RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.16

Straight lines Ex 23.16 Q1

Determine between parallel lines

$ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is

$$\left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right|$$

(i) $4x - 3y - 9 = 0$ and $4x - 3y - 24 = 0$

Distance between the two parallel lines is

$$\left|\frac{-24 - (-9)}{\sqrt{4^2 + 3^2}}\right| = \left|\frac{-24 + 9}{5}\right|$$

$= 3$ units.

(ii) Distance between $8x + 15y - 34 = 0$ and $8x + 15y + 31 = 0$

is $$\left|\frac{-34 - 31}{\sqrt{8^2 + 15^2}}\right| = \frac{65}{17} \text{ units}$$

(iii) Distance between $y = mx + c$ and $y = mx + d$

is $$\left|\frac{c - d}{\sqrt{m^2 + 1}}\right|$$

(iv) Distance between $4x + 3y - 11 = 0$ and $8x + 6y = 15$

is $$\left|\frac{-11 - 15}{\sqrt{4^2 + 3^2}}\right| = \frac{7}{10} \text{ units.}$$

Straight lines Ex 23.16 Q2

The two sides of square are

$5x - 12y - 65 = 0$ and $5x - 12y + 26 = 0$

The distance between these two parallel sides $\left(\text{as both have slope } \frac{5}{12}\right)$ is

$$\left|\frac{-65 - 26}{\sqrt{5^2 + 12^2}}\right| = \left|\frac{-91}{13}\right| = 7 \text{ units.}$$

And all sides of square are equal.

$\therefore$ Area of the square is $7 \times 7 = 49$ sq units.

Straight lines Ex 23.16 Q3

Let the required equation be $y = mx + c$ where m is slope of the line which is equal to slope of $x + 7y + 2 = 0 \left(\text{i.e} \frac{-1}{7}\right)$ as the two lines are parallel.

$\therefore$ The required equation is $y = \frac{-1}{7}x + c$ which is a unit distance from $(1,1)$.

$$\therefore \quad \left|\frac{7(1) + (1) - 7c}{\sqrt{49 + 1}}\right| = 1$$

$8 - 7c = \sqrt{50}$

$$64 + 49c^2 - 112c = 50$$

$$49c^2 - 112C - 14 = 0$$

$$7c^2 - 16c - 2 = 0$$

$$C = \frac{6 \pm 5\sqrt{2}}{7} \qquad \left[\text{using } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$\therefore$ The required equation is.

$$y = \frac{-1}{7}x + \frac{6 \pm 5\sqrt{2}}{7}$$

or $\quad 7y + x + 6 \pm 5\sqrt{2} = 0$

Straight lines Ex 23.16 Q4

Since the coefficient of x and y in the equations $2x+3y-19=0$, $2x+3y-6=0$ and $2x+3y+7=0$ are same, therefore all the lines are parallel.

Distance between parallel lines is $d=\left|\frac{c_2-c_1}{\sqrt{a^2+b^2}}\right|$, where $ax+by+c_1=0$ and $ax+by+c_2=0$ are the lines parallel to each other.

Distance between the lines $2x+3y-19=0$ and $2x+3y-6=0$ is

$$d_1=\left|\frac{-19+6}{\sqrt{2^2+3^2}}\right|=\left|\frac{13}{\sqrt{13}}\right|=\sqrt{13}$$

Distance between the lines $2x+3y+7=0$ and $2x+3y-6=0$ is

$$d_2=\left|\frac{7+6}{\sqrt{2^2+3^2}}\right|=\left|\frac{13}{\sqrt{13}}\right|=\sqrt{13}$$

Since the distances of both the lines $2x+3y+7=0$ and $2x+3y-19=0$ from the line $2x+3y-6=0$ are equal, therefore the lines are equidistant.

Straight lines Ex 23.16 Q5

The equation of lines are

$$3x+2y-\frac{7}{3}=0 \qquad \text{---(i)}$$

$$3x+2y+6=0 \qquad \text{---(ii)}$$

Let equation of mid way be $3x+2y+\lambda=0$ ---(iii)

Then, distance between (i) and (iii) and (ii) and (iii) should be equal.

$$\left|\frac{\lambda+\frac{7}{3}}{\sqrt{9+4}}\right|=\left|\frac{\lambda-6}{\sqrt{9+4}}\right|$$

$$\Rightarrow \quad \lambda+\frac{7}{3}=-\pi+6$$

$$\Rightarrow \quad \lambda=\frac{11}{6}$$

$\therefore$ The required line is $3x + 2y + \frac{11}{6} = 0$ or $18x + 12y + 11 = 0$.

Straight lines Ex 23.16 Q6

Clearly, the slope of each of the given lines is same equal to $-\frac{3}{4}$.

Hence, the line $3x + 4y + 2 = 0$ is parallel to each of the given lines.

Putting $y = 0$ in $3x + 4y + 2 = 0$, we get $x = -\frac{2}{3}$.

So, the coordinates of a point on $3x + 4y + 2 = 0$ are $\left(-\frac{2}{3}, 0\right)$.

The distance d_1 between the lines $3x + 4y + 2 = 0$ and $3x + 4y + 5 = 0$ is given by

$$d_1 = \left|\frac{3\left(-\frac{2}{3}\right) + 4(0) + 5}{\sqrt{3^2 + 4^2}}\right| = \frac{3}{5}$$

The distance d_2 between the lines $3x + 4y + 2 = 0$ and $3x + 4y - 5 = 0$ is given by

$$d_2 = \left|\frac{3\left(-\frac{2}{3}\right) + 4(0) - 5}{\sqrt{3^2 + 4^2}}\right| = \frac{7}{5}$$

$$\frac{d_1}{d_2} = \frac{\frac{3}{5}}{\frac{7}{5}} = \frac{3}{7}$$

So $3x + 4y + 2 = 0$ divides the distance between the lines $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$ in the ratio 3:7.