RD Sharma Solutions Class 11 Maths Chapter 19 Ex 19.1

Arithematic Progressions Ex 19.1 Q1

$a_n = n^2 - n + 1$ ---(i) is the given sequence

Then, first 5 terms are a_1, a_2, a_3, a_4 and a_5

$$a_1 = (1)^2 - 1 + 1 = 1$$
$$a_2 = (2)^2 - 2 + 1 = 3$$
$$a_3 = (3)^2 - 3 + 1 = 7$$
$$a_4 = (4)^2 - 4 + 1 = 13$$
$$a_5 = (5)^2 - 5 + 1 = 21$$

First 5 terms 1, 3, 7, 13 and 21.

Arithematic Progressions Ex 19.1 Q2

$a_n = n^3 - 6n^2 + 11n - 6 \qquad n \in N.$

The first three terms are a_1, a_2 and a_3

$$a_1 = (1)^3 - 6(1)^2 + 11(1) - 6 = 0$$
$$a_2 = (2)^3 - 6(2)^2 + 11(2) - 6 = 0$$
$$a_3 = (3)^3 - 6(3)^2 + 11(3) - 6 = 0$$

$\therefore$ The 1st 3 terms are zero.

and

$$a_n = n^3 - 6n^2 + 11n - 6$$
$$= (n-2)^3 - (n-2) \text{ is positive as } n \geq 4$$

$\therefore$ a_n is always positive.

Arithematic Progressions Ex 19.1 Q3

$a_n = 3a_{n-1} + 2$ for $n > 1$

$$\therefore \quad a_2 = 3a_{2-1} + 2 = 3a_1 + 2$$
$$= 3(3) + 2 = 11 \qquad [\therefore a_1 = 3]$$
$$a_3 = 3a_{3-1} + 2 = 3a_2 + 2$$
$$= (11) + 2 = 35 \qquad [\therefore a_2 = 11]$$
$$a_4 = 3a_{4-1} + 2 = 3a_2 + 2$$
$$= 3(35) + 2 = 107 \qquad [\because a_3 = 35]$$

$\therefore$ The first four terms of A.P are 3, 11, 35, 107.

Arithematic Progressions Ex 19.1 Q4

(i) $a_1 = 1,\ a_n = a_{n-1} + 2,\ n \geq 2$

$a_2 = a_{2-1} + 2 = a_{1+2} = 3$ $\quad [\because a_1 = 1]$

$a_3 = a_{3-1} + 2 = a_2 + 2 = 5$ $\quad [\because a_2 = 3]$

$a_4 = a_{4-1} + 2 = a_3 + 2 = 7$ $\quad [\because a_3 = 5]$

$a_5 = a_{5-1} + 2 = a_4 + 2 = 9$ $\quad [\because a_4 = 7]$

$\therefore$ The first 5 terms of series are $1, 3, 5, 7, 11$.

(ii) $a_1 = a_2 = 1$

$a_n = a_{n-1} + a_{n-2}$ $\qquad n > 2$

$\Rightarrow$ $a_3 = a_{3-1} + a_{3-2}$

$= a_2 + a_1 = 1 + 1 = 2$

$\Rightarrow$ $a_4 = a_{4-1} + a_{4-2}$

$= a_3 + a_2 = 2 + 1 = 3$

$\Rightarrow$ $a_5 = a_{5-1} + a_{5-2}$

$= a_4 + a_3 = 5$

$\therefore$ The given sequence is $1, 1, 3, 5$.

(iii) $a_1 = a_2 = 2$

$a_n = a_{n-1} - 1 \quad n > 2$

$\Rightarrow$ $a_3 = a_{3-1} - 1$

$= a_2 - 1$

$= 2 - 1 = 1$

$\Rightarrow$ $a_4 = a_{4-1} - 1$

$= a_3 - 1 = 1 - 1 = 0$

$\Rightarrow$ $a_5 = a_{5-1} - 1$

$= 0 - 1 = -1$

$\therefore$ The first 5 terms of the sequence are $2, 2, 1, 0, -1$.

Arithematic Progressions Ex 19.1 Q5

$$a_n = a_{n-1} + a_{n-2} \qquad \text{for } n > 2$$

$\Rightarrow \quad a_3 = a_{3-1} + a_{3-2} = a_2 + a_1 = 1 + 1 = 2$

$\Rightarrow \quad a_4 = a_{4-1} + a_{4-2} = a_3 + a_2 = 2 + 1 = 3$

$\Rightarrow \quad a_5 = a_{5-1} + a_{5-2} = a_4 + a_3 = 3 + 2 = 5$

$\Rightarrow \quad a_6 = a_{6-1} + a_{6-2} = a_5 + a_4 = 5 + 31 = 8$

$\therefore$ For $n = 1$

$$\frac{a_{n+1}}{a_n} = \frac{a_2}{a_1} = \frac{1}{1} = 1$$

For $n = 2$

$$\frac{a^3}{a_2} = \frac{2}{1} = 2$$

For $n = 3$

$$\frac{a_4}{a_3} = \frac{3}{2} = 1.5$$

For $n = 4$ and $n = 5$

$$\frac{a_5}{a_4} = \frac{5}{3} \quad \text{and} \quad \frac{a_6}{a_5} = \frac{8}{5}$$

$\therefore$ The required series is $1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \ldots$

Arithematic Progressions Ex 19.1 Q6(i)

$3, -1, -5, -9 \ldots$

$a_1 = 3,\ a_2 = -1,\ a_3 = -5,\ a_4 = -9$

$a_2 - a_1 = -1 - 3 = -4$

$a_3 - a_2 = -5 - (-1) = -4$

$a_4 - a_3 = -9(-5) = -4$

$\therefore$ Common difirence is $d = -4$

$a_4 - a_3 = a_3 - a_2 = a$

$\therefore$ The given sequence is a A.P

$\therefore \quad a_5 = 3 + (5-1)(-4) = -13$

$a_6 = 3 + (6-1)(-4) = -17$

$a_7 = 3 + (7-1)(-4) = -21$

Arithematic Progressions Ex 19.1 Q6(ii)

$-1, \frac{1}{4}, \frac{3}{2}, \frac{11}{4} \ldots$

$a_1 = -1,\ a_2 = \frac{1}{4},\ a_3 = \frac{3}{2},\ a_4 = \frac{11}{4}$

$a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = \frac{5}{4}$

$\therefore$ Common difference is $d = \frac{5}{4}$

$\therefore$ The given sequence is A.P

$a_5 = -1 + (5-1)\frac{5}{4} = 4$

$a_6 = -1 + (6-1)\frac{5}{4} = \frac{21}{4}$

$a_7 = -1 + (7-1)\frac{5}{4} = \frac{26}{4} = \frac{13}{2}$

Arithematic Progressions Ex 19.1 Q6(iii)

(iii) $\sqrt{2},\ 3\sqrt{2},\ 5\sqrt{2},\ 7\sqrt{2}\ldots$

$$a_1 = \sqrt{2},\ a_2 = 3\sqrt{2},\ a_3 = 5\sqrt{2},\ a_4 = 7\sqrt{2}$$

$$a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = 2\sqrt{2}$$

$\therefore$ The common difference is $2\sqrt{2}$

and the given sequence is A.P

$$a_5 = \sqrt{2} + 2\sqrt{2}(5-1) = 9\sqrt{2}$$

$$a_6 = \sqrt{2} + 2\sqrt{2}(6-1) = 11\sqrt{2}$$

$$a_7 = \sqrt{2} + 2\sqrt{2}(7-1) = 13\sqrt{2}$$

Arithematic Progressions Ex 19.1 Q6(iv)

9, 7, 5, 3, ...

$$a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = -2$$

$\therefore$ The common difference is -2

and the given sequence is A.P

$$a_5 = 9 + (-2)(5-1) = 1$$

$$a_6 = 9 + (-2)(6-1) = -1$$

$$a_7 = 9 + (-2)(7-1) = -3$$

Arithematic Progressions Ex 19.1 Q7

$a_n = 2n + 7$

$$a_1 = 2(1) + 7 = 9$$

$$a_2 = 2(2) + 7 = 11$$

$$a_3 = 2(3) + 7 = 13$$

Here, $a_3 - a_2 = a_2 - a_1 = 2$

$\therefore$ The given sequence is A.P

$$a_7 = 2(7) + 7 = 21$$

7th term is 21.

Arithematic Progressions Ex 19.1 Q8

$a_n = 2n^2 + n + 1$

$$a_1 = 2(1)^2 + (1) + 1 = 4$$

$$a_2 = 2(2)^2 + (2) + 1 = 11$$

$$a_3 = 2(3)^2 + (3) + 1 = 21$$

$$a_3 - a_2 \neq a_2 - a_1$$

$\therefore$ The given sequence is not as A.P as consequtive terms do not have a common difference.

RD Sharma Solutions Class 11 Maths Chapter 19 Ex 19.2

Arithematic Progressions Ex 19.2 Q1

(i) 10th term of A.P $1, 4, 7, 10, \ldots$

Here, 1st term $= a_1 = 1$

and common difference $d = 4 - 1 = 3$

We know $a_n = a_1 + (n-1)d$

$$\therefore \quad a_{10} = a_1 + (10-1)d$$
$$= 1 + (10-1)3 \Rightarrow 28$$

(ii) To find 18th term of A.P $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$

Here, 1st term $a_1 = \sqrt{2}$

and $\quad d =$ common difference $= 2\sqrt{2}$

$$\therefore \quad a_n = a_1 + (n-1)d$$
$$a_{18} = \sqrt{2} + 2\sqrt{2}(17) = 35\sqrt{2}$$

(iii) Find nth term of A.P $13, 8, 3, -2$

Here, $\quad a_1 = 13$

$$d = -5$$
$$\therefore \quad a_n = a + (n-1)d$$
$$= 13 + (n-1)(-5)$$
$$= -5n + 18$$

Arithematic Progressions Ex 19.2 Q2

It is given that the sequence $\langle a_n \rangle$ is an A.P

$$\therefore \quad a_n = a + (n-1)d \qquad \text{---(i)}$$

Similarly from (i)

$$a_{m+n} = a + (m+n-1)d \qquad \text{---(ii)}$$
$$a_{m-n} = a + (m-n-1)d \qquad \text{---(iii)}$$

Adding (ii) and (iii)

$$a_{m+n} + a_{m-n} = \left(a + (m+n-1)d\right) + \left(a + (m-n-1)d\right)$$
$$= 2a + (m+n-1+m-n-1)d$$
$$= 2a + 2d(m-1)$$
$$= 2\left(a + (m-1)d\right)$$
$$= 2a_m \text{ Hence proved.}$$

Arithematic Progressions Ex 19.2 Q3

(i) Let nth term of A.P = 248

$\therefore \quad a_n = 248 = a + (n-1)d$

$\Rightarrow \quad 248 = 3 + (n-1)5$

$\therefore \quad n = 50$

$\therefore$ 50th term of the given A.P is 248.

(ii) Which term of A.P 84, 80, 76 is 0?

Let nth term of A.P be 0

Then, $a_n = 0 = a + (n-1)d$

$0 = 84 + (n-1)(-4)$

$\therefore \quad n = 22$

$\therefore$ 22nd term of the given A.P is 0.

(iii) Which term of A.P is 4, 9, 14, ... is 254?

Let nth term of A.P be 254

$a_n = a + (n-1)d$

$254 = 4 + (n-1)5$

$\therefore \quad n = 51$

$\therefore$ 51st term of the given A.P is 254.

Arithematic Progressions Ex 19.2 Q4

(i) Is 68 a term of A.P 7, 10, 13, ...?

Here, $a = 7$

and $x = 10 - 7 = 3$

$\therefore \quad a_n$ term is $= a + (n-1)d$

$= 7 + (n-1)3$

Let 68 be nth temr of A.P

Then,

$68 = 7 + 3(n-1)$

$\Rightarrow \quad 68 = 7 + 3n - 3$

$\Rightarrow \quad 68 - 4 = 3n$

$\Rightarrow \quad 64 = 3n$

$\Rightarrow \quad n = \dfrac{64}{3}$

Which is not a natural number.

$\therefore$ 68 is nota term of given A.P.

(ii) Is 302 a term of A.P 3, 8, 13

Let 302 be nth ter, pf tje given A.P

Here, $302 = 3 + (n-1)5$

$\dfrac{299}{5} = (n-1)$

$n = \dfrac{304}{5}$

Which is not a natural number.

$\therefore$ 302 is not a term of given A.P.

Arithematic Progressions Ex 19.2 Q5

(i) The given sequence is $24, 23\frac{1}{4}, 22\frac{1}{2}, 21\frac{3}{4}, \ldots$

Here, $a = 24$

$$d = 23\frac{1}{4} - 24 = \frac{93-96}{4} = \frac{-3}{4}$$

Let nth term be the 1st negative term.

$$a_n < 0$$
$$a + (n-1)d < 0$$
$$24 - \frac{3}{4}(n-1) < 0$$
$$96 - 3n + 3 < 0$$
$$99 < 3n$$
$$33 < n \quad \text{or} \quad n > 33$$

$\therefore$ 34th term is 1st negative term.

(ii) The given sequence is $12+8i,\ 11+6i,\ 10+4i, \ldots$

Here, $a = 12+8i$

$$d = -1-2i$$

Then, $a_n = a + (n-1)d$

$$= 12 + 8i + (n-1)(-1-2i)$$
$$= (13-n) + i(10-2n)$$

Let nth term be purely real the $(10-2n) = 0$ or $n = 5$

So, 5th term is purely real.

Let nth term be purely imaginary. Then, $13 - n = 0$

$\therefore \quad n = 13$

So, 13th term is purely imaginary.

Arithematic Progressions Ex 19.2 Q6

(i) The given A.P is $7, 10, 13, \ldots 43$.

Let there be n terms,

then, n term = 43

or $\quad 43 = a_n = a + (n-1)d$

$\Rightarrow \quad 43 = 7 + (n-1)3$

$\Rightarrow \quad n = 13$

Thus, there are 13 terms in the given sequence.

(ii) The given A.P is $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, \ldots, \frac{10}{3}$?

Let there be n terms

then, nth term = $\frac{10}{3}$

or $\quad \frac{10}{3} = a_n = a + (n-1)d$

$\Rightarrow \quad \frac{10}{3} = -1 + (n-1)\left(\frac{-5}{6} + 1\right)$

$\Rightarrow \quad n = 27$

Thus, there are 27 terms in the given sequence.

Arithematic Progressions Ex 19.2 Q7

Given: $a = 5$

$d = 3$

a_n = last term = 80

Let there be n terms

$\therefore \quad a_n = 80 = a + (n-1)d$

$80 = 5 + (n-1)3$

$\Rightarrow \quad n = 26$

$\therefore$ Thus, thre are 26 terms in the given sequence.

Arithematic Progressions Ex 19.2 Q8

Given that:

$a_6 = 19 = a + (6-1)d$ ---(i)

$a_{17} = 41 = a + (17-1)d$ ---(ii)

Solving (i) and (ii), we get

$a = 9$ and $d = 2$

$$\begin{aligned} \therefore \quad a_{40} &= a + (40-1)d \\ &= 9 + (40-1)2 \\ &= 9 + 39(2) \\ &= 87 \end{aligned}$$

40th term of the given sequence is 87.

Arithematic Progressions Ex 19.2 Q9

Given:

$$a_9 = 0$$

$$\therefore \quad a + 8d = 0$$

$$a = -8d \qquad \text{---(i)}$$

$$a_{19} = a + (19 - 1)d$$
$$= a + 18d$$
$$= -8d + 18d \qquad [\because a = -8d \text{ from (i)}]$$
$$= 10d \qquad \text{---(ii)}$$

$$a_{29} = a + (29 - 1)d$$
$$= -8d + 28d \qquad [\because a = -8d \text{ from (i)}]$$
$$= 20d \qquad \text{---(iii)}$$

From (ii) and (iii)

$a_{29} = 2a_{19}$ Hence proved.

Arithematic Progressions Ex 19.2 Q10

Given:

$$10a_{10} = 15a_{15}$$

$$\Rightarrow \quad 10(a + (10 - 1)d) = 15(a + (15 - 1)d)$$

$$\Rightarrow \quad 10a + 90d = 15a + 210d$$

$$\Rightarrow \quad 5a + 120d = 0$$

$$\Rightarrow \quad a + 24d = 0 \qquad \text{---(i)}$$

$$a_{25} = a + (25 - 1)d$$
$$= a + 24d$$
$$= 0 \qquad [\because \text{from (i) } a + 24d = 0]$$

Hence proved.

Arithematic Progressions Ex 19.2 Q11

Given:

$$a_{10} = 41 = a + 9d \qquad \text{---(i)}$$

$$a_{18} = 73 = a + 17d \qquad \text{---(ii)}$$

Solving (i) and (ii)

$$a + 9d = 41$$
$$a + 17d = 73$$

We get $a = 5$ and $d = 4$

$$\therefore \quad a_{26} = a + (26 - 1)d$$
$$= 5 + 25(4)$$
$$= 105$$

26th term of the given A.P is 105.

Arithematic Progressions Ex 19.2 Q12

Given:

$$a_{24} = 2a_{10}$$

$\Rightarrow \quad a + 23d = 2(a + 9d)$

$\Rightarrow \quad a = 5d \qquad$ ---(i)

$$a_{72} = a + (72 - 1)d$$

$= a + 71d \qquad [\because a = 5d \text{ from (i)}]$

$\Rightarrow \quad = 76d \qquad$ ---(ii)

$$a_{34} = a + (34 - 1)d$$

$= 5d + 33d \qquad [\because a = 5d \text{ from (i)}]$

$= 38d \qquad$ ---(iii)

From (ii) and (iii)

$a_{72} = 2a_{34} \qquad$ Hence proved.

Arithematic Progressions Ex 19.2 Q13

Given:

$$a_{m+1} = 2a_{n+1}$$

$\Rightarrow \quad a + (m + 1 - 1)d = 2\left(a + (n + 1 - 1)d\right)$

$\Rightarrow \quad a + md = 2a + 2nd$

$\Rightarrow \quad a = (m - 2n)d \qquad$ ---(i)

Then,

$$a_{3m+1} = a + (3m + 1 - 1)d$$

$= a + 3md$

$= 3d - 2nd + 3md$

$= 2(2m - n)d \qquad$ ---(ii)

$$a_{m+n+1} = a + (m + n + 1 - 1)d$$

$= md - 2nd + md + nd$

$= (2m - n)d \qquad$ ---(iii)

From (ii) and (iii)

$a_{2m+1} = 2a_{m+n+1} \qquad$ Hence proved.

Arithematic Progressions Ex 19.2 Q14

The given A.P is 9, 7, 5, ... and 15, 12, 9

Here,

$a = 9 \qquad A = 15$

$d = -2 \qquad D = 3$

Let $a_n = A_n$ for same n.

$\Rightarrow \quad a + (n - 1)d = A + (n - 1)d$

$\Rightarrow \quad 9 + (n - 1)(-2) = 15 + (n - 1)3$

$\Rightarrow \quad n = 7$

$\therefore$ 7th term of both the A.P is same.

Arithematic Progressions Ex 19.2 Q15

(i) A.P is $3, 5, 7, 9, \ldots, 201$.

Here, $a = 3$

$d = 2$

nth term from the end is $l - (n-1)d$

i.e. $201 - (n-1)2$ or $203 - 2n$ ---(i)

12th term from end is

$203 - 2(12) = 179$

(ii) A.P is $3, 8, 13, \ldots, 253$.

Then, 12th term from end is $l - (n-1)d$ i.e.,

$= 253 - (12-1)5$

$= 253 - 55$

$= 198$

(iii) A.P is $1, 4, 7, 10, \ldots, 88$

Then, 12th term from end is $l - (n-1)d$

$= 88 - (12-1)3$

$= 88 - 33$

$= 55$

Arithematic Progressions Ex 19.2 Q16

Given,

$a = 3a_1$ ---(i)

$a_7 = 2a_3 + 1$ ---(ii)

Expanding (i) and (ii)

$a + 3d = 2a$

$\therefore \quad 2a = 3d$ or $a = \frac{3d}{2}$ ---(iii)

$a + 6d = 2a + 4d + 1$

$a + 1 = 2d$ ---(iv)

From (iii) and (iv)

$a = 3$ and $d = 2$

$\therefore$ 1st term of the given A.P is 3, and common difference is 2.

Arithematic Progressions Ex 19.2 Q17

$$a_6 = a + 5d = 12 \quad \text{---(i)}$$
$$a_8 = a + 7d = 22 \quad \text{---(ii)}$$

Solving (i) and (ii)

$$a = -13 \text{ and } d = 5$$

Then,

$$a_n = a + (n-1)d$$
$$= -13 + (n-1)5$$
$$= 5n - 18$$

and

$$a_2 = a + (2-1)d$$
$$= -13 + 5$$
$$= -8$$

Arithematic Progressions Ex 19.2 Q18

The first two digit number divisible by 3 is 12.
and last two digit number divisible by 3 is 99.

So, the required series is 12, 15, 18, ... 99.
Let there be n terms then nth term = 99

$\Rightarrow \quad 99 = a + (n-1)d$

$\Rightarrow \quad 99 = 12 + (n-1)3$

$\Rightarrow \quad n = 30$

30 two digit numbers are divisible by 3.

Arithematic Progressions Ex 19.2 Q19

Given,

$$n = 60$$
$$a = 7$$
$$l = 125$$

$\therefore \quad a + (n-1)d = 125$

$$7 + (59)d = 125$$
$$d = 2$$

$\therefore \quad a_{32} = a + (32-1)d$

$$= 7 + (31)2$$
$$= 69$$

32nd term is 69.

Arithematic Progressions Ex 19.2 Q20

$$a_4 + a_8 = 24 \qquad [\text{Given}]$$

$$\Rightarrow \quad (a + 3d) + (a + 7d) = 24$$

$$\Rightarrow \quad a + 5d = 12 \qquad \text{---(i)}$$

$$a_6 + a_{10} = 34$$

$$\Rightarrow \quad (a + 5d) + (a + 9d) = 34$$

$$\Rightarrow \quad a + 7d = 17 \qquad \text{---(ii)}$$

From (i) and (ii)

$$a = \frac{-1}{2} \text{ and } d = \frac{5}{2}$$

$\therefore$ 1st term is $\frac{-1}{2}$ and common difference is $\frac{5}{2}$.

Arithematic Progressions Ex 19.2 Q21

The nth term from starting

$$= a_n = aa + (n-1)d \qquad \text{---(i)}$$

The nth term from end

$$= l - (n-1)d \qquad \text{---(ii)}$$

Adding (i) and (ii), we get

Sum of nth term from begining and nth term from the end

$$= a + (n-1)d + l - (n-1)d$$

$$= a + l \text{ Hence proved.}$$

Arithematic Progressions Ex 19.2 Q22

$$\frac{a_4}{a_7} = \frac{2}{3} \qquad \text{[Given]}$$

$$\Rightarrow \quad \frac{a+3d}{a+6d} = \frac{2}{3} \qquad =$$

$$\Rightarrow \quad 3a + 9d = 2a + 12d$$

$$\Rightarrow \quad a = 3d \qquad \text{---(i)}$$

$$\frac{a_6}{a_8} = \frac{a+5d}{a+7d}$$

$$\Rightarrow \quad = \frac{3d+5d}{3d+7d} \qquad [\because 3d \text{ from (i)}]$$

$$\Rightarrow \quad = \frac{8d}{10d}$$

$$\Rightarrow \quad = \frac{4}{5}$$

$$\frac{a_6}{a_8} = \frac{4}{5}$$

Arithematic Progressions Ex 19.2 Q23

$$\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + \ldots + \sec\theta_{n-1} \sec\theta_n = \frac{\tan\theta_n - \tan\theta_1}{\sin d}$$

$$\theta_2 - \theta_1 = \theta_3 - \theta_2 = \ldots\ldots\ldots = d$$

$$\sec\theta_1 \sec\theta_2 = \frac{1}{\cos\theta_1 \cos\theta_2} = \frac{\sin d}{\sin d\,(\cos\theta_1 \cos\theta_2)}$$

$$= \frac{\sin(\theta_2 - \theta_1)}{\sin d\,(\cos\theta_1 \cos\theta_2)}$$

$$= \frac{\sin\theta_2 \cos\theta_1 - \cos\theta_2 \sin\theta_1}{\sin d\,(\cos\theta_1 \cos\theta_2)}$$

$$= \frac{1}{\sin d}\left[\frac{\sin\theta_2 \cos\theta_1}{(\cos\theta_1 \cos\theta_2)} - \frac{\cos\theta_2 \sin\theta_1}{(\cos\theta_1 \cos\theta_2)}\right]$$

$$= \frac{1}{\sin d}\left[Tan\theta_2 - Tan\theta_1\right]$$

Similarly, $\sec\theta_2 \sec\theta_3 = \frac{1}{\sin d}\left[Tan\theta_3 - Tan\theta_2\right]$

If we add up all terms, we get

$$= \frac{1}{\sin d}\left[Tan\theta_2 - Tan\theta_1 + Tan\theta_3 - Tan\theta_2 + \ldots\ldots\ldots\ldots + Tan\theta_n - Tan\theta_{n-1}\right]$$

$$= \frac{1}{\sin d}\left[Tan\theta_n - Tan\theta_1\right]$$

Hence Proved

RD Sharma Solutions Class 11 Maths Chapter 19 Ex 19.3

Arithematic Progressions Ex 19.3 Q1

Let the 3rd term of A.P be

$$a-d, a, a+d$$

Then,

$$a-d+a+a+d = 21$$

$$3a = 21$$

$$\therefore \quad a = 7$$

and

$$(a-d)(a+d) = a+6$$

$$a^2-d^2 = a+6$$

$$7^2-d^2 = 7+6 \qquad [\because a = 7]$$

$$d^2 = 36$$

$$d = \pm 6$$

Since d can't be negative, therefore

$\therefore$ The A.P is 1, 7, 13.

Arithematic Progressions Ex 19.3 Q2

Let the 3 numbers in A.P are

$$a-d, a, a+d$$

Then,

$$a-d+a+a+d = 27$$

$$3a = 27$$

$$\therefore \quad a = 9 \qquad \text{---(i)}$$

and

$$(a-d)(a)(a+d) = 648$$

$$(9-d)\,9\,(9-d) = 648 \qquad [\because a = 9]$$

$$9^2-d^2 = 72$$

$$\therefore \quad d = 3 \qquad \text{---(ii)}$$

$\therefore$ The given sequence is 6, 9, 12.

Arithematic Progressions Ex 19.3 Q3

Let the four numbers in A.P be

$$a-3d, a-d, a+d, a+3d$$

$$(a-3d)+(a-d)+(a+d)+(a+3d) = 50$$

$$4a = 50$$

$$a = \frac{25}{2} \qquad \text{---(i)}$$

and

$$(a+3d) = 4(a-3d)$$

$$\frac{25+6d}{2} = 50-12d$$

$$30d = 75$$

$$d = \frac{25}{10} = \frac{5}{2} \qquad \text{---(ii)}$$

$\therefore$ The required sequence is 5, 10, 15, 20.

Arithematic Progressions Ex 19.3 Q4

Let three numbers be $a-d, a, a+d$

Then,

$$a-d+a+a+d=12$$
$$3a=12$$
$$a=4$$

and

$$(a-d)^3+a^3+(a+d)^3=\pm 288$$
$$a^3+d^3+3ad(a+d)+a^3+a^3-a^3-3ad(a-d)-288$$
$$\Rightarrow \quad 2a^3+3a^2d+3ad^2-3a^2d+3ad^2=288$$
$$\Rightarrow \quad 2a^3+3a^2d^2=288$$
$$\Rightarrow \quad 128+48d^2=288$$
$$\therefore \quad d=\pm 2$$

$\therefore$ The required sequence is 2, 4, 6 or 6, 4, 2.

Arithematic Progressions Ex 19.3 Q5

Let 3 numbers in A.P be

$$a-d, a \text{ and } a+d$$
$$\Rightarrow \quad (a-d)+(a)+(a+d)=24$$
$$3a=24$$
$$a=8$$

and

$$(a-d)(a)(a+d)=440$$
$$8^2-d^2=55$$
$$d=3$$

$\therefore$ The required sequence is 5, 8, 11.

Arithematic Progressions Ex 19.3 Q6

Let the four angle be

$$a-3d, a-d, a+d, a+3d$$

Then,

$$\text{sum of alll angles}=360°$$
$$a-3d+a-d+a+d+a+3d=360°$$
$$4a=360°$$
$$a=90° \qquad \text{---(i)}$$

and

$$(a-d)-(a-3d)=10$$
$$2d=10$$
$$d=5$$

$\therefore$ The angle of the given quadrilateral are 75°, 85°, 95° and 105°.

RD Sharma Solutions Class 11 Maths Chapter 19 Ex19.4

Arithematic Progressions Ex 19.4 Q1

(i) $50, 46, 42, \ldots, 10$ terms

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2 \times 50 + (10-1)(-4)\right]$$
$$= 320$$

(ii) $13, 5, \ldots, 12$ terms

$$S_{12} = \frac{12}{2}\left[2 \times 1 + (12-1)2\right]$$
$$= 6 \times 24 = 144$$

(iii) $3, \frac{9}{2}, 6, \frac{15}{2}, \ldots, 25$ terms

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{25} = \frac{25}{2}\left(2 \times 3 + 24 \times \frac{3}{2}\right)$$
$$= 525$$

(iv) $41, 36, 31, \ldots, 12$ terms

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{25} = \frac{25}{2}\left[2 \times 41 + (11)(-5)\right]$$
$$= 162$$

(v) $a+b, a-b, a-3b, \ldots$ to 22 terms

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{22} = \frac{22}{2}\left[2a + 2b + 21(-2b)\right]$$
$$= 22a - 440b$$

(vi) $(x-y)^2, (x^2+y^2), (x+y)^2, \ldots, x$ terms

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$= \frac{n}{2}\left[2\left(x^2 + y^2 - 2xy\right) + (x-1)(-2xy)\right]$$
$$= n\left[(x-y)^2 + (n-1)xy\right]$$

$\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}$, ...to n terms

nth term in above sequence is $\frac{(2n-1)x - ny}{x+y}$

Sum of n terms is given by

$$\frac{1}{x+y}\left[x + 3x + 5x + \ldots + (2n-1)x - (y + 2y + 3y \ldots + ny)\right]$$

$$= \frac{1}{x+y}\left[\frac{n}{2}(2x + (n-1)2x) - \frac{n(n+1)y}{2}\right]$$

$$= \frac{\quad}{2(x+y)}\left[2n^2x - 2n^2y - ny\right]$$

Arithematic Progressions Ex 19.4 Q2

(i) $2 + 5 + 8 + \ldots + 182.$

a_n term of given A.P is 182

$a_n = a + (n-1)d = 182$

$\Rightarrow \quad 182 = 2 + (n-1)3$

or $\quad n = 61$

Then,

$$S_n = \frac{n}{2}[a + l]$$

$$= \frac{61}{2}[2 + 182]$$

$$= 61 \times 92$$

$$= 5612$$

(ii) $101 + 99 + 97 + \ldots + 47$

a_n term of A.P of n terms is 47.

$\therefore \quad 47 = a + (n-1)d$

$47 = 101 + (n-1)(-2)$

or $\quad n = 28$

Then,

$$S_n = \frac{n}{2}[a + l]$$

$$= \frac{28}{2}[101 + 47]$$

$$= 14 \times 148$$

$$= 2072$$

(iii) $(a-b)^2 + \left(a^2 + b^2\right) + (a+b)^2 + \ldots + \left[(a+b)^2 + 6ab\right]$

Let number of terms be n

Then,

$a_n = (a+b)^2 + 6ab$

$\Rightarrow \quad (a-b)^2 + (n-1)(2ab) = (a+b)^2 + 6ab$

$\Rightarrow \quad a^2 + b^2 - 2ab + 2abn - 2ab = a^2 + b^2 + 2ab + 6ab$

$\Rightarrow \quad n = 6$

Then,

$$S_n = \frac{n}{2}[a + l]$$

$$S_6 = \frac{6}{2}\left[a^2 + b^2 - 2ab + a^2 + b^2 + 2ab + 6ab\right]$$

$$= 6\left[a^2 + b^2 + 3ab\right]$$

Arithematic Progressions Ex 19.4 Q3

A.P formed is $1, 2, 3, 4, \ldots, n$.

Here,

$$a = 1$$
$$d = 1$$
$$l = n$$

So sum of n terms $= S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$$= \frac{n}{2}\left[2 + (n-1)1\right]$$

$$= \frac{n(n+1)}{2}$$ is the sum of first n natural numbers.

Arithematic Progressions Ex 19.4 Q4

The natural numbers which are divisible by 2 or 5 are:

$2+4+5+6+8+10+\cdots+100 = (2+4+6+\cdots+100)+(5+15+25+\cdots+95)$ Now

$(2+4+6+\cdots+100)$ and $(5+15+25+\cdots+95)$ are AP with common difference 2 and 10 respectively.

Therefore

$$2+4+6+\cdots+100 = 2\frac{50}{2}(1+50)$$
$$= 2550$$

Again

$$5+15+25+\cdots+95 = 5(1+3+5+\cdots+19)$$
$$= 5\left(\frac{10}{2}\right)(1+19)$$
$$= 500$$

Therefore the sum of the numbers divisible by 2 or 5 is:

$$2+4+5+6+8+10+\cdots+100 = 2550+500$$
$$= 3050$$

Arithematic Progressions Ex 19.4 Q5

The series of n odd natural numbers are $1, 3, 5, \ldots, n$

Where n is odd natural number

Then, sum of n terms is

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$= \frac{n}{2}\left[2(1) + (n-1)(2)\right]$$
$$= n^2$$

The sum of n odd natural numbers is n^2.

Arithematic Progressions Ex 19.4 Q6

The series so formed is $101, 103, 105, \ldots, 199$

Let number of terms be n

Then,

$$a_n = a + (n-1)d = 199$$

$$\Rightarrow \quad 199 = 101 + (n-1)2$$

$$\Rightarrow \quad n = 50$$

The sum of n terms $= S_n = \frac{n}{2}[a+l]$

$$S_{50} = \frac{50}{2}[101+199]$$

$$= 7500$$

The sum of odd numbers between 100 and 200 is 7500.

Arithematic Progressions Ex 19.4 Q7

The odd numbers betwwen 1 and 100 divisible by 3 are $3, 9, 15, \ldots, 999$

Let the number of terms be n then, nth term is 999.

$$a_n = a(n-1)d$$

$$999 = 3 + (n-1)6$$

$$\Rightarrow \quad n = 167$$

The sum of n terms

$$S_n = \frac{n}{2}[a+l]$$

$$\Rightarrow \quad S_{167} = \frac{167}{2}[3+999]$$

$$= 83667$$ Hence proved.

Arithematic Progressions Ex 19.4 Q8

The required series is $85, 90, 95, \ldots, 715$

Let there be n terms in the A.P

Then,

$$n\text{th term} = 715$$

$$715 = 85 + (n-1)5$$

$$n = 127$$

Then,

$$S_n = \frac{n}{2}[a+l]$$

$$S_{127} = \frac{127}{2}[85+715]$$

$$= 50800$$

Arithematic Progressions Ex 19.4 Q9

The series of integers divisble by 7 between 50 and 500 are

$$56, 63, 70, \ldots, 497$$

Let the number of terms be n then, nth term = 497

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad 497 = 56 + (n-1)7$$

$$\Rightarrow \quad n = 64$$

The sum $S_n = \frac{n}{2}[a + l]$

$$\Rightarrow \quad S_{64} = \frac{64}{2}[56 + 497]$$

$$= 32 \times 553$$

$$= 17696$$

Arithematic Progressions Ex 19.4 Q10

All even integers will have common difference = 2

$$\therefore \quad \text{A.P is } 102, 104, 106, \ldots, 998$$

$$t_n = a + (n-1)d$$

$$t_n = 998, a = 102, d = 2$$

$$998 = 102 + (n-1)(2)$$

$$998 = 102 + 2n - 2$$

$$998 - 100 = 2n$$

$$2n = 898$$

$$n = 449$$

S_{449} can be calculated by

$$S_n = \frac{n}{2}[a + l]$$

$$= \frac{449}{2}[102 + 998]$$

$$= \frac{449}{2} \times 1100$$

$$= 449 \times 550$$

$$= 246950$$

Arithematic Progressions Ex 19.4 Q11

The series formed by all the integers between 100 and 550 which are divisible by 9 is

$$108, 117, 123, \ldots, 549$$

Let there be n terms in the A.P then, the nth term is 549

$$549 = a + (n-1)d$$

$$549 = 108 + (n-1)9$$

$$\Rightarrow \quad n = 50$$

Then,

In the given series $3+5+7+9+\ldots$ to $3n$

Here,

$a = 3$

$d = 2$

Number of terms $= 3n$

The sum of n term is

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$\Rightarrow \quad S_{3n} = \frac{3n}{2}\left[6+(3n-1)2\right]$$

$$= 3n(2n+3)$$

Arithematic Progressions Ex 19.4 Q13

The first number between 100 and 800 which on division by 16 leaves the remainder 7 is 112 and last number is 791.

Thus, the series so formed is $103, 119, \ldots, 791$

Let number of terms be n, then

nth term $= 791$

Then,

$$a_n = a+(n-1)d$$

$$\Rightarrow \quad 791 = 103+(n-1)16$$

$$\Rightarrow \quad n = 44$$

Then, sum of all terms of the given series is

$$S_{43} = \frac{44}{2}\left[103+791\right]$$

$$= \frac{44\times 894}{2}$$

$$= 19668$$

Arithematic Progressions Ex 19.4 Q14

(i) $25+22+19+16+\ldots+x=115$

Here, sum of the given series of say n terms is 115

So, the nth term $= x$

Here, $a = 25$ and $d = 22 - 25 = -3$

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad x = 25 - 3(n-1)$

$\Rightarrow \quad x = 28 - 3n \qquad$ ---(i)

The sum of n terms

$$S_n = \frac{n}{2}[a + l]$$

$\Rightarrow \quad 115 = \frac{n}{2}[25 + 28 - 3n]$

$\Rightarrow \quad 230 = 53n - 3n^2$

$\Rightarrow \quad 3n^2 - 53n - 230 = 0$

$\Rightarrow \quad 3n^2 - 30n - 23n - 230 = 0$

$\Rightarrow \quad n = 10$ or $\frac{23}{3}$

But n can't be function

$\therefore \quad n = 10 \qquad$ ---(ii)

From (i) and (ii)

$x = 28 - 3n$

$= 28 - 3(10)$

$= -2$

$x = -2$

(ii) $1 + 4 + 7 + 10 + \ldots + x = 590$

Here, $a = 1$

$d = 4 - 1 = 3$

Let there be n terms so the nth term $= x$

$\Rightarrow \quad x = 1 + (n-1)3 \qquad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad x = 3n - 2 \qquad$ ---(i)

and

$S_n = 590 \qquad$ [Given]

$\Rightarrow \quad \frac{n}{2}[a + l] = 590$

$\Rightarrow \quad \frac{n}{2}[1 + 3n - 2] = 590 \qquad [\because l = x = 3n - 2]$

$\Rightarrow \quad 3n^2 - n - 1080 = 0$

$\Rightarrow \quad 3n^2 - 60n + 59n - 1080 = 0$

$\Rightarrow \quad 3n(n-20) + 59(n-20) = 0$

$\Rightarrow \quad n = 20 \qquad$ ---(ii)

From (i) and (ii)

$x = 3n - 2$

$= 3(20) - 2$

$= 58$

$x = 58$

Arithematic Progressions Ex 19.4 Q15

Sum first n terms of the given AP is

$S_n = 3n^2 + 2n$

$S_{n-1} = 3(n-1)^2 + 2(n-1)$

$a_n = S_n - S_{n-1}$

$a_n = 3n^2 + 2n - 3(n-1)^2 - 2(n-1)$

$a_n = 6n - 1$

$a_r = 6r - 1$

r^{th} term is $6r - 1$.

Arithematic Progressions Ex 19.4 Q16

Given,

$$a_1 = -14 = a + 0d \qquad \text{---(i)}$$
$$a_5 = 2 = a + 4d \qquad \text{---(ii)}$$

Solving (i) and (ii)

$$a_1 = a = -14 \text{ and } d = 4$$

Let ther be n terms then sum of there n terms = 40

$\therefore \quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\Rightarrow \quad 40 = \frac{n}{2}\left[-28 + (n-1)4\right]$

$\Rightarrow \quad 4n^2 - 32n - 80 = 0$

or $\quad n = 10$ or -2

But n can't be negative

$\therefore \quad n = 10$

The given A.P has 10 terms.

Arithematic Progressions Ex 19.4 Q17

Given,

$$a_7 = 10$$
$$S_{14} - S_7 = 17 \qquad \text{---(i)}$$
$$\therefore \quad S_{14} = 17 + S_7 = 17 + 10 = 27 \qquad \text{---(ii)}$$

From (i) and (ii)

$S_7 = \frac{7}{2}\left[2a + (7-1)d\right] \qquad \left[\text{Using } S_n = \frac{n}{2}\left[2a + (n-1)d\right]\right]$

$\Rightarrow \quad 10 = 7a + 21d \qquad \text{---(iii)}$

and

$S_{14} = \frac{14}{2}\left[2a + 13d\right]$

$\Rightarrow \quad 27 = 28a + 182d \qquad \text{---(iv)}$

Solving (iii) and (iv)

$$a = 1 \text{ and } d = \frac{1}{7}$$

$\therefore$ The required A.P is

$$1, 1 + \frac{1}{7}, 1 + \frac{2}{7}, 1 + \frac{3}{7}, \ldots, +\infty$$

or $\quad 1, \frac{8}{7}, \frac{9}{7}, \frac{10}{7}, \frac{11}{7}, \ldots, \infty$

Arithematic Progressions Ex 19.4 Q18

Given,

$$a_3 = 7 = a + 2d \qquad \text{---(i)}$$

$$a_7 = 3a_3 + 2$$

$$\therefore \quad a_7 = 3(7) + 2 \qquad [\because a_3 = 7]$$

$$= 23 = a + 6d \qquad \text{---(ii)}$$

solving (i) and (ii)

$$a = -1,\ d = 4$$

Then, sum of 20 terms of this A.P

$$\Rightarrow \quad S_{20} = \frac{20}{2}[2 + (20-1)4] \qquad \left[\text{Using } S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$$= 10 \times 74$$

$$= 740$$

First term is -1 common defference $= 4$, sum of 20 terms $= 740$.

Arithematic Progressions Ex 19.4 Q19

Given,

$$a = 2$$

$$l = 50$$

$$\therefore \quad l = a + (n-1)d$$

$$50 = 2 + (n-1)d$$

$$(n-1)d = 48 \qquad \text{---(i)}$$

S_n of all n terms is given 442

$$\therefore \quad S_n = \frac{n}{2}[a + l]$$

$$442 = \frac{n}{2}[2 + 50]$$

$$\text{or} \quad n = 17 \qquad \text{---(ii)}$$

From (i) and (ii)

$$d = \frac{48}{n-1} = \frac{48}{16} = 3$$

The common difference is 3.

Arithematic Progressions Ex 19.4 Q20

Let no. of terms be 2n

Odd terms sum=24=$T_1+T_3+...+T_{2n-1}$

Even terms sum=30=$T_2+T_4+...+T_{2n}$

Subtract above two equations

nd=6

$$T_{2n}=T_1+\frac{21}{2}$$

$$T_{2n}-a=\frac{21}{2}$$

$$(2n-1)d=\frac{21}{2}$$

$$12-\frac{21}{2}=d=\frac{3}{2}$$

$$\Rightarrow n=6\times\frac{2}{3}=4$$

$$Total\ terms=2n=8$$

Subtitute above values in equation of

sum of even terms or odd terms, we get

$$a=\frac{3}{2}$$

So series is $\frac{3}{2},3,\frac{9}{2}$.......

Arithematic Progressions Ex 19.4 Q21

Let a be the first term of the AP and d is the common difference. Then

$$S_n=\frac{n}{2}(2a+(n-1)d)$$

$$n^2p=\frac{n}{2}(2a+(n-1)d)$$

$$np=\frac{1}{2}[2a+(n-1)d]$$

$$2np=2a+(n-1)d \qquad ......(1)$$

Again

$$S_m=\frac{m}{2}(2a+(m-1)d)$$

$$m^2p=\frac{m}{2}(2a+(m-1)d)$$

$$mp=\frac{1}{2}[2a+(m-1)d]$$

$$2mp=2a+(m-1)d \qquad ......(2)$$

Now subtract (1) from (2)

$$2p(m-n)=(m-n)d$$

$$d=2p$$

Therefore

$$2mp=2a+(m-1)\cdot 2p$$

$$2a=2p$$

$$a=p$$

The sum up to p terms will be:

$$S_p=\frac{p}{2}(2a+(p-1)d)$$

$$=\frac{p}{2}(2p+(p-1)\cdot 2p)$$

$$=\frac{p}{2}(2p+2p^2-2p)$$

$$=p^3$$

Hence it is shown.

Arithematic Progressions Ex 19.4 Q22

$$a_{12}=a+11d=-13 \qquad ---(i) \qquad [\text{Given}]$$

$$s_4=\frac{4}{2}(2a+3d)=24 \qquad ---(ii) \qquad [\text{Given}]$$

From (i) and (ii)

$d=-2$ and $a=9$

Then,

Sum of irst 10 terms is

$$S_{10}=\frac{10}{2}[2\times 9+(9)(-2)] \qquad \left[\text{Using } S_n=\frac{n}{2}[2a+(n-1)d]\right]$$

$$=0$$

Sum of first 10 terms is zero.

Arithematic Progressions Ex 19.4 Q23

$$a_5 = a + 4d = 30 \quad ---(i) \quad [\text{Given}]$$
$$a_{12} = a + 11d = 65 \quad ---(ii) \quad [\text{Given}]$$

From (i) and (ii)

$d = 5$ and $a = 10$

Then,

Sum of irst 20 terms is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad S_{20} = \frac{20}{2}[2 \times 10 + (20-1)5]$$
$$= 1150$$

Sum of first 20 terms is 1150.

Arithematic Progressions Ex 19.4 Q24

Here,

$$a_k = 5k + 1$$
$$a_1 = 5 + 1 = 6$$
$$a_2 = 5(2) + 1 = 11$$
$$a_3 = 5(3) + 1 = 16$$
$$d = 11 - 6 = 16 - 11 = 5$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{n}{2}[2(6) + (n-1)(5)]$$
$$= \frac{n}{2}[12 + 5n - 5]$$
$$S_n = \frac{n}{2}(5n + 7)$$

Arithematic Progressions Ex 19.4 Q25

sum of all two digit numbers which when divided by 4,

yields 1 as remainder, $\Rightarrow$ all 4n+1 terms with $n \geq 3$

13,17,21,............97

$n = 22, a = 13, d = 4$

$$sum of\ terms = \frac{22}{2}[26 + 21 \times 4] = 11 \times 110 = 1210$$

Arithematic Progressions Ex 19.4 Q26

Sum of terms 25, 22, 19,......, is 116

$$\frac{n}{2}\left[50+(n-1)(-3)\right]=116$$

$$\frac{n}{2}\left[53-3n\right]=116$$

$$53n-3n^2=232$$

$$3n^2-53n+232=0$$

$$3n^2-29n-24n+232=0$$

$$n(3n-29)-8(3n-29)=0$$

$$(3n-29)(n-8)=0$$

$$\Rightarrow n=8 or \frac{29}{3}$$

n cannot be in fraction, so n=8

last term=25-7×3=4

Arithematic Progressions Ex 19.4 Q27

Let the number of terms is n.
Now the sum of the series is:

$$1+3+5+\cdots+2001$$

Here $l=2001$ and $d=2$.
Therefore

$$l=a+(n-1)d$$
$$2001=1+(n-1)\cdot 2$$
$$2(n-1)=2000$$
$$n-1=1000$$
$$n=1001$$

Therefore the sum of the series is:

$$S=\frac{1001}{2}\left[2+(1001-1)2\right]$$
$$=1001^2$$
$$=1002001$$

Arithematic Progressions Ex 19.4 Q28

Let the number of terms to be added to the series is n.
Now $a=-6$ and $d=0.5$.
Therefore

$$-25=\frac{n}{2}\left[2(-6)+(n-1)(0.5)\right]$$
$$-50=n\left[-12+0.5n-0.5\right]$$
$$-12.5n+0.5n^2+50=0$$
$$n^2-25n+100=0$$
$$n=20,5$$

Therefore the value of n will be either 20 or 5.

Arithematic Progressions Ex 19.4 Q29

Here the first term $a = 2$. Let the common difference is d.
Now

$$\frac{5}{2}\left[2a+(5-1)d\right]=\frac{1}{4}\left[\frac{5}{2}\left[2(a+5d)+(5-1)d\right]\right]$$

$$\frac{5}{2}\left[2\cdot 2+4d\right]=\frac{5}{8}\left[2\cdot 2+14d\right]$$

$$10+10d=\frac{5}{2}+\frac{35}{4}d$$

$$\frac{5}{4}d=-7.5$$

$$d=-6$$

The 20^{th} term will be:

$$a+(n-1)d=2+(20-1)(-6)$$

$$=-112$$

Hence it is shown.

Arithematic Progressions Ex 19.4 Q30

$$S_{(2n+1)}=S_1=\frac{(2n+1)}{2}\left[2a+(2n+1-1)d\right]$$

$$S_1=\frac{(2n+1)}{2}\left[2a+2nd\right]$$

$$=(2n+1)(a+nd) \qquad \text{---(i)}$$

Sum of odd terms = S_2

$$S_2=\frac{(n+1)}{2}\left[2a+(n+1-1)(2d)\right]$$

$$=\frac{(n+1)}{2}\left[2a+2nd\right]$$

$$S_2=(n+1)(a+nd) \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$S_1:S_2=(2n+1)(a+nd):(n+1)(a+nd)$$

$$S_1:S_2=(2n+1);(n+1)$$

Arithematic Progressions Ex 19.4 Q31

Here,

$$S_n = 3n^2 \qquad \text{---(i)} \qquad [\text{Given}]$$

Where n is number of term

$$\therefore \quad S_n = \frac{n}{2}\left[2a + (n-1)d\right] \qquad \text{---(ii)}$$

From (i) and (ii)

$$3n^2 = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$6n = 2a + nd - d$$

Equating both sides

$$6n = nd$$

$$\therefore \quad d = 6 \qquad \text{---(iii)}$$

and

$$0 = 2a - d$$

or $\quad d = 2a \qquad$ ---(iv)

From (iii) and (iv)

$$a = 3 \text{ and } d = 6$$

$\therefore$ The required A.P is $3, 9, 15, 21, \ldots, \infty$

Arithematic Progressions Ex 19.4 Q32

$$S_n = nP + \frac{1}{2}n(n-1)Q \qquad [\text{Given}]$$

$$S_n = \frac{n}{2}\left[2P + (n-1)Q\right] \qquad \text{---(i)}$$

We know

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] \qquad \text{---(ii)}$$

Where a = first term and d = common difference comparing (i) and (ii)

$$d = Q$$

$\therefore$ The common difference is Q.

Arithematic Progressions Ex 19.4 Q33

Let sum of n terms of two A.P be S_n and $S'n$.

Then, $S_n = 5n + 4$ and $S'_n = 9n + 16$ respectively.

Then, if ratio of sum of n terms of 2A.P is giben, then the ratio of there nth ther is obtained by replacing n by $(2n-1)$.

$$\frac{a_n}{a_n'} = \frac{5(2n-1)+4}{9(2n-1)+16}$$

$\therefore$ Ratio of there 18th term is

$$\frac{a_{18}}{a'_{18}} = \frac{5(2\times 18-1)+4}{9(2\times 18-1)+16}$$

$$= \frac{5\times 35+4}{9\times 35+16}$$

$$= \frac{179}{321}$$

Arithematic Progressions Ex 19.4 Q34

Let sum of n term of 1 A.P series be S_n are other S_n

The, $S_n = 7n+2$ ---(i).

$S_n = n+4$ ---(ii)

If the ratio of sum of n terms of 2 A.P is given, then the ratio of there nth term is obtained by replacing n by $(2n-1)$.

$$\frac{a_n}{a_{n'}} = \frac{7(2n-1)+2}{(2n-1)+4}$$

Putting $n = 5$ to get the ratio of 5th term, we get

$$\frac{a_5}{a'5} = \frac{7(2\times 5-1)+2}{(2\times 5-1)+4} = \frac{65}{13} = \frac{5}{1}$$

The ratio is 5 : 1.

RD Sharma Solutions Class 11 Maths Chapter 19 Ex 19.5

Arithematic Progressions Ex 19.5 Q1(i)

$\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ will be in A.P if $\frac{c+a}{b} - \frac{b+c}{a} = \frac{a+b}{c} - \frac{c+a}{b}$

if $\quad \frac{ca+a^2-b^2-cb}{ab} = \frac{ab+b^2-c^2-ac}{bc}$

LHS $\quad \Rightarrow \frac{ca+a^2-b^2-cb}{ab}$

$\Rightarrow \quad \frac{c^2a+a^2c-b^2c-c^2b}{abc}$

$\Rightarrow \quad \frac{c(a-b)[a+b+c]}{abc} \qquad$ ---(i)

RHS $\quad \Rightarrow \frac{ab+b^2-c^2-ac}{bc}$

$\Rightarrow \quad \frac{a^2b+ab^2-ac^2-a^2c}{abc}$

$\Rightarrow \quad \frac{a(b-c)[a+b+c]}{abc} \qquad$ ---(ii)

and since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

$c(b-a) = a(b-c) \qquad$ ---(iii)

$\therefore \quad$ LHS = RHS and the given terms are in A.P.

Arithematic Progressions Ex 19.5 Q1(ii)

$a(b+c), b(c+a), c(a+b)$ are in A.P if $b(c+a) - a(b+c) = c(a+b) - b(c+a)$

LHS $\quad = b(c+a) - a(b+c)$

$= bc + ab - ab - ac$

$= c(b-a)$ ---(i)

RHS $= c(a+b) - b(c+a)$

$= ca + cb - bc - ba$

$= a(c-b)$ ---(ii)

and $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$\therefore$ $\frac{1}{a} - \frac{1}{b} = \frac{1}{b} - \frac{1}{c}$

or $c(b-a) = a(c-b)$ ---(iii)

From (i), (ii) and (iii)

$a(b+c), b(c+a), c(a+b)$ are in A.P

Arithematic Progressions Ex 19.5 Q2

$\frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}$ are in A.P if $\frac{b}{a+c} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{a+c}$

LHS $= \frac{b}{a+c} - \frac{a}{b+c}$

$\Rightarrow \frac{b^2 + bc - a^2 - ac}{(a+c)(b+c)}$

$\Rightarrow \frac{(b-a)(a+b+c)}{(a+c)(b+c)}$ ---(i)

RHS $= \frac{a}{a+b} - \frac{b}{a+c}$

$\Rightarrow \frac{ca + c^2 - b^2 - ab}{(a+b)(b+c)}$

$$\Rightarrow \quad \frac{(c-b)(a+b+c)}{(a+b)(b+c)} \qquad \text{---(ii)}$$

and $\quad a^2, b^2, c^2$ are in A.P

$$\therefore \quad b^2 - a^2 = c^2 - b^2 \qquad \text{---(iii)}$$

Substituting $b^2 - a^2$ with $c^2 - b^2$

(i) = (ii)

$$\therefore \quad \frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b} \text{ are in A.P}$$

Arithematic Progressions Ex 19.5 Q3(i)

$a^2(b+c), b^2(c+a), c^2(a+b)$ are in A.P.

If $b^2(c+a) - a^2(b+c) = c^2(a+b) - b^2(a+c)$

$$\Rightarrow \quad b^2c + b^2a - a^2b - a^2c = c^2a + c^2b - b^2a - b^2c$$

Given, $b - a = c - b \qquad [a, b, c \text{ are in A.P}]$

$$c\left(b^2 - a^2\right) + ab(b-a) = a\left(c^2 - b^2\right) + bc(c-b)$$

$$(b-a)(ab+bc+ca) = (c-b)(ab+bc+ca)$$

Cancelling $ab + bc + ca$ from both sides

$$b - a = c - b$$

$2b = c + a$ which is true

Hence, $a^2(b+c), (c+a)b^2$ and $c^2(a+b)$ are also in A.P.

Arithematic Progressions Ex 19.5 Q3(ii)

(ii) T.P $b+c-a, c+a-b, a+b-c$ are in A.P.

$b+c-a, c+a-b, a+b-c$ are in A.P only if $(c+a-b)-(b+c-a)=(a+b-c)-(c+a-b)$

$$\text{LHS} \Rightarrow (c+a-b)-(b+c-a)$$
$$\Rightarrow \quad 2a-2b \qquad \text{---(i)}$$

$$\text{RHS} \Rightarrow (a+b-c)-(c+a-b)$$
$$\Rightarrow \quad 2b-2c \qquad \text{---(ii)}$$

Since,

a, b, c are in A.P

$\therefore \quad b-a=c-b$

or $\quad a-b=b-c \qquad$ ---(iii)

From (i), (ii) and (iii)

LHS = RHS

Thus, given numbers

$b+c-a, c+a-b, a+b-c$ are in A.P

Arithematic Progressions Ex 19.5 Q3(iii)

To prove $bc-a^2, ca-b^2, ab-c^2$ are in A.P

$$\left(ca-b^2\right)-\left(bc-a^2\right)=\left(ab-c^2\right)-\left(ca-b^2\right)$$

$$\text{LHS} = \left(a-b^2-bc+a^2\right)$$
$$= (a-b)[a+b+c] \qquad \text{---(i)}$$

$$\text{RHS} = ab - c^2 - ca + b^2$$
$$= (b-c)[a+b+c] \qquad \text{---(ii)}$$

and since a, b, c are in ab

$$b - c = a - b$$

$\therefore$ LHS = RHS

and

Thus, $bc - a^2, ca - b^2, ab - c^2$ are in A.P

Arithematic Progressions Ex 19.5 Q4

(i) If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\text{LHS} = \frac{1}{b} - \frac{1}{a}$$
$$= \frac{a-b}{ab} = \frac{c(a-b)}{abc} \qquad \text{---(i)}$$

$$\text{RHS} = \frac{1}{c} - \frac{1}{b}$$
$$= \frac{a(b-c)}{abc} \qquad \text{---(ii)}$$

Since, $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P

$$\frac{b+c}{a} - \frac{c+a}{b} = \frac{c+a}{b} - \frac{a+b}{c}$$

$$\frac{b^2 + cb - ac - a^2}{ab} = \frac{c^2 + ac - ab - b^2}{bc}$$

$$\Rightarrow \quad \frac{(b-a)(a+b+c)}{ab} = \frac{(c-b)(a+b+c)}{bc}$$

$$\text{or} \quad \frac{a(b-c)}{abc} = \frac{c(a-b)}{abc} \qquad \text{---(iii)}$$

From (i), (ii) and (iii)

LHS = RHS

Hence, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

(ii) If bc, ca, ab are in A.P

Then,

$$ca - bc = ab - ca$$

$$c(a-b) = a(b-c) \qquad \text{---(i)}$$

If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\Rightarrow \quad c(a-b) = a(b-c) \qquad \text{---(ii)}$$

Thus, the condition necessary to prove bc, ca, ab in A.P is fullfilled.

Thus, bc, ca, ab, are in A.P.

Arithematic Progressions Ex 19.5 Q5

(i) If $(a-c)^2 = 4(a-b)(b-c)$

Then,

$$a^2 + c^2 - 2ac = 4\left(ab - b^2 - ac + bc\right)$$

$$\Rightarrow \quad a^2 + c^2 4b^2 + 2ac - 4ab - 4bc = 0$$

$$\Rightarrow \quad (a+c-2b)^2 = 0 \qquad \left[\text{Using } (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc\right]$$

$$\therefore \quad a + c - 2b = 0$$

$$\text{or} \quad a + c = 2b$$

and since,

$$a, b, c \text{ are in A.P} \qquad [\text{Given}]$$

$$a + c = 2b$$

Hence proved.

$$(a-c)^2 = 4(a-b)(b-c)$$

(ii) If $a^2 + c^2 + 4ac = 2(ab + bc + ca)$

Then,

$$a^2 + c^2 + 2ac - 2ab - 2bc = 0$$

$$\text{or} \quad (a+c-b)^2 - b^2 = 0 \qquad \left[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc\right]$$

$$\text{or} \quad b = a + c - b$$

$$\text{or} \quad 2b = a + c$$

$$b = \frac{a+c}{2}$$

and since,

$$a, b, c \text{ are in A.P}$$

$$b = \frac{a+c}{2}$$

Thus, $a^2 + c^2 + 4ac = 2(ab + bc + ca)$ Hence proved.

(iii) If $a^3 + c^3 + 6abc = 8b^3$

$$\text{or} \quad a^3 + c^3 - (2b)^3 + 6abc = 0$$

$$\text{or} \quad a^3 + (-2b)^3 + c^3 + 3 \times a \times (-2b) \times c = 0$$

$$\therefore \quad (a - 2b + c) = 0 \qquad \left[\begin{array}{l}\because x^3 + y^3 + z^3 + 3xyz = 0 \\ \text{or if } x + y + z = 0\end{array}\right]$$

or $\quad a + c = 2b$

$$a - b = c - b$$

and since, a, b, c are in A.P

Thus, $a - b = c - b$

Hence proved. $a^3 + c^3 + 6abc = 8b^3$

Arithematic Progressions Ex 19.5 Q6

Here,

$$a\left(\frac{1}{b}+\frac{1}{c}\right),\ b\left(\frac{1}{c}+\frac{1}{a}\right),\ c\left(\frac{1}{a}+\frac{1}{b}\right) \text{ are in A.P.}$$

$$\Rightarrow \quad a\left(\frac{1}{b}+\frac{1}{c}\right)+1,\ b\left(\frac{1}{c}+\frac{1}{a}\right)+1,\ c\left(\frac{1}{a}+\frac{1}{b}\right)+1 \text{ are in A.P.}$$

$$\Rightarrow \quad \left(\frac{ac+ab+bc}{bc}\right),\ \left(\frac{ab+bc+ac}{ac}\right),\ \left(\frac{cb+ac+ab}{ab}\right) \text{ are in A.P.}$$

$$\Rightarrow \quad \frac{1}{bc},\ \frac{1}{ac},\ \frac{1}{ab} \text{ are in A.P.}$$

$$\Rightarrow \quad \frac{abc}{bc},\ \frac{abc}{ac},\ \frac{abc}{ab} \text{ are in A.P.}$$

$$\Rightarrow \quad a,\ b,\ c \text{ are in A.P.}$$

RD Sharma Solutions Class 11 Maths Chapter 19 Ex 19.6

Arithematic Progressions Ex 19.6 Q1

(i) 7 and 13

Let A be the arithematic mean of 7 and 13.

Then,

$$7, A, 13 \text{ ar ein A.P}$$

$$\Rightarrow \quad A - 7 = 13 - A$$

$$\Rightarrow \quad A = \frac{13+7}{2} = 10$$

$\therefore$ A.M is 10.

(ii) 12 and -8

Let A be the arithematic mean of 12 and -8

Then,

$$12, A, -8 \text{ are in A.P}$$

$$\Rightarrow \quad A - 12 = -8 - A$$

$$\Rightarrow \quad A = \frac{12+(-8)}{2} = 2$$

$\therefore$ A.M is 2.

(iii) $(x-y)$ and $(x+y)$

Let A be the arithematic mean of $(x-y)$ and $(x+y)$

Then,

$$(x-y), A, (x+y) \text{ are in A.P}$$

$$\Rightarrow \quad A - (x-y) = (x+y) - A$$

$$\Rightarrow \quad A = \frac{(x-y)+(x+y)}{2} = \frac{2x}{2} = x$$

$\therefore$ A.M is x.

Arithematic Progressions Ex 19.6 Q2

Let A_1, A_2, A_3, A_4 be the 4 A.M.s between 4 and 19

Then,

$4, A_1, A_2, A_3, A_4, 19$ are in A.P of 6 terms

$A_n = a + (n-1)d$

$a_6 = 19 = 4 + (6-1)d$

or $\quad d = 3 \qquad$ ---(i)

Now,

$A_1 = a + d = 4 + 3 = 7$

$A_2 = A_1 + d = 7 + 3 = 10$

$A_3 = A_2 + d = 10 + 3 = 13$

$A_4 = A_3 + d = 13 + 3 = 16$

The 4 A.M.s between 4 and 19 are $7, 10, 13, 16$.

Arithematic Progressions Ex 19.6 Q3

$2, a_1, a_2, a_3, a_4, a_5, a_6, a_7, 17$

$17 = a + 8d$

$a = 2 \Rightarrow d = \frac{15}{8}$

$a_1 = 2 + \frac{15}{8} = \frac{31}{8}$

$a_2 = \frac{31}{8} + \frac{15}{8} = \frac{46}{8}$

so we get our final series as

$2, \frac{31}{8}, \frac{46}{8}, \frac{61}{8}, \frac{76}{8}, \frac{91}{8}, \frac{106}{8}, \frac{121}{8}, \frac{136}{8} = 17$

Arithematic Progressions Ex 19.6 Q4

Let $A_1, A_2, A_3, A_4, A_5, A_6$ be the 6 AM's between 15 and -13

Then,

$15, A_1, A_2, A_3, A_4, A_5, A_6, -13$ are in A.P of 8 terms

Here, $-13 = a_8 = a + 7d$

$\Rightarrow \quad -13 = 15 + 7d$

or $\quad d = -4 \qquad$ ---(i)

$\therefore \quad A_1 = a + d = 15 - 4 = 11$

$A_2 = a + 2d = 15 - 2(4) = 7$

$A_3 = a + 3d = 15 - 4(3) = 3$

$A_4 = a + 4d = 15 - 4(4) = -1$

$A_5 = a + 5d = 15 - 4(5) = -5$

$A_6 = a + 6d = 15 - 4(6) = -9$

The 6 A.M.s between 15 and -13 are $11, 7, 3, -1, -5$ and -9.

Arithematic Progressions Ex 19.6 Q5

Let the n A.M's between 3 and 17 be $A_1, A_2, A_3, \ldots, A_n$

Then,

ATQ

$$\frac{A_n}{A_1} = \frac{3}{1} \qquad \text{---(i)}$$

We know that

$3, A_1, A_2, A_3, \ldots, A_n, 17$ are in A.P of $n+2$ terms

So, 17 is the $(n+2)$ th terms.

i.e. $\quad 17 = 3 + (n + 2 - 1)d \qquad$ [Using $a_n = a + (n-1)d$]

or $\quad d = \dfrac{14}{(n+1)} \qquad$ ---(ii)

$\therefore \quad A_n = 3 + (n + 1 - 1)d$

$$= 3 + \frac{14n}{n+1} = \frac{17n+3}{n+1} \qquad \text{---(iii)}$$

$$A_1 = 3 + d = \frac{3n+17}{n+1} \qquad \text{---(iv)}$$

From (i), (iii) and iv

$$\frac{A_n}{A_1} = \frac{17n+3}{3n+17} = \frac{3}{1}$$

$\therefore \quad n = 6$

There are 6 A'M between 3 and 17.

Arithematic Progressions Ex 19.6 Q6

Let there be n A.M between 7 and 71 and let the A.M's be $A_1, A_2, A_3, \dots, A_n$.

So,

$7, A_1, A_2, A_3, \dots, A_n, 71$ are in A.P of $(n+2)$ terms

$A_5 = a_6 = a + 5d = 27$ [Given]

$\Rightarrow \quad a + 5d = 27$

$\Rightarrow \quad d = 4$ $[\because a = 7]$ ---(i)

The $(n+2)$ th term of A.P is 71

$\therefore \quad a_{n+2} = 7 = a + (n+2-1)d$

or $\quad n = 15$

There are 15 AM's between 7 and 71.

Arithematic Progressions Ex 19.6 Q7

Let $A_1, A_2, A_3, A_4, \dots, A_n$ be the n AMs inserted between two number a and b.

Then,

$a, A_1, A_2, A_3, A_4, \dots, A_n, b$ are in A.P

So, the mean of a and b

$$\text{A.M} = \frac{a+b}{2}$$

The mean of A_1 and A_n

$$\text{A.M} = \frac{a+d+b-d}{2} = \frac{a+b}{2}$$

Similarly mean of A_2 and A_{n-1}

$$\text{A.M} = \frac{a+2d+b-2d}{2} = \frac{a+b}{2}$$

Similarly we observe the means is equidistant from begining and the end is constant $\frac{a+b}{2}$.

The AM is $\frac{a+b}{2}$.

Arithematic Progressions Ex 19.6 Q8

Here,

A_1 is the A.M of x and y,

and A_2 is the A.M of y and z.

Then,

$$A_1 = \frac{x+y}{2} \quad \text{---(i)} \qquad \left[\because \text{AM} = \frac{a+b}{2}\right]$$

$$A_2 = \frac{y+z}{2} \quad \text{---(ii)}$$

Let A.M be the arithematic mean of A_1 and A_2

Then,

$$\text{A.M} = \frac{A_1 + A_2}{4}$$

$$= \frac{x+y+y+z}{4}$$

$$= \frac{x+2y+z}{4} \quad \text{---(iii)}$$

Since, 4, y, z are in A.P [Given]

$$y = \frac{x+a}{2} \quad \text{---(iv)}$$

From (iii) and (iv)

$$\text{A.M} = \frac{\left(\frac{x+a}{2}\right) + \left(\frac{2y}{2}\right)}{2} = \frac{y+y}{2} = y$$

Hence, proved A.M between A_1 and A_2 is y.

Arithematic Progressions Ex 19.6 Q9

$8, a_1, a_2, a_3, a_4, a_5, 26$

$a = 8$

$a + 6d = 26$

$\Rightarrow d = \frac{18}{6} = 3$

So series is 8, 11, 14, 17, 20, 23, 26

RD Sharma Solutions Class 11 Maths Chapter 19 Ex 19.7

Arithematic Progressions Ex 19.7 Q1

Let the amount saved by the man in first year be x.

Then,

ATQ

$x+(x+100)+(x+200)+\ldots+(x+900)=16500$

As his saving increased by Rs 100 every year.

$\therefore \quad 10x+100+200+\ldots+900=16500 \quad$ ---(i)

Here,

$100+200+300+\ldots+900$ form a seried of

$a=100,\ d=100$ and $n=9$

So,

$$S_n=\frac{n}{2}[a+l]$$

$$S_9=\frac{9}{2}[100+900]=4500 \quad \text{---(ii)}$$

From (i) and (ii)

$10x+(4500)=16500$

$10x=12000$

or $\quad x=1200$

The man saved Rs 1200 in the first year.

Arithematic Progressions Ex 19.7 Q2

Let the man save Rs 200 in n numbers of years.

Then,

ATQ

$32+36+40+\ldots=200$

It rorms a series of n terms, with $a=32$ and $d=4$

$$\therefore \quad S_n=\frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow \quad 200=\frac{n}{2}[2(32)+(n-1)4]$$

$\Rightarrow \quad 400=60n+4n^2$

$\Rightarrow \quad n^2+15n-100=0$

$\Rightarrow \quad n=5$ or -20

But, $n \neq -20$ [It can't be negative]

$\therefore$ $n = 5$

The man will save Rs 200 in 5 years.

Arithematic Progressions Ex 19.7 Q3

Let the 40 annual instalments form an alithmetic series of common diference d and first instalment a

Then, series so firmed is

$$a + (a + d) + (a + 2d) + \ldots = 3600$$

or $s_n = \frac{n}{2}[2a + (n - 1)d]$

or $3600 = 20[2a + 39d]$

$2a + 39d = 180$ ---(i)

and sum of first 30 terms is $\frac{2}{3}$ of 3600

$= 2400$

$\Rightarrow$ $2400 = \frac{30}{2}[2a + (29)d]$

or $2a + 29d = 160$ ---(ii)

From (i) and (ii)

$a = 51$

The first installment paid by this man is Rs 51.

Arithematic Progressions Ex 19.7 Q4

Let the number of Radio manufactured increase by x each year and number of radio manufacture in first year be a. So, A.P formed ATQ is

$$a, a + x, a + 2x, \ldots$$

Here,

$a_3 = a + 2x = 600$ ---(i)

$a_7 = a + 6x = 700$ ---(ii)

From (i) and (ii)

$a = 550,\ x = 25$

(i) 550 Radio's were manufactured in the first year.

(ii) The total produce in 7 years is sum of produce in the first 7 years.

$$S_7 = \frac{7}{2}[550 + 700] \qquad \left[\because S_n = \frac{n}{2}[a + l]\right]$$

$$= 4375$$

4375 Radio's were manufactured in first 7 years.

(iii) The product in 10th year

$$a_{10} = a + 9d$$

$$= 550 + 9(25) = 775$$

775 Radio's were manufactured in the 10th year.

Arithematic Progressions Ex 19.7 Q5

There are 25 trees at equal distance of 5 m in a line with a well(w), and the distance of the well from the nearesst tree = 10 m.

Thus,

The total distance travelled by gardener to tree 1 and back is 2×10 m = 20 m

Similarly for all the 25 trees.

The distance covered by gardener is

$$= 2[10 + (10 + 5) + (10 + 2 \times 5) + (10 + 3 \times 5) + \ldots + (10 + 23 \times 5)] \qquad \text{---(i)}$$

This forms a series of 1st term $a = 10$, common difference $d = 5$ and $n = 25$

$\therefore \quad 10 + (10 + 5) + (10 + 2 \times 5) + \ldots + (10 + 24 \times 5)$

$$\Rightarrow \quad S_{25} = \frac{25}{2}[2 \times 10 + (24)5] = 25[10 + 60] = 1750 \text{ m} \qquad \text{---(ii)}$$

From (i) and (ii)

Total distance = 2×1750 m = 3500 m.

Arithematic Progressions Ex 19.7 Q6

The man counts at the rate of Rs 180 per minute for half an hour. After this he counts at the rate of Rs 3 less every minute than preceding minute.

Then, the amount counted in first 30 minute

$$= \text{Rs } 180 \times 30 = \text{Rs } 5400 \qquad \text{---(i)}$$

The amount left to be counted after 30 minute

$= \text{Rs } 10710 - 5400 = \text{Rs } 5310$ ---(ii)

ATQ

A.P formed is $(180 - 3) + (180 - 2 \times 3) + \ldots = 5310$

Let time taken to count 5310 be t

Then,

$$S_t = \frac{t}{2}\left[(180 - 3) + (t - 1)(-3)\right]$$

$$5310 = \frac{t}{2}\left[200 - 3t\right]$$

or $t = 59$ minute

Thus, the total time taken by the man to count Rs 10710 is $(59 + 30) = 89$ minutes.

Arithematic Progressions Ex 19.7 Q7

The piece of equipment deprecites 15% in first year i.e., $\frac{15}{100} \times 600,000 = \text{Rs } 90,000$

$\therefore$ Value after 1st year $= 600,000 - 90,000$

$= \text{Rs } 510,000$ ---(i)

The equipment deprecites at the rate 135% in 2nd year i.e., $\frac{135}{1000} \times 600,000 = 81000$

$\therefore$ Value after 2nd year $= 81000$

The value after 3rd year $= \frac{12}{100} \times 600000 = 72000$

The total depreciation in 10 years

$$\Rightarrow \quad S_{10} = \frac{10}{2}\left[2 \times 81000 + (9)(-9000)\right]$$

$$= 5\left[81000\right] \qquad \left[\text{Using } S_n = \frac{n}{2}\left[2a + (n-1)d\right]\right]$$

$$= 405000$$

$\therefore$ The cost of machine after 10 years = Rs 600000 − 405000

$$= 105000.$$

Arithematic Progressions Ex 19.7 Q8

Total cost of tractor

$$= 6000 + \left[(500 + 12\% \text{ of } 6000 \text{ for } 1 \text{ year}) + (500 + 12\% \text{ of } 5500 \text{ } 1 \text{ year}) + \ldots.. + 12 \text{ times}\right]$$

$$= 6000 + 6000 + \frac{12}{100}(6000 + 5500 + \ldots\ldots + 12 \text{ times})$$

$$= 12000 + \frac{12}{100}\left[\frac{12}{2}(6000 + 5000)\right]$$

$$= 12000 + \frac{12}{100} \times \frac{12}{2} \times 6500$$

$$= 12000 + (72 \times 65)$$

$$= 12000 + 4680$$

$$= 16680$$

Total cost of tractor = Rs. 16680

Arithematic Progressions Ex 19.7 Q9

Total cost of Scooter

$$= \text{Rs}4000 + \left[\begin{array}{l}\{\text{Rs } 1000 + \text{S.I. on Rs Rs } 18000 \text{ for } 1 \text{ year}\} \\ +\{\text{Rs } 1000 + \text{S.I. on Rs Rs } 17000 \text{ for } 1 \text{ year}\} \\ +\ldots.. + 18 \text{ times}\end{array}\right]$$

$$= (4000 + 18000) + \text{S.I. for } 1 \text{ year on } (18000 + 17000 + \ldots..\text{to } 18 \text{ times})$$

$$= 22000 + \text{S.I. for } 1 \text{ year on } \left\{\frac{18}{2}(18000 + 1000)\right\}$$

$$= 22000 + 9(19000) \times \frac{10}{100}$$

$$= 22000 + 17100$$

$$= \text{Rs } 39100$$

Total cost of Scooter = Rs. 39100

Arithematic Progressions Ex 19.7 Q10

First year the person income is: 300,000

Second year his income will be: $300,000 + 10,000 = 310,000$

⋮

⋮

This way he receives the amount after 20 years will be:

$$300,000 + 310,000 + \cdots + 490,000$$

This is an AP with first term $a = 300000$ and common difference $d = 10,000$.

Therefore

$$S = \frac{20}{2}\left[2 \cdot 300000 + (20-1)10000\right]$$

$$= 10\left[600000 + 190000\right]$$

$$= 7900000$$

Arithematic Progressions Ex 19.7 Q11

In 1^{st} installment the man paid 100 rupees.

In 2^{nd} installment the man paid $(100+5) = 105$ rupees.

⋮

⋮

Likewise he pays up to the 30^{th} installment as follows:

$$100 + 105 + \cdots + (100 + 5 \times 29)$$

This is an AP with $a = 100$ and common difference $d = 5$.

Therefore at the 30^{th} installment the amount he will pay

$$T_{30} = 100 + (30-1)(5)$$

$$= 100 + 145$$

$$= 245$$

Arithematic Progressions Ex 19.7 Q12

Suppose carpenter took n days to finish his job.

First day carpenter made five frames

$a_1 = 5$

Each day after first day he made two more frames
d=2

$\therefore$ On n^{th} day frames made by carpenter are,

$a_n = a_1 + (n-1)d$

$\Rightarrow a_n = 5 + (n-1)2$

Sum of all the frames till n^{th} day is

$S = \frac{n}{2}[a_1 + a_n]$

$192 = \frac{n}{2}[5 + 5 + (n-1)2]$

$192 = 5n + n^2 - n$

$n^2 + 4n - 192 = 0$

$(n+16)(n-12) = 0$

$n = -16$ or $n = 12$

But number of days cannot be negative hence $n = 12$.

The carpenter took 12 days to finish his job.

Arithematic Progressions Ex 19.7 Q13

We know that sum of interior angles of a polygon with n sides is given by,

$a_n = 180°(n - 2)$

Sum of interior angles of a polygon with 3 sides is given by,

$a_3 = 180°(3 - 2) = 180°$............(i)

Sum of interior angles of a polygon with 7 sides is given by,

$a_4 = 180°(4 - 2) = 360°$............(ii)

Sum of interior angles of a polygon with 5 sides is given by,

$a_5 = 180°(5 - 2) = 540°$............(iii)

From eq^n (i), eq^n (ii) and eq^n (iii) we get,

$a_4 = 360° = 180° + 180° = a_3 + 180° = a_3 + d$

$a_5 = 540° = 180° + 360° = a_3 + 2d$

Hence the sums of the interior angles of polygons with 3, 4, 5, 6,... sides form an arithmetic progression.

Sum of interior angles of 21 sided polygon

$= 180°(21 - 2)$

$= 3420°$