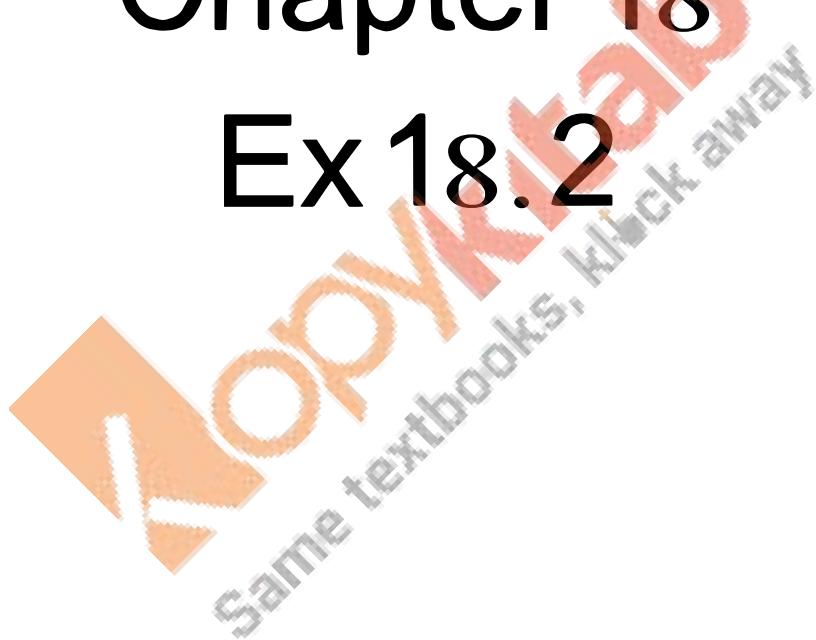


**RD Sharma
Solutions
Class 11 Maths
Chapter 18
Ex 18.2**



Binomial Theorem Ex 18.2 Q1

$$T_{r+1} = T_r = (-1)^r {}^n C_r x^{n-r} y^r$$

$$T_{11} = T_{10+1} = (-1)^{10} {}^{25} C_{10} (2x)^{15} \left(\frac{1}{x^2}\right)^{10} = {}^{25} C_{10} \left(\frac{2^{15}}{x^5}\right) = \frac{25!}{10!5!} 2^{15} x^{15} \times x^{-20}$$

11th term from the end = $(26 - 11 + 1) = 16^{\text{th}}$ from beginning.

$$\Rightarrow T_{16} = T_{15+1} = (-1)^{15} {}^{25} C_{15} (2x)^{10} \left(\frac{1}{x^2}\right)^{15} = {}^{25} C_{15} \frac{2^{10}}{x^{20}}$$

Binomial Theorem Ex 18.2 Q2

$$T_n = T_{r+1} = (-1)^r x^{n-r} y^r \times {}^{10} C_r$$

$$n = 7, r = 6, x = 3x^2, y = \frac{1}{x^3}$$

$$T_7 = T_{6+1} = (-1)^6 {}^{10} C_6 (3x^2)^4 \left(\frac{1}{x^3}\right)^6 = {}^{10} C_6 3^4 x^8 \times \frac{1}{x^{18}} = {}^{10} C_6 \times \frac{81}{x^{10}} = \frac{210 \times 81}{x^{10}} = \frac{17010}{x^{10}}$$

Binomial Theorem Ex 18.2 Q3

Fifth term from the end is

$(11-5+1) = 7^{\text{th}}$ term from beginning

$$\begin{aligned} T_7 &= T_{6+1} = (-1)^r {}^n C_r x^{n-r} y^r \\ &= (-1)^6 {}^{10} C_6 (3x)^4 \left(\frac{1}{x^2}\right)^6 = {}^{10} C_6 \times 3^4 \times \frac{x^4}{x^{12}} = \frac{210 \times 81}{x^8} = \frac{17010}{x^8} \end{aligned}$$

Binomial Theorem Ex 18.2 Q4

$$T_N = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$N = 8, r = 7, x = x^{3/2} y^{1/2}, y = x^{1/2} y^{3/2}, n = 10$$

$$T_8 = T_{7+1} = (-1)^7 {}^{10} C_7 \left(x^{3/2} y^{1/2}\right)^3 \left(x^{1/2} y^{3/2}\right)^7 = -{}^{10} C_7 x^{9/2} \times x^{7/2} \times y^{3/2} y^{21/2} = -120 x^8 y^{12}$$

Binomial Theorem Ex 18.2 Q5

$$T_N = T_{r+1} = {}^n C_r x^{n-r} y^r$$

$$N = 7, r = 6, n = 8, x = \frac{4x}{5}, y = \frac{5}{2x}$$

$$T_7 = T_{6+1} = {}^8 C_6 \left(\frac{4x}{5}\right)^2 \left(\frac{5}{2x}\right)^6 = 28 \times \frac{4^2}{5^2} \times x^4 \times \frac{5^6}{2^6 \times x^6} = \frac{28}{4} \times \frac{5^4}{x^4} = \frac{7 \times 5 \times 125}{x^4} = \frac{4375}{x^4}$$

Binomial Theorem Ex 18.2 Q6

Term from the beginning

$$T_N = T_{r+1} = {}^n C_r x^{n-r} y^r \quad \text{--- (i)}$$

$$N = 4, r = 3, n = 9, x = x, y = \frac{2}{x}$$

$$T_4 = T_{3+1} = {}^9 C_3 x^6 \left(\frac{2}{x}\right)^3 = \frac{9 \times 8 \times 7}{3 \times 2} x^3 \times 8 = 672 x^3$$

4th term from the end = 7th term from beginning

Using (i)

$$N = 7, r = 6, n = 9, x = x, y = \frac{2}{x}$$

$$T_7 = T_{6+1} = {}^9 C_6 x^3 \left(\frac{2}{x}\right)^6 = \frac{9 \times 8 \times 7}{3 \times 2} \times \frac{2^6}{x^3} = \frac{5376}{x^3}$$

Binomial Theorem Ex 18.2 Q7

$$T_N = T_{r+1} = (-1)^r {}^n C_2 x^{n-r} y^r$$

4th term from the end = 7th term from beginning

$$N = 7, r = 6, n = 9, x = \frac{4x}{5}, y = \frac{5}{2x}$$

$$T_7 = T_{6+1} = (-1)^6 {}^9 C_6 \left(\frac{4x}{5}\right)^3 \left(\frac{5}{2x}\right)^6 = \frac{9 \times 8 \times 7}{3 \times 2} \times \frac{4^3 \times 5^6}{5^3 \times 2^6} \times \frac{x^3}{x^6} = \frac{9 \times 8 \times 7 \times 5^3}{6 \times x^3} = \frac{9 \times 8 \times 7 \times 125}{6 \times x^3} = \frac{10500}{x^3}$$

Binomial Theorem Ex 18.2 Q8

7th term from the end = 3rd term from beginning

$$T_N = T_{r+1} = (-1)^r {}^n C_2 x^{n-r} y^r$$

$$N = 3, r = 2, n = 8, x = 2x^2, y = \frac{3}{2x}$$

$$T_3 = T_{2+1} = (-1)^2 {}^8 C_2 (2x^2)^6 \left(\frac{3}{2x}\right)^2 = \frac{8 \times 7}{2} \times \frac{2^6 \times 3^2 \times x^{12}}{2^2 \times x^2} = 8 \times 7 \times 9 \times 8 \times x^{10} = 4032x^{10}$$

Binomial Theorem Ex 18.2 Q9(i)

$$x^{10} \text{ in } \left(2x^2 - \frac{1}{x}\right)^{20}$$

$$T_r = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$(-1)^r {}^{20} C_r (2x^2)^{20-r} \left(\frac{1}{x}\right)^r$$

Coefficient of x^{10} is

$$(-1)^r {}^{20} C_r 2^{20-r} x^{40-2r} x^{-r} \quad \text{--- (i)}$$

$$\Rightarrow x^{40-3r} = x^{10}$$

$$\Rightarrow 10 - 3r = 10$$

$$3r = 30$$

$$r = 10$$

Substituting $r = 10$ in (i)

$$= \frac{(-1)^{10} {}^{20} C_{10} 2^{10}}{}^{20} C_{10} 2^{10}$$

Binomial Theorem Ex 18.2 Q9(ii)

$$x^7 \text{ in } \left(x - \frac{1}{x^2}\right)^{40}$$

$$T_r = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$= (-1)^r {}^{40} C_r x^{40-r} \left(\frac{1}{x^2}\right)^r$$

$$= (-1)^r {}^{40} C_r x^{40-r-2r}$$

$$\Rightarrow x^7 = x^{40-3r}$$

$$7 = 40 - 3r$$

$$3r = 33$$

$$r = 11$$

$$= (-1)^{11} {}^{40} C_{11} \text{ is coeff of } x^7$$

$$= - {}^{40} C_{11}$$

Binomial Theorem Ex 18.2 Q9(iii)

$$\begin{aligned}
x^{-15} \text{ in } & \left(3x^2 - \frac{a}{3x^3}\right)^{10} \\
& (-1)^r {}^{10}C_r \left(3x^2\right)^{10-r} \left(\frac{a}{3x^3}\right)^r \\
& (-1)^r {}^{10}C_r \frac{3^{10-r} a^r}{3^r} x^{20-2r-3r} \\
\Rightarrow & x^{20-5r} = x^{-15} \\
20-5r &= -15 \\
35 &= 5r \\
r &= 7 \\
& (-1)^7 {}^{10}C_7 \frac{3^3 a^7}{3^7} \\
& -\frac{40}{27} a^7
\end{aligned}$$

Binomial Theorem Ex 18.2 Q9(iv)

$$\begin{aligned}
x^9 \text{ in expansion of } & \left(x^2 - \frac{1}{3x}\right)^9 \\
T_n = T_{r+1} &= (-1)^r {}^nC_r x^{n-r} y^r \\
&= (-1)^r {}^9C_r \left(x^2\right)^{9-r} \left(\frac{1}{3x}\right)^r \\
&= (-1)^r {}^9C_r x^{\frac{1}{3r}} x^{18-2r-r} \\
\Rightarrow & x^{18-3r} = x^9 \\
18-3r &= 9 \\
3r &= 9 \\
r &= 3 \\
&= (-1)^3 {}^9C_3 \frac{1}{3^3} \\
&= -\frac{9 \times 8 \times 7}{3 \times 2 \times 9 \times 3} \\
&= \frac{-28}{9}
\end{aligned}$$

Binomial Theorem Ex 18.2 Q9(v)

$$\begin{aligned}
x^m \text{ in expansion of } & \left(x + \frac{1}{x}\right)^n \\
T_n = {}^nC_r x^{n-r} y^r &= {}^nC_r x^{n-r} \left(\frac{1}{x}\right)^r \\
x^{n-2r} &= x \\
n-2r &= m \\
r &= \frac{n-m}{2} \\
{}^nC_{\frac{n-m}{2}} &= \frac{n!}{\left(\frac{n-m}{2}\right)! \left(\frac{n+m}{2}\right)!}
\end{aligned}$$

Binomial Theorem Ex 18.2 Q9(vi)

$$\begin{aligned}
(1-2x^3+3x^5)\left(1+\frac{1}{x}\right)^4 &= (1-2x^3+3x^5) \left[{}^4C_0 + {}^4C_1 \frac{1}{x} + {}^4C_2 \left(\frac{1}{x}\right)^2 + {}^4C_3 \left(\frac{1}{x}\right)^3 + {}^4C_4 \left(\frac{1}{x}\right)^4 + \right. \\
&\quad \left. {}^4C_5 \left(\frac{1}{x}\right)^5 + {}^4C_6 \left(\frac{1}{x}\right)^6 + {}^4C_7 \left(\frac{1}{x}\right)^7 + {}^4C_8 \left(\frac{1}{x}\right)^8 \right] \\
&= -(2x^3) \left({}^4C_1 \left(\frac{1}{x}\right)^2 \right) + \left(3x^5 \times {}^4C_4 \left(\frac{1}{x}\right)^4 \right) \\
&= -(56) + (210) \\
&= -112 + 168 \\
&= 154
\end{aligned}$$

Binomial Theorem Ex 18.2 Q9(vii)

$$\begin{aligned}
(a-2b)^{12} &= {}^{12}C_0 a^{12} - {}^{12}C_1 a^{11}(2b)^1 + {}^{12}C_2 a^{10}(2b)^2 - {}^{12}C_3 a^9(2b)^3 + \dots - {}^{12}C_7 a^5(2b)^7 + \dots \\
&= -\frac{12!}{7!5!} \times 128 \\
&= -\frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2} \times 128 \\
&= \mathbf{-101376}
\end{aligned}$$

Binomial Theorem Ex 18.2 Q9(viii)

$$\begin{aligned}
(1-3x+7x^2)(1-x)^{16} &= (1-3x+7x^2) \left({}^{16}C_0 - {}^{16}C_1 x + {}^{16}C_2 x^2 + \dots + {}^{16}C_{16} x^{16} \right) \\
\therefore \text{Coefficient of } x \text{ in } (1-3x+7x^2)(1-x)^{16} &= 1 \times (-{}^{16}C_1) - 3 \times (-{}^{16}C_0) \\
&= -16 - 3 \\
&= -19
\end{aligned}$$

Binomial Theorem Ex 18.2 Q10

$$\begin{aligned}
T_n = T_{r+1} &= {}^nC_r x^{n-r} y^r \\
&= {}^{21}C_r \left(\left(\frac{x}{\sqrt{y}} \right)^{\frac{1}{3}} \right)^{21-r} \left(\left(\frac{y}{x^{\frac{1}{3}}} \right)^{\frac{1}{2}} \right)^r \\
&= {}^{21}C_r \left(\frac{x^{\frac{7-r}{3}}}{y^{\frac{7-r}{6}}} \right) \frac{y^{\frac{r}{2}}}{x^{\frac{r}{6}}} \\
&\quad \frac{x^{\frac{7-r}{3}-\frac{r}{6}}}{y^{\frac{7-r}{6}-\frac{r}{2}}} \\
\Rightarrow \quad x \frac{42-2r-r}{6} &= y \frac{21-r-3r}{6}
\end{aligned}$$

Since x and y have same power

$$\begin{aligned}
\frac{42-3r}{6} &= \frac{-(21-4r)}{6} \\
42+21 &= 4r+3r \\
63 &= 7r \\
r &= 9
\end{aligned}$$

Term is 10th

$$(t_n = t_{r+1})$$

Binomial Theorem Ex 18.2 Q11

$$(-1)^r {}^{20}C_r \left(2x^2\right)^{20-r} \left(\frac{1}{x}\right)^r$$

$$x^{40-2r} x^{-r} = x^9$$

$$40 - 3r = 9$$

$$31 = 3r$$

$$r = \frac{31}{3}$$

r can not be in fraction

∴ There is no term involving x^9 .

Binomial Theorem Ex 18.2 Q12

Any term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$ is

$$T_n = T_{r+1} = {}^nC_r x^{n-r} y^r$$

$$= {}^{12}C_r \left(x^2\right)^{12-r} \left(\frac{1}{x}\right)^{12}$$

$$= {}^{12}C_r x^{24-2r} x^{-12}$$

$$x^{24-2r} = x^{-1}$$

$$12 - 2r = -1$$

$$2r = 13$$

$$r = \frac{13}{2}$$

r can not be a fraction, therefore there is no term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$

having the term $x^{-\frac{1}{2}}$.

Binomial Theorem Ex 18.2 Q13(i)

$$\left(\frac{2}{3}x - \frac{3}{2x}\right)^{20}$$

Here, $n = 20$ which is an even number so, $\left(\frac{20}{2} + 1\right)^{\text{th}}$ i.e., 11th term is the middle term.

We know that,

$$T_n = T_{r+1} = (-1)^r {}^nC_r x^{n-r} y^r$$

$$n = 20, r = 10, x = \frac{2}{3}x, y = \frac{2}{3x}$$

$$T_{11} = T_{10+1} = (-1)^{10} {}^{20}C_{10} \left(\frac{2}{3}x\right)^{10} \left(\frac{3}{2x}\right)^{10}$$

$$= {}^{20}C_{10} \frac{2^{10}}{3^{10}} x^{10} \times \frac{3^{10}}{2^{10}} x^{-10}$$

$$= {}^{20}C_{10}$$

Binomial Theorem Ex 18.2 Q13(ii)

Here, $n = 12$, which is even number.

so, $\left(\frac{12}{2} + 1\right)$ th term i.e., 7th term is the middle term.

Hence, the middle term = $T_7 = T_{6+1}$

$$\begin{aligned}\therefore T_7 &= T_{6+1} = {}^{12}C_6 \times \left(\frac{a}{x}\right)^{12-6} \times (bx)^6 \\&= {}^{12}C_6 \left(\frac{a}{x}\right)^6 \times (bx)^6 \\&= \frac{12!}{(12-6)! 6!} \times \frac{a^6}{x^6} \times b^6 x^6 \\&= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6!}{(6 \times 5 \times 4 \times 3 \times 2 \times 1)} \times a^6 b^6 \\&= 924 \times a^6 b^6\end{aligned}$$

\therefore The middle term = $924 \times a^6 b^6$.

Binomial Theorem Ex 18.2 Q13(iii)

$$\left(x^2 - \frac{2}{x}\right)^{10}$$

Here, $n = 10$

$\therefore \left(\frac{n}{2} + 1\right)^{\text{th}} = \left(\frac{10}{2} + 1\right)^{\text{th}} = 6^{\text{th}}$ term is the middle term.

The term formula is

$$\begin{aligned}T_n - T_{n+1} &= (-1)^{r-a} C_r x^{r-a} y^a \\T_6 - T_{5+1} &= (-1)^{5-10} C_5 \left(x^2\right)^{10-5} \left(\frac{2}{x}\right)^5 \\&= -{}^{10}C_5 x^{20-10} \frac{2^5}{x^5} \\&= \frac{-10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \times 2^5 x^5 \\&= -8064 x^5\end{aligned}$$

Binomial Theorem Ex 18.2 Q13(iv)

$$\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$$

Here $n = 10$, which is even, therefore it has 11 terms

\therefore middle term is $\left(\frac{n}{2} + 1\right) = 6^{\text{th}}$ term

$$\begin{aligned}T_n - T_{n+1} &= (-1)^{r-a} C_r x^{r-a} y^a \\T_6 - T_{5+1} &= (-1)^{5-10} C_5 \left(\frac{x}{a}\right)^{10-5} \left(\frac{a}{x}\right)^5 \\&= -\frac{10!}{5! 5!} \times \frac{x^5}{a^5} \times a^5 \times x^{-5} \\&= -252\end{aligned}$$

Binomial Theorem Ex 18.2 Q14(i)

$$\left(3x - \frac{x^3}{6}\right)^9$$

Here, $n = 9$, which is odd number

$\therefore \left(\frac{9+1}{2}\right)^{\text{th}}$ and $\left(\frac{9+1}{2}+1\right)^{\text{th}}$ i.e., 5th, 6th term are the middle term.

Here, the term formula is

$$T_5 = T_{4+1} = (-1)^4 {}^9C_4 (3x)^5 \left(\frac{x^3}{6}\right)^4$$

$$= {}^9C_4 \frac{3^5}{6^4} \times x^5 \times x^{12}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 3^5}{4 \times 3 \times 2 \times 3^4 \times 2^4} x^{17}$$

$$= \frac{189}{8} x^{17}$$

$$T_6 = T_{5+1} = (-1)^5 {}^9C_5 (3x)^4 \left(\frac{x^3}{6}\right)^5$$

$$= -\frac{9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \times \frac{3^4}{6^5} \times x^4 \times x^{15}$$

$$= -\frac{9 \times 8 \times 7 \times 6 \times 3^4}{5 \times 4 \times 3 \times 2 \times 3^5 \times 2^5} x^{19}$$

$$= -\frac{21}{16} x^{19}$$

Binomial Theorem Ex 18.2 Q14(ii)

$$\left(3x^2 - \frac{1}{x}\right)^7$$

Here, $n = 7$, which is odd

$\therefore \left(\frac{7+1}{2}\right)^{\text{th}}$ and $\left(\frac{7+1}{2}+1\right)^{\text{th}} = 4^{\text{th}}, 5^{\text{th}}$ term are middle term or $\left(2x^2 - \frac{1}{x}\right)^7$

$$T_n = T_{r+1} = (-1)^r {}^nC_r x^{n-r} y^r$$

$$T_4 = T_{3+1} = (-1)^3 {}^7C_3 \left(2x^2\right)^{7-3} \left(\frac{1}{x}\right)^3$$

$$= -{}^7C_3 \frac{2^4 x^8}{x^3}$$

$$= -560x^5$$

$$T_5 = T_{4+1} = (-1)^4 {}^7C_4 \left(2x^2\right)^{7-4} \left(\frac{1}{x}\right)^4$$

$$= {}^7C_4 \frac{2^3 x^6}{x^4}$$

$$= {}^7C_4 \frac{7 \times 6 \times 5 \times 8}{3 \times 2} x^2$$

$$= 280x^2$$

Binomial Theorem Ex 18.2 Q14(iii)

$$\left(3x - \frac{2}{x^2}\right)^{15}$$

7th and 8th terms are middle terms

$$\binom{15}{7}(3x)^8\left(-\frac{2}{x^2}\right)^7, \binom{15}{8}(3x)^7\left(-\frac{2}{x^2}\right)^8$$

$$\frac{-6435 \times 3^8 \times 2^7}{x^6}, \frac{6437 \times 3^7 \times 2^8}{x^9}$$

Binomial Theorem Ex 18.2 Q14(iv)

$$\left(x^4 - \frac{1}{x^3}\right)^{11}$$

Here, $n = 11$, which is odd number

$\therefore \left(\frac{11+1}{2}\right)^{\text{th}}$ and $\left(\frac{11+1}{2} + 1\right)^{\text{th}} = 6^{\text{th}}$, 7th term are the middle terms in $\left(x^4 - \frac{1}{x^3}\right)^{11}$

The term formula is

$$T_r = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$T_6 = T_{5+1} = (-1)^5 {}^{11} C_5 (x^4)^{11-5} \left(\frac{1}{x^3}\right)^5 \\ = -{}^{11} C_5 x^{24} \frac{1}{x^{15}}$$

$$= \frac{-11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} x^9$$

$$= -11 \times 3 \times 2 \times 7 x^9$$

$$= -462 x^9$$

$$T_7 = T_{6+1} = (-1)^6 {}^{11} C_6 (x^4)^{11-6} \left(\frac{1}{x^3}\right)^6 \\ = 462 \frac{x^{20}}{x^{18}} \\ = 462 x^2$$

Binomial Theorem Ex 18.2 Q15(i)

$$\left(x - \frac{1}{x}\right)^{10}$$

Here, $n = 10$, which is even, \therefore it has 11 terms

\therefore middle term is $\left(\frac{n}{2} + 1\right) = 6^{\text{th}}$ term

$$T_r = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$T_6 = T_{5+1} = (-1)^5 {}^{10} C_5 (x)^{10-5} \left(\frac{1}{x}\right)^5 \\ = \frac{-10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} x^5 \\ = -3 \times 2 \times 7 \times 6 \\ = -252$$

Binomial Theorem Ex 18.2 Q15(ii)

$$(1 - 2x + x^2)^n$$

Here, n is odd, $\therefore (1 - 2x + x^2)$ has $n+1 = \text{even term}$

\therefore middle term is $\left(\frac{n+1}{2}\right)^{\text{th}}$ term

$$T_n = T_{r+1} = {}^n C_r x^{n-r} y^r$$

$$\frac{T_{n+1}}{2} = T_{\frac{n}{2}} = {}^n C_{\frac{n}{2}} (1 - 2x)^{n-\frac{n}{2}} (x^2)^{\frac{n}{2}}$$

$$= \frac{n!}{\frac{n}{2}! \frac{n}{2}!} (1 - 2x)^{\frac{n}{2}} x^{\frac{2n}{2}}$$

$$= \frac{(2n)!}{(n!)^2} (-1)^n x^n$$

$$[\because (1-x)^n = 1 - nx]$$

Binomial Theorem Ex 18.2 Q15(iii)

$$(1 + 3x + 3x^2 + x^3)^{2n}$$

This expansion is $((1+x)^3)^{2n} = (1+x)^{6n}$

Since $6n$ is even \therefore it has $6n+1 = \text{odd terms}$ has middle term is

$$\left(\frac{6n}{2} + 1\right)^{\text{th}} = (4n)^{\text{th}} \text{ term}$$

$$T_n = T_{r+1} = {}^n C_r x^{n-r} y^r$$

$$T_{4n} = T_{3n+1} = {}^{6n} C_{3n} (1)^{6n-3n} (x)^{3n}$$

$$= \frac{(6n)!}{(3n)!(3n)!} x^{3n} \quad [\because 1^{6n-3n} = 1]$$

Binomial Theorem Ex 18.2 Q15(iv)

$$\left(2x - \frac{x^2}{4}\right)^9$$

4th and 5th terms are middle terms

$$\binom{9}{4}(2x)^5 \left(-\frac{x^2}{4}\right)^4 + \binom{9}{5}(2x)^4 \left(-\frac{x^2}{4}\right)^5$$

$$\frac{63}{4}x^{13}, -\frac{63}{32}x^{14}$$

Binomial Theorem Ex 18.2 Q15(v)

$$\left(x - \frac{1}{x}\right)^{2n+1}$$

$2n+1$ is odd hence this expansion will have $2n+2$ – even terms.

Hence, middle terms is $\frac{2n+1}{2} = n+1, n+2$

Term formula is

$$T_r = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$\begin{aligned} T_{n+1} &= T_{n+2} = (-1)^n {}^{2n+1} C_n (x)^{2n+1-n} \left(\frac{1}{x}\right)^n \\ &= (-1)^n {}^{2n+1} C_n x^{n+1-n} \\ &= (-1)^n {}^{2n+1} C_n x \end{aligned}$$

$$\begin{aligned} T_{n+2} &= T_{n+1+1} = (-1)^{n+1} {}^{2n+1} C_{n+1} (x)^{2n+1-(n+1)} \left(\frac{1}{x}\right)^{n+1} \\ &= (-1)^{n+1} {}^{2n+1} C_{n+1} x^{-1} \\ &= (-1)^{n+1} {}^{2n+1} C_{n+1} \frac{1}{x} \\ &= (-1)^{n+1} {}^{2n+1} C_n \frac{1}{x} \quad [\because {}^n C_r = {}^n C_{r-1}] \end{aligned}$$

Binomial Theorem Ex 18.2 Q15(vi)

$$\left(3 - \frac{x^2}{6}\right)^7$$

Here $n = 7$, which is odd

\therefore middle term is $\binom{7+1}{2}$ and $\binom{7+1}{2} + 1 = 4^{\text{th}}, 5^{\text{th}}$ terms

$$T_3 = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$T_4 = T_{3+1} = (-1)^{3+1} {}^7 C_3 (3)^{7-3} \left(\frac{x^2}{6}\right)^3$$

$$= -\frac{7!}{3!4!} \times 3^4 \times \frac{x^9}{6^3}$$

$$= -\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 81 \times \frac{x^9}{216}$$

$$= -\frac{105}{8} x^9$$

And

$$T_5 = T_{r+1} = (-1)^r {}^n C_r x^{n-r} y^r$$

$$T_6 = T_{4+1} = (-1)^{4+1} {}^7 C_4 (3)^{7-4} \left(\frac{x^2}{6}\right)^4$$

$$= -\frac{7!}{4!3!} \times 3^3 \times \frac{x^{12}}{6^4}$$

$$= -\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 27 \times \frac{x^{12}}{1296}$$

$$= \frac{35}{48} x^{12}$$

Binomial Theorem Ex 18.2 Q15(vii)

$$\left(\frac{x}{3} + 9y\right)^{10}$$

Here $n=10$, which is even, therefore it has 11 terms

∴ middle term is $\binom{n}{2} = 6^{\text{th}}$ term

$$T_6 = T_{6+1} = (-1)^{6+1} C_6 x^{10-6} y^6$$

$$T_6 = T_{6+1} = (-1)^{6+1} C_6 \left(\frac{x}{3}\right)^{10-6} (9y)^6$$

$$= -\frac{10!}{5!5!} \times \frac{x^6}{3^6} \times 9^6 \times y^6$$

$$= 61236x^6y^6$$

Binomial Theorem Ex 18.2 Q15(viii)

For the given binomial expansion $n = 12$.

So middle term is $\binom{12}{2} = 7^{\text{th}}$ term.

$$T_7 = {}^{12}C_6 (2ax)^{12-6} \left(-\frac{b}{x^2}\right)^6$$

$$T_7 = {}^{12}C_6 (2ax)^6 \left(\frac{b}{x^2}\right)^6$$

$$T_7 = {}^{12}C_6 (2^6 a^6 x^6) \left(\frac{b^6}{x^{12}}\right)$$

$$T_7 = {}^{12}C_6 \left(\frac{2^6 a^6 b^6}{x^6}\right)$$

$$\text{Middle term is } {}^{12}C_6 \left(\frac{2^6 a^6 b^6}{x^6}\right).$$

Binomial Theorem Ex 18.2 Q15(ix)

For the given binomial expansion $n = 9$,

So middle terms are $\binom{9+1}{2} = 5^{\text{th}}$ term and $\binom{9+3}{2} = 6^{\text{th}}$ term.

$$T_5 = {}^9C_4 \left(\frac{p}{x}\right)^{9-4} \left(\frac{x}{p}\right)^4$$

$$T_5 = {}^9C_4 \left(\frac{p}{x}\right)^5 \left(\frac{x}{p}\right)^4$$

$$T_5 = {}^9C_4 \left(\frac{p}{x}\right)$$

$$T_6 = {}^9C_5 \left(\frac{p}{x}\right)^{9-5} \left(\frac{x}{p}\right)^5$$

$$T_6 = {}^9C_5 \left(\frac{p}{x}\right)^4 \left(\frac{x}{p}\right)^5$$

$$T_6 = {}^9C_5 \left(\frac{x}{p}\right)$$

The middle terms are ${}^9C_4 \left(\frac{p}{x}\right)$ and ${}^9C_5 \left(\frac{x}{p}\right)$.

Binomial Theorem Ex 18.2 Q15(x)

For the given binomial expansion $n = 10$.

So middle term is $\binom{10}{2} + 1 = 6^{\text{th}}$ term.

$$T_6 = {}^{10}C_5 \left(\frac{x}{a}\right)^{10-5} \left(-\frac{a}{x}\right)^5$$

$$T_6 = - {}^{10}C_5 \left(\frac{x}{a}\right)^5 \left(\frac{a}{x}\right)^5$$

$$T_6 = - {}^{10}C_5 = -252$$

Middle term is -252 .

Binomial Theorem Ex 18.2 Q16(i)

$$\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$$

In expansion

$$\begin{aligned} T_{r+1} &= {}^9C_r \left(\frac{3x^2}{2}\right)^{9-r} \left(-\frac{1}{3x}\right)^r \\ &= {}^9C_r \left(\frac{3}{2}\right)^{9-r} (x^{18-2r}) \left(\frac{-1}{3}\right)^r x^{-r} \end{aligned}$$

Let T_{r+1} be independent of x

$$18 - 3r = 0 \text{ or } r = 6$$

\therefore Required term

$$\begin{aligned} \Rightarrow T_{r+1} &= T_{6+1} = T_7 = {}^9C_6 \left(\frac{3}{2}\right)^{9-6} \left(\frac{-1}{3}\right)^6 x^{18-3(6)} \\ &= 84 \left(\frac{27}{8}\right) \left(\frac{1}{179}\right) x^0 = \frac{7}{18} \end{aligned}$$

Binomial Theorem Ex 18.2 Q16(ii)

$$\left(2x + \frac{1}{3x^2}\right)^9$$

4th term is independent of x

$$\binom{9}{3} (2x)^6 \left(\frac{1}{3x^2}\right)^3 = \binom{9}{3} \frac{64}{27}$$

Binomial Theorem Ex 18.2 Q16(iii)

$$T_{r+1} = (-1)^r {}^nC_r \left(2x^2\right)^{25-r} \left(\frac{3}{x^3}\right)^r = (-1)^r {}^nC_r 2^{25-r} 3^r x^{50-2r-3r}$$

Term independent of $x \Rightarrow x^0$

$$\Rightarrow x^{50-5r} = x^0 \Rightarrow 50 - 5r = 0 \Rightarrow r = 10$$

$$\therefore t_{11} = (-1)^{10} {}^{25}C_{10} 2^{15} \times 3^{10} = {}^{25}C_{10} 2^{15} 3^{10}$$

Binomial Theorem Ex 18.2 Q16(iv)

$$\left(3x - \frac{2}{x^2}\right)^{15}$$

$$\begin{aligned} T_{r+1} &= (-1)^r {}^{15}C_r \left(3x\right)^{15-r} \left(\frac{2}{x^2}\right)^r \\ &= (-1)^r {}^{15}C_r 3^{15-r} 2^r x^{15-r-2r} \end{aligned}$$

Term independent of $x \Rightarrow x^0$

$$\Rightarrow x^{15-3r} = x^0$$

$$15 - 3r = 0 \Rightarrow r = 5$$

$$\therefore t_6 = (-1)^5 {}^{15}C_5 3^{10} 2^5$$

$$\begin{aligned} &= -\frac{15!}{5!10!} 3^{10} 2^5 = -\frac{15 \times 14 \times 13 \times 12 \times 11}{120} 3^{10} 2^5 \\ &= -3003 \times 3^{10} \times 2^5 \end{aligned}$$

Binomial Theorem Ex 18.2 Q16(v)

$$\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\frac{3}{2x^2}\right)^r$$

$$= {}^{10}C_r x^{\frac{5-r}{2}-2r} 3^r \times 3^{-\frac{5+r}{2}} \times 2^{-r}$$

Independent of $x \Rightarrow x^0$

$$x \frac{10-r-4r}{r} = x^0$$

$$10-5r=0$$

$$r=2$$

$$t_3 = {}^{10}C_2 3^{2-5+1} 2^{-2}$$

$$= {}^{10}C_2 3^{-3} 2^{-2}$$

$$= \frac{10!}{2!8!} \times \frac{1}{36} = \frac{10 \times 9}{2 \times 36} = \frac{5}{4}$$

Binomial Theorem Ex 18.2 Q16(vi)

$$\left(x - \frac{1}{x^2}\right)^{3n}$$

$$T_{r+1} = (-1)^r {}^{3n}C_r x^{3n-r} \left(\frac{1}{x^2}\right)^r$$

$$= (-1)^r {}^{3n}C_r x^{3n-r-2r}$$

Independent of $x \Rightarrow x^0$

$$x^{3n-3r} = x^0 \Rightarrow r=n$$

$$= (-1)^n {}^{3n}C_r$$

Binomial Theorem Ex 18.2 Q16(vii)

We have,

$$\left(\frac{1}{2}x^{\frac{1}{3}} + x^{-\frac{1}{5}} \right)^8$$

Let $(r+1)^{\text{th}}$ term be independent of x .

$$\begin{aligned} T_{r+1} &= {}^8C_r \left(\frac{1}{2}x^{\frac{1}{3}} \right)^{8-r} \left(x^{-\frac{1}{5}} \right)^r \\ &= {}^8C_r \left(\frac{1}{2} \right)^{8-r} \times \left(x^{\frac{1}{3}} \right)^{8-r} \times \left(\frac{1}{x^{\frac{1}{5}}} \right)^r \\ &= {}^8C_r \left(\frac{1}{2} \right)^{8-r} \times (x)^{\frac{8-r}{3}} \times \left(\frac{1}{x^{\frac{1}{5}}} \right) \\ &= {}^8C_r \left(\frac{1}{2} \right)^{8-r} \times (x)^{\frac{8-r-r}{3}} \\ &= {}^8C_r \left(\frac{1}{2} \right)^{8-r} \times (x)^{\frac{40-5r-3r}{15}} \\ &= {}^8C_r \left(\frac{1}{2} \right)^{8-r} \times (x)^{\frac{40-8r}{15}} \end{aligned}$$

If it is independent of x , we must have

$$\frac{40-8r}{15} = 0$$

$$\Rightarrow 8r = 40$$

$$\Rightarrow r = 5$$

\therefore The term independent of $x = T_6$

Now,

$$\begin{aligned} T_6 &= {}^8C_r \left(\frac{1}{2}x^{\frac{1}{3}} \right)^{8-5} \left(x^{-\frac{1}{5}} \right)^5 \\ &= 56 \times \left(\frac{1}{2} \right)^3 \\ &= 56 \times \frac{1}{8} \\ &= 7 \end{aligned}$$

Hence, required term = 7

Binomial Theorem Ex 18.2 Q16(viii)

$$\begin{aligned} & (1+x+2x^3) \left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^9 \\ &= (1+x+2x^3) \left[\left(\frac{3}{2}x^2 \right)^9 - {}^9C_1 \left(\frac{3}{2}x^2 \right)^8 \frac{1}{3x} \dots + {}^9C_6 \left(\frac{3}{2}x^2 \right)^3 \left(\frac{1}{3x} \right)^6 - {}^9C_7 \left(\frac{3}{2}x^2 \right)^2 \left(\frac{1}{3x} \right)^7 \right] \end{aligned}$$

In the second bracket, we have to search the term so x^0 and $\frac{1}{x^3}$ which when multiplying by 1 and $2x^3$ is first bracket will give the term independent of x . The term containing $\frac{1}{x}$ will not occur in second bracket.

The term independent of x

$$\begin{aligned} &= 1 \left[{}^9C_6 \frac{3^3}{2^3} \times \frac{1}{3^6} \right] - 2x^3 \left[{}^9C_7 \frac{3^3}{2^3} \times \frac{1}{3^7} \times \frac{1}{x^3} \right] \\ &= \left[\frac{9 \times 8 \times 7}{1 \times 2 \times 3} \times \frac{1}{8 \times 27} \right] - 2 \left[\frac{9 \times 8}{1 \times 2} \times \frac{1}{4 \times 243} \right] \\ &= \frac{7}{18} - \frac{2}{27} \\ &= \frac{17}{54} \end{aligned}$$

Required term = $\frac{17}{54}$

Binomial Theorem Ex 18.2 Q16(ix)

We have,

$$\left(\sqrt[3]{x} + \frac{1}{2\sqrt[3]{x}}\right)^{18}, x > 0$$

Let $(r+1)^{\text{th}}$ term be independent of x .

$$\begin{aligned}\therefore T_{r+1} &= {}^{18}C_r \left(\sqrt[3]{x}\right)^{18-r} \times \left(\frac{1}{2\sqrt[3]{x}}\right)^r \\ &= {}^{18}C_r \left(x^{\frac{1}{3}}\right)^{18-r} \times \left(\frac{1}{2}\right)^r \times \left(\frac{1}{x^{\frac{r}{3}}}\right) \\ &= {}^{18}C_r \left(x^{\frac{8-r}{3}}\right) \times \left(\frac{1}{r}\right) \times \left(\frac{1}{2}\right)^r \\ &= {}^{18}C_r \left(x^{\frac{18-r}{3}}\right) \times \left(\frac{1}{2}\right)^r \\ &= {}^{18}C_r \left(x^{\frac{18-r}{3}}\right) \times \left(\frac{1}{2}\right)^r\end{aligned}$$

If it is independent of x , we must have

$$\frac{18-2r}{3} = 0$$

$$\Rightarrow 18 = 2r$$

$$\Rightarrow r = 9$$

\therefore Term independent of $x = T_{9+1} = T_{10}$

Now,

$$\begin{aligned}T_{10} &= {}^{18}C_9 \left(\sqrt[3]{x}\right)^{18-9} \left(\frac{1}{2\sqrt[3]{x}}\right)^9 \\ &= {}^{18}C_9 \left(\sqrt[3]{x}\right)^9 \times \frac{1}{2^9} \times \left(\frac{1}{\sqrt[3]{x}}\right)^9 \\ &= \frac{{}^{18}C_9}{2^9}\end{aligned}$$

Hence, required term = $\frac{{}^{18}C_9}{2^9}$.

Binomial Theorem Ex 18.2 Q16(x)

$$\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^6$$

In expansion

$$\begin{aligned}T_{r+1} &= {}^6C_r \left(\frac{3x^2}{2}\right)^{6-r} \left(-\frac{1}{3x}\right)^r \\ &= {}^6C_r \left(\frac{3}{2}\right)^{6-r} \left(x^{12-3r}\right) \left(-\frac{1}{3}\right)^r\end{aligned}$$

Let T_{r+1} be independent of x ,

$$12-3r = 0 \text{ or } r = 4$$

\therefore Required term

$$\begin{aligned}\Rightarrow T_{r+1} &= T_{4+1} = T_5 = {}^6C_4 \left(\frac{3}{2}\right)^{6-4} \left(-\frac{1}{3}\right)^4 x^{12-3(4)} \\ &= 15 \left(\frac{9}{4}\right) \left(\frac{1}{81}\right) x^0 = \frac{5}{12}\end{aligned}$$

Binomial Theorem Ex 18.2 Q17

We know that the coefficient of r th term in the expansion of $(1+x)^n$ is ${}^nC_{r-1}$
∴ Coefficient of $(2r+4)$ th term of the expansion $(1+x)^{18} = {}^{18}C_{2r+4-1} = {}^{18}C_{2r+3}$

and, coefficient of $(r-2)$ th term of the expansion $(1+x)^{18} = {}^{18}C_{r-2-1} = {}^{18}C_{r-3}$

It is given that these coefficients are equal.

$$\therefore {}^{18}C_{2r+3} = {}^{18}C_{r-3}$$

$$\Rightarrow 2r+3 = r-3 \text{ or, } 2r+3+r-3 = 18 \quad \left[\because {}^nC_r = {}^nC_s \Rightarrow r = s \text{ or, } r+s = n \right]$$

$$\Rightarrow r = -6 \text{ or, } 3r = 18$$

$$\Rightarrow r = -6 \text{ or, } r = 6$$

$$\Rightarrow r = 6$$

$\left[\because r = -6 \text{ is not possible} \right]$

Binomial Theorem Ex 18.2 Q18

$$(1+x)^{43}$$

$$\binom{43}{2r} = \binom{43}{r+1}$$

$$2r+r+1=43$$

$$3r=42$$

$$r=14$$

Binomial Theorem Ex 18.2 Q19

Now, Coefficient of $(r+1)$ th term in the expansion of $(4+x)^{n+1} = {}^{n+1}C_{r+1-1} = {}^{n+1}C_r$

and, Coefficient of r th term in $(1+x)^n +$ Coefficient of $(r+1)$ th term in $(1+x)^n$

$$\begin{aligned}&= {}^nC_{r-1} + {}^nC_{r+1-1} \\&= {}^nC_{r-1} + {}^nC_r \\&= \frac{n!}{\{n-(r-1)\}!(r-1)!} + \frac{n!}{(n-r)!(r)!} \\&= \frac{n!}{\{n-r+1\}!(r-1)!} + \frac{n!}{(n-r)!(r)!} \\&= \frac{n!}{(n-r+1)(n-r)!(r-1)!} + \frac{n!}{(n-r)!(r-1)!(r)} \\&= \frac{n!}{(n-r+1)(n-r)!(r-1)!} + \frac{n!}{(n-r)!(r-1)!(r)} \\&= \frac{n!}{(n-r)!(r-1)!} \left[\frac{1}{n-r+1} + \frac{1}{r} \right] \\&= \frac{n!}{(n-r)!(r-1)!} \left[\frac{r+n-r+1}{(n-r+1)r} \right] \\&= \frac{n!}{(n-r)!(r-1)!} \left[\frac{n+1}{(n-r+1)r} \right] \\&= \frac{n!(n+1)}{(n-r)!(n-r+1)(r-1)!r} \\&= \frac{(n+1)!}{(n-r+1)!r!} \\&= \frac{(n+1)!}{(n+1-r)!r!} \\&= {}^{n+1}C_r\end{aligned}$$

$$\therefore {}^{n+1}C_r = {}^nC_{r-1} + {}^nC_r$$

The coefficient of $(r+1)$ th term in the expansion of $(1+x)^{n+1}$ is equal to the sum of the coefficients of r th and $(r+1)$ th terms in the expansion of $(1+x)^n$.

We have,

$$\left(x + \frac{1}{x}\right)^{2n}$$

Let $(r+1)^{\text{th}}$ term be independent of x .

$$\begin{aligned} \therefore T_{r+1} &= {}^{2n}C_r (x)^{2n-r} \left(\frac{1}{x}\right)^r \\ &= {}^{2n}C_r (x)^{2n-r-r} \\ &= {}^{2n}C_r x^{2n-2r} \end{aligned}$$

If it is independent of x , we must have,

$$2n - 2r = 0$$

$$\Rightarrow 2n = 2r$$

$$\Rightarrow r = n$$

\therefore Term independent of $x = T_{n+1}$

Now,

$$\begin{aligned} T_{n+1} &= {}^{2n}C_n (x - 1)^{2n-n} \left(\frac{1}{x}\right)^n \\ &= {}^{2n}C_n \\ &= \frac{(2n)!}{(2n-n)!n!} \\ &= \frac{(2n)!}{n!n!} \\ &= \frac{(2n)(2n-1)(2n-2)\dots 5 \times 4 \times 3 \times 2 \times 1}{n!n!} \\ &= \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\} \{2 \times 4 \times 6 \times \dots 2n\}}{n!n!} \\ &= \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\} \times 2^n \{1 \times 2 \times 3 \times \dots n\}}{n!n!} \\ &= \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\} \times 2^n \times n!}{n!n!} \\ &= 2^n \times \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\}}{n!} \end{aligned}$$

\therefore The term independent of $x = \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\}}{n!} \times 2^n$ Hence proved.

We have,

$$(1+x)^n$$

Now,

$$\text{Coefficient of 5th term} = {}^nC_{5-1} = {}^nC_4$$

$$\text{Coefficient of 5th term} = {}^nC_{6-1} = {}^nC_5$$

$$\text{and, Coefficient of 5th term} = {}^nC_{7-1} = {}^nC_6$$

It is given that these coefficients are in A.P.

$$\therefore 2^nC_5 = {}^nC_4 + {}^nC_6$$

$$\Rightarrow 2 \left[\frac{n!}{(n-5)!5!} \right] = \frac{n!}{(n-4)!4!} + \frac{n!}{(n-6)!6!}$$

$$\Rightarrow \frac{2}{(n-5)!5!} = \frac{1}{(n-4)!4!} + \frac{1}{(n-6)!6!}$$

$$\Rightarrow \frac{2}{(n-5)(n-6)!5 \times 4!} = \frac{1}{(n-4)(n-5)(n-6)!4!} + \frac{1}{(n-6)!6 \times 5 \times 4!}$$

$$\Rightarrow \frac{2}{(n-5) \times 5} = \frac{1}{(n-4)(n-5)} + \frac{1}{6 \times 5}$$

$$\Rightarrow \frac{2}{5(n-5)} - \frac{1}{30} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \frac{12-(n-5)}{30(n-5)} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \frac{12-n+5}{30} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \frac{17-n}{30} = \frac{1}{n-4}$$

$$\Rightarrow 17n - 68 - n^2 + 4n = 30$$

$$\Rightarrow 21n - 68 - n^2 - 30 = 0$$

$$\Rightarrow 21n - n^2 - 98 = 0$$

$$\Rightarrow n^2 - 21n + 98 = 0$$

$$\Rightarrow n^2 - 7n - 14n + 98 = 0$$

$$\Rightarrow n(n-7) - 17(n-7) = 0$$

$$\Rightarrow (n-7)(n-14) = 0$$

$$\Rightarrow n = 7 \text{ or, } n = 14$$

We have,

$$(1+x)^{2n}$$

Now,

$$\text{Coefficient 2nd term} = {}^{2n}C_{2-1} = {}^{2n}C_1$$

$$\text{Coefficient 3rd term} = {}^{2n}C_{3-1} = {}^{2n}C_2$$

$$\text{and, Coefficient 4th term} = {}^{2n}C_{4-1} = {}^{2n}C_3$$

It is given that these coefficients are in A.P.

$$\therefore 2{}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$$

$$\Rightarrow 2 = \frac{{}^{2n}C_1}{{}^{2n}C_2} + \frac{{}^{2n}C_3}{{}^{2n}C_2}$$

$$\Rightarrow 2 = \frac{2}{2n-2+1} + \frac{2n-3+1}{3}$$

$$\Rightarrow 2 = \frac{2}{2n-1} + \frac{2n-2}{3}$$

$$\Rightarrow 2 = \frac{6 + (2n-1)(2n-2)}{3(2n-1)}$$

$$\Rightarrow 6(2n-1) = 6 + 4n^2 - 4n - 2n + 2$$

$$\Rightarrow 12n - 6 = 8 + 4n^2 - 6n$$

$$\Rightarrow 4n^2 - 6n - 12n + 8 + 6 = 0$$

$$\Rightarrow 4n^2 - 18n + 14 = 0$$

$$\Rightarrow 2(2n^2 - 9n + 7) = 0$$

$$\Rightarrow 2n^2 - 9n + 7 = 0 \quad \text{Hence proved.}$$

We have,

$$(1+x)^n$$

Let the three consecutive terms are r th, $(r+1)$ th and $(r+2)$ th i.e., T_r, T_{r+1} and T_{r+2}

\therefore Coefficients of r th term = ${}^n C_{r-1} = 220$

Coefficients of $(r+1)$ th term = ${}^n C_{r+1-1} = {}^n C_r = 495$

and, Coefficients of $(r+2)$ th term = ${}^n C_{r+2-1} = {}^n C_{r+1} = 792$

Now,

$$\frac{{}^n C_{r+1}}{{}^n C_r} = \frac{792}{495}$$

$$\Rightarrow \frac{n-(r+1)+1}{r+1} = \frac{792}{495} \quad \left[\because \frac{{}^n C_r}{{}^n C_{r-1}} = \frac{n-r+1}{r} \right]$$

$$\Rightarrow \frac{n-r}{r+1} = \frac{792}{495}$$

$$= \frac{72}{45}$$

$$= \frac{8}{5}$$

$$\Rightarrow \frac{n-r}{r+1} = \frac{8}{5}$$

$$\Rightarrow 5n - 5r = 8r + 8$$

$$\Rightarrow 5n - 5r - 8r = 8$$

$$\Rightarrow 5n - 13r = 8$$

$$\text{and, } \frac{{}^n C_r}{{}^n C_{r-1}} = \frac{495}{220}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{495}{220}$$
$$= \frac{45}{20}$$
$$= \frac{9}{4}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{9}{4}$$

$$\Rightarrow 4n - 4r + 4 = 9r$$

$$\Rightarrow 4n - 4r - 9r = -4$$

$$\Rightarrow 4n - 13r = -4 \quad \text{---(ii)}$$

Subtracting equation (ii) from equation (i),

$$n = 8 + 4$$

$$\Rightarrow n = 12$$

We have,

$$(1+x)^n$$

∴ Coefficients of 2nd term = ${}^nC_{2-1} = {}^nC_1$

Coefficients of 3rd term = ${}^nC_{3-1} = {}^nC_2$

and, Coefficients of 4th term = ${}^nC_{4-1} = {}^nC_3$

It is given that these coefficients are in A.P.

$$\therefore 2^n C_2 = {}^nC_1 + {}^nC_3$$

$$\Rightarrow 2 = \frac{{}^nC_1}{{}^nC_2} + \frac{{}^nC_3}{{}^nC_2}$$

$$\Rightarrow 2 = \frac{2}{n-2+1} + \frac{n-3+1}{3} \quad \left[\because \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-r}{r+1} \right]$$

$$\Rightarrow 2 = \frac{2}{n-1} + \frac{n-2}{3}$$

$$\Rightarrow 2 = \frac{6 + (n-1)(n-2)}{3(n-1)}$$

$$\Rightarrow 6(n-1) = 6 + n^2 - 2n - n + 2$$

$$\Rightarrow 6n - 6 = 8 + n^2 - 3n$$

$$\Rightarrow n^2 - 3n - 6n + 8 + 6 = 0$$

$$\Rightarrow n^2 - 9n + 14 = 0$$

$$\Rightarrow n^2 - 7n - 2n + 14 = 0$$

$$\Rightarrow n(n-7) - 2(n-7) = 0$$

$$\Rightarrow (n-2)(n-7) = 0$$

$$\Rightarrow n = 7$$

$[\because n-2 \neq 0]$

Binomial Theorem Ex 18.2 Q25

We have,

$$(1+x)^n$$

Coefficients of p th term = ${}^nC_{p-1}$

and, Coefficients of q th term = ${}^nC_{q-1}$

It is given that, these coefficients are equal.

$$\therefore {}^nC_{p-1} = {}^nC_{q-1}$$

$$\Rightarrow p-1 = q-1 \text{ or, } p-1+q-1 = n$$

$\left[\because {}^nC_r = {}^nC_s \Rightarrow r = s \text{ or, } r+s = n \right]$

$$\Rightarrow p-q = 0 \text{ or, } p+q = n+2$$

$$\therefore p+q = n+2 \quad \text{Hence proved.}$$

Binomial Theorem Ex 18.2 Q26

We have,

$$(1+x)^n$$

Let the three consecutive terms are T_r , T_{r+1} and T_{r+2}

$$\therefore \text{Coefficients of } T_r = {}^n C_{r-1} = 56$$

$$\text{Coefficients of } T_{r+1} = {}^n C_{r+1-1} = {}^n C_r = 70$$

$$\text{and, Coefficients of } T_{r+2} = {}^n C_{r+2-1} = {}^n C_{r+1} = 56$$

Now,

$$\frac{{}^n C_{r+1}}{{}^n C_r} = \frac{56}{70}$$

$$\Rightarrow \frac{n-(r+1)+1}{r+1} = \frac{4}{5} \quad \left[\because \frac{{}^n C_r}{{}^n C_{r-1}} = \frac{n-r+1}{r} \right]$$

$$\Rightarrow \frac{n-r}{r+1} = \frac{4}{5}$$

$$\Rightarrow 5n - 5r = 4r + 4$$

$$\Rightarrow 5n - 9r = 4 \quad \text{---(i)}$$

and,

$$\frac{{}^n C_r}{{}^n C_{r-1}} = \frac{70}{56}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{5}{4}$$

$$\Rightarrow 4n - 4r + 4 = 5r$$

$$\Rightarrow 4n - r = -4 \quad \text{---(ii)}$$

Subtracting equation (ii) from (i), we get

$$n = 4 + 4 = 8$$

Put $n = 8$ in equation (i), we get

$$5 \times 8 - 9r = 4$$

$$\Rightarrow -9r = 4 - 40$$

$$\Rightarrow r = 4$$

\therefore Three consecutive terms are 4th, 5th and 6th.

We are given,

$$T_3 = a, T_4 = b, T_5 = c, T_6 = d$$

We have to prove that

$$\begin{aligned} \frac{b^2 - ac}{c^2 - bd} &= \frac{5a}{3c} \\ \Rightarrow \frac{b^2 - ac}{a} &= \frac{5}{3} \left[\frac{c^2 - bd}{c} \right] \\ \Rightarrow \frac{1}{b} \left[\frac{b^2 - ac}{a} \right] &= \frac{5}{3} \left[\frac{c^2 - bd}{bc} \right] \\ \Rightarrow \frac{b}{a} - \frac{c}{b} &= \frac{5}{3} \left[\frac{c}{b} - \frac{d}{c} \right] \end{aligned}$$

---(i)

Now we know,

$$a = {}^n C_2 x^{n-2} \alpha^2$$

$$b = {}^n C_3 x^{n-3} \alpha^3$$

$$c = {}^n C_4 x^{n-4} \alpha^4$$

$$d = {}^n C_5 x^{n-5} \alpha^5$$

Putting these values in equation (i), we get

$$\begin{aligned} \frac{{}^n C_3 x^{n-3} \alpha^3}{{}^n C_2 x^{n-2} \alpha^2} - \frac{{}^n C_4 x^{n-4} \alpha^4}{{}^n C_3 x^{n-3} \alpha^3} &= \frac{5}{3} \left[\frac{{}^n C_4 x^{n-4} \alpha^4}{{}^n C_3 x^{n-3} \alpha^3} - \frac{{}^n C_5 x^{n-5} \alpha^5}{{}^n C_4 x^{n-4} \alpha^4} \right] \\ \Rightarrow \left[\frac{{}^n C_3}{{}^n C_2} - \frac{{}^n C_4}{{}^n C_3} \right] \alpha &= \frac{5\alpha}{3x} \left[\frac{{}^n C_4}{{}^n C_3} - \frac{{}^n C_5}{{}^n C_4} \right] \end{aligned}$$

We know that,

$$\frac{{}^n C_r}{{}^n C_{r-1}} = \frac{n-r+1}{r}$$

∴ The given equation above becomes,

$$\begin{aligned} \left[\frac{n-2}{3} - \frac{n-3}{4} \right] &= \frac{5}{3} \left[\frac{n-3}{4} - \frac{n-4}{5} \right] \\ \Rightarrow \frac{4n-8-3n+9}{3 \times 4} &= \frac{5n-15-4n+16}{3 \times 4} \\ \Rightarrow \frac{n+1}{12} &= \frac{n+1}{12} \end{aligned}$$

Which is true.

Hence proved.

Suppose the binomial is $(x+a)^n$

We are given,

$$T_6 = a, T_7 = b, T_8 = c, T_9 = d$$

We have to prove that

$$\begin{aligned} & \frac{b^2 - ac}{c^2 - bd} = \frac{4a}{3c} \\ \Rightarrow & \frac{b^2 - ac}{a} = \frac{4}{3} \left[\frac{c^2 - bd}{c} \right] \\ \Rightarrow & \frac{1}{b} \left[\frac{b^2 - ac}{a} \right] = \frac{4}{3} \left[\frac{c^2 - bd}{bc} \right] \\ \Rightarrow & \frac{b}{a} - \frac{c}{b} = \frac{4}{3} \left[\frac{c}{b} - \frac{d}{c} \right] \end{aligned}$$

---(i)

Now we know,

$$a = {}^n C_5 x^{n-5} a^5$$

$$b = {}^n C_6 x^{n-6} a^6$$

$$c = {}^n C_7 x^{n-7} a^7$$

$$d = {}^n C_8 x^{n-8} a^8$$

Putting these values in equation (i), we get

$$\begin{aligned} & \frac{{}^n C_6 x^{n-6} a^6}{{}^n C_5 x^{n-5} a^5} - \frac{{}^n C_7 x^{n-7} a^7}{{}^n C_6 x^{n-6} a^6} = \frac{4}{3} \left[\frac{{}^n C_7 x^{n-7} a^7}{{}^n C_6 x^{n-6} a^6} - \frac{{}^n C_8 x^{n-8} a^8}{{}^n C_7 x^{n-7} a^7} \right] \\ \Rightarrow & \left[\frac{{}^n C_6}{{}^n C_5} - \frac{{}^n C_7}{{}^n C_6} \right] a = \frac{4a}{3} \left[\frac{{}^n C_7}{{}^n C_6} - \frac{{}^n C_8}{{}^n C_7} \right] \end{aligned}$$

We know that,

$$\frac{{}^n C_r}{{}^n C_{r-1}} = \frac{n-r+1}{r}$$

∴ The given equation above becomes,

$$\begin{aligned} & \left[\frac{n-5}{6} - \frac{n-6}{7} \right] = \frac{4}{3} \left[\frac{n-6}{7} - \frac{n-7}{8} \right] \\ \Rightarrow & \frac{7n-35-6n+36}{6 \times 7} = \frac{8n-48-7n+49}{3 \times 7 \times 2} \\ \Rightarrow & \frac{n+1}{42} = \frac{n+1}{42} \end{aligned}$$

Which is true.

Hence proved.

We have,

$$(1+x)^n$$

Let the three consecutive terms are T_r, T_{r+1} and T_{r+2}

$$\therefore \text{Coefficients of } r\text{th term} = {}^nC_{r-1} = 76$$

$$\text{Coefficients of } (r+1)\text{th term} = {}^nC_{r+1-1} = {}^nC_r = 95$$

$$\text{and, Coefficients of } (r+2)\text{th term} = {}^nC_{r+2-1} = {}^nC_{r+1} = 76$$

Now,

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{76}{95}$$

$$\Rightarrow \frac{n-(r+1)+1}{r+1} = \frac{76}{95}$$

$$\left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r} \right]$$

$$\Rightarrow \frac{n-r-1}{r+1} = \frac{4}{5}$$

$$\Rightarrow \frac{n-r}{r+1} = \frac{4}{5}$$

$$\Rightarrow 5n - 5r = 4r + 4$$

$$\Rightarrow 5n - 5r - 4r = 4$$

$$\Rightarrow 5n - 9r = 4$$

and,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{95}{76}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{5}{4}$$

$$\Rightarrow 4n - 4r + 4 = 5r$$

$$\Rightarrow 4n - 9r = -4$$

---(ii)

Subtracting equation (ii) from (i), we get

$$n = 4 + 4$$

$$\Rightarrow n = 8$$

Binomial Theorem Ex 18.2 Q30

It is given that,

$$T_6 = 112, T_7 = 7, T_8 = \frac{1}{4}$$

$$\therefore T_6 = {}^nC_{n-5} x^{n-5} \times a^5 = 112$$

$$T_7 = {}^nC_{n-6} x^{n-6} \times a^6 = 7$$

$$\text{and, } T_8 = {}^nC_{n-7} x^{n-7} \times a^7 = \frac{1}{4}$$

Now,

$$\frac{T_7}{T_6} = \frac{{}^nC_{n-6} x^{n-6} \times a^6}{{}^nC_{n-5} x^{n-5} \times a^5} = \frac{7}{112}$$

$$\Rightarrow \frac{{}^nC_{n-6}}{ {}^nC_{n-5}} \times \frac{a}{x} = \frac{1}{16}$$

$$\Rightarrow \frac{n-6+1}{n-(n-5)+1} \times \frac{a}{x} = \frac{1}{16}$$

$$\Rightarrow \frac{n-5}{6} \times \frac{a}{x} = \frac{1}{16}$$

$$\Rightarrow \frac{a}{x} = \frac{6}{16} \times \frac{1}{n-5}$$

$$\Rightarrow \frac{a}{x} = \frac{3}{8} \times \frac{1}{(n-5)}$$

and,

$$\frac{T_8}{T_7} = \frac{{}^nC_{n-7} x^{n-7} \times a^7}{{}^nC_{n-6} x^{n-6} \times a^6} = \frac{\frac{1}{4}}{7}$$

$$\Rightarrow \frac{T_8}{T_7} = \frac{{}^nC_{n-7}}{ {}^nC_{n-6}} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \frac{{}^nC_{n-7}}{ {}^nC_{n-6}} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \frac{n-7+1}{n-(n-6)+1} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \frac{n-6}{7} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \frac{a}{x} = \frac{1}{4(n-6)}$$

$$\left[\because \frac{{}^nC_r}{ {}^nC_{r-1}} = \frac{n-r+1}{r} \right]$$

---(i)

---(ii)

Comparing equation (i) and (ii), we get

$$\frac{3}{8} \times \frac{1}{(n-5)} = \frac{1}{4(n-6)}$$

$$\Rightarrow \frac{3}{2} \times \frac{1}{(n-5)} = \frac{1}{(n-6)}$$

$$\Rightarrow 3(n-6) = 2(n-5)$$

$$\Rightarrow 3n - 18 = 2n - 10$$

$$\Rightarrow 3n - 2n = 18 - 10$$

$$\Rightarrow n = 8$$

Putting $n = 8$ in equation (ii), we get

$$\frac{a}{x} = \frac{1}{4(8-6)}$$

$$\Rightarrow \frac{a}{x} = \frac{1}{8}$$

$$\Rightarrow x = 8a$$

Now,

$$T_6 = 112$$

$$\Rightarrow {}^n C_{n-5} \times x^{n-5} \times a^5 = 112$$

$$\Rightarrow {}^8 C_3 \times x^3 \times a^5 = 112$$

$$\Rightarrow {}^8 C_3 \times (8a)^3 \times a^5 = 112$$

$$\Rightarrow \frac{8!}{(8-3)!3!} \times 8^3 \times a^8 = 112$$

$$\Rightarrow \frac{8!}{5!3!} \times 512 \times a^8 = 112$$

$$\Rightarrow \frac{8 \times 7 \times 6 \times 5!}{5!3!} \times 512 \times a^8 = 112$$

$$\Rightarrow a^8 = \frac{112}{56 \times 512}$$

$$\Rightarrow a^8 = \frac{2}{512}$$

$$\Rightarrow a^8 = \frac{1}{256}$$

$$\Rightarrow a^8 = \left(\frac{1}{2}\right)^8$$

$$\Rightarrow a = \frac{1}{2}$$

Putting $a = \frac{1}{2}$ in $x = 8a$, we get

$$x = 8 \times \frac{1}{2} = 4$$

Hence, $x = 4$, $a = \frac{1}{2}$ and $n = 8$.

It is given that

$$T_2 = 240$$

$$T_3 = 720$$

$$T_4 = 1080$$

$$\therefore T_2 = {}^n C_1 \times x^{n-1} \times a = 240$$

$$T_3 = {}^n C_2 \times x^{n-2} \times a^2 = 720$$

$$\text{and, } T_4 = {}^n C_3 \times x^{n-3} \times a^3 = 1080$$

Now,

$$\frac{T_4}{T_3} = \frac{{}^n C_3 \times x^{n-3} \times a^3}{{}^n C_2 \times x^{n-2} \times a^2} = \frac{1080}{720}$$

$$\Rightarrow \frac{{}^n C_3 a}{{}^n C_2 x} = \frac{3}{2}$$

$$\Rightarrow \frac{n-3+1}{2+1} \times \frac{a}{x} = \frac{3}{2}$$

$$\Rightarrow \frac{n-2}{3} \times \frac{a}{x} = \frac{3}{2}$$

$$\Rightarrow \frac{a}{x} = \frac{9}{2(n-2)}$$

and,

$$\frac{T_3}{T_2} = \frac{{}^n C_2 \times x^{n-2} \times a^2}{{}^n C_1 \times x^{n-1} \times a} = \frac{720}{240}$$

$$\Rightarrow \frac{{}^n C_2 a}{{}^n C_1 x} = 3$$

$$\Rightarrow \frac{n-2+1}{2} \times \frac{a}{x} = 3$$

$$\Rightarrow \frac{n-1}{2} \times \frac{a}{x} = 3$$

$$\Rightarrow \frac{a}{x} = \frac{6}{n-1}$$

--(i)

--(ii)

Comparing equation (i) and equation (ii), we get

$$\frac{6}{n-1} = \frac{9}{2(n-2)}$$

$$\Rightarrow 12(n-2) = 9(n-1)$$

$$\Rightarrow 12n - 24 = 9n - 9$$

$$\Rightarrow 3n = 24 - 9$$

$$\Rightarrow 3n = 15$$

$$\Rightarrow n = 5$$

Putting $n = 5$ in equation (ii), we get

$$\frac{a}{x} = \frac{6}{5-1}$$

$$\Rightarrow \frac{a}{x} = \frac{6}{4}$$

$$\Rightarrow \frac{a}{x} = \frac{3}{2}$$

$$\Rightarrow a = \frac{3}{2}x$$

Now,

$$T_2 = {}^nC_1 \times x^{n-1} \times a = 240$$

$$\Rightarrow {}^5C_1 \times x^4 \times \left(\frac{3}{2}x\right) = 240 \quad \left[\because n = 5 \text{ and } a = \frac{3}{2}x\right]$$

$$\Rightarrow x^5 = \frac{240 \times 2}{5 \times 3}$$

$$\Rightarrow x^5 = 32$$

$$\Rightarrow x^5 = 2^5$$

$$\Rightarrow x = 2$$

Putting $x = 2$ in $a = \frac{3}{2}x$, we get

$$a = \frac{3}{2} \times 2 = 3$$

Hence, $x = 2$, $a = 3$ and $n = 5$.

Binomial Theorem Ex 18.2 Q32

It is given that

$$T_1 = 729$$

$$T_2 = 7290$$

and, $T_3 = 30375$

$$\therefore T_1 = {}^n C_0 \times a^n = 729$$

$$T_2 = {}^n C_{n-1} \times a^{n-1} \times b = 7290$$

$$\text{and, } T_3 = {}^n C_{n-2} \times a^{n-2} \times b^2 = 30375$$

Now,

$$\frac{T_2}{T_1} = \frac{{}^n C_{n-1} \times a^{n-1} \times b}{{}^n C_0 \times a^n} = \frac{7290}{729}$$

$$\Rightarrow \frac{{}^n C_{n-1} \times a^{n-1} \times b}{{}^n C_0 \times a^n} = 10$$

$$\Rightarrow \frac{{}^n C_{n-1} \times b}{1} = 10$$

$$\Rightarrow \frac{n!}{(n-n+1)!(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \frac{n!}{(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \frac{n(n-1)!}{(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \frac{b}{a} = \frac{10}{n}$$

and,

$$\frac{T_3}{T_2} = \frac{{}^n C_{n-2} \times a^{n-2} \times b^2}{{}^n C_{n-1} \times a^{n-1} \times b} = \frac{30375}{7290}$$

$$\Rightarrow \frac{{}^n C_{n-2} \times b}{{}^n C_{n-1}} = \frac{25}{6}$$

$$\Rightarrow \frac{n-2+1}{n-(n-1)+1} \times \frac{b}{a} = \frac{26}{6}$$

$$\Rightarrow \frac{n-1}{2} \times \frac{b}{a} = \frac{25}{6}$$

$$\Rightarrow \frac{b}{a} = \frac{25}{6} \times \frac{2}{(n-1)}$$

--- ()

$$\left[\because \frac{{}^n C_r}{{}^n C_{r-1}}} = \frac{n-r+1}{r} \right]$$

$$\Rightarrow \frac{b}{a} = \frac{25}{6} \times \frac{2}{(n-1)}$$

$$\Rightarrow \frac{b}{a} = \frac{25}{3(n-1)}$$

---(ii)

Comparing equation (i) and equation (ii), we get

$$\frac{10}{n} = \frac{25}{3(n-1)}$$

$$\Rightarrow 30(n-1) = 25n$$

$$\Rightarrow 30n - 30 = 25n$$

$$\Rightarrow 5n = 30$$

$$\Rightarrow n = 6$$

Now,

$$T_1 = {}^n C_0 \times a^n = 729$$

$$\Rightarrow a^n = 729$$

$$\Rightarrow a^6 = 729$$

$[\because n = 6]$

$$\Rightarrow a^6 = 3^6$$

$$\Rightarrow a = 3$$

Putting $a = 3$ in $n = 6$ in equation (i), we get

$$\frac{b}{3} = \frac{10}{6}$$

$$\Rightarrow b = \frac{10}{2} = 5$$

Hence, $a = 3$, $b = 5$ and $n = 6$.

Binomial Theorem Ex 18.2 Q33

We have,

$$(3+ax)^9 = {}^9 C_0 \times 3^9 + {}^9 C_1 \times 3^8 \times (ax)^1 + {}^9 C_2 \times 3^7 \times (ax)^2 + {}^9 C_3 \times 3^6 \times (ax)^3 + \dots$$

$$\therefore \text{Coefficient of } x^2 = {}^9 C_2 \times 3^7 \times a^2$$

$$\text{and, } \text{Coefficient of } x^3 = {}^9 C_3 \times 3^6 \times a^3$$

$$\text{Now, Coefficient of } x^2 = \text{Coefficient of } x^3$$

$$\Rightarrow {}^9 C_2 \times 3^7 \times a^2 = {}^9 C_3 \times 3^6 \times a^3$$

$$\Rightarrow 36 \times 3^7 \times a^2 = 84 \times 3^6 \times a^3$$

$$\Rightarrow a = \frac{36 \times 3^7}{84 \times 3^6} = \frac{9}{7}$$

Binomial Theorem Ex 18.2 Q34

We have,

$$(1+2a)^4 (2-a)^5$$

Now,

$$(1+2a)^4 = {}^4C_0 + {}^4C_1 2a + {}^4C_2 (2a)^2 + {}^4C_3 (2a)^3 + {}^4C_4 (2a)^4$$

$$\text{and, } (2-5)^5 = {}^5C_0 \times 2^5 + {}^5C_1 \times 2^4 (-a) + {}^5C_2 \times 2^3 (-a)^2 + {}^5C_3 \times 2^2 (-a)^3 + {}^5C_4 \times 2 (-a)^4 + {}^5C_5 (-a)^5 \\ = {}^5C_0 \times 2^5 - {}^5C_1 \times 2^4 \times a + {}^5C_2 \times 2^3 \times a^2 - {}^5C_3 \times 2^2 \times a^3 + {}^5C_4 \times 2 \times a^4 - {}^5C_5 \times a^5$$

$$\therefore (1+2a)^4 (2-a)^5 = \left[{}^4C_0 + {}^4C_1 2a + {}^4C_2 (2a)^2 + {}^4C_3 (2a)^3 + {}^4C_4 (2a)^4 \right] \left[{}^5C_0 \times 2^5 - {}^5C_1 \times 2^4 \times a + {}^5C_2 \times 2^3 \times a^2 - {}^5C_3 \times 2^2 \times a^3 + {}^5C_4 \times 2 \times a^4 - {}^5C_5 \times a^5 \right]$$

$$\therefore \text{Coefficients of } a^4 = 2 {}^5C_4 - {}^4C_1 \times 2 \times {}^5C_3 \times 2^2 + {}^4C_2 (2)^2 \times {}^5C_2 \times 2^3 - {}^4C_3 (2)^3 \times {}^5C_1 \times 2^4 + {}^4C_4 (2)^4 \times {}^5C_0 \times 2^5 \\ = 2 \times 5 - 8 \times 4 \times 10 + 32 \times 6 \times 10 - 128 \times 4 \times 5 + 512 \times 1 \times 1 \\ = 10 - 320 + 1920 - 2560 + 512 \\ = 2442 - 2880 \\ = -438$$

$$\therefore \text{Coefficients of } a^4 = -438.$$

Binomial Theorem Ex 18.2 Q35

$$\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$$

$$\binom{10}{2} \left(\sqrt{x}\right)^8 \left(-\frac{k}{x^2}\right)^2 = 405$$

$$45k^2 = 405$$

$$k^2 = 9$$

$$k = 3$$

Binomial Theorem Ex 18.2 Q36

$$(y^{1/2} + x^{1/3})^n$$

$$\binom{n}{n-2} (y^{1/2})^2 (x^{1/3})^{n-2}$$

$$\binom{n}{n-2} = 45$$

$$n(n-1) = 90$$

$$n^2 - 10n + 9n - 90$$

$$n(n-10) + 9(n-10) = 0$$

$$n = -9 \text{ or } 10$$

n cannot be negative. So, $n = 10$

$$6\text{th term} \binom{10}{5} (y^{1/2})^5 (x^{1/3})^5 = 252 y^{\frac{5}{2}} x^{\frac{5}{3}}$$

Binomial Theorem Ex 18.2 Q37

$$\left(\frac{p}{2} + 2\right)^8$$

$$\binom{8}{4} \left(\frac{p}{2}\right)^4 2^4 = 1120$$

$$70p^4 = 1120$$

$$p^4 = 16$$

$$p = 2$$

Binomial Theorem Ex 18.2 Q38

$$\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$$

7th term from beginning is

$$\binom{n}{6} (\sqrt[3]{2})^{n-6} \left(\frac{1}{\sqrt[3]{3}}\right)^6$$

7th term from end is

$$\binom{n}{n-6} (\sqrt[3]{2})^6 \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}$$

$$\text{Given } \frac{\text{7th term from beginning}}{\text{7th term from end}} = \frac{\binom{n}{6} (\sqrt[3]{2})^{n-12} \left(\frac{1}{\sqrt[3]{3}}\right)^{12-n}}{\binom{n}{n-6}}$$

$$= \frac{\binom{n}{6} (\sqrt[3]{2})^{n-12} (\sqrt[3]{3})^{n-12}}{\binom{n}{n-6}}$$

$$= \frac{\binom{n}{6} (6)^{\frac{n-12}{3}}}{\binom{n}{n-6}} = \frac{1}{6}$$

$$\frac{n-12}{3} = -1$$

$$n = 12 - 3 = 9$$

Binomial Theorem Ex 18.2 Q39

Seventh term from the beginning and end in the binomial expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{2}}\right)^n$ are equal,

$$\Rightarrow T_7 = T_{n-6}$$

$$\Rightarrow {}^nC_6 \left(\sqrt[3]{2}\right)^6 \left(\frac{1}{\sqrt[3]{2}}\right)^{n-6} = {}^nC_{n-6} \left(\sqrt[3]{2}\right)^{n-6} \left(\frac{1}{\sqrt[3]{2}}\right)^6$$

$$\Rightarrow \left(\sqrt[3]{2}\right)^6 \left(\frac{1}{\sqrt[3]{2}}\right)^{n-6} = \left(\sqrt[3]{2}\right)^{n-6} \left(\frac{1}{\sqrt[3]{2}}\right)^6$$

$$\Rightarrow \left(\frac{1}{\sqrt[3]{2}}\right)^{2n-12} = \left(\frac{1}{\sqrt[3]{2}}\right)^{12}$$

$$\Rightarrow 2n - 12 = 12$$

$$\Rightarrow n = 12$$

