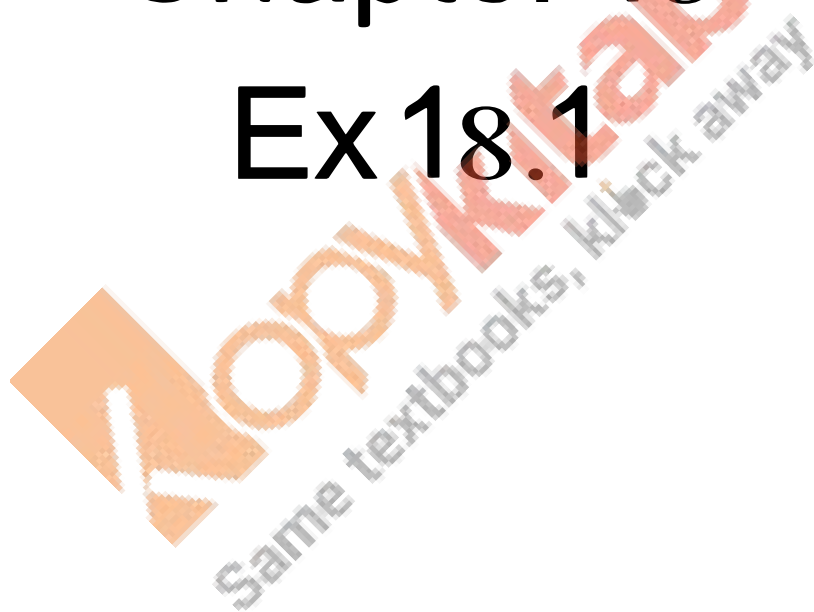


RD Sharma
Solutions
Class 11 Maths
Chapter 18
Ex 18.1



Binomial Theorem Ex 18.1 Q1(i)

The expansion of $(x+y)^n$ has $n+1$ term so, the expansion of $(2x+3y)^5$ has 6 terms.

Using binomial theorem, we have

$$\begin{aligned}(2x+3y)^5 &= {}^5C_0(2x)^5(3y)^0 + {}^5C_1(2x)^4(3y)^1 + {}^5C_2(2x)^3(3y)^2 + {}^5C_3(2x)^2(3y)^3 \\ &\quad + {}^5C_4(2x)(3y)^4 + {}^5C_5(2x)^0(3y)^5 \\ &= 2^5x^5 + 5 \times 2^4 \times 3 \times x^4 \times y + 10 \times 2^3 \times 3^2 \times x^3 \times y^2 + 10 \times 2^2 \times 3^3 \times x^2 \times y^3 \\ &\quad + 5 \times 2 \times 3^4 \times x \times y^4 + 3^5y^5 \\ &= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5\end{aligned}$$

Binomial Theorem Ex 18.1 Q1(ii)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $(2x-3y)^4$ has 5 terms.

Using binomial theorem, we have

$$\begin{aligned}(2x-3y)^4 &= {}^4C_0(2x)^4(3y)^0 - {}^4C_1(2x)^3(3y)^1 + {}^4C_2(2x)^2(3y)^2 - {}^4C_3(2x)^1(3y)^3 + {}^4C_4(2x)^0(3y)^4 \\ &= 2^4x^4 - 4 \times 2^3 \times 3 \times x^3y + 6 \times 2^2 \times 3^2 \times x^2y^2 - 4 \times 2 \times 3^3 \times xy^3 + 3^4y^4 \\ &= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4\end{aligned}$$

Binomial Theorem Ex 18.1 Q1(iii)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(x - \frac{1}{x}\right)^6$ has 7 term.

Using binomial theorem, we get

$$\begin{aligned}\left(x - \frac{1}{x}\right)^6 &= {}^6C_0x^6\left(\frac{1}{x}\right)^0 - {}^6C_1x^5\left(\frac{1}{x}\right) + {}^6C_2x^4\left(\frac{1}{x}\right)^2 - {}^6C_3x^3\left(\frac{1}{x}\right)^3 + {}^6C_4x^2\left(\frac{1}{x}\right)^4 - {}^6C_5x\left(\frac{1}{x}\right)^5 + {}^6C_6x^0\left(\frac{1}{x}\right)^6 \\ &= x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6}\end{aligned}$$

Binomial Theorem Ex 18.1 Q1(iv)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $(1-3x)^7$ has 8 term.

Using binomial theorem to expand, we get

$$\begin{aligned}(1-3x)^7 &= {}^7C_0(1)^7(3x)^0 - {}^7C_1(3x) + {}^7C_2(3x)^2 - {}^7C_3(3x)^3 + {}^7C_4(3x)^4 - {}^7C_5(3x)^5 + {}^7C_6(3x)^6 + {}^7C_7(3x)^7 \\&= 1 - 21x + 21 \times 9x^2 - 35 \times 3^3x^3 + 35 \times 3^4x^4 - 21 \times 3^5x^5 + 7 \times 3^6x^6 - 3^7x^7 \\&= 1 - 21x + 189x^2 - 945x^3 + 2835x^4 - 5103x^5 + 5103x^6 - 2187x^7\end{aligned}$$

Binomial Theorem Ex 18.1 Q1(v)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(ax - \frac{b}{x}\right)^6$ has 7 terms.

Using binomial theorem to expand, we get

$$\begin{aligned}\left(ax - \frac{b}{x}\right)^6 &= {}^6C_0(ax)^6\left(\frac{b}{x}\right)^0 - {}^6C_1(ax)^5\left(\frac{b}{x}\right) + {}^6C_2(ax)^4\left(\frac{b}{x}\right)^2 - {}^6C_3(ax)^3\left(\frac{b}{x}\right)^3 + {}^6C_4(ax)^2\left(\frac{b}{x}\right)^4 - {}^6C_5(ax)\left(\frac{b}{x}\right)^5 \\&\quad + {}^6C_6(ax)^0\left(\frac{b}{x}\right)^6 \\&= a^6x^6 - 6a^5x^5\frac{b}{x} + 15a^4x^4\frac{b^2}{x^2} - 20a^3b^3 + 15a^2\frac{b^4}{x^2} - 6a\frac{b^5}{x^4} + \frac{b^6}{x^6} \\&= a^6x^6 - 6a^5x^4b + 15a^4b^2x^2 - 20a^3b^3 + 15\frac{a^2b^4}{x^2} - 6\frac{ab^5}{x^4} + \frac{b^6}{x^6}\end{aligned}$$

Binomial Theorem Ex 18.1 Q1(vi)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right)^6$ has 7 terms.

Using binomial theorem to expand, we get

$$\begin{aligned}\left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right)^6 &= {}^6C_0 \left(\sqrt{\frac{x}{a}}\right)^6 \left(\sqrt{\frac{a}{x}}\right)^0 - {}^6C_1 \left(\sqrt{\frac{x}{a}}\right)^5 \left(\sqrt{\frac{a}{x}}\right)^1 + {}^6C_2 \left(\sqrt{\frac{x}{a}}\right)^4 \left(\sqrt{\frac{a}{x}}\right)^2 - {}^6C_3 \left(\sqrt{\frac{x}{a}}\right)^3 \left(\sqrt{\frac{a}{x}}\right)^3 \\ &\quad + {}^6C_4 \left(\sqrt{\frac{x}{a}}\right)^2 \left(\sqrt{\frac{a}{x}}\right)^4 - {}^6C_5 \left(\sqrt{\frac{x}{a}}\right) \left(\sqrt{\frac{a}{x}}\right)^5 + {}^6C_6 \left(\sqrt{\frac{x}{a}}\right)^0 \left(\sqrt{\frac{a}{x}}\right)^6 \\ &= \left(\frac{x}{a}\right)^{\frac{1}{2} \times 6} - 6 \left(\frac{x}{a}\right)^{\frac{1}{2} \times 5} \left(\frac{a}{x}\right)^{\frac{1}{2}} + 15 \left(\frac{x}{a}\right)^{\frac{1}{2} \times 4} \left(\frac{a}{x}\right)^{2 \times \frac{1}{2}} - 20 \left(\frac{x}{a}\right)^{3 \times \frac{1}{2}} \left(\frac{a}{x}\right)^{3 \times \frac{1}{2}} + 15 \left(\frac{x}{a}\right)^{2 \times \frac{1}{2}} \left(\frac{a}{x}\right)^{4 \times \frac{1}{2}} \\ &\quad - 6 \left(\frac{x}{a}\right)^{\frac{1}{2}} \left(\frac{a}{x}\right)^{5 \times \frac{1}{2}} + \left(\frac{a}{x}\right)^{6 \times \frac{1}{2}} \\ &= \frac{x^3}{a^3} - 6 \frac{x^{\frac{5}{2}} a^{\frac{1}{2}}}{a^{\frac{5}{2} \times \frac{1}{2}}} + 15x \frac{x^2 \times a}{a^2 \times x} - 20x \frac{x^{\frac{3}{2}} a^{\frac{3}{2}}}{a^{\frac{3}{2} \times \frac{3}{2}}} + 15x \frac{x \times a^2}{a \times x^2} - 6x \frac{x^{\frac{1}{2}} a^{\frac{5}{2}}}{a^{\frac{1}{2} \times \frac{5}{2}}} + \frac{a^3}{x^3} \\ &= \frac{x^3}{a^3} - \frac{6x^2}{a^2} + \frac{15x}{a} - 20 + \frac{15a}{x} - \frac{6a^2}{x^2} + \frac{a^3}{x^3}\end{aligned}$$

Binomial Theorem Ex 18.1 Q1(vii)

$$\begin{aligned}& \left(\sqrt[3]{x} - \sqrt[3]{a}\right)^6 \\ &= \binom{6}{0} \left(\sqrt[3]{x}\right)^6 \left(-\sqrt[3]{a}\right)^0 + \binom{6}{1} \left(\sqrt[3]{x}\right)^5 \left(-\sqrt[3]{a}\right)^1 + \binom{6}{2} \left(\sqrt[3]{x}\right)^4 \left(-\sqrt[3]{a}\right)^2 \\ & \quad + \binom{6}{3} \left(\sqrt[3]{x}\right)^3 \left(-\sqrt[3]{a}\right)^3 + \binom{6}{4} \left(\sqrt[3]{x}\right)^2 \left(-\sqrt[3]{a}\right)^4 + \binom{6}{5} \left(\sqrt[3]{x}\right)^1 \left(-\sqrt[3]{a}\right)^5 \\ & \quad + \binom{6}{6} \left(\sqrt[3]{x}\right)^0 \left(-\sqrt[3]{a}\right)^6 \\ &= x^2 - 6x^{\frac{5}{3}} a^{\frac{1}{3}} + 15x^{\frac{4}{3}} a^{\frac{2}{3}} - 20ax + 15x^{\frac{2}{3}} a^{\frac{4}{3}} - 6x^{\frac{1}{3}} a^{\frac{5}{3}} + a^2\end{aligned}$$

Binomial Theorem Ex 18.1 Q1(viii)

Let $y = 1+2x$, then

$$(1+2x-3x^2)^5 = (y-3x^2)^5$$

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $(y-3x^2)^5$ has 6 terms.

Using binomial theorem to expand, we get

$$\begin{aligned}(y-3x^2)^5 &= {}^5C_0 y^5 (3x^2)^0 - {}^5C_1 y^4 (3x^2)^1 + {}^5C_2 y^3 (3x^2)^2 - {}^5C_3 y^2 (3x^2)^3 + {}^5C_4 y (3x^2)^4 - {}^5C_5 y^0 (3x^2)^5 \\&= y^5 - 5y^4 \cdot 3x^2 + 10y^3 \cdot 9x^4 - 10y^2 (27x^6) + 5y \cdot 81x^8 - 243x^{10}\end{aligned}$$

Now,

$$\begin{aligned}y^5 &= (1+2x)^5 = {}^5C_0 + {}^5C_1(2x)^1 + {}^5C_2(2x)^2 + {}^5C_3(2x)^3 + {}^5C_4(2x)^4 + {}^5C_5(2x)^5 \\y^4 &= (1+2x)^4 = {}^4C_0 + {}^4C_1(2x)^1 + {}^4C_2(2x)^2 + {}^4C_3(2x)^3 + {}^4C_4(2x)^4 \\y^3 &= (1+2x)^3 = {}^3C_0 + {}^3C_1(2x) + {}^3C_2(2x)^2 + {}^3C_3(2x)^3 \\y^2 &= (1+2x)^2 = {}^2C_0 + {}^2C_1(2x) + {}^2C_2(2x)^2 \\y &= (1+2x)\end{aligned}$$

Substituting the values of powers of y in the equation above, we get,

$$\begin{aligned}(1+2x-3x^2)^5 &= [{}^5C_0 + {}^5C_1(2x)^1 + {}^5C_2(2x)^2 + {}^5C_3(2x)^3 + {}^5C_4(2x)^4 + {}^5C_5(2x)^5] \\&\quad - 15x^2 [{}^4C_0 + {}^4C_1(2x)^1 + {}^4C_2(2x)^2 + {}^4C_3(2x)^3 + {}^4C_4(2x)^4] \\&\quad + 90x^4 [{}^3C_0 + {}^3C_1(2x) + {}^3C_2(2x)^2 + {}^3C_3(2x)^3] - 270x^6 \\&\quad [{}^2C_0 + {}^2C_1(2x) + {}^2C_2(2x)^2 + 5 \times 81x^8(1+2x) - 243x^{10}] \\&= 10 + 10x + 10 \times 4x^2 + 10 \times 8x^3 + 5 \times 16x^4 + 32x^5 - 15x^2 - 120x^3 \\&\quad - 180x^4 + 480x^5 - 240x^6 + 90x^4 + 540x^5 + 1080x^6 + 720x^7 - 270x^6 \\&\quad - 1080x^7 - 1080x^8 + 405x^8 + 810x^9 - 243x^{10} \\&= 1 + 10x + 25x^2 - 40x^3 + 92x^5 + 570x^6 - 360x^7 - 675x^8 + 810x^9 - 243x^{10}\end{aligned}$$

Let $y = x + 1$, then

$$\left(x + 1 - \frac{1}{x}\right)^3 = \left(y - \frac{1}{x}\right)^3$$

The expansion of $(x + y)^n$ has $n + 1$ terms so the expansion of $\left(y - \frac{1}{x}\right)^3$ has 4 terms.

Using binomial theorem to expand, we get

$$\begin{aligned}\left(y - \frac{1}{x}\right)^3 &= {}^3C_0 y^3 \left(\frac{1}{x}\right)^0 - {}^3C_1 y^2 \left(\frac{1}{x}\right) + {}^3C_2 y \left(\frac{1}{x}\right)^2 - {}^3C_3 y^0 \left(\frac{1}{x}\right)^3 \\ &= y^3 - 3y^2 \times \frac{1}{x} + 3y \times \frac{1}{x^2} - \frac{1}{x^3}\end{aligned}$$

Putting $y = x + 1$, we get

$$\begin{aligned}\left(x + 1 - \frac{1}{x}\right)^3 &= (x + 1)^3 - 3(x + 1)^2 \times \frac{1}{x} + 3(x + 1) \times \frac{1}{x^2} - \frac{1}{x^3} \\ &= x^3 + 1 + 3x^2 + 3x - 3x - \frac{3}{x} - 6 + \frac{3}{x} + \frac{3}{x^2} - \frac{1}{x^3} \\ &= x^3 + 3x^2 - 5 + \frac{3}{x^2} - \frac{1}{x^3}\end{aligned}$$

Binomial Theorem Ex 18.1 Q1(x)

Let $y = 1 - 2x$, then

$$(1 - 2x + 3x^2)^3 = (y + 3x^2)^3$$

The expansion of $(x + y)^n$ has $n + 1$ terms so the expansion of $(y + 3x^2)^3$ has 4 terms.

Using binomial theorem to expand, we get

$$\begin{aligned}(y + 3x^2)^3 &= {}^3C_0 y^3 (3x^2)^0 + {}^3C_1 y^2 (3x^2)^1 + {}^3C_2 y (3x^2)^2 + {}^3C_3 y^0 (3x^2)^3 \\ &= y^3 + 3y^2 (3x^2) + 3y (9x^2) + (27x^6)\end{aligned}$$

Substituting $y = 1 - 2x$, we get,

$$\begin{aligned}(1 - 2x + 3x^2)^3 &= (1 - 2x)^3 + 3(1 + 4x^2 - 4x)(3x^2) + 3(1 - 2x)(9x^2) + (27x^6) \\ &= 1 - 8x^3 - 6x + 12x^2 + 9x^2 + 36x^4 - 36x^3 + 27x^2 - 54x^3 + 27x^6 \\ &= 1 - 6x + 21x^2 - 44x^3 + 63x^4 - 54x^5 + 27x^6\end{aligned}$$

Binomial Theorem Ex 18.1 Q2(i)

$$\begin{aligned}
 & (\sqrt{x+1} + \sqrt{x-1})^6 + (\sqrt{x+1} - \sqrt{x-1})^6 \\
 &= {}^6C_0(\sqrt{x+1})^6 + {}^6C_1(\sqrt{x+1})^5(\sqrt{x-1}) + {}^6C_2(\sqrt{x+1})^4(\sqrt{x-1})^2 - {}^6C_3(\sqrt{x+1})^3(\sqrt{x-1})^3 \\
 &+ {}^6C_4(\sqrt{x+1})^2(\sqrt{x-1})^4 + {}^6C_5(\sqrt{x+1})(\sqrt{x-1})^5 + {}^6C_6(\sqrt{x-1})^6 - {}^6C_0(\sqrt{x+1})^6 - \\
 &{}^6C_1(\sqrt{x+1})^5(\sqrt{x-1}) + {}^6C_2(\sqrt{x+1})^4(\sqrt{x-1})^2 - {}^6C_3(\sqrt{x+1})^3(\sqrt{x-1})^3 + \\
 &{}^6C_4(\sqrt{x+1})^2(\sqrt{x-1})^4 - {}^6C_5(\sqrt{x+1})(\sqrt{x-1})^5 + {}^6C_6(\sqrt{x-1})^6 \\
 &= 2[(x+1)^3 + 15(x+1)^2(x-1) + 15(x+1)(x-1)^2 + (x-1)^3] \\
 &= 2 \left[\begin{array}{c} x^3 + 1 + 3x + 3x^2 + 15x^3 - 15x^2 + 15x - 15 + 30x^2 - 30x \\ + 15x^3 + 15x^2 + 15x + 15 - 30x^2 - 30x + x^3 - 1 - 3x^2 + 3x \end{array} \right] \\
 &= 64x^3 - 48x \\
 &= 16x(4x^2 - 3)
 \end{aligned}$$

Binomial Theorem Ex 18.1 Q2(ii)

$$\begin{aligned}
 & (x + \sqrt{x^2 - 1})^6 + (x - \sqrt{x^2 - 1})^6 \\
 &= 2 \left[{}^6C_0x^6 + {}^6C_2x^4(\sqrt{x^2 - 1})^2 + {}^6C_4x^2(\sqrt{x^2 - 1})^4 + {}^6C_6(\sqrt{x^2 - 1})^6 \right] \\
 &= 2 \left[x^6 + 15x^4(x^2 - 1) + 15x^2(x^2 - 1)^2 + (x^2 - 1)^3 \right] \\
 &= 2 \left[x^6 + 15x^6 - 15x^4 + 15x^6 + 15x^2 - 30x^4 + x^6 - 1 - 3x^4 + 3x^2 \right] \\
 &= 64x^6 - 96x^4 + 36x^2 - 2
 \end{aligned}$$

Binomial Theorem Ex 18.1 Q2(iii)

$$\begin{aligned}
 & (1 + 2\sqrt{x})^5 + (1 - 2\sqrt{x})^5 \\
 &= 2 \left[{}^5C_0 + {}^5C_2(2\sqrt{x})^2 + {}^5C_4(2\sqrt{x})^4 \right]
 \end{aligned}$$

$$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$$

$$= {}^6C_0(\sqrt{2})^6 + {}^6C_1(\sqrt{2})^5 + {}^6C_2(\sqrt{2})^4 + {}^6C_3(\sqrt{2})^3 + {}^6C_4(\sqrt{2})^2 + {}^6C_5(\sqrt{2}) + {}^6C_6 + {}^6C_0(\sqrt{2})^6 - {}^6C_1(\sqrt{2})^5 + {}^6C_2(\sqrt{2})^4 - {}^6C_3(\sqrt{2})^3 + {}^6C_4(\sqrt{2})^2 - {}^6C_5(\sqrt{2}) + {}^6C_6(\sqrt{2})^0$$

$$= 2[2^3 + 15 \times 2^2 + 15 \times 2 + 1]$$

$$= 2[8 + 60 + 30 + 1] = 2(99) = 198$$

Binomial Theorem Ex 18.1 Q2(v)

$$(3+\sqrt{2})^5 - (3-\sqrt{2})^5$$

$$= 2[{}^5C_1(3)^4(\sqrt{2})^1 + {}^5C_3(3)^2(\sqrt{2})^3 + {}^5C_5(\sqrt{2})^5]$$

$$= 2[5 \times 81 \times \sqrt{2} + 10 \times 9 \times 2\sqrt{2} + 4\sqrt{2}]$$

$$= 2[405\sqrt{2} + 180\sqrt{2} + 4\sqrt{2}]$$

$$= 2[589\sqrt{2}]$$

$$= 1178\sqrt{2}$$

Binomial Theorem Ex 18.1 Q2(vi)

$$(2+\sqrt{3})^7 + (2-\sqrt{3})^7$$

$$= 2[{}^7C_0 2^7 + {}^7C_2 2^5(\sqrt{3})^2 + {}^7C_4 2^3(\sqrt{3})^4 + {}^7C_6 2(\sqrt{3})^6]$$

$$= 2[128 + 21 \times 32 \times 3 + 35 \times 8 \times 9 + 7 \times 2 \times 27]$$

$$= 2[128 + 2016 + 2520 + 378]$$

$$= 2[5042]$$

$$= 10084$$

Binomial Theorem Ex 18.1 Q2(vii)

$$\begin{aligned}
 & (\sqrt{3}+1)^5 - (\sqrt{3}-1)^5 \\
 &= 2 \left[{}^5C_1(\sqrt{3})^4 + {}^5C_3(\sqrt{3})^2 + {}^5C_5 \right] \\
 &= 2 [5 \times 9 + 10 \times 3 + 1] \\
 &= 2 [45 + 30 + 1] \\
 &= 2 [76] \\
 &= 152
 \end{aligned}$$

Binomial Theorem Ex 18.1 Q2(viii)

$$\begin{aligned}
 & (0.99)^5 + (1.01)^5 \\
 &= (1 - .01)^5 + (1 + .01)^5 \\
 &= 2 \left[{}^5C_1 + {}^5C_3(.01)^2 + {}^5C_5(.01)^5 \right] \\
 &= 2 \left[5 + 10 \times \frac{1}{10^4} + \frac{1}{10^{10}} \right] \\
 &= 2 \left[5 + \frac{1}{1000} + \frac{1}{10^{10}} \right] \\
 &= 2.0020001
 \end{aligned}$$

Binomial Theorem Ex 18.1 Q2(ix)

$$\begin{aligned}
 & (\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6 \\
 &= 2 \left[{}^6C_1(\sqrt{3})^5(\sqrt{2}) + {}^6C_3(\sqrt{3})^3(\sqrt{2})^3 + {}^6C_5(\sqrt{3})(\sqrt{2})^5 \right] \\
 &= 2 \left[6 \times \sqrt{6} \times 9 + 20 \times 3\sqrt{3} \times 2\sqrt{2} + 6 \times \sqrt{3} \times 4\sqrt{2} \right] \\
 &= 2 \left[54\sqrt{6} + 120\sqrt{6} + 24\sqrt{6} \right] \\
 &= 2 \left[198\sqrt{6} \right] \\
 &= 396\sqrt{6}
 \end{aligned}$$

Binomial Theorem Ex 18.1 Q2(x)

$$\left\{a^2 + \sqrt{a^2 - 1}\right\}^4 + \left\{a^2 - \sqrt{a^2 - 1}\right\}^4$$

$$\text{Let } a^2 = A, \quad \sqrt{a^2 - 1} = B$$

$$\begin{aligned} & (A+B)^4 + (A-B)^4 \\ &= B^4 + {}^4C_1AB^3 + {}^4C_2A^2B^2 + {}^4C_3A^3B + A^4 + B^4 - {}^4C_1AB^3 + {}^4C_2A^2B^2 - {}^4C_3A^3B + A^4 \\ &= 2\{A^4 + {}^4C_2A^2B^2 + B^4\} \\ &= 2\{A^4 + 6A^2B^2 + B^4\} \\ &= 2\{a^8 + 6a^4(a^2 - 1) + (a^2 - 1)^2\} \\ &= 2[a^8 + 6a^6 - 6a^4 + a^4 + 1 - 2a^2] \end{aligned}$$

$$\left\{a^2 + \sqrt{a^2 - 1}\right\}^4 + \left\{a^2 - \sqrt{a^2 - 1}\right\}^4 = 2a^8 + 12a^6 - 10a^4 - 4a^2 + 2$$

Binomial Theorem Ex 18.1 Q3

We have,

$$\begin{aligned} & (a+b)^4 - (a-b)^4 \\ &= [{}^4C_0a^4b^0 + {}^4C_1a^3b^1 + {}^4C_2a^2b^2 + {}^4C_3a^1b^3 + {}^4C_4a^0b^4] \\ & \quad - [{}^4C_0a^4b^0 - {}^4C_1a^3b^1 + {}^4C_2a^2b^2 - {}^4C_3a^1b^3 + {}^4C_4a^0b^4] \\ &= [{}^4C_0a^4(-b)^0 + {}^4C_1a^3(-b)^1 + {}^4C_2a^2(-b)^2 + {}^4C_3a^1(-b)^3 + {}^4C_4a^0(-b)^4] \\ & \quad - [{}^4C_0a^4(-b)^0 + {}^4C_1a^3(-b)^1 + {}^4C_2a^2(-b)^2 + {}^4C_3a^1(-b)^3 + {}^4C_4a^0(-b)^4] \\ &= [{}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4] - [{}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4] \\ &= {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4 - {}^4C_0a^4 + {}^4C_1a^3b - {}^4C_2a^2b^2 + {}^4C_3ab^3 - {}^4C_4b^4 \\ &= 2[{}^4C_1a^3b + {}^4C_3ab^3] \\ &= 2[4a^3b + 4ab^3] \\ &= 8[a^3b + ab^3] \end{aligned}$$

$$\therefore (a+b)^4 - (a-b)^4 = 8(a^3b + ab^3) \quad \text{---(i)}$$

Putting $a = \sqrt{3}$ and $b = \sqrt{2}$ in equation (i), we get

$$\begin{aligned} (\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 &= 8[(\sqrt{3})^3 \times \sqrt{2} + (\sqrt{3}) \times (\sqrt{2})^3] \\ &= 8[3\sqrt{6} + 2\sqrt{6}] \\ &= 8 \times 5\sqrt{6} \\ &= 40\sqrt{6} \end{aligned}$$

$$\therefore (\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 = 40\sqrt{6}.$$

Binomial Theorem Ex 18.1 Q4

We have,

$$\begin{aligned} & (x+1)^6 - (x-1)^6 \\ &= \left[{}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x^1 + {}^6C_6x^0 \right] \\ &+ \left[{}^6C_0x^6(-1)^0 + {}^6C_1x^5(-1)^1 + {}^6C_2x^4(-1)^2 + {}^6C_3x^3(-1)^3 + {}^6C_4x^2(-1)^4 + {}^6C_5x^1(-1)^5 + {}^6C_6x^0(-1)^6 \right] \\ &= \left[{}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6 - {}^6C_0x^6 - {}^6C_1x^5 + {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 \right. \\ &\quad \left. - {}^6C_5x + {}^6C_6 \right] \\ &= 2 \left[{}^6C_0x^6 + {}^6C_2x^4 + {}^6C_4x^2 + {}^6C_6 \right] \\ &= 2 \left[x^6 + 15x^4 + 15x^2 + 1 \right] \end{aligned}$$

$$\therefore (x+1)^6 + (x-1)^6 = 2 \left[x^6 + 15x^4 + 15x^2 + 1 \right] \quad \text{---(i)}$$

Putting $x = \sqrt{2}$ in equation (i), we get

$$\begin{aligned} (x+1)^6 + (x-1)^6 &= 2 \left[(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1 \right] \\ &= 2[8 + 60 + 30 + 1] \\ &= 2[99] \\ &= 198 \end{aligned}$$

$$\therefore (x+1)^6 + (x-1)^6 = 198$$

Binomial Theorem Ex 18.1 Q5(i)

We have,

$$\begin{aligned} (96)^3 &= (100 - 4)^3 \\ &= {}^3C_0 \times 100^3 + {}^3C_1 \times 100^2 \times (-4) + {}^3C_2 \times 100 \times (-4)^2 + {}^3C_3 \times (-4)^3 \\ &= 100^3 - 3 \times 100^2 \times 4 + 3 \times 100 \times 4^2 - 4^3 \\ &= 1000000 - 120000 + 4800 - 64 \\ &= 1004800 - 120064 \\ &= 884736 \end{aligned}$$

$$\therefore (96)^3 = 884736$$

Binomial Theorem Ex 18.1 Q5(ii)

We have,

$$\begin{aligned} (102)^5 &= (100 + 2)^5 \\ &= {}^5C_0 \times 100^5 + {}^5C_1 \times 100^4 \times 2 + {}^5C_2 \times 100^3 \times 2^2 + {}^5C_3 \times 100^2 \times 2^3 + {}^5C_4 \times 100 \times 2^4 + {}^5C_5 \times 2^5 \\ &= 100^5 + 5 \times 100^4 \times 2 + 10 \times 100^3 \times 2^2 + 10 \times 100^2 \times 2^3 + 5 \times 100 \times 2^4 + 2^5 \\ &= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32 \\ &= 11040808032 \end{aligned}$$

$$\therefore (102)^5 = 11040808032$$

Binomial Theorem Ex 18.1 Q5(iii)

We have,

$$\begin{aligned}
 (101)^4 &= (100+1)^4 \\
 &= {}^4C_0 \times 100^4 + {}^4C_1 \times 100^3 + {}^4C_2 \times 100^2 + {}^4C_3 \times 100 + {}^4C_4 \\
 &= 100^4 + 4 \times 100^3 + 6 \times 100^2 + 4 \times 100 + 1 \\
 &= 100000000 + 4000000 + 60000 + 400 + 1 \\
 &= 104060401
 \end{aligned}$$

$$\therefore (101)^4 = 104060401$$

Binomial Theorem Ex 18.1 Q5(iv)

We have,

$$\begin{aligned}
 (98)^5 &= (100-2)^5 \\
 &= {}^5C_0 \times 100^5 + {}^5C_1 \times 100^4 \times (-2) + {}^5C_2 \times 100^3 \times (-2)^2 + {}^5C_3 \times 100^2 \times (-2)^3 + {}^5C_4 \times 100 \times (-2)^4 + {}^5C_5 \times (-2)^5 \\
 &= {}^5C_0 \times 100^5 - {}^5C_1 \times 100^4 \times 2 + {}^5C_2 \times 100^3 \times 4 - {}^5C_3 \times 100^2 \times 8 + {}^5C_4 \times 100 \times 16 - {}^5C_5 \times 32 \\
 &= 100^5 - 10 \times 100^4 + 40 \times 100^3 - 80 \times 100^2 + 80 \times 100 - 32 \\
 &= 10000000000 - 1000000000 + 40000000 - 800000 + 8000 - 32 \\
 &= 10040008000 - 1000800032 \\
 &= 9039207968
 \end{aligned}$$

$$\therefore (98)^5 = 9039207968$$

Binomial Theorem Ex 18.1 Q6

$$\begin{aligned}
 2^{3n} - 7n - 1 &= 2^{3(n)} - 7(n) - 1 \\
 &= 8^n - 7n - 1 \\
 &= (1+7)^n - 7n - 1 \\
 &= ({}^nC_0 + {}^nC_1(7)^1 + {}^nC_2(7)^2 + \dots + {}^nC_n(7)^n) - 7n - 1 \\
 &= (1 + 7n + 49{}^nC_2 + \dots + 49(7)^{n-2}) - 7n - 1 \\
 &= 49({}^nC_2 + \dots + 7^{n-2})
 \end{aligned}$$

$\therefore 2^{3n} - 7n - 1$ is divisible by 49

Hence, proved

Binomial Theorem Ex 18.1 Q7

$$\begin{aligned}
 3^{2n+2} - 8n - 9 &= 3^{2(n+1)} - 8n - 9 \\
 &= 9^{n+1} - 8n - 9 \\
 &= (1+8)^{n+1} - 8n - 9 \\
 &= ({}^{n+1}C_0 + {}^{n+1}C_1 8^1 + {}^{n+1}C_2 8^2 + \dots + {}^{n+1}C_{n+1} 8^{n+1}) - 8n - 9 \\
 &= (1 + 8(n+1) + 64{}^{n+1}C_2 + \dots + 64(8)^{n-1}) - 8n - 9 \\
 &= 64({}^{n+1}C_2 + \dots + 8^{n-1})
 \end{aligned}$$

Thus, $3^{2n+2} - 8n - 9$ is divisible by 64.

Binomial Theorem Ex 18.1 Q8

$$\begin{aligned}
 & 3^{3n} - 26n - 1 \\
 &= (3^3)^n - 26n - 1 \\
 &= 27^n - 26n - 1 \\
 &= (1 + 26)^n - 26n - 1 \\
 &= \left({}^nC_0 + {}^nC_1(26)^1 + {}^nC_2(26)^2 + \dots + {}^nC_n(26)^n \right) - 26n - 1 \\
 &= \left(1 + 26n + 676 {}^nC_2 + \dots + 676(26)^{n-2} \right) - 26n - 1 \\
 &= 676 \left({}^nC_2 + \dots + (26)^{n-2} \right)
 \end{aligned}$$

$\therefore 3^{3n} - 26n - 1$ is divisible for $n \in \mathbb{N}$.

Hence, proved

Binomial Theorem Ex 18.1 Q9

We have,

$$\begin{aligned}
 (1.1)^{10000} &= (1 + 0.1)^{10000} \\
 &= {}^{10000}C_0 + {}^{10000}C_1(0.1) + {}^{10000}C_2(0.1)^2 + \dots + {}^{10000}C_{10000}(0.1)^{10000} \\
 &= 1 + 10000 \times (0.1) + \text{other positive terms} \\
 &= 1 + 1000 + \text{other positive terms} \\
 &= 1001 + \text{other positive terms} > 1000
 \end{aligned}$$

$$\therefore (1.1)^{10000} > 1000$$

Binomial Theorem Ex 18.1 Q10

$$\begin{aligned}
 (1.2)^{4000} &= (1 + 0.2)^{4000} \\
 &= {}^{4000}C_0(0.2)^0(1)^{4000} + {}^{4000}C_1 \times (0.2)^1 \times 1^{3999} + \dots + {}^{4000}C_{4000}(0.2)^{4000}1^0 \\
 &= 1 + 4000 \times 0.2 \times 1 + \dots + (0.2)^{4000} \\
 &= 1 + 800 + \dots + (0.2)^{4000}
 \end{aligned}$$

Here, we clearly observe $(1.2)^{4000}$ is less than 801 thus, $(1.2)^{4000} < 800$.

Binomial Theorem Ex 18.1 Q11

$$\begin{aligned}
 (1.01)^{10} + (1 - 0.01)^{10} &= (1 + 0.01)^{10} + (1 - 0.01)^{10} \\
 &= \left({}^{10}C_1 + {}^{10}C_2 \frac{1}{10^2} + {}^{10}C_3 \frac{1}{10^3} \dots + {}^{10}C_{10} \frac{1}{10^{10}} \right) + \left({}^{10}C_1 - {}^{10}C_2 \frac{1}{10^2} + {}^{10}C_3 \frac{1}{10^3} - {}^{10}C_4 \frac{1}{10^4} + \dots \right) \\
 &= 2 \left({}^{10}C_1 - {}^{10}C_3 \frac{1}{10^3} + {}^{10}C_5 \frac{1}{10^5} + {}^{10}C_7 \frac{1}{10^7} + {}^{10}C_9 \frac{1}{10^9} \right) \\
 &= 2 \left(10 + \frac{10!}{3!7!} \frac{1}{1000} + \frac{10!}{5!5!} \frac{1}{(10)^5} + \frac{10!}{7!3!} \times \frac{1}{10^7} + \frac{10!}{9!1!} \frac{1}{10^9} \right) \\
 &= 2 \left(10 + \frac{9 \times 8}{3 \times 2 \times 1000} + \frac{9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 10^5} + \frac{9 \times 8}{3 \times 2 \times 10^7} + \frac{1}{10^8} \right) \\
 &= 2.0090042
 \end{aligned}$$

Binomial Theorem Ex 18.1 Q12

$$\begin{aligned}
 2^{4n+4} - 15n - 16 &= 2^{4(n+1)} - 15n - 15 - 1 \\
 &= (16)^{(n+1)} - 15(n+1) - 1 \\
 &= (1+15)^{n+1} - 15(n+1) - 1 \\
 &= \left[{}^{n+1}C_0 + {}^{n+1}C_1(15) + {}^{n+1}C_2(15)^2 + \dots + {}^{n+1}C_{n+1}(15)^{n+1} \right] - 15(n+1) - 1 \\
 &= \left[1 + 15(n+1) + {}^{n+1}C_2(15)^2 + \dots + {}^{n+1}C_{n+1}(15)^{n+1} \right] - 15(n+1) - 1 \\
 &= 225 \left[{}^{n+1}C_2 + \dots + {}^{n+1}C_{n+1}(15)^{n-1} \right] \\
 &= 225 \times \text{natural number}
 \end{aligned}$$