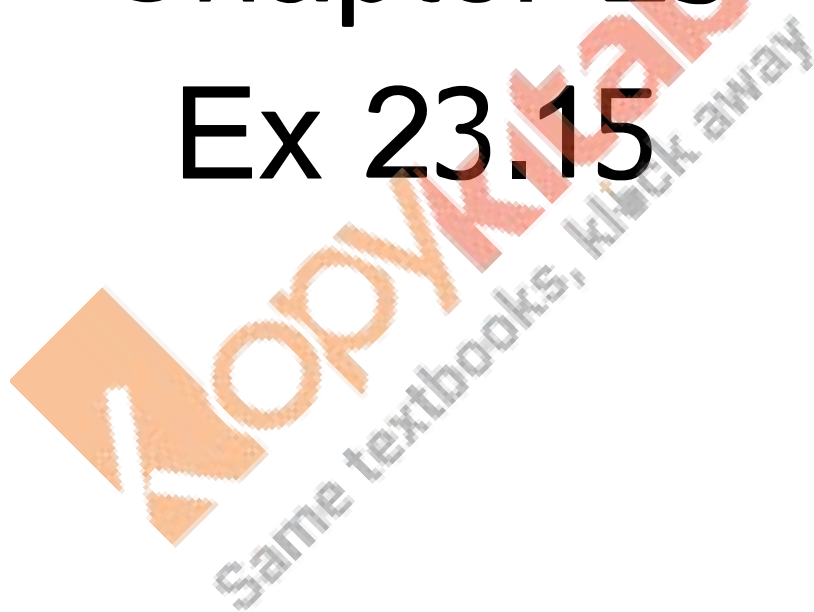


RD Sharma
Solutions
Class 11 Maths
Chapter 23
Ex 23.15



Straight lines Ex 23.15 Q1

Distance of a point (x_1, y_1) from $ax + by + c = 0$ is

$$= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Here, $a = 3$, $b = -5$, $c = 7$, $x_1 = 4$, $y_1 = 5$

$$\begin{aligned}\therefore \text{Distance} &= \frac{|3(4) - 5(5) + 7|}{\sqrt{3^2 + 5^2}} \\ &= \frac{|12 - 25 + 7|}{\sqrt{9 + 25}} = \frac{|6|}{\sqrt{34}} \text{ units.}\end{aligned}$$

Straight lines Ex 23.15 Q2

Equation of line passing through $(\cos \theta, \sin \theta)$ and $(\cos \phi, \sin \phi)$ is

$$y - \sin \phi = \left(\frac{\sin \phi - \sin \theta}{\cos \phi - \cos \theta} \right) (x - \cos \phi)$$

$$y - \sin \phi = \left(\frac{2 \cos \frac{\theta + \phi}{2} \sin \frac{\phi - \theta}{2}}{-2 \sin \frac{\theta + \phi}{2} \sin \frac{\phi - \theta}{2}} \right) (x - \cos \phi)$$

$$y - \sin \phi = -\cot \left(\frac{\theta + \phi}{2} \right) (x - \cos \phi)$$

$$x \cot \left(\frac{\theta + \phi}{2} \right) + y - \sin \phi - \cos \phi \cot \left(\frac{\theta + \phi}{2} \right) = 0$$

Distance of this line from origin,

$$\begin{aligned}&= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \\ &= \frac{|0 + 0 - \sin \phi - \cos \phi \cot \left(\frac{\theta + \phi}{2} \right)|}{\sqrt{\left(\cos \left(\frac{\theta + \phi}{2} \right) \right)^2 + 1}} \\ &= \frac{\sin \phi + \cos \phi \cot \left(\frac{\theta + \phi}{2} \right)}{\operatorname{cosec} \left(\frac{\theta + \phi}{2} \right)}\end{aligned}$$

$$\begin{aligned}
 &= \sin \phi \sin \left(\frac{\theta + \phi}{2} \right) + \cos \phi \cos \left(\frac{\theta + \phi}{2} \right) \\
 &= \cos \left(\frac{\theta + \phi}{2} - \phi \right) \\
 &= \cos \left(\frac{\theta + \phi - 2\phi}{2} \right)
 \end{aligned}$$

$$D = \cos \left(\frac{\theta - \phi}{2} \right)$$

Straight lines Ex 23.15 Q3

Line formed from joining $(a \cos \alpha, a \sin \alpha)$ and $(a \cos \beta, a \sin \beta)$

$$\begin{aligned}
 \Rightarrow y - a \sin \beta &= \frac{a \sin \beta - a \sin \alpha}{a \cos \beta - a \cos \alpha} \times (x - a \cos \beta) \\
 \Rightarrow y - a \sin \beta &= \frac{2 \sin \left(\frac{\beta - \alpha}{2} \right) \cos \left(\frac{\beta + \alpha}{2} \right)}{-2 \sin \left(\frac{\beta - \alpha}{2} \right) \sin \left(\frac{\beta + \alpha}{2} \right)} \times (x - a \cos \beta) \\
 \Rightarrow y - a \sin \beta &= -\cot \left(\frac{\beta + \alpha}{2} \right) (x - a \cos \beta) \\
 \Rightarrow y + \cot \left(\frac{\alpha + \beta}{2} \right) x - a \cos \beta \cot \left(\frac{\beta + \alpha}{2} \right) - a \sin \beta &= 0
 \end{aligned}$$

Then, the length of perpendicular

$$\begin{aligned}
 \Rightarrow & \frac{|0(y) + 0 - a \cos \beta \cot \left(\frac{\beta + \alpha}{2} \right) - a \sin \beta|}{\sqrt{1 + \cot^2 \left(\frac{\alpha + \beta}{2} \right)}} \\
 \Rightarrow & \frac{a \cos \beta \cot \left(\frac{\alpha + \beta}{2} \right) + a \sin \beta}{\operatorname{cosec} \left(\frac{\alpha + \beta}{2} \right)} \\
 \Rightarrow & a \cos \beta \cos \left(\frac{\alpha + \beta}{2} \right) + a \sin \beta \sin \left(\frac{\alpha + \beta}{2} \right) \\
 \Rightarrow & a \cos \left(\frac{\alpha - \beta}{2} \right) \quad \left[\text{using } \cos A \cos B + \sin A \sin B = \cos (A - B) \right]
 \end{aligned}$$

Hence, proved.

Straight lines Ex 23.15 Q4

Let (h, k) be the point on the line $2x + 11y - 5 = 0$

$$\Rightarrow 2h + 11k - 5 = 0 \quad \text{--- (1)}$$

Let p and q be length of perpendicular from (h, k) on lines $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$ so,

$$\begin{aligned}
 p &= q \\
 \frac{24h + 7k - 20}{\sqrt{(24)^2 + (7)^2}} &= \frac{4h - 3k - 2}{\sqrt{(4)^2 + (-3)^2}} \\
 \frac{24h + 7k - 20}{\sqrt{576 + 49}} &= \frac{4h - 3k - 2}{\sqrt{25}} \\
 \frac{24h + 7k - 20}{25} &= \frac{4h - 3k - 2}{5}
 \end{aligned}$$

$$24h + 7k - 20 = 20h - 15k - 10$$

$$4h = -22k + 10$$

$$4 \left(\frac{5 - 11k}{4} \right) = -22k + 10 \quad \left[\text{Using equation (1)} \right]$$

$$10 - 22k = -22k + 10$$

$$LHS = RHS$$

So,

Distance $24x + 7y = 20$ and $4x - 3y - 2 = 0$ from any point on the line $2x + 11y - 5 = 0$ is equal.

Straight lines Ex 23.15 Q5

The point of intersection of two lines can be calculated by solving the equations

Solving $2x + 3y = 21$ and $3x - 4y + 11 = 0$, we get the point of intersection as $P(3, -5)$

Distance of P from $8x - 6y + 5 = 0$ is

Here, $a = 8$, $b = -6$, $c = 5$, $x_1 = 3$, $y_1 = -5$

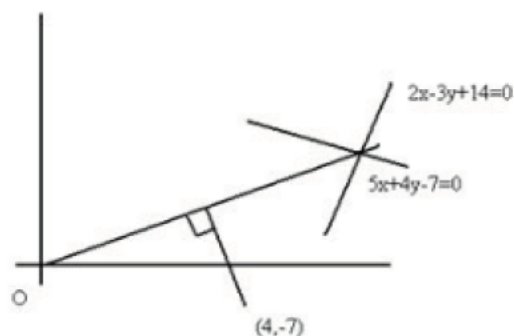
$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow \frac{|8(3) - 6(-5) + 5|}{\sqrt{64 + 36}}$$

$$\Rightarrow \frac{|24 + 30 + 5|}{\sqrt{100}} = \frac{|59|}{10}$$

$$\Rightarrow \frac{59}{10}$$

Straight lines Ex 23.15 Q6



The point of intersection of the lines $2x - 3y + 14 = 0$ and $5x + 4y - 7 = 0$ can be found out by solving these equations.

Solving these equations we get, $x = -\frac{35}{23}$ and $y = \frac{252}{69}$

Equation of line joining origin and the point $\left(-\frac{35}{23}, \frac{252}{69}\right)$

is $y = mx$, where $m = \frac{\frac{252}{69}}{-\frac{35}{23}} = -\frac{12}{5}$

Therefore the equation of required line is $y = -\frac{12x}{5}$

$$12x + 5y = 0$$

Perpendicular distance from $(4, -7)$ to $12x + 5y = 0$ is

$$p = \frac{|12(4) + 5(-7)|}{\sqrt{12^2 + (-5)^2}} = \frac{13}{13} = 1$$

Straight lines Ex 23.15 Q7

Any point on x-axis is $(\pm a, 0)$
 (x_1, y_1)

Perpendicular distance from a line $bx + ay = ab$ is

$$\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right| = a$$

Where,

$$a = b, b = a, c = -ab, x_1 = \pm a, y_1 = 0$$

$$= \left| \frac{b(x) + a(0) - ab}{\sqrt{a^2 + b^2}} \right| = a$$

$$a = 0 \quad \text{or}$$

$$\frac{b(x) + a(0) - ab}{\sqrt{a^2 + b^2}} = a$$

$$\frac{b}{a}x = \pm \sqrt{a^2 + b^2} + b$$

$$x = \frac{a}{b} \left(b \pm \sqrt{a^2 + b^2} \right)$$

$$x = 0$$

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