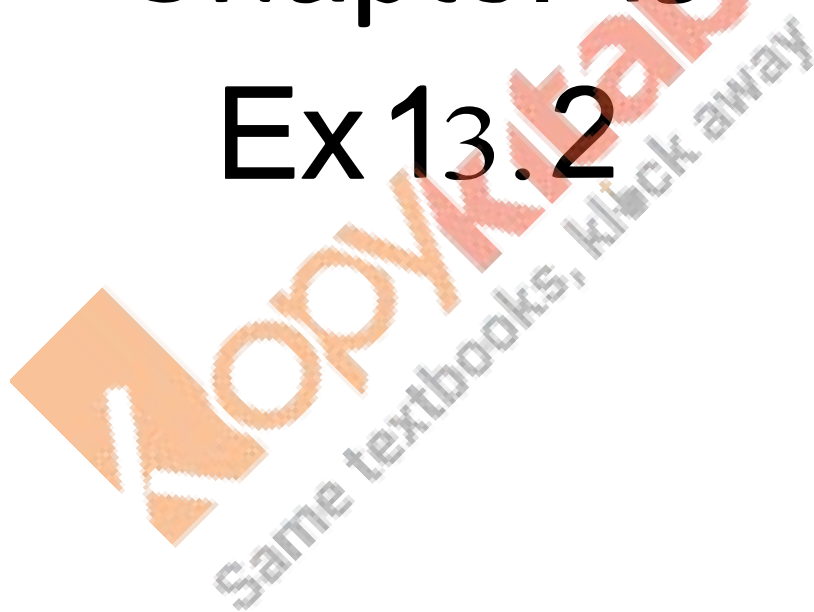


RD Sharma  
Solutions  
Class 11 Maths  
Chapter 13  
Ex 13.2



**Complex Numbers Ex 13.2 Q1(i)**

$$\begin{aligned}
 (1+i)(1+2i) &= 1 \times (1+2i) + i(1+2i) \\
 &= 1 + 2i + i + 2i^2 \\
 &= 1 + 3i - 2 \\
 &= -1 + 3i
 \end{aligned}$$

$$\therefore (1+i)(1+2i) = -1 + 3i$$

**Complex Numbers Ex 13.2 Q1(ii)**

$$\begin{aligned}
 \frac{3+2i}{-2+i} &= \frac{3+2i}{-2+i} \times \frac{-2-i}{-2-i} && [\text{Rationalising the denominator}] \\
 &= \frac{3(-2-i) + 2i(-2-i)}{(-2)^2 - (i)^2} && [\because (a+ib)(a-ib) = a^2 + b^2] \\
 &= \frac{-6 - 3i - 4i + 2}{4 + 1} && [\because -i^2 = 1] \\
 &= \frac{-4 - 7i}{5} \\
 &= -\frac{4}{5} - \frac{7}{5}i
 \end{aligned}$$

$$\therefore \frac{3+2i}{-2+i} = -\frac{4}{5} - \frac{7}{5}i$$

**Complex Numbers Ex 13.2 Q1(iii)**

$$\begin{aligned}
 \frac{1}{(2+i)^2} &= \frac{1}{2^2 + (i)^2 + 2 \times 2 \times i} \\
 &= \frac{1}{4 - 1 + 4i} \\
 &= \frac{1}{3 + 4i} \\
 &= \frac{1}{(3+4i)} \times \frac{(3-4i)}{(3-4i)} && [\text{on rationalising the denominator}] \\
 &= \frac{3-4i}{3^2 + 4^2} && [\because (a+ib)(a-ib) = a^2 + b^2] \\
 &= \frac{3-4i}{25} \\
 &= \frac{3}{25} - \frac{4}{25}i
 \end{aligned}$$

$$\therefore \frac{1}{(2+i)^2} = \frac{3}{25} - \frac{4}{25}i$$

**Complex Numbers Ex 13.2 Q1(iv)**

$$\begin{aligned}\frac{1-i}{1+i} &= \frac{(1-i)}{(1+i)} \times \frac{(1-i)}{(1-i)} && \text{(Rationalising the denominator)} \\ &= \frac{(1-i)^2}{1^2+1^2} && [\because (a+ib)(a-ib) = a^2+b^2] \\ &= \frac{1^2+i^2-2 \times i \times 1}{2} \\ &= \frac{-2i}{2} \\ &= -i \\ &= 0-i\end{aligned}$$

$$\therefore \frac{1-i}{1+i} = 0-i$$

### Complex Numbers Ex 13.2 Q1(v)

$$\begin{aligned}\frac{(2+i)^3}{2+3i} &= \frac{2^3+i^3+3 \times 2 \times i(2+i)}{2+3i} && [\because (a+b)^3 = a^3+b^3+3ab(a+b)] \\ &= \frac{(8-i+6i(2+i))}{2+3i} \times \frac{(2-3i)}{2-3i} && \text{(On rationalising the denominator)} \\ &= \frac{(8-i+12i+6i^2)(2-3i)}{2^2+3^2} \\ &= \frac{(8-6+11i)(2-3i)}{4+9} && [\because i^2 = -1] \\ &= \frac{(2+11i)(2-3i)}{13} \\ &= \frac{4-6i+22i+33}{13} \\ &= \frac{37+16i}{13} \\ &= \frac{37}{13} + \frac{16}{13}i\end{aligned}$$

$$\therefore \frac{(2+i)^3}{2+3i} = \frac{37}{13} + \frac{16}{13}i$$

### Complex Numbers Ex 13.2 Q1(vi)

$$\begin{aligned}
 \frac{(1+i)(1+\sqrt{3}i)}{1-i} &= \frac{1(1+\sqrt{3}i) + i(1+\sqrt{3}i)}{1-i} \\
 &= \frac{(1+\sqrt{3}i+i-\sqrt{3})}{1-i} \quad (\because i^2 = -1) \\
 &= \frac{(1-\sqrt{3})+i(1+\sqrt{3})}{1-i} \times \frac{(1+i)}{(1+i)} \quad (\text{Rationalising the denominator}) \\
 &= \frac{(1-\sqrt{3})(1+i) + i(1+\sqrt{3})(1+i)}{1^2 + 1^2} \\
 &= \frac{1+i-\sqrt{3}(1+i) + i(1+i+\sqrt{3}(1+i))}{2} \\
 &= \frac{1+i-\sqrt{3}-\sqrt{3}i+i(1+i+\sqrt{3}+\sqrt{3}i)}{2} \\
 &= \frac{1-\sqrt{3}+i-\sqrt{3}i+i-1+\sqrt{3}i-\sqrt{3}}{2} \\
 &= \frac{-2\sqrt{3}+2i}{2} \\
 &= -\sqrt{3}+i
 \end{aligned}$$

$$\therefore \frac{(1+i)(1+\sqrt{3}i)}{1-i} = -\sqrt{3}+i$$

#### Complex Numbers Ex 13.2 Q1(vii)

$$\begin{aligned}
 \frac{2+3i}{4+5i} &= \frac{2+3i}{4+5i} \times \frac{(4-5i)}{(4-5i)} \quad (\text{rationalising the denominator}) \\
 &= \frac{2(4-5i) + 3i(4-5i)}{4^2 + 5^2} \\
 &= \frac{8-10i+12i+15}{16+25} \quad (\because i^2 = -1) \\
 &= \frac{23+2i}{41} \\
 &= \frac{23}{41} + \frac{2}{41}i
 \end{aligned}$$

$$\therefore \frac{2+3i}{4+5i} = \frac{23}{41} + \frac{2}{41}i$$

#### Complex Numbers Ex 13.2 Q1(viii)

$$\begin{aligned}
 \frac{(1-i)^3}{1-i^3} &= \frac{1^3 - i^3 - 3 \times 1 \times i(1-i)}{1-(-i)} \quad \left[ \begin{array}{l} \because (a-b)^3 = a^3 - b^3 - 3ab(a-b) \\ \text{and } i^3 = -i \end{array} \right] \\
 &= \frac{1 - (-i) - 3i(1-i)}{1+i} \\
 &= \frac{1+i-3i-3}{1+i} \\
 &= \frac{-2-2i}{1+i} \\
 &= \frac{-2(1+i)}{1+i} \\
 &= -2 \\
 &= -2+0i
 \end{aligned}$$

$$\therefore \frac{(1-i)^3}{1-i^3} = -2+0i$$

#### Complex Numbers Ex 13.2 Q1(ix)

$$\begin{aligned}
 (1+2i)^{-3} &= \frac{1}{(1+2i)^3} & \left( \because z^{-3} = \frac{1}{z^3} \right) \\
 &= \frac{1}{1^3 + (2i)^3 + 3 \times 1 \times 2i(1+2i)} \\
 &= \frac{1}{1^3 + 2^3 \times i^3 + 6i(1+2i)} \\
 &= \frac{1}{1 - 8i + 6i - 12} & (\because i^3 = -i \text{ and } i^2 = -1) \\
 &= \frac{1}{-11 - 2i} \\
 &= \frac{1}{-11 - 2i} \times \frac{(-11 + 2i)}{(-11 + 2i)} \\
 &= \frac{-11 + 2i}{(-11)^2 + 2^2} \\
 &= \frac{-11 + 2i}{121 + 4} \\
 &= \frac{-11}{125} + \frac{2}{125}i
 \end{aligned}$$

$$\therefore (1+2i)^{-3} = \frac{-11}{125} + \frac{2}{125}i$$

### Complex Numbers Ex 13.2 Q1(x)

$$\begin{aligned}
 \frac{3-4i}{(4-2i)(1+i)} &= \frac{3-4i}{4(1+i)-2i(1+i)} \\
 &= \frac{3-4i}{4+4i-2i+2} \\
 &= \frac{3-4i}{6+2i} \\
 &= \frac{3-4i}{6+2i} \times \frac{6-2i}{6-2i} \\
 &= \frac{3(6-2i)-4i(6-2i)}{6^2+2^2} \\
 &= \frac{18-6i-24i-8}{36+4} \\
 &= \frac{10-30i}{40} \\
 &= \frac{10(1-3i)}{40} \\
 &= \frac{1-3i}{4} \\
 &= \frac{1}{4} - \frac{3}{4}i
 \end{aligned}$$

$$\therefore \frac{3-4i}{(4-2i)(1+i)} = \frac{1}{4} - \frac{3}{4}i$$

### Complex Numbers Ex 13.2 Q1(xi)

$$\begin{aligned}
 \left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) &= \frac{(1+i-2(1-4i))}{(1-4i)(1+i)} \times \frac{3-4i}{5+i} \\
 &= \frac{(1+i-2+8i)}{1(1+i)-4i(1+i)} \times \frac{3-4i}{5+i} \\
 &= \frac{(1+i-2+8i)}{1(1+i)-4i(1+i)} \times \frac{3-4i}{5+i} \\
 &= \frac{(-1+9i)}{(1+i-4i+4)} \times \frac{3-4i}{5+i} \\
 &= \frac{-1(3-4i)+9i(3-4i)}{(5-3i)(5+i)} \\
 &= \frac{-3+4i+27i+36}{5(5+i)-3i(5+i)} \\
 &= \frac{33+31i}{25+5i-15i+3} \\
 &= \frac{33+31i}{28-10i} \\
 &= \frac{(33+31i)}{28-10i} \times \frac{(28+10i)}{28+10i} \\
 &= \frac{33 \times 28 + 33 \times 10i + 31i \times 28 + 31i \times 10i}{28^2 + 10^2} \\
 &= \frac{924 + 330i + 868i - 310}{784 + 100} \\
 &= \frac{614 + 1198i}{884} \\
 &= \frac{614}{884} + \frac{1198}{884}i \\
 &= \frac{307}{442} + \frac{599}{442}i
 \end{aligned}$$

$$\therefore \left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \frac{307}{442} + \frac{599}{442}i$$

### Complex Numbers Ex 13.2 Q1(xii)

We have

$$\begin{aligned}
 \frac{5+\sqrt{2}i}{1-\sqrt{2}i} &= \frac{5+\sqrt{2}i}{1-\sqrt{2}i} \times \frac{1+\sqrt{2}i}{1+\sqrt{2}i} \\
 &= \frac{5(1+\sqrt{2}i)+\sqrt{2}i(1+\sqrt{2}i)}{1+2} \\
 &= \frac{5+5\sqrt{2}i+\sqrt{2}i-2}{3} \\
 &= \frac{3+6\sqrt{2}i}{3} \\
 &= 1+2\sqrt{2}i
 \end{aligned}$$

$$\text{Therefore, } \frac{5+\sqrt{2}i}{1-\sqrt{2}i} = 1+2\sqrt{2}i$$

### Complex Numbers Ex 13.2 Q2(i)

We have  $(x + iy)(2 - 3i) = 4 + i$

$$\Rightarrow x(2 - 3i) + iy(2 - 3i) = 4 + i$$

$$\Rightarrow 2x - 3xi + 2yi + 3y = 4 + i$$

$$\Rightarrow 2x + 3y + i(-3x + 2y) = 4 + i$$

Equating the real and imaginary parts we get

$$2x + 3y = 4 \dots\dots\dots(i)$$

$$-3x + 2y = 1 \dots\dots\dots(ii)$$

Multiplying (i) by 3 and (ii) by 2 and adding

$$6x - 6x - 9y + 4y = 12 + 2$$

$$\Rightarrow 13y = 14$$

$$\Rightarrow y = \frac{14}{13}$$

Substituting the value of  $y$  in (i), we get

$$2x + 3 \times \frac{14}{13} = 4$$

$$\Rightarrow 2x + \frac{42}{13} = 4$$

$$\Rightarrow 2x = 4 - \frac{42}{13}$$

$$\Rightarrow 2x = \frac{52 - 42}{13}$$

$$\Rightarrow 2x = \frac{10}{13}$$

$$\Rightarrow x = \frac{5}{13}$$

Hence

$$x = \frac{5}{13} \text{ and } y = \frac{14}{13}$$

$$(3x - 2iy)(2 + i)^2 = 10(1 + i)$$

$$\Rightarrow (3x - 2iy)(2^2 + i^2 + 2 \times 2 \times i) = 10 + 10i$$

$$\Rightarrow (3x - 2iy)(4 - 1 + 4i) = 10 + 10i$$

$$\Rightarrow 3x(3 + 4i) - 2iy(3 + 4i) = 10 + 10i$$

$$\Rightarrow 9x + 12xi - 6yi + 8y = 10 + 10i$$

$$\Rightarrow 9x + 8y + i(12x - 6y) = 10 + 10i$$

Equating the real and imaginary parts we get

$$9x + 8y = 10 \dots\dots\dots (i)$$

$$12x - 6y = 10 \dots\dots\dots (ii)$$

Multiplying (i) by 6 and (ii) by 8 and adding

$$54x + 96y + 48y - 48y = 60 + 80$$

$$\Rightarrow 150x = 140$$

$$\Rightarrow x = \frac{140}{150}$$

$$\Rightarrow x = \frac{14}{15}$$

Substituting value of x in (i) we get

$$9 \times \frac{14}{15} + 8y = 10$$

$$\Rightarrow \frac{42}{5} + 8y = 10$$

$$\Rightarrow 8y = 10 - \frac{42}{5}$$

$$\Rightarrow 8y = \frac{50 - 42}{5}$$

$$\Rightarrow 8y = \frac{8}{5}$$

$$\Rightarrow y = \frac{1}{5}$$

$$\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$$

$$\Rightarrow \frac{(3-i)((1+i)x - 2i) + (3+i)((2-3i)y + i)}{(3+i)(3-i)} = i$$

$$\Rightarrow \frac{(3-i)(1+i)x - 2i(3-i) + (3+i)(2-3i)y + i(3+i)}{3^2 + 1^2} = i$$

$$\Rightarrow \frac{(3+3i-i+1)x - 6i-2 + (6-9i+2i+3)y + 3i-1}{9+1} = i$$

$$\Rightarrow \frac{(4+2i)x - 6i-2 + (9-7i)y + 3i-1}{10} = i$$

$$\Rightarrow 4x + 2ix - 6i - 2 + 9y - 7iy + 3i - 1 = 10i$$

$$\Rightarrow 4x + 9y - 3 + i(2x - 7y - 3) = 10i$$

Equating real and imaginary parts we get

$$4x + 9y - 3 = 0 \dots\dots\dots (i)$$

$$\text{and } 2x - 7y - 3 = 10$$

$$i.e. 2x - 7y = 13 \dots\dots\dots (ii)$$

Multiplying (i) by 7, (ii) by 9 and adding we get

$$28x + 18x + 63y - 63y = 117 + 21$$

$$\Rightarrow 46x = 117 + 21$$

$$\Rightarrow 46x = 138$$

$$\Rightarrow x = \frac{138}{46} = 3$$

Substituting the value of  $x = 3$  in (i), we get

$$4 \times 3 + 9y = 3$$

$$\Rightarrow 9y = -9$$

$$\Rightarrow y = \frac{-9}{9}$$

$$\Rightarrow y = -1$$

Hence

$$x = 3, y = -1$$

$$(1+i)(x+iy) = 2-5i$$

$$\Rightarrow 1(x+iy) + i(x+iy) = 2-5i$$

$$\Rightarrow x+iy+ix-y = 2-5i$$

$$\Rightarrow x-y+i(x+y) = 2-5i$$

Equating real and imaginary parts we get

$$x-y = 2 \dots\dots\dots (i)$$

$$x+y = -5 \dots\dots\dots (ii)$$

Adding (i) and (ii) we get

$$2x = 2-5$$

$$\Rightarrow 2x = -3$$

$$\Rightarrow x = \frac{-3}{2}$$

Substituting the value of  $x$  in (i), we get

$$\frac{-3}{2} - y = 2$$

$$\Rightarrow \frac{-3}{2} - 2 = y$$

$$\Rightarrow y = \frac{-3-4}{2}$$

$$\Rightarrow y = \frac{-7}{2}$$

Hence

$$x = \frac{-3}{2}, y = \frac{-7}{2}$$

### Complex Numbers Ex 13.2 Q3(i)

If  $z = x + iy$  is a complex number, then the conjugate of  $z$  denoted by  $\bar{z}$  is defined as  $\bar{z} = x - iy$

$$\text{let } z = 4 - 5i$$

$$\Rightarrow \bar{z} = 4 + 5i$$

### Complex Numbers Ex 13.2 Q3(ii)

$$\text{let } z = \frac{1}{3+5i}$$

$$= \frac{1}{3+5i} \times \frac{(3-5i)}{(3-5i)} \quad (\text{On rationalising the denominator})$$

$$= \frac{3-5i}{3^2+5^2}$$

$$\Rightarrow z = \frac{3-5i}{9+25}$$

$$\text{So } \bar{z} = \frac{3+5i}{34}$$

$$= \frac{3}{34} + \frac{5}{34}i$$

### Complex Numbers Ex 13.2 Q3(iii)

$$\begin{aligned}
 \text{let } z &= \frac{1}{1+i} \\
 &= \frac{1}{1+i} \times \frac{(1-i)}{(1-i)} \\
 &= \frac{1-i}{1^2+1^2} \\
 &= \frac{1-i}{2}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \bar{z} &= \frac{1+i}{2} \\
 &= \frac{1}{2} + \frac{1}{2}i
 \end{aligned}$$

**Complex Numbers Ex 13.2 Q3(iv)**

$$\begin{aligned}
 \text{let } z &= \frac{(3-i)^2}{2+i} \\
 &= \frac{3^2+i^2-2 \times 3 \times i}{2+i} \\
 &= \frac{9-1-6i}{2+i} \\
 &= \frac{8-6i}{2+i} \\
 &= \frac{8-6i}{2+i} \times \frac{2-i}{2-i} \\
 &= \frac{8(2-i)-6i(2-i)}{2^2+1^2} \\
 &= \frac{16-8i-12i-6}{4+1} \\
 &= \frac{10-20i}{5}
 \end{aligned}$$

$$\Rightarrow z = 2 - 4i$$

Hence

$$\bar{z} = 2 + 4i$$

**Complex Numbers Ex 13.2 Q3(v)**

$$\begin{aligned}
 \text{let } z &= \frac{(1+i)(2+i)}{3+i} \\
 &= \frac{2+i+i(2+i)}{3+i} \\
 &= \frac{2+i+2i-1}{3+i} \\
 &= \frac{1+3i}{3+i} \\
 &= \frac{(1+3i)(3-i)}{(3+i)(3-i)} \\
 &= \frac{3-i+3i(3-i)}{3^2+1^2} \\
 &= \frac{3-i+9i+3}{9+1} \\
 &= \frac{6+8i}{10} \\
 &= \frac{2(3+4i)}{10} \\
 \Rightarrow z &= \frac{3+4i}{5}
 \end{aligned}$$

Hence

$$\begin{aligned}
 \bar{z} &= \frac{3-4i}{5} \\
 &= \frac{3}{5} - \frac{4}{5}i
 \end{aligned}$$

#### Complex Numbers Ex 13.2 Q3(vi)

$$\begin{aligned}
 \text{let } z &= \frac{(3-2i)(2+3i)}{(1+2i)(2-i)} \\
 &= \frac{3(2+3i)-2i(2+3i)}{2-i+2i(2-i)} \\
 &= \frac{6+9i-4i+6}{2-i+4i+2} \\
 &= \frac{12+5i}{4+3i} \\
 &= \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i} \\
 &= \frac{12(4-3i)+5i(4-3i)}{4(4-3i)+3i(4-3i)} \\
 &= \frac{48-36i+20i+15}{16-12i+12i+9} \\
 &= \frac{63-16i}{16+9} \\
 \Rightarrow z &= \frac{63-16i}{25}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \bar{z} &= \frac{63+16i}{25} \\
 &= \frac{63}{25} + \frac{16}{25}i
 \end{aligned}$$

#### Complex Numbers Ex 13.2 Q4(i)

If  $z = x + iy$  is a complex number, then the multiplicative inverse of  $z$ , denoted by  $z^{-1}$  or  $\frac{1}{z}$

$$\begin{aligned}\text{is defined as } z^{-1} &= \frac{1}{z} \\ &= \frac{1}{x + iy} \\ &= \frac{1}{x + iy} \times \frac{x - iy}{x - iy} \\ &= \frac{x - iy}{x^2 + y^2} \\ &= \frac{x}{x^2 + y^2} - \frac{y}{x^2 + y^2} i\end{aligned}$$

Given

$$z = 1 - i$$

$$\begin{aligned}\therefore z^{-1} &= \frac{1}{1^2 + 1^2} - \frac{(-1)}{1^2 + 1^2} i \\ &= \frac{1}{2} + \frac{1}{2} i\end{aligned}$$

#### Complex Numbers Ex 13.2 Q4(ii)

$$\begin{aligned}\text{let } z &= (1 + i\sqrt{3})^2 \\ &= 1^2 + (i\sqrt{3})^2 + 2 \times 1 \times i\sqrt{3} \\ &= 1 - 3 + 2\sqrt{3}i \\ &= -2 + 2\sqrt{3}i\end{aligned}$$

$$\begin{aligned}\therefore z^{-1} &= \frac{-2}{(-2)^2 + (2\sqrt{3})^2} - \frac{2\sqrt{3}i}{(-2)^2 + (2\sqrt{3})^2} \\ &= \frac{-2}{4 + 12} - \frac{2\sqrt{3}i}{4 + 12} \\ &= \frac{-2}{16} - \frac{2\sqrt{3}i}{16} \\ &= \frac{-1}{8} - \frac{\sqrt{3}i}{8}\end{aligned}$$

#### Complex Numbers Ex 13.2 Q4(iii)

$$\text{let } z = 4 - 3i$$

$$\begin{aligned}\text{Then } z^{-1} &= \frac{4}{4^2 + (-3)^2} - \frac{(-3)}{4^2 + (-3)^2} i \\ &= \frac{4}{16 + 9} + \frac{3}{16 + 9} i \\ &= \frac{4}{25} + \frac{3}{25} i\end{aligned}$$

#### Complex Numbers Ex 13.2 Q4(iv)

$$\text{let } z = \sqrt{5} + 3i$$

$$\begin{aligned}\text{Then } z^{-1} &= \frac{\sqrt{5}}{(\sqrt{5})^2 + (3)^2} - \frac{3}{(\sqrt{5})^2 + (3)^2} i \\ &= \frac{\sqrt{5}}{5 + 9} - \frac{3}{5 + 9} i \\ &= \frac{\sqrt{5}}{14} - \frac{3}{14} i\end{aligned}$$

#### Complex Numbers Ex 13.2 Q5

If  $z = x + iy$  then  $|z| = \sqrt{x^2 + y^2}$

We have,

$$z_1 = 2 - i, z_2 = 1 + i$$

$$\begin{aligned} z_1 + z_2 &= 2 - i + 1 + i \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{And } z_1 - z_2 &= 2 - i - 1 - i \\ &= 1 - 2i \end{aligned}$$

$$\begin{aligned} \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} &= \frac{3 + 1}{1 - 2i + i} \\ &= \frac{4}{1 - i} \\ &= \frac{4}{1 - i} \times \frac{1 + i}{1 + i} \\ &= \frac{4(1 + i)}{1^2 + 1^2} \\ &= \frac{4(1 + i)}{2} \\ &= 2(1 + i) \end{aligned}$$

$$\begin{aligned} \therefore \left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right| &= |2(1 + i)| \\ &= 2|1 + i| \\ &= 2 \times \sqrt{1^2 + 1^2} \\ &= 2 \times \sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

$$(\because |z_1 z_2| = |z_1| \times |z_2|)$$

(i)

$$\begin{aligned}\frac{z_1 z_2}{z_1} &= \frac{z_1 z_2}{z_1} \times \frac{z_1}{z_1} && \text{(rationalising the denominator)} \\&= \frac{(z_1)^2 z_2}{z_1 z_1} \\&= \frac{(2-i)^2 (-2+i)}{|z_1|^2} && (\because z \bar{z} = |z|^2) \\&= \frac{(2^2 + i^2 - 2 \times 2 \times i) (-2+i)}{|2-i|^2} \\&= \frac{(4 - 1 - 4i) (-2+i)}{2^2 + (-1)^2} \\&= \frac{(3 - 4i) (-2+i)}{4+1} \\&= \frac{3(-2+i) - 4i(-2+i)}{5} \\&= \frac{-6 + 3i + 8i + 4}{5} \\&= \frac{-2 + 11i}{5}\end{aligned}$$

$$\begin{aligned}\therefore \operatorname{Re}\left(\frac{z_1 z_2}{z_1}\right) &= \operatorname{Re}\left(\frac{-2}{5} + \frac{11i}{5}\right) \\&= \frac{-2}{5}\end{aligned}$$

(ii)

$$\begin{aligned}\frac{1}{z_1 z_1} &= \frac{1}{|z_1|^2} \\&= \frac{1}{|2-i|^2} \\&= \frac{1}{2^2 + (-1)^2} \\&= \frac{1}{4+1} \\&= \frac{1}{5}, \text{ which is purely real}\end{aligned}$$

$$\therefore \operatorname{Im}\left(\frac{1}{z_1 z_1}\right) = 0$$

$$\begin{aligned}
 \text{let } z &= \frac{1+i}{1-i} - \frac{1-i}{1+i} \\
 &= \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)} \\
 &= \frac{1^2 + i^2 + 2 \times 1 \times i - (1^2 + i^2 - 2 \times 1 \times i)}{1^2 + 1^2} \\
 &= \frac{1 - 1 + 2i - (1 - 1 - 2i)}{2} \\
 &= \frac{2i + 2i}{2} \\
 &= \frac{4i}{2} \\
 \Rightarrow z &= 2i
 \end{aligned}$$

$$\begin{aligned}
 \therefore |z| &= |2i| \\
 &= 2|i| & (\because |z_1 z_2| = |z_1| \times |z_2|) \\
 &= 2 \times 1 & (\because |i| = 1) \\
 &= 2
 \end{aligned}$$

### Complex Numbers Ex 13.2 Q8

$$x + iy = \frac{a+ib}{a-ib}$$

$$\Rightarrow \overline{(x+iy)} = \overline{\left(\frac{a+ib}{a-ib}\right)} \quad (\text{on taking conjugate both sides})$$

$$\begin{aligned}
 \Rightarrow x - iy &= \frac{\overline{(a+ib)}}{\overline{(a-ib)}} & \left( \because \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}} \right) \\
 &= \frac{a-ib}{a+ib}
 \end{aligned}$$

$$\therefore (x+iy)(x-iy) = \frac{a+ib}{a-ib} \times \frac{a-ib}{a+ib}$$

$$\Rightarrow x^2 + y^2 = 1$$

proved

### Complex Numbers Ex 13.2 Q9

For  $n = 1$ , we have,

$$\begin{aligned}\left(\frac{1+i}{1-i}\right)^1 &= \frac{1+i}{1-i} \\ &= \frac{1+i}{1-i} \times \frac{1+i}{1+i} \\ &= \frac{(1+i)^2}{1^2+1^2} \\ &= \frac{1^2+i^2+2 \times 1 \times i}{2} \\ &= \frac{2i}{2} \quad (\because i^2 = -1) \\ &= i, \text{ which is not real}\end{aligned}$$

For  $n = 2$ , we have

$$\begin{aligned}\left(\frac{1+i}{1-i}\right)^2 &= i^2 \quad \left(\because \frac{1+i}{1-i} = i \text{ from above}\right) \\ &= -1, \text{ which is real}\end{aligned}$$

Hence the least positive integral value of  $n$  is 2.

### Complex Numbers Ex 13.2 Q10

$$\begin{aligned}\text{let } z &= \frac{1+i \cos \theta}{1-2i \cos \theta} \\ &= \frac{1+i \cos \theta}{1-2i \cos \theta} \times \frac{1+2i \cos \theta}{1+2i \cos \theta} \\ &= \frac{1+2i \cos \theta + i \cos \theta (1+2i \cos \theta)}{1^2 + (2 \cos \theta)^2} \\ &= \frac{1+2i \cos \theta + i \cos \theta - 2 \cos^2 \theta}{1+4 \cos^2 \theta} \\ &= \frac{1-2 \cos^2 \theta + 3i \cos \theta}{1+4 \cos^2 \theta} \\ &= \frac{1-2 \cos^2 \theta}{1+4 \cos^2 \theta} + \frac{3 \cos \theta}{1+4 \cos^2 \theta} i\end{aligned}$$

we know that  $z$  is purely real if and only if  $\text{Im } z = 0$

$$\begin{aligned}\therefore \frac{3 \cos \theta}{1+4 \cos^2 \theta} &= 0 \quad (\because z \text{ is given to be purely real}) \\ \Rightarrow 3 \cos \theta &= 0 \\ \Rightarrow \cos \theta &= 0 \\ \Rightarrow \cos \theta &= \cos \frac{\pi}{2}\end{aligned}$$

$\therefore$  The general solution is given by

$$\theta = 2n\pi \pm \frac{\pi}{2}, n \in \mathbb{Z}$$

### Complex Numbers Ex 13.2 Q11

$$\begin{aligned}
 \text{let } z &= \frac{(1+i)^n}{(1-i)^{n-2}} \\
 &= \frac{(1+i)^n}{(1-i)^n} (1-i)^2 \\
 &= \left(\frac{1+i}{1-i}\right)^n \times (1-i)^2 \\
 &= i^n (1+i^2 - 2 \times 1 \times i) \quad \left(\because \frac{1+i}{1-i} = i, \text{ using problem 10}\right) \\
 &= i^n (1 - 1 - 2i) \\
 &= -2i \times i^n \\
 &= -2i^{n+1}
 \end{aligned}$$

$\therefore$  For  $n = 1$

$$\begin{aligned}
 z &= -2i^{1+1} \\
 &= -2i^2 \\
 &= 2, \text{ which is a real number}
 \end{aligned}$$

$\therefore$  The smallest positive integer value of  $n$  is 1.

### Complex Numbers Ex 13.2 Q12

$$\begin{aligned}
 \left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 &= x + iy \\
 \Rightarrow \left(\frac{(1+i)(1+i)}{(1-i)(1+i)}\right)^3 - \left(\frac{(1-i)(1-i)}{(1+i)(1-i)}\right)^3 &= x + iy \text{ [Rationalizing the denominator]} \\
 \Rightarrow \left(\frac{1+2i-1}{1+1}\right)^3 - \left(\frac{1-2i-1}{1+1}\right)^3 &= x + iy \\
 \Rightarrow \left(\frac{2i}{2}\right)^3 - \left(\frac{-2i}{2}\right)^3 &= x + iy \\
 \Rightarrow i^3 - (-i)^3 &= x + iy \\
 \Rightarrow -i - i &= x + iy \\
 \Rightarrow -2i &= x + iy
 \end{aligned}$$

Comparing the real and imaginary parts,

$$(x, y) = (0, 2)$$

### Complex Numbers Ex 13.2 Q13

$$\frac{(1+i)^2}{2-i} = x + iy$$

$$\Rightarrow \frac{(1+2i-1)}{2-i} = x + iy$$

$$\Rightarrow \frac{2i}{2-i} = x + iy$$

$$\Rightarrow \frac{2i(2+i)}{(2-i)(2+i)} = x + iy \text{ [Rationalizing the denominator]}$$

$$\Rightarrow \frac{2(2i-1)}{4+1} = x + iy$$

$$\Rightarrow \frac{4i-2}{5} = x + iy$$

$$\Rightarrow -\frac{2}{5} + i\frac{4}{5} = x + iy$$

Comparing the real and imaginary parts, we get

$$x = -\frac{2}{5}, y = \frac{4}{5}$$

$$x + y = \frac{2}{5}$$

**Complex Numbers Ex 13.2 Q14**

$$\left(\frac{1-i}{1+i}\right)^{100} = a + ib$$

$$\Rightarrow \left(\frac{(1-i)(1-i)}{(1+i)(1-i)}\right)^{100} = a + ib \text{ [Rationalizing the denominator]}$$

$$\Rightarrow \left(\frac{(1-2i-1)}{(1+1)}\right)^{100} = a + ib$$

$$\Rightarrow \left(\frac{-2i}{2}\right)^{100} = a + ib$$

$$\Rightarrow (-i)^{100} = a + ib$$

$$\Rightarrow 1 = a + ib$$

Comparing, we get  $(a,b) = (1,0)$

**Complex Numbers Ex 13.2 Q15**

$$a = \cos \theta + i \sin \theta$$

$$\frac{1+a}{1-a}$$

$$= \frac{1+\cos \theta+i \sin \theta}{1-\cos \theta-i \sin \theta}$$

$$= \frac{(1+\cos \theta+i \sin \theta)(1-\cos \theta+i \sin \theta)}{(1-\cos \theta-i \sin \theta)(1-\cos \theta+i \sin \theta)} \quad [\text{Rationalizing the denominator}]$$

$$= \frac{(1+\cos \theta+i \sin \theta)(1-\cos \theta+i \sin \theta)}{(1-\cos \theta)^2-(i \sin \theta)^2}$$

$$= \frac{(1+i \sin \theta)^2-\cos^2 \theta}{1-2 \cos \theta+\cos^2 \theta+\sin^2 \theta}$$

$$= \frac{1+2i \sin \theta-\sin^2 \theta-\cos^2 \theta}{1-2 \cos \theta+\cos^2 \theta+\sin^2 \theta}$$

$$= \frac{1+2i \sin \theta-1}{1-2 \cos \theta+\cos^2 \theta+\sin^2 \theta} \quad [\because \cos^2 \theta+\sin^2 \theta=1]$$

$$= \frac{2i \sin \theta}{1-2 \cos \theta+1}$$

$$= \frac{2i \sin \theta}{2-2 \cos \theta}$$

$$= \frac{i \sin \theta}{1-\cos \theta}$$

$$= \frac{i 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}}$$

$$= \frac{i \cos \frac{\theta}{2}}{\sin \frac{\theta}{2}} = i \cot \frac{\theta}{2}$$

### Complex Numbers Ex 13.2 Q16(i)

We have,

$$x = \frac{3-5i}{2}$$

$$\Rightarrow 2x = 3-5i$$

$$\Rightarrow 2x-3 = -5i$$

$$\Rightarrow (2x-3)^2 = (-5i)^2$$

$$\Rightarrow 4x^2+9-12x = -25$$

$$\Rightarrow 4x^2-12x+34 = 0$$

$$\Rightarrow 2(2x^2-6x+17) = 0$$

$$\Rightarrow 2x^2-6x+17 = 0 \quad \dots\dots\dots(i)$$

$$\therefore 2x^3+2x^2-7x+72$$

$$= x(2x^2-6x+17) + 6x^2-17x+2x^2-7x+72 \quad (\text{adding and subtracting } 6x^2 \text{ and } 17x)$$

$$= x \times 0 + 8x^2-24x+72 \quad (\text{using (i)})$$

$$= 4(2x^2-6x+17) + 4$$

$$= 4 \times 0 + 4 \quad (\text{using (i)})$$

$$= 4$$

### Complex Numbers Ex 13.2 Q16(ii)

We have,

$$x = 3 + 2i$$

$$\Rightarrow x - 3 = 2i$$

$$\Rightarrow (x - 3)^2 = (2i)^2$$

$$\Rightarrow x^2 + 3^2 - 2 \times 3 \times x = -4$$

$$\Rightarrow x^2 + 9 - 6x + 4 = 0$$

$$\Rightarrow x^2 - 6x + 13 = 0 \quad \dots\dots\dots(i)$$

Now,

$$x^4 - 4x^3 + 4x^2 + 8x + 44$$

$$= x^2(x^2 - 6x + 13) + 6x^2 - 13x^2 - 4x^3 + 4x^2 + 8x + 44 \quad \text{(adding and subtracting } 6x^3 \text{ and } 13x^2 \text{)}$$

$$= x^2 \times 0 + 2x^3 - 9x^2 + 8x + 44 \quad \text{(using (i))}$$

$$= 2x(x^2 - 6x + 13) + 12x^2 - 26x - 9x^2 + 8x + 44 \quad \text{(adding and subtracting } 12x^2 \text{ and } 26x \text{)}$$

$$= 2x \times 0 + 3x^2 - 18x + 44 \quad \text{(using (i))}$$

$$= 3(x^2 - 6x + 13) + 5 \quad \text{(using (i))}$$

$$= 3 \times 0 + 5$$

$$= 5$$

### Complex Numbers Ex 13.2 Q16(iii)

We have,

$$x = -1 + i\sqrt{2}$$

$$\Rightarrow x + 1 = i\sqrt{2}$$

$$\Rightarrow (x + 1)^2 = (i\sqrt{2})^2 \quad \text{(squaring both sides)}$$

$$\Rightarrow x^2 + 1 + 2x = -2$$

$$\Rightarrow x^2 + 2x + 3 = 0 \dots\dots\dots(i)$$

Now,

$$x^4 + 4x^3 + 6x^2 + 4x + 9$$

$$= x^2(x^2 + 2x + 3) + 2x^3 + 3x^2 + 4x + 9$$

$$= x^2 \times 0 + 2x(x^2 + 2x + 3) - x^2 - 2x + 9 \quad \text{(using (i))}$$

$$= 2x \times 0 - (x^2 + 2x + 3) + 3 + 9 \quad \text{(using (i) and adding and subtracting 3)}$$

$$= -0 + 3 + 9 \quad \text{(using (i))}$$

$$= 12$$

### Complex Numbers Ex 13.2 Q16(iv)

We have,

$$x = \frac{1+i}{\sqrt{2}}$$

$$\Rightarrow \sqrt{2}x = 1+i$$

$$\Rightarrow (\sqrt{2}x)^2 = (1+i)^2 \quad \text{(squaring both sides)}$$

$$\begin{aligned}\Rightarrow 2x^2 &= 1^2 + (i)^2 + 2 \times 1 \times i \\ &= 1 - 1 + 2i\end{aligned}$$

$$\Rightarrow 2x^2 = 2i$$

$$\Rightarrow x^2 = i$$

$$\Rightarrow (x^2)^2 = (i)^2 \quad \text{(squaring both sides)}$$

$$\Rightarrow x^4 = -1$$

$$\Rightarrow x^4 + 1 = 0 \quad \dots\dots\dots(i)$$

Now

$$x^6 + x^4 + x^2 + 1$$

$$= x^6 + x^2 + x^4 + 1$$

$$= x^2(x^4 + 1) + 1(x^4 + 1)$$

$$= x^2 \times 0 + 1 \times 0 \quad \text{(using (i))}$$

$$= 0$$

### Complex Numbers Ex 13.2 Q16(v)

$$x = (-2 - \sqrt{3}i)$$

$$x^2 = (-2 - \sqrt{3}i)^2 = 4 + 4\sqrt{3}i + 3i^2 = 1 + 4\sqrt{3}i$$

$$x^3 = (1 + 4\sqrt{3}i)(-2 - \sqrt{3}i) = -2 - 8\sqrt{3}i - \sqrt{3} - 12i^2 = 10 - 9\sqrt{3}i$$

$$x^4 = (1 + 4\sqrt{3}i)^2 = 1 + 8\sqrt{3}i + 48i^2 = -47 + 8\sqrt{3}i$$

$$\begin{aligned}2x^4 + 5x^3 + 7x^2 - x + 41 &= 2(-47 + 8\sqrt{3}i) + 5(10 - 9\sqrt{3}i) + 7(1 + 4\sqrt{3}i) - (-2 - \sqrt{3}i) + 41 \\ &= -94 + 16\sqrt{3}i + 50 - 45\sqrt{3}i + 7 + 28\sqrt{3}i + 2 + \sqrt{3}i + 41 \\ &= (-94 + 50 + 7 + 2 + 41) + (16\sqrt{3}i - 45\sqrt{3}i + 28\sqrt{3}i + \sqrt{3}i) \\ &= 6 + 0 \\ &= 6\end{aligned}$$

### Complex Numbers Ex 13.2 Q17

$$\begin{aligned}
 & (1-i)^n \left(1 - \frac{1}{i}\right)^n \\
 &= (1-i)^n \left(\frac{i-1}{i}\right)^n \\
 &= \left\{ \frac{(1-i)(i-1)}{i} \right\}^n \\
 &= \left\{ \frac{(1-i)(1-i)}{-i} \right\}^n \\
 &= \left\{ \frac{(1-i)^2}{-i} \right\}^n \\
 &= \left\{ \frac{1-2i-1}{-i} \right\}^n \\
 &= \left\{ \frac{-2i}{-i} \right\}^n = 2^n
 \end{aligned}$$

### Complex Numbers Ex 13.2 Q18

$$\begin{aligned}
 (1+i)z &= (1-i)\bar{z} \\
 \Rightarrow z &= \frac{(1-i)\bar{z}}{(1+i)} \\
 \Rightarrow z &= \frac{(1-i)(1-i)\bar{z}}{(1+i)(1-i)} \quad [\text{Rationalizing the denominator}] \\
 \Rightarrow z &= \frac{(1-2i-1)\bar{z}}{(1+1)} \\
 \Rightarrow z &= \frac{-2i}{2}\bar{z} \\
 \Rightarrow z &= -i\bar{z}
 \end{aligned}$$

### Complex Numbers Ex 13.2 Q19

$$\operatorname{Re}(z^2) = 0, |z| = 2$$

$$\text{Let } z = x + iy$$

$$z^2 = 0$$

$$\Rightarrow (x + iy)^2 = 0$$

$$\Rightarrow x^2 + 2ixy - y^2 = 0$$

$$\Rightarrow x^2 - y^2 = 0 \dots (i), \text{ which is the real part of } (x + iy)^2.$$

$$|z| = 2$$

$$\Rightarrow \sqrt{x^2 + y^2} = 2$$

$$\Rightarrow x^2 + y^2 = 4 \dots (ii)$$

Adding (i) and (ii), we get

$$2x^2 = 4$$

$$\Rightarrow x^2 = 2$$

$$\Rightarrow x = \pm\sqrt{2}, y = \pm\sqrt{2}$$

$$x + iy = \sqrt{2} + i\sqrt{2}$$

$$= -\sqrt{2} - i\sqrt{2}$$

$$= \sqrt{2} - i\sqrt{2}$$

$$= -\sqrt{2} + i\sqrt{2}$$

**Kopykitab**  
Same textbooks, click away

$$\text{let } z = x + iy,$$

$$\frac{z-1}{z+1}$$

$$= \frac{x+iy-1}{x+iy+1}$$

$$= \frac{x-1+iy}{x+1+iy}$$

$$= \frac{(x-1+iy)(x+1-iy)}{(x+1+iy)(x+1-iy)} \quad [\text{Rationalizing the denominator}]$$

$$= \frac{(x-1+iy)(x+1-iy)}{(x+1)^2 - (iy)^2}$$

$$= \frac{x^2 + x - ixy - x - 1 + iy + ixy + iy + y^2}{x^2 + 2x + 1 + y^2}$$

$$= \frac{x^2 - 1 + 2iy + y^2}{x^2 + 2x + 1 + y^2}$$

$$= \frac{x^2 + y^2 - 1}{x^2 + 2x + 1 + y^2} + i \frac{2y}{x^2 + 2x + 1 + y^2}$$

$\therefore$  It is a purely imaginary no. therefore real part = 0

$$\frac{x^2 + y^2 - 1}{x^2 + 2x + 1 + y^2} = 0$$

$$\Rightarrow x^2 + y^2 - 1 = 0$$

$$\Rightarrow x^2 + y^2 = 1$$

$$\Rightarrow \sqrt{x^2 + y^2} = 1$$

$$\Rightarrow |z| = 1$$

$$\text{Let } z_1 = x_1 + iy_1, z_2 = x_2 + iy_2$$

$$|z_1| = 1 \Rightarrow x_1^2 + y_1^2 = 1$$

$$z_2 = \frac{z_1 - 1}{z_1 + 1}$$

$$x_2 + iy_2 = \frac{x_1 + iy_1 - 1}{x_1 + iy_1 + 1}$$

$$\Rightarrow x_2 + iy_2 = \frac{x_1 - 1 + iy_1}{x_1 + 1 + iy_1}$$

$$\Rightarrow x_2 + iy_2 = \frac{(x_1 - 1 + iy_1)(x_1 + 1 - iy_1)}{(x_1 + 1 + iy_1)(x_1 + 1 - iy_1)} \text{ [Rationalizing the denominator]}$$

$$\Rightarrow x_2 + iy_2 = \frac{(x_1 - 1)(x_1 + 1) - iy_1(x_1 - 1) + iy_1(x_1 + 1) + y_1^2}{(x_1 + 1)^2 - (iy_1)^2}$$

$$\Rightarrow x_2 + iy_2 = \frac{x_1^2 - 1 + y_1^2 - iy_1x_1 + iy_1 + iy_1x_1 + y_1^2}{(x_1 + 1)^2 - (iy_1)^2}$$

$$\Rightarrow x_2 + iy_2 = \frac{x_1^2 + y_1^2 - 1 + 2iy_1}{(x_1 + 1)^2 - (iy_1)^2}$$

$$\Rightarrow x_2 + iy_2 = \frac{1 - 1 + 2iy_1}{(x_1 + 1)^2 - (iy_1)^2} [\because x_1^2 + y_1^2 = 1]$$

$$\Rightarrow x_2 + iy_2 = \frac{2iy_1}{(x_1 + 1)^2 - (iy_1)^2} [\because x_1^2 + y_1^2 = 1]$$

Since there is no real part in the RHS, therefore  $x_2 = 0$ .

The real part of the  $z_2 = 0$ .

### Complex Numbers Ex 13.2 Q22

$$\text{Let } z = x + iy$$

$$|z+1| = z + 2(1+i)$$

$$\Rightarrow |x+iy+1| = x+iy+2+2i$$

$$\Rightarrow \sqrt{(x+1)^2 + y^2} = (x+2) + i(y+2)$$

Comparing real and imaginary parts, we get

$$x+2 = \sqrt{x^2 + 2x+1+y^2} \text{ and } y+2=0$$

$$y+2=0$$

$$\Rightarrow y = -2$$

$$\& (x+2)^2 = x^2 + 2x+1+y^2$$

$$\Rightarrow x^2 + 4x+4 = x^2 + 2x+1+y^2$$

$$\Rightarrow 2x+3 = y^2$$

$$\Rightarrow 2x+3 = (-2)^2$$

$$\Rightarrow 2x+3 = 4$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

$$\therefore z = x+iy = \frac{1}{2} - i2$$

### Complex Numbers Ex 13.2 Q23

$$\text{Let } z = x + iy$$

$$|z| = z + 1 + 2i$$

$$\Rightarrow |x + iy| = x + iy + 1 + 2i$$

$$\Rightarrow \sqrt{x^2 + y^2} = (x + 1) + i(y + 2)$$

$$\Rightarrow x^2 + y^2 = (x + 1)^2 + 2i(x + 1)(y + 2) - (y + 2)^2 \quad [\text{Squaring both sides}]$$

$$\Rightarrow x^2 + y^2 = x^2 + 2x + 1 + 2i(xy + 2x + y + 2) - (y^2 + 4y + 4)$$

$$\Rightarrow 2y^2 - 2x + 4y + 4 = 2i(xy + 2x + y + 2)$$

$$\Rightarrow y^2 - x + 2y + 2 = i(xy + 2x + y + 2)$$

$$\Rightarrow (y^2 - x + 2y + 2) - i(xy + 2x + y + 2) = 0$$

Comparing we get,

$$(xy + 2x + y + 2) = 0$$

$$\Rightarrow (x + 1)(y + 2) = 0$$

$$\Rightarrow x = -1 \text{ \& } y = -2$$

$$\text{Also, } (y^2 - x + 2y + 2) = 0$$

$$\text{Taking } x = -1, (y^2 - (-1) + 2y + 2) = 0$$

$$\Rightarrow (y^2 + 2y + 3) = 0$$

Does not have a solution since roots will be imaginary

$$\text{Taking } y = -2, (4 - x - 4 + 2) = 0$$

$$\Rightarrow x = 2$$

$$\therefore z = x + iy = 2 - 2i$$

### Complex Numbers Ex 13.2 Q24

$$(1 + i)^{2n} = (1 - i)^{2n}$$

$$\Rightarrow \left( \frac{1 + i}{1 - i} \right)^{2n} = 1$$

$$\Rightarrow \left( \frac{(1 + i)(1 + i)}{(1 - i)(1 + i)} \right)^{2n} = 1 \quad [\text{Rationalizing the denominator}]$$

$$\Rightarrow \left( \frac{1 + 2i - 1}{1 + 1} \right)^{2n} = 1$$

$$\Rightarrow \left( \frac{2i}{2} \right)^{2n} = 1$$

$$\Rightarrow i^{2n} = 1$$

$$\therefore n = 2$$

### Complex Numbers Ex 13.2 Q25

$$\begin{aligned}
 |z_1 + z_2 + z_3| &= \left| \frac{z_1 \bar{z_1}}{z_1} + \frac{z_2 \bar{z_2}}{z_2} + \frac{z_3 \bar{z_3}}{z_3} \right| \\
 &= \left| \frac{|z_1|^2}{z_1} + \frac{|z_2|^2}{z_2} + \frac{|z_3|^2}{z_3} \right| \\
 &= \left| \frac{1}{\bar{z_1}} + \frac{1}{\bar{z_2}} + \frac{1}{\bar{z_3}} \right| \dots\dots\dots [\because |z_1| = |z_2| = |z_3| = 1] \\
 &= \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right| \\
 &= 1
 \end{aligned}$$

### Complex Numbers Ex 13.2 Q26

Let  $z = x + iy$

$$z^2 = (x + iy)^2 = x^2 - y^2 + 2xyi$$

$$|z|^2 = z\bar{z} = (x + iy)(x - iy) = x^2 + y^2$$

$$z^2 + |z|^2 = 0$$

$$x^2 - y^2 + 2xyi + x^2 + y^2 = 0$$

$$2x^2 + 2xyi = 0$$

$$\Rightarrow 2x^2 = 0 \text{ and } 2xy = 0$$

$$\Rightarrow x = 0 \text{ and } y \in \mathbb{R}$$

$$\therefore z = 0 + iy \text{ where } y \in \mathbb{R}$$

**Kopykitab**  
Same textbooks, click away