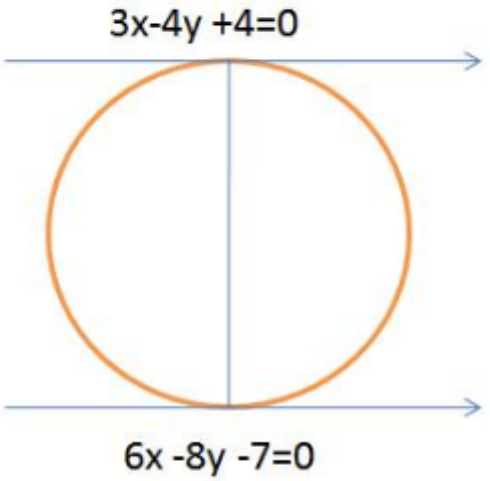
Circles Ex 24.1 Q17

Slope of $3x-4y+4=0$ is $\frac{4}{3}$

Slope of $6x-8y-7=0$ is $\frac{8}{6}=\frac{4}{3}$

Slope of $3x-4y+4=0$ and $6x-8y-7=0$ are same.

Hence two lines are parallel and are shown in figure.



Rewriting $6x-8y-7=0$, we get,

$3x-4y-\frac{7}{2}=0$

Perpendicular distance between two lines =

$$\left|\frac{4+\frac{7}{2}}{\sqrt{9+16}}\right|$$

$$=\left|\frac{15}{10}\right|$$

$$=\frac{3}{2}\text{ units}$$

Circles Ex 24.1 Q18

$$x=\frac{2at}{1+t^2}, y=a\left(\frac{1-t^2}{1+t^2}\right)$$

$$x^2+y^2=\frac{4a^2t^2}{(1+t^2)^2}+\frac{a^2(1-t^2)^2}{(1+t^2)^2}$$

$$=\frac{4a^2t^2+a^2(1-2t^2+t^4)}{(1+t^2)^2}$$

$$=\frac{4a^2t^2+a^2-2a^2t^2+a^2t^4}{(1+t^2)^2}$$

$$=\frac{2a^2t^2+a^2+a^2t^4}{(1+t^2)^2}$$

$$=\frac{a^2(1+2t^2+t^4)}{(1+t^2)^2}$$

$x^2+y^2=a^2$ is equation of a circle.

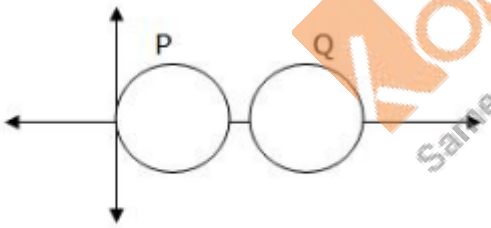
Circles Ex 24.1 Q19

Given circle is $x^2+y^2-2x-2y+1=0$
Rewriting the equation, we get,

$$x^2-2x+1+y^2-2y+1=1$$

$$(x-1)^2+(y-1)^2=1 \quad \ldots\ldots\ldots(1)$$

The given circle has its centre at (1, 1) and radius =1
from (1).When circle is rolled on X-axis, it center moves horizontally through distance
$=2\pi$.
Figure shows circle with centre (1, 1) at P. After rolling it on X-axis, it takes the position Q.
The coordinates of it's centre become $(1, 1+2\pi)$.
Radius of the circle at Q = 1.



Hence, equation of new circle is
$[x-(1+2\pi)^2]+(y-1)^2$

Circles Ex 24.1 Q20

The centre O lies on the line $x - 4y = -7$ and the perpendicular bisector MO of AB.
The coordinates of $Mare\ (1, 4)$.

Thus, the equation of MO is $x = 1$

Point of intersection of $x - 4y = -7\ and\ x = 1$ is

$$O = (1, 2)$$

Also the radius of circle is

$$AO = \sqrt{(1+3)^2 + (2-4)^2}$$

$$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

Thus the equation of circle is

$$(x-1)^2 + (y-2)^2 = 20$$

$$\Rightarrow \quad x^2 + y^2 - 2x - 4y - 15 = 0$$

Circles Ex 24.1 Q21

The line $2x - y + 1 = 0$ touches the circle at $A(2, 5)$. The centre of circle lies on the line $m : x + y = 9$.

Now AO is perpendicular to $2x - y + 1 = 0$

$\therefore$ equation of AO is

$$x + 2y = d \ldots\ldots\ldots (3)$$

But AO passes through $A(2, 5)$

$$\therefore\ d = 12$$

$\therefore$ equation of AO is

$$x + 2y = 12 \ldots\ldots\ldots\ldots (4)$$

The point of intersection of $x + y = 9$ and $x + 2y = 12$ is $(6, 3)$ which is the centre of the circle.

Radius = $AO = \sqrt{(6-2)^2 + (3-5)^2} = \sqrt{16+4} = \sqrt{20}$

Hence, equation of circle is

$$(x-6)^2 + (y-3)^2 = 20$$