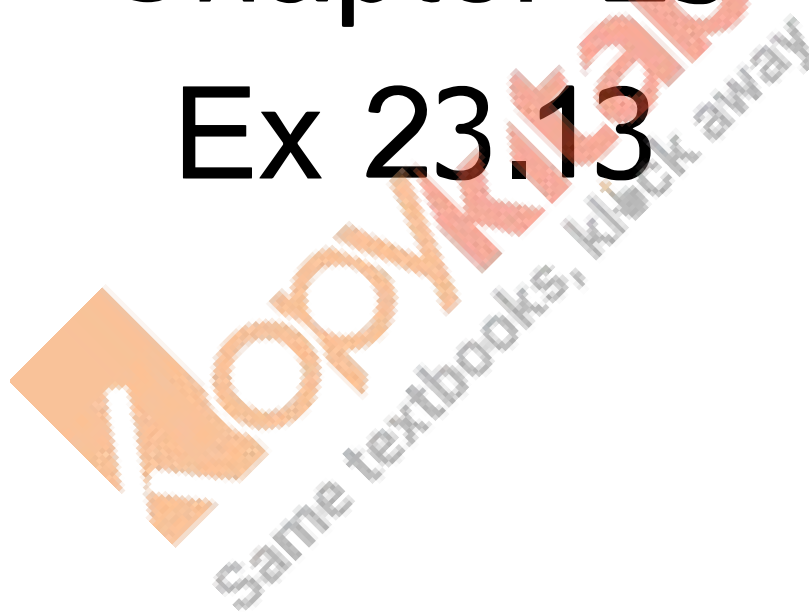


RD Sharma
Solutions
Class 11 Maths
Chapter 23
Ex 23.13



Straight lines Ex 23.13 Q1(i)

Writing the equation in the form

$$y = mx + c$$

$$3x + y + 12 = 0$$

$$y = -3x - 12$$

$$\Rightarrow m_1 = -3$$

Also

$$x + 2y - 1 = 0$$

$$2y = 1 - x$$

$$y = \frac{1}{2} - \frac{x}{2}$$

$$\Rightarrow m_2 = \frac{-1}{2}$$

Angle between the lines

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{-3 - \left(\frac{-1}{2}\right)}{1 + (-3)\left(\frac{-1}{2}\right)} \right|$$

$$= \left| \frac{-3 + \frac{1}{2}}{1 + \frac{3}{2}} \right| = \left| \frac{\frac{-6 + 1}{2}}{\frac{2 + 3}{2}} \right|$$

$$= \left| \frac{-5}{5} \right| = 1$$

$$\Rightarrow \text{angle} = \frac{\pi}{4}$$

Straight lines Ex 23.13 Q(ii)

Finding slopes of the lines by converting the equation in the form

$$y = mx + c$$

$$3x - y + 5 = 0$$

$$\Rightarrow y = 3x + 5$$

$$\Rightarrow m_1 = 3$$

Also

$$x - 3y + 1 = 0$$

$$3y = x + 1$$

$$y = \frac{x}{3} + \frac{1}{3}$$

$$\Rightarrow m_2 = \frac{1}{3}$$

Thus angle between the lines is

$$\tan \theta = \left| \frac{m_1 - m_2}{m_1 m_2} \right|$$

$$= \left| \frac{3 - \frac{1}{3}}{1 + 3 \times \frac{1}{3}} \right| = \left| \frac{\frac{9-1}{3}}{1+1} \right|$$

$$= \left| \frac{\frac{8}{3}}{2} \right| = \left| \frac{8}{6} \right| = \frac{4}{3}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{4}{3} \right)$$

Straight lines Ex 23.13 Q(iii)

To find angle between the lines, convert the equations in the form

$$y = mx + c$$

$$3x + 4y - 7 = 0$$

$$\Rightarrow 4y = -3x + 7$$

$$y = \frac{-3}{4}x + \frac{7}{4}$$

$$\Rightarrow m_1 = \frac{-3}{4}$$

Also, $4x - 3y + 5 = 0$

$$\Rightarrow 3y = 4x + 5$$

$$\Rightarrow y = \frac{4}{3}x + \frac{5}{3}$$

$$\Rightarrow m_2 = \frac{4}{3}$$

The angle between the lines is given by $\tan \theta = \frac{|m_1 - m_2|}{1 + m_1 m_2}$

$$= \frac{\left| \frac{-3}{4} - \frac{4}{3} \right|}{1 + \frac{(-3)}{4} \left(\frac{4}{3} \right)} = \frac{\left| \frac{-3}{4} - \frac{4}{3} \right|}{1 - 1}$$

$$\Rightarrow \theta = \frac{\pi}{2} \text{ or } 90^\circ$$

Straight lines Ex 23.13 Q(iv)

To find angle convert the equation in the form $y = mx + c$

$$x - 4y = 3$$

$$\Rightarrow 4y = x - 3$$

$$y = \frac{x}{4} - \frac{3}{4}$$

$$\Rightarrow m_1 = \frac{1}{4}$$

$$\begin{aligned}\text{Also, } 6x - y &= 11 \\ y &= 6x - 11 \\ \Rightarrow m_2 &= 6\end{aligned}$$

Thus, angle between the lines is

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{\frac{1}{4} - 6}{1 + \frac{1}{4} \times 6} \right|$$

$$= \left| \frac{-\frac{23}{4}}{1 + \frac{3}{2}} \right| = \left| \frac{-\frac{23}{4}}{\frac{5}{2}} \right|$$

$$\theta = \tan^{-1} \left(\frac{23}{10} \right)$$

Straight lines Ex 23.13 Q(v)

Converting the equation in the form

$$y = mx + c$$

$$y = \frac{(mn + n^2)}{m^2 - mn}x + \frac{n^3}{(m^2 - mn)}$$

$$\Rightarrow m_1 = \frac{mn + n^2}{m^2 - mn}$$

$$\text{Also, } y = \frac{(mn - n^2)}{nm + m^2}x + \frac{m^3}{nm + m^2}$$

$$\Rightarrow m_2 = \frac{mn - n^2}{nm + m^2}$$

Thus, angle between 2 lines is $\tan \theta$

$$\begin{aligned}
 \Rightarrow \quad \tan \theta &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\
 &= \left| \frac{\left(\frac{mn + n^2}{m^2 - mn} \right) - \left(\frac{mn - n^2}{nm + m^2} \right)}{1 + \left(\frac{mn + n^2}{m^2 - mn} \right) \left(\frac{mn - n^2}{nm + m^2} \right)} \right| \\
 &= \left| \frac{m^2 n^2 + m^3 n + n^3 m + n^2 m^2 - m^3 n + m^2 n^2 + n^2 m^2 - mn^3}{m^3 n + m^4 - m^2 n^2 - m^3 n + m^2 n^2 - mn^3 + mn^3 - n^4} \right| \\
 &= \left| \frac{4m^2 n^2}{m^4 - n^4} \right| \\
 \Rightarrow \quad \theta &= \tan^{-1} \left| \frac{4m^2 n^2}{m^4 - n^4} \right|
 \end{aligned}$$

Straight lines Ex 23.13 Q2

Slope of line $2x - y + 3 = 0$

$$\text{is } \frac{-2}{-1} = \frac{(\text{coefficient of } x)}{(\text{coefficient of } y)} = 2$$

$$\therefore m_1 = 2 \quad \text{--- (i)}$$

Slope of line $x + y + 2 = 0$

$$\text{is } \frac{-1}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } y)}$$

$$\therefore m_2 = -1 \quad \text{--- (ii)}$$

Acute angle between lines

$$\begin{aligned}
 \theta &= \tan^{-1} \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\
 &= \tan^{-1} \left| \frac{2 - (-1)}{1 - (2)(-1)} \right| \\
 &= \tan^{-1} \left| \frac{3}{1 - 2} \right| = \tan^{-1} \left| \frac{3}{-1} \right| = \tan^{-1} |3|
 \end{aligned}$$

Straight lines Ex 23.13 Q3

Let $ABCD$ be a quadrilateral

$$AB = \sqrt{(0-2)^2 + (2+1)^2}$$

Using distance formula

$$\begin{aligned} & \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13} \end{aligned}$$

$$BC = \sqrt{(2-0)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}$$

$$CD = \sqrt{(4-2)^2 + (0-3)^2} = \sqrt{4+9} = \sqrt{13}$$

$$DA = \sqrt{(4-2)^2 + (0+1)^2} = \sqrt{4+1} = \sqrt{5}$$

Since opposite sides (AB and CD) and (BC and DA) are equal

$\therefore$ The given quadrilateral is a parallelogram.

Straight lines Ex 23.13 Q4

The equation between the points

$$\begin{matrix} (2, 0) & \text{and} & (0, 3) \\ (x_1, y_1) & & (x_2, y_2) \end{matrix}$$

$$\text{Slope of line} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{3-0}{0-2} = \frac{-3}{2}$$

Also, slope of line $x + y = 1$

Converting in the form $y = mx + c$

$$y = 1 - x$$

$$\Rightarrow m_2 = -1$$

Thus, $\tan \theta =$ angle between the lines

$$\Rightarrow \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$= \left| \frac{\frac{-3}{2} - (-1)}{1 + \left(\frac{-3}{2}\right)(-1)} \right| = \left| \frac{\frac{-3}{2} + 1}{1 + \frac{3}{2}} \right|$$

$$= \left| \frac{\frac{-3+2}{2}}{\frac{2+3}{2}} \right| = \left| \frac{\frac{-1}{2}}{\frac{5}{2}} \right| = \frac{1}{5}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{1}{5} \right)$$

Straight lines Ex 23.13 Q5

Let l_1 be the line joining AO and

Let l_2 be the line joining BO

Then, line l_1 is $y - 0 = \left(\frac{0 - x_1}{0 - y_1} \right) (x - 0)$

$$yy_1 = x_1x = 0$$

Then, $m_1 = \frac{x_1}{y_1}$

Then line l_2 is $y - 0 = \left(\frac{0 - x_2}{0 - y_2} \right) (x - 0)$

Then, $m_2 = \frac{x_2}{y_2}$

$$\begin{aligned} \text{Then, } \tan \theta &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{\frac{x_1}{y_1} - \frac{x_2}{y_2}}{1 + \frac{x_1}{y_1} \frac{x_2}{y_2}} \right| \\ &= \left| \frac{x_1 y_2 - y_1 x_2}{y_1 y_2 + x_1 x_2} \right| \end{aligned}$$

From triangle,

$$\begin{aligned} AC &= \sqrt{(AB)^2 + (BC)^2} \\ &= \sqrt{(m_1^2 + m_2^2 - 2m_1 m_2) + (1 + m_1 m_2)^2} \\ &= \sqrt{m_1^2 + m_2^2 - 2m_1 m_2 + 1 + m_1^2 m_2^2 + 2m_1 m_2} \\ &= \sqrt{m_1^2 + m_2^2 + 1 + m_1^2 m_2^2} \end{aligned}$$

$$\begin{aligned} \cos \theta &= \frac{BC}{AC} = \frac{1 + m_1 m_2}{\sqrt{m_1^2 + m_2^2 + m_1^2 m_2^2 + 1}} \\ &= \frac{1 + \frac{x_1}{y_1} \frac{x_2}{y_2}}{\sqrt{\frac{x_1^2}{y_1^2} + \frac{x_2^2}{y_2^2} + \frac{x_1^2 x_2^2}{y_1^2 y_2^2} + 1}} \\ &= \frac{\frac{y_1 y_2 + x_1 x_2}{y_1 y_2}}{\sqrt{\frac{x_1^2 y_2^2 + x_2^2 y_1^2 + x_1^2 x_2^2 + y_1^2 y_2^2}{y_1^2 y_2^2}}} \\ &= \frac{y_1 y_2 + x_1 x_2}{\sqrt{x_1^2 (y_2^2 + x_2^2) + y_1^2 (y_2^2 + x_2^2)}} \\ &= \frac{y_1 y_2 + x_1 x_2}{\sqrt{x_1^2 + y_1^2} \sqrt{y_2^2 + x_2^2}} \end{aligned}$$

Hence proved.