RD Sharma Solutions Class 11 Maths Chapter 24 Ex 24.1

Circles Ex 24.1 Q1(i)

The general equation of circle is $(x-a)^2+(y-b)^2=r^2$ ....(A)

where (a, b) are centre and r is radius

$$\therefore \quad (x+2)^2+(y-3)^2=4^2$$

$$\Rightarrow \quad (x+2)^2+(y-3)^2=16$$

Circles Ex 24.1 Q1(ii)

The general equation of circle is $(x-a)^2+(y-b)^2=r^2$ ....(A)

where (a, b) are centre and r is radius

From (A)

$$(x-a)^2+(y-b)^2=\left(\sqrt{a^2+b^2}\right)^2$$

$$\Rightarrow \quad x^2-2ax+a^2+y^2-2bx+b^2=a^2+b^2$$

$$\Rightarrow \quad x^2+y^2-2ax-2by=0$$

Circles Ex 24.1 Q1(iii)

The general equation of circle is $(x-a)^2+(y-b)^2=r^2$ ....(A)

where (a,b) are centre and r is radius

From (A)

$$(x-0)^2+(y+1)^2=1^2$$

$$\Rightarrow \quad x^2+y^2+2y+1=1$$

$$\Rightarrow \quad x^2+y^2+2y=0$$

Circles Ex 24.1 Q1(iv)

The general equation of circle is $(x-a)^2+(y-b)^2=r^2$ ....(A)

where (a,b) are centre and r is radius

From (A)

$$(x-a\cos\alpha)^2+(y-a\sin\alpha)^2=a^2$$

$$\Rightarrow \quad x^2-2a\cos\alpha x+y^2-2a\sin\alpha y+a^2\left(\cos^2\alpha+\sin^2\alpha\right)=a^2$$

$$\Rightarrow \quad x^2+y^2-2a\cos\alpha x-2a\sin\alpha y=0$$

Circles Ex 24.1 Q1(v)

The general equation of circle is $(x-a)^2+(y-b)^2=r^2$ ....(A)

where (a,b) are centre and r is radius

From (A)

$$(x-a)^2+(y-a)^2=\left(\sqrt{2}a\right)^2$$

$$\Rightarrow \quad x^2-2ax+a^2+y^2-2ay+a^2=2a^2$$

$$\Rightarrow \quad x^2+y^2-2ax-2ay=0$$

Circles Ex 24.1 Q2

The general equation of circle is $(x-a)^2+(y-b)^2=r^2$

or $x^2+y^2-2ax-2by+a^2+b^2=r^2$ ....(A)

Where (a,b) is the centre and r be the radius of the circle.

(i) $(x-1)^2+y^2=4$

$\Rightarrow \quad (x-1)^2+(y-0)^2=2^2$

Comparing with (A) we get,

$(1,0)$ is the centre

2 is the radius

(ii) $(x+5)^2+(y+1)^2=9$

$\Rightarrow (x+5)^2+(y+1)^2=3^2$

Comparing with (A), we get

centre $=(-5,-1)$

radius $=3$

(iii) $x^2+y^2-4x+6y=5$

$\Rightarrow (x^2-4x+4)+(y^2+6y+9)=5+4+9$

$\Rightarrow (x-2)^2+(y+3)^2=(3\sqrt{2})^2$

Comparing with (A), we get

centre $=(2,-3)$

radius $=3\sqrt{2}$

(iv) $x^2+y^2-x+2y=3$

$\Rightarrow \left(x^2-x+\frac{1}{4}\right)+(y^2+2y+1)=3+\frac{1}{4}+1$

$\Rightarrow \left(x-\frac{1}{2}\right)^2+(y+1)^2=\left(\frac{\sqrt{17}}{2}\right)^2$

Comparing with (A), we get

centre $=\left(\frac{1}{2},-1\right)$

radius $=\frac{\sqrt{17}}{2}$

Circles Ex 24.1 Q3

We know that the equation of circle whose centre in (a,b) and radius r is

$$(x-a)^2+(y-b)^2=r^2 \text{ ....(1)}$$

We have centre $=(1,2)$

$\therefore \quad (x-1)^2+(y-2)^2=r^2$ ....(2)

Also, circle passes through $(4,6)$

$\therefore \quad (4-1)^2+(6-2)^2=r^2$

$\Rightarrow \quad 9+16=r^2$

$\Rightarrow \quad r=5$

Thus, equation of required circle in

$$(x-1)^2+(y-2)^2=5^2$$

or

$$x^2+y^2-2x-4y-20=0$$

Circles Ex 24.1 Q4

The given equations of lines are

$$x + 3y = 0 \ldots\ldots\ldots\ldots\ldots(1)$$
$$2x - 7y = 0 \ldots\ldots\ldots\ldots(2)$$
$$x + y = -1 \ldots\ldots\ldots\ldots\ldots(3)$$
$$x - 2y = -4 \ldots\ldots\ldots\ldots(4)$$

The general equation of circle with centre (a, b) and radius r is

$$(x - a)^2 + (y - b)^2 = r^2 \ldots\ldots\ldots(A)$$

centre of (A) is the point of intersection of (iii) & (iv)

$\therefore$ centre $= (-2, 1)$

$\therefore (A)$

$\Rightarrow \quad (x + 2)^2 + (y - 1)^2 = r^2 \ldots\ldots\ldots(B)$

Also, (A) passes through point of intersection of (1) & (2), that is through $P = (0, 0)$

$\therefore \quad 2^2 + (-1)^2 = r^2 \quad \Rightarrow \quad r = \sqrt{5}$

Thus, the equation of required circle is

$$(x + 2)^2 + (y - 1)^2 = 5$$

or

$$x^2 + y^2 + 4x - 2y = 0$$

Circles Ex 24.1 Q5

The genral equation of circle, with centre (a, b) and radius r in

$$(x - a)^2 + (y - b)^2 = r^2 \ldots\ldots\ldots(A)$$

Now,

According to the question

centre $= (0, 6)$ and radius $= 4$

$\therefore (A)$

$\Rightarrow \quad (x - 0)^2 + (y - 6)^2 = 4^2$

$\Rightarrow \quad x^2 + y^2 - 12y + 20 = 0$

Circles Ex 24.1 Q6

The equation of two diameters of the circle $(x - a)^2 + (y - b)^2 = r^2 \ldots\ldots\ldots(A)$

is $2x + y = 6 \ldots\ldots\ldots\ldots(1)$

$3x + 2y = 4 \ldots\ldots\ldots(2)$

The point of intersection of (i) & (2) is $C = (8, -10)$, which is the centre of circle.

Also, radius $= 10$

$\therefore (A)$

$\Rightarrow \quad (x - 8)^2 + (y + 10)^2 = 10^2$

$\Rightarrow \quad x^2 + y^2 - 16x + 20y + 64 = 0$

Circles Ex 24.1 Q7(i)

The circle touches the *axes* at $(0,6)$ and $(6,0)$ respectively

Thus, the centre of circle will be $(6,6)$ (as shown in fig)

and radius $= OA = \sqrt{(6-0)^2 + (6-6)^2} = \sqrt{36} = 6$ (by distance formulla)

$\therefore$ the equation of circle will be $(x-6)^2 + (y-6)^2 = 6^2$

$\Rightarrow \quad x^2 + y^2 - 12x - 12y + 36 = 0$

Circles Ex 24.1 Q7(ii)

The circle touches the $x-axis$ at $A = (5,0)$ and has radius 6 unit

Thus,

centre $= (5, b)$

By distance formulla $OA = 6$

$\Rightarrow \quad \sqrt{(5-5)^2 + (b-0)^2} = 6$

$\Rightarrow \quad b = 6 \qquad \Rightarrow$ centre $= (5,6)$

so, the equation of required circle is

$(x-5)^2 + (y-6)^2 = 6^2$

$\Rightarrow \quad x^2 + y^2 - 10x - 12y + 25 = 0$

Circles Ex 24.1 Q7(iii)

The circle touches both the *axis* at $A = (a,0)$ and $B = (0,a)$

so, the centre of circle will be (a,a) and radius $= a$.

so, the equation of circle is $(x-a)^2 + (y-a)^2 = a^2$ .... (A)

Now,

(A) Passes through $P(2,1)$

$\therefore \quad (2-a)^2 + (1-a)^2 = a^2$

$\Rightarrow \quad 4 - 4a + a^2 + 1 - 2a + a^2 = a^2$

$\Rightarrow \quad 5 - 6a + a^2 = 0$

$\Rightarrow \quad (a-5)(a-1) = 0$

$\Rightarrow \quad a = 5$ or 1

Thus the equation of circle will be

$x^2 - 10x + y^2 - 10y + 25 = 0, \qquad x^2 + y^2 - 2x - 2y + 1 = 0$

Circles Ex 24.1 Q7(iv)

The circle passes through origin $(0,0)$ and has radius = 17 units
Also, the ordinate of centre is -15 then assume abssisa is a.

$\therefore \quad OC = 17$

$\Rightarrow \quad \sqrt{(a-0)^2 + (0+15)^2} = 17 \quad$ (by distance formulla)

$\Rightarrow \quad \sqrt{a^2 + 225} = 17$

$\Rightarrow \quad a^2 + 225 = 289$

$\Rightarrow \quad a^2 = 64 \qquad \Rightarrow \quad a = \pm 8$

$\therefore$ centre = $(\pm 8, -15)$

Thus, the equation of circle will be,

$$(x \pm 8)^2 + (y+15)^2 = 17^2$$

$$\Rightarrow \quad x^2 + y^2 \mp 16x + 30y = 0$$

Circles Ex 24.1 Q8

The centre of the required circle in $(3,4)$ and the circle touches the line $5x + 12y = 1$

so, radius = OA = Perpendicular distance of O to $5x + 12y = 1$
[$\because$ radius is perpendicular to the tangent]

$$\Rightarrow \quad OA = \frac{5 \times 3 + 12 \times 4 - 1}{\sqrt{5^2 + 12^2}}$$

$$= \frac{62}{13}$$

Thus the equation of circle will be,

$$(x-3)^2 + (y-4)^2 = \left(\frac{62}{13}\right)^2$$

$$\Rightarrow \quad 169\left[x^2 + y^2 - 6x - 8y\right] + 25 \times 169 = 3844$$

$$\Rightarrow \quad 169\left[x^2 + y^2 - 6x - 8y\right] + 381 = 0$$

Circles Ex 24.1 Q9

The required circle touches $A(a,0)$ and $B(0,a)$ on the *axes*
so the centre = (a,a) P radius = a

Also, the centre lies on $x - 2y = 3$

$\Rightarrow \quad a - 2a = 3$

$\Rightarrow \quad -a = 3 \qquad \Rightarrow a = -3$

$\therefore$ centre = $(-3,-3)$ and radius = 3.

Thus the equation of circle is

$$(x+3)^2 + (y+3)^2 = 3^2$$

$$\Rightarrow \quad x^2 + y^2 + 6x + 6y + 9 = 0$$

Circles Ex 24.1 Q10

We heve,

$$2x - 3y = -4 \ldots\ldots\ldots (1)$$
$$3x + 4y = 5 \ldots\ldots\ldots\ldots (2)$$

The point of intersection of (1) & (2) is

$$P = \left(\frac{-1}{17}, \frac{66}{51}\right) \text{or } P = \left(\frac{-1}{17}, \frac{22}{17}\right)$$

According to the equation centre = $\left(\frac{-1}{17}, \frac{22}{17}\right)$

Also, the circle passes through $0(0, 0)$

$$\therefore \quad r = OC = \sqrt{\left(0 + \frac{1}{17}\right)^2 + \left(0 - \frac{22}{17}\right)^2}$$

$$= \sqrt{\frac{1}{289} + \frac{484}{289}} = \frac{\sqrt{485}}{17}$$

Thus, the required equation of circle is

$$\left(x + \frac{1}{17}\right)^2 + \left(y - \frac{22}{17}\right)^2 = \frac{485}{289}$$

Circles Ex 24.1 Q11

We are given that a circle has radius 4 and touches the coordinate *axes* in 1^{st} quadrant.

Thus the centre = $(4, 4)$

Now C_2 and C_3 are the images of C_1 with respect to $y = 0$ and $x = 0$

so, for C_2

centre = $(-4, 4)$ and radius = 4

Thus the equation of circle C_2 is $(x + 4)^2 + (y - 4)^2 = 4^2$

$$\Rightarrow \quad x^2 + y^2 + 8x - 8y + 16 = 0$$

And for C_3

centre = $(4, -4)$ and radius = 4

Thus, the equation of circle C_3 is $(x - 4)^2 (y + 4)^2 = 4^2$

$$\Rightarrow \quad x^2 + y^2 - 8x + 8y + 16 = 0$$

Circles Ex 24.1 Q12

The circle touches $y - axis$ at $C(0,3)$ and makes an intercept AB of 8 units on $x - axis$.

$\therefore \quad AB = 8$

Let M be the mid-point of AB

We know $OM = 3$ and $AM = 4$ $\quad \left(\because AM = \frac{1}{2} AB\right)$

$\therefore$ In $\triangle AOM$,

$$AO^2 = AM^2 + MO^2 = 4^2 + 3^2 = 25$$

$\therefore \quad AO = 5$

Also, $\quad AO = CO = 5 \quad$ (radius)

$\therefore$ centre $= O = (5,3)$

Thus, the equation of circle is

$$(x-5)^2 + (y-3)^2 = 5^2$$

$\Rightarrow \quad x^2 + y^2 \pm 10x - 6y + 9 = 0$

Circles Ex 24.1 Q13

Here, circle is passing through two point $(0, 3), (0, -3)$ and radius is 5

Equation of circle is,

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-h)^2 + (y-k)^2 = 25 ------(1)$$

It is passing through $(0, 3)$

$$(0-h)^2 + (3-k)^2 = 25$$

$$h^2 + (3-k)^2 = 25 --------(2)$$

It is also passing through $(0, -3)$

$$(0-h)^2 + (-3-k)^2 = 25$$

$$h^2 + (3+k)^2 = 25 --------(3)$$

$[(2)-(3)]$,

$$(3-k)^2 - (3+k)^2 = 0$$

$$(3-k+3+k)(3-k-3-k) = 0$$

$$6(-2k) = 0$$

$$k = 0$$

Put $k = 0$ in equation (2)

$$h^2 + (3-0)^2 = 25$$

$$h^2 = 25 - 9$$

$$h^2 = 16$$

$$h = \pm 4$$

Put $h = 4$ and $k = 0$ in equation (1),

$$(x-4)^2 + y^2 = 25$$

$$x^2 - 8x + y^2 = 9$$

Put $h = -4$ and $k = 0$ in equation (1)

$$(x+4)^2 + y^2 = 25$$

$$x^2 \pm 8x + y^2 = 9$$

Circles Ex 24.1 Q14

Area of given circle is =154

$\pi r^2 = 154$

$\frac{22}{7} r^2 = 154$

$r^2 = 154 \times \frac{7}{22}$

$r^2 = 49$

$r = 7$

The intersection point of 2x-3y = 5 and 3x-4y = 7 is
The centre of the circle.

Solving simultaneous equations
2x-3y = 5 and 3x-4y = 7 we get,

Centre of circle as (1, -1)

Equation of circle with centre (1, -1) and radius = 7 is,

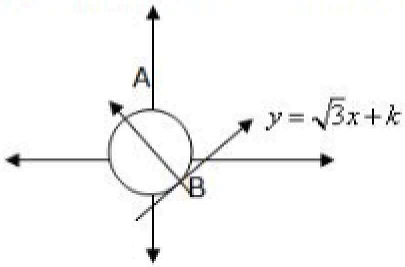
$(x-1)^2 + (y+1)^2 = 7^2$

$x^2 - 2x + 1 + y^2 + 2y + 1 = 49$

$x^2 - 2x + y^2 + 2y = 47$

Circles Ex 24.1 Q15

Centre is (0,0) and radius = 4 as shown in figure.



AB be a line passing through centre of circle. Tangent $y=\sqrt{3}x+k$ touches the circle at B(a,b)

$a^2+b^2=16$...............(1)

AB is perpendicular to tangent.

Slope of AB $=-\frac{1}{\sqrt{3}}$

Equation of AB is

$y=-\frac{1}{\sqrt{3}}x$ ....[AB passes through centre (0,0)]

$b=-\frac{1}{\sqrt{3}}a$ ....(2)

Substituting (2) in (1), we get,

$$a^2+\frac{1}{3}a^2=16$$

$$\frac{4a^2}{3}=16$$

$$a=\pm 2\sqrt{3}$$

$$b=\mp 2$$

B(a,b) is on $y=\sqrt{3}x+k$

$$\mp 2=\pm\sqrt{3}(2\sqrt{3})+k$$

$$\pm 2=\mp 6+k$$

$$k=\pm 8$$

Circles Ex 24.1 Q16

Intersection of 3x + y = 14 and 2x + 5y = 18 is Obtained by solving two equations.

x = 4 and y = 2

Point (4,2) is on circle, hence it's distance from centre (1,-2)

= Radius

$$=\sqrt{(1-4)^2+(-2-2)^2}$$

$$=\sqrt{9+16}$$

$$=5$$

Equation of the circle with centre (4,2) and radius 5 is,

$$(x-1)^2+(y+2)^2=25$$

$$x^2-2x+1+y^2+4y+4=25$$

$$x^2+y^2-2x+4y-20=0$$

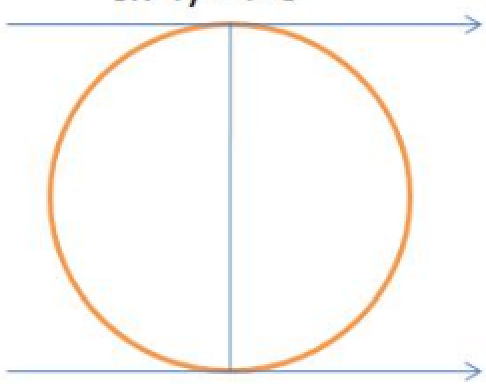
Circles Ex 24.1 Q17

Slope of $3x - 4y + 4 = 0$ is $\frac{4}{3}$

Slope of $6x - 8y - 7 = 0$ is $\frac{8}{6} = \frac{4}{3}$

Slope of $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are same.

Hence two lines are parallel and are shown in figure.



Rewriting $6x - 8y - 7 = 0$, we get,

$3x - 4y - \frac{7}{2} = 0$

Perpendicular distance between two lines =

$\left| \frac{4 + \frac{7}{2}}{\sqrt{9+16}} \right|$

$= \left| \frac{15}{10} \right|$

$= \frac{3}{2}$ units

Circles Ex 24.1 Q18

$x = \frac{2at}{1+t^2}, y = a\left(\frac{1-t^2}{1+t^2}\right)$

$x^2 + y^2 = \frac{4a^2t^2}{(1+t^2)^2} + \frac{a^2(1-t^2)^2}{(1+t^2)^2}$

$= \frac{4a^2t^2 + a^2(1 - 2t^2 + t^4)}{(1+t^2)^2}$

$= \frac{4a^2t^2 + a^2 - 2a^2t^2 + a^2t^4}{(1+t^2)^2}$

$= \frac{2a^2t^2 + a^2 + a^2t^4}{(1+t^2)^2}$

$= \frac{a^2(1 + 2t^2 + t^4)}{(1+t^2)^2}$

$x^2 + y^2 = a^2$ is equation of a circle.

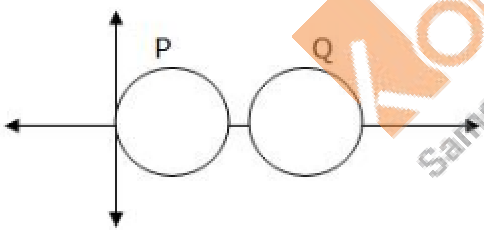
Circles Ex 24.1 Q19

Given circle is $x^2+y^2-2x-2y+1=0$
Rewriting the equation, we get,

$$x^2-2x+1+y^2-2y+1=1$$

$$(x-1)^2+(y-1)^2=1 \ldots\ldots\ldots(1)$$

The given circle has its centre at (1, 1) and radius =1 from (1).When circle is rolled on X-axis, it center moves horizontally through distance
$=2\pi$.
Figure shows circle with centre (1, 1) at P. After rolling it on X-axis, it takes the position Q.
The coordinates of it's centre become $(1, 1+2\pi)$.
Radius of the circle at Q = 1.



Hence, equation of new circle is

$$[x-(1+2\pi)^2]+(y-1)^2$$

Circles Ex 24.1 Q20

The centre O lies on the line $x - 4y = -7$ and the perpendicular bisector MO of AB.
The coordinates of M are $(1, 4)$.

Thus, the equation of MO is $x = 1$

Point of intersection of $x - 4y = -7$ and $x = 1$ is

$$O = (1, 2)$$

Also the radius of circle is

$$AO = \sqrt{(1+3)^2 + (2-4)^2}$$

$$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

Thus the equation of circle is

$$(x-1)^2 + (y-2)^2 = 20$$

$$\Rightarrow \quad x^2 + y^2 - 2x - 4y - 15 = 0$$

Circles Ex 24.1 Q21

The line $2x - y + 1 = 0$ touches the circle at $A(2, 5)$. The centre of circle lies on the line $m: x + y = 9$.

Now AO is perpendicular to $2x - y + 1 = 0$

$\therefore$ equation of AO is

$$x + 2y = d \text{ ....} (3)$$

But AO passes through $A(2, 5)$

$$\therefore d = 12$$

$\therefore$ equation of AO is

$$x + 2y = 12 \text{ ....} (4)$$

The point of intersection of $x + y = 9$ and $x + 2y = 12$ is $(6, 3)$ which is the centre of the circle.

Radius $= AO = \sqrt{(6-2)^2 + (3-5)^2} = \sqrt{16+4} = \sqrt{20}$

Hence, equation of circle is

$$(x-6)^2 + (y-3)^2 = 20$$