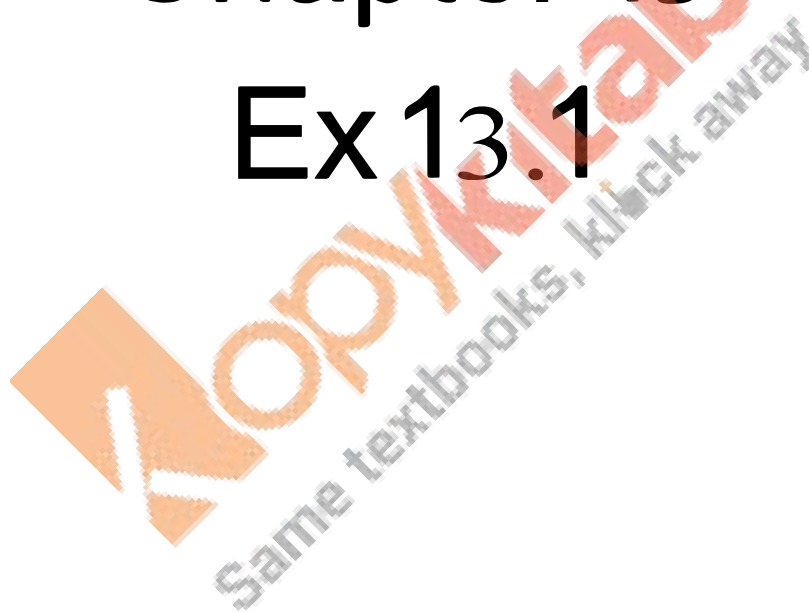


RD Sharma
Solutions
Class 11 Maths
Chapter 13
Ex 13.1



Complex numbers Ex 13.1 Q1(i)

We know that $i = \sqrt{-1}$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q , so that

$$n = 4p + q, 0 \leq q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$

$$= (i^4)^p \times i^q$$

$$= 1^p \times i^q$$

$$= i^q \quad [\because 1^{p-1}]$$

Hence $i^n = i^q$, where $0 \leq q < 4$

$$\therefore i^{457} = i^{4 \times 114} \times i^1$$

$$= i^1$$

$$= i$$

Complex numbers Ex 13.1 Q1(ii)

We know that $i = \sqrt{-1}$

$$i^2 = -1$$

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$$i^4 = 1$$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q , so that

$$n = 4p + q, 0 \leq q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$

$$= (i^4)^p \times i^q$$

$$= 1^p \times i^q$$

$$= i^q \quad [\because 1^{p-1}]$$

Hence $i^n = i^q$, where $0 \leq q < 4$

$$\therefore i^{528} = i^{4 \times 132}$$

$$= (i^4)^{132}$$

$$= 1^{132}$$

$$= 1$$

$$\therefore (i^{528}) = 1$$

Complex numbers Ex 13.1 Q1(iii)

We know that $i = \sqrt{-1}$

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In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q , so that
$$n = 4p + q, 0 \leq q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$

$$= (i^4)^p \times i^q$$

$$= 1^p \times i^q$$

$$= i^q \quad [\because 1^{p-1}]$$

Hence $i^n = i^q$, where $0 \leq q < 4$

$$\therefore \frac{1}{i^{58}} = \frac{1}{i^{4 \times 14} \times i^2}$$

$$= \frac{1}{1 \times i^2}$$

$$= \frac{1}{-1} \quad [\because i^2 = -1]$$

$$= -1$$

We know that $i = \sqrt{-1}$

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$$i^3 = -i$$

$$i^4 = 1$$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q , so that
$$n = 4p + q, 0 \leq q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$

$$= (i^4)^p \times i^q$$

$$= 1^p \times i^q$$

$$= i^q \quad [\because 1^p = 1]$$

Hence $i^n = i^q$, where $0 \leq q < 4$

$$\begin{aligned}\therefore i^{37} + \frac{1}{i^{67}} &= i^{4 \times 9} \times i^1 + \frac{1}{i^{4 \times 16} \times i^3} \\ &= 1 \times i^1 + \frac{1}{1 \times i^3} \\ &= i + \frac{1}{i^3 \times i} \\ &= i + \frac{i}{i^4} \\ &= i + \frac{i}{1} \quad [\because i^4 = 1] \\ &= 2i\end{aligned}$$

$$\therefore i^{37} + \frac{1}{i^{67}} = 2i$$

We know that $i = \sqrt{-1}$

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In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q , so that
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$$= i^{4p} \times i^q$$

$$= (i^4)^p \times i^q$$

$$= 1^p \times i^q$$

$$= i^q \quad [\because 1^{p-1}]$$

Hence $i^n = i^q$, where $0 \leq q < 4$

$$\begin{aligned} \left(i^{41} + \frac{1}{i^{257}}\right)^9 &= \left(i^{4 \times 10} \times i^1 + \frac{1}{i^{4 \times 64} \times i^1}\right)^9 \\ &= \left(1 \times i + \frac{1}{1 \times i}\right)^9 \\ &= \left(i + \frac{1}{i}\right)^9 \\ &= \left(i + \frac{1}{i \times i} \times i\right)^9 \\ &= \left(i + \frac{i}{-1}\right)^9 \\ &= (i - i)^9 \\ &= 0 \end{aligned}$$

We know that $i = \sqrt{-1}$

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In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q , so that
$$n = 4p + q, 0 \leq q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$

$$= (i^4)^p \times i^q$$

$$= 1^p \times i^q$$

$$= i^q \quad [\because 1^{p-1}]$$

Hence $i^n = i^q$, where $0 \leq q < 4$

$$\begin{aligned}(i^{77} + i^{70} + i^{87} + i^{414})^3 &= (i^{4 \times 19} \times i^1 + i^{4 \times 17} \times i^2 + i^{4 \times 21} \times i^3 + i^{4 \times 103} \times i^2)^3 \\&= (1 \times i + 1 \times i^2 + 1 \times i^3 + 1 \times i^2)^3 \\&= (i - 1 - i - 1)^3 \\&= (-2)^3 \\&= -8\end{aligned}$$

$$\therefore (i^{77} + i^{70} + i^{87} + i^{414})^3 = -8$$

Complex numbers Ex 13.1 Q1(vii)

We know that $i = \sqrt{-1}$

$$i^2 = -1$$

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In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q , so that
$$n = 4p + q, 0 \leq q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$

$$= (i^4)^p \times i^q$$

$$= 1^p \times i^q$$

$$= i^q \quad [\because 1^{p-1}]$$

Hence $i^n = i^q$, where $0 \leq q < 4$

$$\begin{aligned}\therefore i^{30} + i^{40} + i^{60} &= i^{4 \times 7} \times i^2 + i^{4 \times 10} + i^{4 \times 15} \\&= 1 \times i^2 + 1 + 1 \\&= -1 + 1 + 1 \\&= 1\end{aligned}$$

$$\therefore i^{30} + i^{40} + i^{60} = 1$$

Complex numbers Ex 13.1 Q1(viii)

We know that $i = \sqrt{-1}$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q , so that
$$n = 4p + q, 0 \leq q < 4$$

$$\text{Then } i^n = i^{4p+q}$$

$$= i^{4p} \times i^q$$

$$= (i^4)^p \times i^q$$

$$= 1^p \times i^q$$

$$= i^q \quad [\because 1^{p-1}]$$

Hence $i^n = i^q$, where $0 \leq q < 4$

$$\begin{aligned} i^{49} + i^{68} + i^{89} + i^{110} &= i^{4 \times 12} \times i^1 + i^{4 \times 17} + i^{4 \times 22} \times i^1 + i^{4 \times 27} \times i^2 \\ &= 1 \times i + 1 + 1 \times i + 1 \times i^2 \\ &= i + 1 + i - 1 \\ &= 2i \end{aligned}$$

$$\therefore i^{49} + i^{68} + i^{89} + i^{110} = 2i$$

Complex numbers Ex 13.1 Q2

$$\begin{aligned} 1 + i^{10} + i^{20} + i^{30} &= 1 + i^{4 \times 2} \times i^2 + i^{4 \times 5} + i^{4 \times 7} \times i^2 \\ &= 1 + 1 \times i^2 + 1 + 1 \times i^2 \\ &= 1 - 1 + 1 - 1 \\ &= 0, \text{ which is real number} \end{aligned}$$

Complex numbers Ex 13.1 Q3(i)

$$\begin{aligned} i^{49} + i^{68} + i^{89} + i^{110} &= i^{4 \times 12} \times i^1 + i^{4 \times 17} + i^{4 \times 22} \times i^1 + i^{4 \times 27} \times i^2 \\ &= 1 \times i + 1 + 1 \times i + 1 \times i^2 \\ &= i + 1 + i - 1 \\ &= 2i \end{aligned}$$

$$\therefore i^{49} + i^{68} + i^{89} + i^{110} = 2i$$

Complex numbers Ex 13.1 Q3(ii)

$$\begin{aligned} i^{30} + i^{80} + i^{120} &= i^{4 \times 7} \times i^2 + i^{4 \times 20} + i^{4 \times 30} \\ &= 1 \times i^2 + 1 + 1 \\ &= -1 + 1 + 1 \\ &= 1 \end{aligned}$$

$$\therefore i^{30} + i^{80} + i^{120} = 1$$

Complex numbers Ex 13.1 Q3(iii)

$$i + i^2 + i^3 + i^4 = 1 + (-1) + (-i) + 1$$

$$= 0$$

$$\therefore i + i^2 + i^3 + i^4 = 0$$

Complex numbers Ex 13.1 Q3(iv)

$$i^5 + i^{10} + i^{15} = i^{4 \times 1} \times i^1 + i^{4 \times 2} \times i^2 + i^{4 \times 3} \times i^3$$

$$= 1 \times i + 1 \times i^2 + 1 \times i^3$$

$$= i - 1 - i$$

$$= -1$$

$$\therefore i^5 + i^{10} + i^{15} = -1$$

Complex numbers Ex 13.1 Q3(v)

$$\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} = \frac{i^{4 \times 148} + i^{147} \times i^2 + i^{4 \times 147} + i^{4 \times 146} \times i^2 + i^{4 \times 146}}{i^{4 \times 145} \times i^2 + i^{4 \times 145} + i^{4 \times 144} \times i^2 + i^{4 \times 144} + i^{4 \times 143} \times i^2}$$

$$= \frac{1 + 1 \times i^2 + 1 + 1 \times i^2 + 1}{1 \times i^2 + 1 + 1 \times i^2 + 1 + 1 \times i^2}$$

$$= \frac{1 - 1 + 1 - 1 + 1}{-1 + 1 - 1 + 1 - 1}$$

$$= \frac{1}{-1}$$

$$= -1$$

$$\therefore \frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} = -1$$

Complex numbers Ex 13.1 Q3(vi)

$$1 + i^2 + i^4 + i^6 + i^8 + \dots + i^{20}$$

$$= 1 + i^2 + i^4 + i^{4 \times 1} \times i^2 + i^{4 \times 2} + i^{4 \times 2} \times i^2 + i^{4 \times 3} + i^{4 \times 3} \times i^2 + i^{4 \times 4} + i^{4 \times 4} \times i^2 + i^{4 \times 5}$$

$$= 1 - 1 + 1 + 1 \times i^2 + 1 + 1 \times i^2 + 1 + 1 \times i^2 + 1 + 1 \times i^2 + 1$$

$$= 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1$$

$$= 1$$

Complex numbers Ex 13.1 Q3(vii)

$$(1+i)^6 + (1-i)^3 = \left[(1+i)^2 \right]^3 + (1-i)^3$$

$$= (1+i^2+2i)^3 + (1-3+3i-i^3)$$

$$= (1-1+2i)^3 + (1-3-3+i)$$

$$= 8i^3 - 2 - 2i$$

$$= -8i - 2 - 2i$$

$$= -2 - 10i$$