RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.1

Straight Lines Ex 23.1 Q1

(i) Angle made with positive x axis is $\frac{-\pi}{4}$.

$\therefore m = \tan\theta = \tan\left(\frac{-\pi}{4}\right) = -1$

(ii) Angle made with positive x axis is $\frac{2\pi}{3}$

$\therefore m = \tan\theta = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

(iii) Angle made with positive x axis is $\frac{3\pi}{4}$

$\therefore m = \tan\theta = \tan\left(\frac{3\pi}{4}\right) = \tan\left(\pi - \frac{\pi}{4}\right) = -1$

(iv) Angle made with positive x axis is $\frac{\pi}{3}$

$\therefore m = \tan\theta = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

Straight Lines Ex 23.1 Q2

(i) $(-3,2)$ and $(1,4)$

slope of line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{1-(-3)} = \frac{2}{4} = \frac{1}{2}$

(ii) $\left(at_1^2, 2at_1\right)$ and $\left(at_2^2, 2at_2\right)$

slope of line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2} = \frac{2}{t_2 + t_1}$

(iii) $(3,-5)$ and $(1,2)$

slope of line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-5)}{1-3} = \frac{7}{-2} = \frac{-7}{2}$

Straight Lines Ex 23.1 Q3(i)

Slope of line joining $(5,6)$ and $(2,3)$

$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-6}{2-5} = \frac{-3}{-3} = 1$

Slope of line joining $(9,-2)$ and $(6,-5)$

$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-(-2)}{6-9} = \frac{-5+2}{-3} = 1$

Here $m_1 = m_2$

$\therefore$ The two lines are parallel.

Straight Lines Ex 23.1 Q3(ii)

Slope of line joining $(-1,1)$ and $(9,5)$

$$m_1 = \frac{5-1}{9-(-1)} = \frac{4}{10} = \frac{2}{5}$$

Slope of line joining $(3,-5)$ and $(8,-3)$

$$m_2 = \frac{-3-(-5)}{8-3} = \frac{-3+5}{5} = \frac{2}{5}$$

Here $m_1 = m_2$

$\therefore$ The two lines are parallel

Straight Lines Ex 23.1 Q3(iii)

Slope of line joining $(6,3)$ and $(1,1)$

$$m_1 = \frac{1-3}{1-6} = \frac{-2}{-5} = \frac{2}{5}$$

Slope of line joining $(-2,5)$ and $(2,-5)$

$$m_2 = \frac{-5-5}{2-(-2)} = \frac{-10}{4} = \frac{-5}{2}$$

Here $m_1 \times m_2 = \frac{2}{5} \times \frac{-5}{2} = -1$

$\therefore$ The lines are perpendicular to each other.

Straight Lines Ex 23.1 Q3(iv)

Slope of line joining $(3,15)$ and $(16,6)$

$$m_1 = \frac{6-15}{16-3} = \frac{-9}{13}$$

Slope of line joining $(-5,3)$ and $(8,2)$

$$m_2 = \frac{2-3}{8-(-5)} = \frac{-1}{13}$$

Here, neither $m_1 = m_2$ nor $m_1 \times m_2 = -1$

$\therefore$ The lines are neither parallel nor perpendicular.

Straight Lines Ex 23.1 Q4

(i) Line bisects first quadrant.

$\Rightarrow$ Angle between line and positive direction of x-axis $= \frac{90°}{2}$

$= 45°$

Slope of line $(m) = \tan\theta$

$m = \tan 45°$

$m = 1$

(ii) Line makes angle of 30° wiht the positve direction of y-axis.

= Angle between line and positive side of axis $= 90° + 30°$

$\theta° = 120°$

$m = \tan 120°$

$m = -\sqrt{3}$

Straight Lines Ex 23.1 Q5(i)

$A(4,8), B(5,12)$ and $C(9,28)$

slope of $AB = \frac{12-8}{5-4} = \frac{4}{1} = 4$

slope of $BC = \frac{28-12}{9-5} = \frac{16}{4} = 4$

slope of $CA = \frac{8-28}{4-9} = \frac{-20}{-5} = 4$

Since all 3 line segments have the same slope, they are parallel.
Since they have a common point B, they are collinear.

Straight Lines Ex 23.1 Q5(ii)

$A(16,-18), B(3,-6)$ and $C(-10,6)$

slope of $AB = \frac{-6-(-18)}{3-16} = \frac{12}{-13}$

slope of $BC = \frac{6-(-6)}{-10-3} = \frac{12}{-13}$

slope of $CA = \frac{6-(-18)}{-10-16} = \frac{12}{-13}$

Since all 3 line segments have the same slope and share a common vertex B, they are collinear.

Straight Lines Ex 23.1 Q6

Slope of line joining $(-1,4)$ and $(0,6)$ is

$m_1 = \frac{6-4}{0-(-1)} = 2$

Slope of line joining $(3,y)$ and $(2,7)$ is

$m_2 = \frac{7-y}{2-3} = y-7$

Since the two lines are parallel $m_1 = m_2$

$\Rightarrow 2 = y-7$

$\Rightarrow y = 9$

Straight Lines Ex 23.1 Q7

(i) If $slope = \tan\theta = 0 \Rightarrow \theta = 0$
When the slope of a line is zero then the line is parallel to x-axis.

(ii) If the slope is positive then $\tan\theta =$ positive $\Rightarrow \theta =$ acute
Thus the line makes an acute angle $\left(0<\theta<\frac{\pi}{2}\right)$ with the positive x-axis.

(iii) When the slope is negative then $\tan\theta =$ negative $\Rightarrow \theta$ is obtuse
Thus the line makes an obtuse angle angle $\left(\theta>\frac{\pi}{2}\right)$ with the positive x-axis.

Straight Lines Ex 23.1 Q8

Slope of line joining $(2,-3)$ and $(-5,1)$

$$m_1 = \frac{1-(-3)}{-5-2} = \frac{4}{-7}$$

Slope of line joining $(7,-1)$ and $(0,3)$

$$m_2 = \frac{3-(-1)}{0-7} = \frac{4}{-7}$$

Since $m_1 = m_2$, the two lines are parallel.

Straight Lines Ex 23.1 Q9

Slope of line joining $(2,-5)$ and $(-2,5)$ is

$$m_1 = \frac{5-(-5)}{-2-2} = \frac{-5}{2}$$

Slope of line joining $(6,3)$ and $(1,1)$

$$m_2 = \frac{1-3}{1-6} = \frac{2}{5}$$

$$m_1 \times m_2 = \frac{-5}{2} \times \frac{2}{5} = -1$$

$\therefore$ The two lines are perpendicular to each other

Straight Lines Ex 23.1 Q10

$$\text{Slope of } AB = \frac{2-4}{1-0} = -2$$

$$\text{Slope of } BC = \frac{3-2}{3-1} = \frac{1}{2}$$

$$\text{slope of } AB \times \text{slope of } BC = -2 \times \frac{1}{2} = -1$$

$\therefore$ Angle between AB and $BC = \frac{\pi}{2}$

$\therefore$ ABC are the vertices of a right angled triangle.

Straight Lines Ex 23.1 Q11

Here $A(-4, -1), B(-2,-4), C(4,0), D(2, 3)$

Slope of $AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4+1}{-2+4}$

$M_{AB} = \frac{-3}{2}$

Slope of $BC = \frac{0+4}{4+2}$

$M_{BC} = \frac{2}{3}$

Slope of $AD = \frac{3+1}{2+4}$

$M_{AD} = \frac{2}{3}$

Slope of $CD = \frac{3-0}{2-4}$

$M_{CD} = \frac{-3}{2}$

$\Rightarrow \quad M_{AB} = M_{CD}$ and $M_{BC} = M_{AD}$

$\Rightarrow \quad AB || CD$ and $BC || AD$

$M_{AB} \times M_{BC} = \frac{-3}{2} \times \frac{2}{3}$

$M_{AB} \times M_{BC} = -1$

$\Rightarrow \quad AB \perp BC$

$M_{BC} \times M_{CD} = \frac{2}{3} \times \frac{-3}{2}$

$M_{BC} \times M_{CD} = -1$

$\Rightarrow \quad BC \perp CD$

Thus,

$AB || CD$ and $BC || AD$

$AB \perp BC$, $BC \perp CD$, $CD \perp AD$

$\Rightarrow \quad ABCD$ is a rectangle

Straight Lines Ex 23.1 Q12

If 3 points lie on a line (ie they are collinear) lines joining these point have the same slope

$\therefore$ slope of AP = slope of PB = slope of BA

$\Rightarrow \frac{b-0}{a-h} = \frac{k-b}{0-a} = \frac{k-0}{0-h} \ldots\ldots\ldots(i)$

$\Rightarrow \frac{k-b}{0-a} = \frac{k-0}{0-h}$

$\Rightarrow -kh + bh = -ka$

$\Rightarrow -1 + \frac{b}{k} = \frac{-a}{h} \qquad$ (dividing by kh)

$\Rightarrow \frac{a}{h} + \frac{b}{k} = 1$

Hence Proved

Straight Lines Ex 23.1 Q13

Let $m_1 = x$, $m_2 = 2x$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\frac{1}{3} = \left|\frac{x - 2x}{1 + 2x^2}\right|$$

Case I:

$$\frac{1}{3} = \frac{x - 2x}{1 + 2x^2}$$

$$2x^2 + 1 = -3x$$

$$2x^2 + 3x + 1 = 0$$

$$2x^2 + 2x + x + 1 = 0$$

$$2x(x + 1) + 1(x + 1) = 0$$

$$(x + 1)(2x + 1) = 0$$

$$x = -1, -\frac{1}{2}$$

Case II:

$$\frac{1}{3} = \left(\frac{-x}{1 + 2x^2}\right)$$

$$\frac{1}{3} = \frac{x}{1 + 2x^2}$$

$$2x^2 + 1 = 3x$$

$$2x^2 - 3x + 1 = 0$$

$$2x^2 - 2x - x + 1 = 0$$

$$2x(x - 1) - 1(x - 1) = 0$$

$$(x - 1)(2x - 1) = 0$$

$$x = 1, \frac{1}{2}$$

Slope of other line is

$$1, \frac{1}{2} \quad \text{or} \quad -1, -\frac{1}{2}$$

Straight Lines Ex 23.1 Q14

Slope of $AB = \frac{97 - 92}{1995 - 1985} = \frac{5}{10} = \frac{1}{2}$

Population (p) in 2010 can be calculated using the slope of AC.

Slope of $AC = \frac{p - 92}{2010 - 1985} = \frac{p - 92}{25} = \frac{1}{2}$ = Slope of AB

$\Rightarrow \quad p - 92 = \frac{25}{2}$

$\Rightarrow \quad 2p - 184 = 25$

$\Rightarrow \quad 2p = 209$

$\Rightarrow \quad p = \frac{209}{2}$

$\therefore \quad p = 104.50$ crores

Straight Lines Ex 23.1 Q15

Let $A(-2,-1)$, $B(4,0)$, $C(3,3)$ and $D(-3,2)$ be a quadrilateral.

$$\text{slop of } AB = \frac{0-(-1)}{4-(-2)} = \frac{1}{6}$$

$$\text{slop of } BC = \frac{3-0}{3-4} = -3$$

$$\text{slop of } CD = \frac{3-2}{3-(-3)} = \frac{1}{6}$$

$$\text{slop of } DA = \frac{2-(-1)}{-3-(-2)} = -3$$

we observe that slope of opposite side of the quadrilateral $ABCD$ are equal.
Hence the quadrilateral $ABCD$ is a parallelogram.

Straight Lines Ex 23.1 Q16

Slope of the line segment joning the points $(3,-1)$ and $(4,-2)$ is

$$m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{-2-(-1)}{4-3} = \frac{-2+1}{4-3} = \frac{-1}{1} = -1$$

Slope of x axis is 0
$\Rightarrow m_2 = 0$

If θ is the angle between x-axis and the line segment then

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$= \left|\frac{-1-0}{1+(-1)(0)}\right|$$

$$= \frac{-1}{1} = -1$$

$\therefore\ \theta = 135^0$

Straight Lines Ex 23.1 Q17

The slope of the line joining $(-2,6)$ and $(4,8)$ is

$$m_1 = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$$

The slope of the line joining $(8,12)$ *and* $(x,24)$ is

$$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$$

Since the lines are perpendicular to each other

$$m_1 \times m_2 = -1$$

$$\frac{1}{3}\times\frac{12}{x-8} = -1$$

$$\Rightarrow\ 4 = 8-x$$

$$\Rightarrow\ x = 4$$

Straight Lines Ex 23.1 Q18

The given points are $A(x,-1), B(2,1)$ and $C(4,5)$

It is given that the points are collinear. So, the area of the triangle that they form must be zero.

Hence,

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0 \quad ---(1)$

Putting the value of $(x_1y_1),(x_2y_2),(x_3y_3)$ in (i)

$x(1-5)+(2)(5-(-1))+4(-1-1)=0$

$-4x+2(5+1)+4(-2)=0$

$-4x+12-8=0$

$-4x=-12+8$

$-4x=-4$

$x=1$

Straight Lines Ex 23.1 Q19

Slope of the line segment joning the points $(3,-1)$ and $(4,-2)$ is

$$m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{-2-(-1)}{4-3}=\frac{-2+1}{4-3}=\frac{-1}{1}=-1$$

Slope of x axis is 0

$\Rightarrow m_2=0$

If θ is the angle between x-axis and the line segment then

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$=\left|\frac{-1-0}{1+(-1)(0)}\right|$$

$$=\frac{-1}{1}=-1$$

$\therefore \theta=135^0$

Straight Lines Ex 23.1 Q20

Let the vertices be $A(-2,-1), B(4,0), C(3,3), D(-3,2)$.

Using slope formula, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, we get:

Slope of AB $(m_1) = \dfrac{0-(-1)}{4-(-2)} = \dfrac{1}{6}$

Slope of CD $(m_2) = \dfrac{2-3}{-3-3} = \dfrac{-1}{-6} = \dfrac{1}{6}$

$\Rightarrow m_1 = m_2 \Rightarrow AB \parallel CD$

Also

Slope of AD $(m_3) = \dfrac{2-(-1)}{-3-(-2)} = \dfrac{3}{-1} = -3$

Slope of BC $(m_4) = \dfrac{3-0}{3-4} = \dfrac{3}{-1} = -3$

$\Rightarrow m_3 = m_4 \Rightarrow AD \parallel BC$

Hence, ABCD is a parrallelogram.

Straight Lines Ex 23.1 Q21

Let $ABCD$ be the given quadrilateral

E is mid point of AB

F is mid point of BC

G is mid point of CD

H is mid point of AD

Using mid point formula $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

Coordinates of $E = \left(\dfrac{4+1}{2}, \dfrac{1+7}{2}\right) = \left(\dfrac{5}{2}, 4\right)$

Coordinates of $F = \left(\dfrac{1-6}{2}, \dfrac{7+0}{2}\right) = \left(\dfrac{-5}{2}, \dfrac{7}{2}\right)$

Coordinates of $G = \left(\dfrac{-6-1}{2}, \dfrac{0-9}{2}\right) = \left(\dfrac{-7}{2}, \dfrac{-9}{2}\right)$

Coordinates of $H = \left(\dfrac{-1+4}{2}, \dfrac{-9+1}{2}\right) = \left(\dfrac{3}{2}, -4\right)$

Now, $EFGH$ is parallelogram if diagonals EG and FH have the same mid-point.

Coordinates of mid-point of $EG = \left(\dfrac{\frac{5-7}{2}}{2}, \dfrac{4-\frac{9}{2}}{2}\right) = \left(\dfrac{-2}{4}, \dfrac{-1}{4}\right) = \left(\dfrac{-1}{2}, \dfrac{-1}{4}\right)$

Coordinates of mid-point of $FH = \left(\dfrac{\frac{-5+3}{2}}{2}, \dfrac{\frac{7-8}{2}}{2}\right) = \left(\dfrac{-2}{4}, \dfrac{-1}{4}\right)$

$\Rightarrow EFGH$ is parallelogram