RD Sharma Solutions Class 11 Maths Chapter 24 Ex 24.3

The Circle Ex 24.3 Q1

We will use equation in diameter form.

$$(x-2)(x+2)+(y+3)(y-4)=0$$

$$x^2-4+y^2-y-12=0$$

$$x^2+y^2-y-16=0 \text{..........}(1)$$

$$2g=0,\ 2f=-1 \qquad \text{............}[\text{From }(1)]$$

$$g=0,\ f=-\frac{1}{2}$$

Centre of the circle is $(0, \frac{1}{2})$

$$r=\sqrt{0+\frac{1}{4}+16}$$

$$r=\frac{\sqrt{65}}{2}$$

The Circle Ex 24.3 Q2

Centre of the circles

$$x^2+y^2+6x-14y-1=0$$

is $(-3,7)$

and $x^2+y^2-4x+10y-2=0$

is $(2,-5)$

Equation of the circle is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$\Rightarrow$ $(x+3)(x-2)+(y-7)(y+5)=0$

$\Rightarrow$ $x^2+3x-2x-6+y^2-7y+5y-35=0$

$\Rightarrow$ $x^2+y^2+x-2y-41=0$

The Circle Ex 24.3 Q3

Let the sides AB, BC, CD and DA of the square $ABCD$ be represented by the equations
$y=3$, $x=6$, $y=6$ and $x=9$ respectively.

Then, coordinates are

$A(6,3)$, $B(9,3)$, $C(9,6)$ and $D(6,6)$.

The equation of the circle with diagonal AC

$$(x-6)(x-9)+(4-3)(4-6)=0$$

$\Rightarrow$ $x^2-6x-9x+54+y^2-3y-6y+18=0$

$\Rightarrow$ $x^2+y^2-15x-9y+72=0$

The equation of the circle with diagonal BD as diameter is

$$(x-9)(x-6)+(y-3)(y-6)=0$$

$\Rightarrow$ $x^2-9x-6x+54+y^2-3y-6y+18=0$

$\Rightarrow$ $x^2+y^2-15x-9y+72=0$

$$x^2+y^2-15x-9y+72=0$$

The Circle Ex 24.3 Q4

The given equation are

$x-3y=4$ ....(i)

$3x+y=22$ ....(ii)

$x-3y=14$ ....(iii)

$3x+y=62$ ....(iv)

Let A, B, C & D are the points of intersection of the lines (i)&(ii), (ii)&(iii), (iii)&(iv) and (iv)&(i)

$\therefore$ $A=(7,1)$, $B=(8,-2)$, $C=(20,2)$ & $D=(19,5)$

AC will be the diameter of the circle

so,

the equation of circle is

$$(x-7)(x-20)+(y-1)(y-2)=0$$

$\Rightarrow$ $x^2+y^2-27x-3y+142=0$

The Circle Ex 24.3 Q5

The line $3x + 4y = 12$
will meet the axis at $A\left(0,3\right)$ & $B\left(4,0\right)$

Since the circle passes through origin & A and B
$\therefore$ AB is a diameter

Thus the equation of circle is

$$\left(x-0\right)\left(x-4\right)+\left(y-3\right)\left(y-0\right)=0$$

$$\Rightarrow \quad x^2+y^2-4x-3y=0$$

The Circle Ex 24.3 Q6

Since the circle is passes through origin and cut intercept a and b on
$x - axis$ & $y - axis$

$\therefore$ Coordinate of circle $A = \left(0,b\right)$ and $B = \left(a,0\right)$
AB is the diameter of circle

$\therefore$ The equation of circle is

$$\left(x-a\right)\left(x-0\right)+\left(y-0\right)\left(y-b\right)=0$$

$$\Rightarrow \quad x^2+y^2\pm ax\pm by=0$$

The Circle Ex 24.3 Q7

Equation of circle in diameter form is,

$$(x+4)\ (x-12)+(y-3)\ (y+1)=0$$

$$x^2-8x-48+y^2-2y-3=0$$

$$x^2-8x-2y+y^2-51=0 \ldots\ldots\ldots\ldots\ldots(1)$$

To find y-intercept, put x=0 in (1),

$$y^2-2y-51=0$$

$$y=\frac{2\pm\sqrt{4+204}}{2}$$

$$y=\frac{2\pm\sqrt{208}}{2}$$

y intercepts are $1\pm4\sqrt{13}$

The Circle Ex 24.3 Q8

The given equations are

$$x^2+2ax-b^2=0\ldots\ldots(i)$$

$$x^2+2px-q^2=0\ldots\ldots(ii)$$

We have, the roots of (i) will give the absissae and the roots of (ii) will give the ordinate of $A\,\&\,B$ respectively.

Now roote of (i)

$$x=\frac{-2a\pm\sqrt{4a^2+4b^2}}{2}=-9\pm\sqrt{a^2+b^2}$$

Roots of (ii)

$$x=\frac{-2p\pm\sqrt{4p^2+4q^2}}{2}=-p\pm\sqrt{p^2+q^2}$$

Coordinates of $A=\left(-a+\sqrt{a^2+b^2},-p\sqrt{p^2+q^2}\right)$

$$B=\left(-a-\sqrt{a^2+b^2},-p-\sqrt{p^2+q^2}\right)$$

so, the equation of circle is

$$\left(x+a-\sqrt{a^2+b^2}\right)\left(x+a+\sqrt{a^2+b^2}\right)+\left(y+p-\sqrt{p^2+q^2}\right)\left(y+p+\sqrt{p^2+q^2}\right)=0$$

$$\Rightarrow\quad x^2+y^2+2ax+2py-\left(a^2+b^2+p^2+q^2\right)+a^2+p^2=0$$

$$\Rightarrow\quad x^2+y^2+2ax+2py-\left(b^2+q^2\right)=0$$

The radius is

$$r=\sqrt{g^2+f^2-c}$$

$$=\sqrt{a^2+p^2-\left(b^2+q^2\right)}$$

$$=\sqrt{a^2+b^2+p^2+q^2}$$

The Circle Ex 24.3 Q9

Here AB and AD are taken as x − axis and y − axis respectively.

Since $ABCD$ is a square thus the coordinate of

$$A=(0,0)$$

$$B=(a,0)\ C=(a,a)$$

$$D=(0,a)$$

$\because$ BD is a diameter

so, the equation of circle is

$$(x-a)(x-0)+(y-0)(y-a)=0$$

$$\Rightarrow\quad x^2+y^2-ax-ay=0$$

so,

$$x^2+y^2-a(x+y)=0$$

The Circle Ex 24.3 Q10

The given equation of line & circle in

$$2x - y + 6 = 0 \ldots\ldots\ldots (i)$$

$$x^2 + y^2 - 2y - 9 = 0 \ldots\ldots (ii)$$

The point of intersection of (i)&(ii) is

$$x^2 + (2x + 6)^2 - 2(2x + 6) - 9 = 0$$

$$\Rightarrow \quad x^2 + 4x^2 + 24x + 36 - 4x - 12 - 9 = 0$$

$$\Rightarrow \quad 5x^2 + 20x + 15 = 0$$

$$\Rightarrow \quad x^2 + 4x + 3 = 0$$

$$\Rightarrow \quad (x + 3)(x + 1) = 0 \quad \Rightarrow \quad x = (-3, -1)$$

$$\therefore \ y = (0, 4)$$

$\therefore \ A = (-3, 0)$ & $B = (-1, 4)$

$\because \ AB$ is a diameter, so the equation of circle is

$$(x + 3)(x + 1) + (y - 0)(y - 4) = 0$$

$$\Rightarrow \quad x^2 + y^2 + 4x - 4y + 3 = 0$$

The Circle Ex 24.3 Q11

The triangle is formed by

$$x = 0 \ldots\ldots\ldots (i)$$

$$y = 0 \ldots\ldots\ldots (ii) \&$$

$$lx + my = 1 \ldots\ldots\ldots (iii)$$

The line (iii) cuts the axis at

$$A = \left(0, \frac{1}{m}\right) \text{ and at } B = \left(\frac{1}{l}, 0\right)$$

Now, AB will be the diameter of circle.

$\therefore$ equation of circle will be,

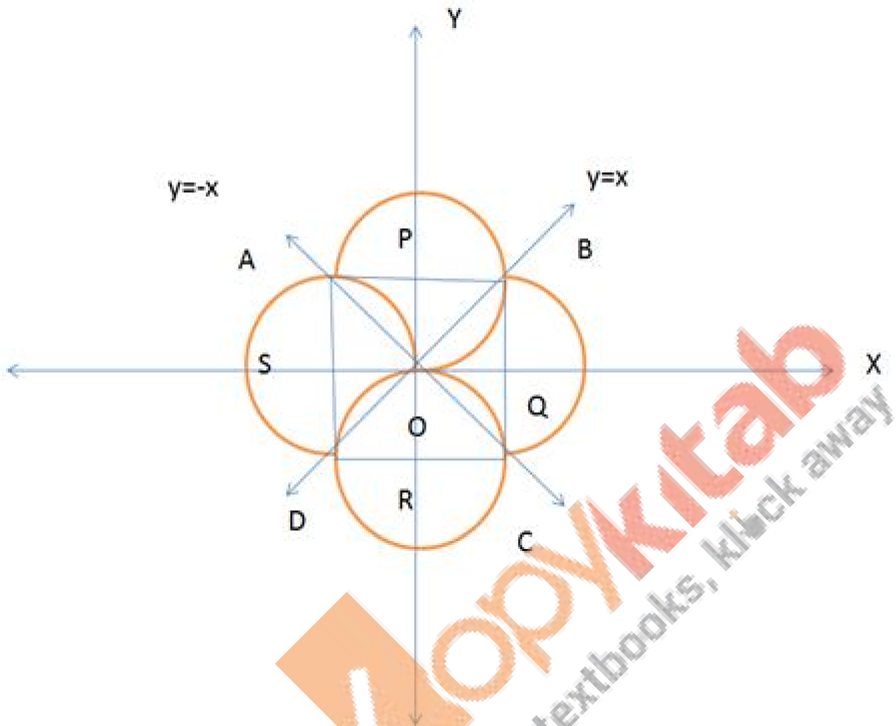
$$\left(x - \tfrac{1}{l}\right)(x - 0) + (y - 0)\left(y - \tfrac{1}{m}\right) = 0$$

$$\Rightarrow \quad x^2 + y^2 - \frac{x}{l} - \frac{y}{m} = 0$$

$$\Rightarrow \quad x^2 + y^2 - \frac{x}{l} - \frac{y}{m} = 0$$

The Circle Ex 24.3 Q12

Figure shows four such circles.



Angle between y=x and y=-x is $\frac{\pi}{2}$

$\therefore$ Angle between OB and OA $=\frac{\pi}{2}$

Hence, AB, BC, CD and AD are diameters of circles.

$\angle BOQ=\frac{\pi}{4}$

$\text{Sin } \angle BOQ=\frac{BQ}{OB}$

$\text{Sin}\ \dfrac{\pi}{4} = \dfrac{BQ}{\sqrt{2}}$

$\dfrac{1}{\sqrt{2}} = \dfrac{BQ}{\sqrt{2}}$

BQ=1

Radius of circles=1=OQ

Coordinates of B is (1,1).

Similarly, coordinates of A(-1,1), C (1,-1), D(-1,-1)

Equation of circle with diameter AB is

$(x+1)(x-1)+(y-1)(y-1)=0$

$x^2-1+y^2-2y+1=0$

$x^2+y^2-2y=0$

Equation of circle with diameter BC is

$(x-1)(x-1)+(y-1)(y+1)=0$

$x^2-2x+1+y^2-1=0$

$x^2+y^2-2x=0$

Equation of circle with diameter CD is

$(x+1)(x-1)+(y+1)(y+1)=0$

$x^2-1+y^2+2y+1=0$

$x^2+y^2+2y=0$

Equation of circle with diameter AD is

$(x+1)(x+1)+(y-1)(y+1)=0$

$x^2+2x+1+y^2-1=0$

$x^2+y^2+2x=0$