)$

So Distance from origin to the above point is

$$\sqrt{\left(\frac{4a}{\tan^2\theta}\right)^2+\left(\frac{4a}{\tan\theta}\right)^2} = \frac{4a}{\tan^2\theta}\sqrt{1+\tan^2\theta} = 4a\cot\theta\,\mathrm{cosec}\,\theta$$

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**Parabola Ex 25.1 Q16**

The vertex and focus of the parabola are $A(0, 4)$ and $F(0,2)$ respectively.

$AF = 2$

As point A and F lie on y-axis, so y-axis is the axis of the parabola.

If the diretrix meets the axis of parabola at point Z, then $AZ = AF = 2$.

$\therefore OZ = OF + FA + AZ = 2 + 2 + 2 = 6$

So equation of diretrix is $y = 6$

Let $P(x, y)$ be any point in the plane of focus and diretrix,

and MP be the perpendicular distance from P to the diretrix,

then P lies on parabola iff $FP = MP$

$$\Leftrightarrow \sqrt{(x-0)^2 + (y-2)^2} = \frac{|y-6|}{1}$$

$$\Leftrightarrow x^2 + (y-2)^2 = (y-6)^2$$

$$\Leftrightarrow x^2 + y^2 - 4y + 4 = y^2 - 12y + 36$$

$$\Leftrightarrow x^2 + 8y = 32$$

$x^2 + 8y = 32$ is the required equation of the parabola.

**Parabola Ex 25.1 Q17**

The line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$.

$\therefore (mx+1)^2 = 4x$

$m^2x^2 + 2mx + 1 = 4x$

$m^2x^2 + (2m-4)x + 1 = 0$

As we know tangent touches the parabola, so the roots of the above quadratic will be equal.

$\Rightarrow D = b^2 - 4ac = 0$

$\Rightarrow (2m-4)^2 - 4(m^2)(1) = 0$

$\Rightarrow 4m^2 + 16 - 16m - 4m^2 = 0$

$\Rightarrow m = 1$