# RD Sharma Solutions Class 11 Maths Chapter 10 Ex 10.2

**Sine and Cosine Formulae and their Applications Ex-10.2 Q1**

The area of a triangle $ABC$ is given by

$$\Delta = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2}\times 5\times 6\sin 60^\circ$$
$$= \frac{15\sqrt{3}}{2}\text{sq.unit}$$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q2**

The area of a triangle $ABC$ is given by

$$\Delta = \frac{1}{2}ab\sin C$$
$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$
$$= \frac{2+3-5}{2\sqrt{6}}$$
$$= 0$$
$$\sin C = \sqrt{1-\cos^2 C}$$
$$= 1$$

Therefore,

$$\Delta = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2}\sqrt{6}$$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q3**

We have, $a = 4, b = 6$ and $c = 8$

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{7}{8}$$
$$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{11}{16}$$
$$\cos C = \frac{a^2+b^2-c^2}{2ab} = -\frac{1}{4}$$
$$8\cos A + 16\cos B + 4\cos C = 8\times\frac{7}{8} + 16\times\frac{11}{16} + 4\times\left(-\frac{1}{4}\right)$$
$$= 17$$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q4**

In any $\triangle ABC$, we have

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$
$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$
$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

we have,

$a = 18, b = 24, c = 30$

Therefore,

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{1152}{1440} = \frac{4}{5}$$
$$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{648}{1080} = \frac{3}{5}$$
$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{0}{864} = 0$$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q5**

$b(c\cos A - a\cos C) = c^2 - a^2$

$RHS$

$= c^2 - a^2$

$= k^2 \sin^2 C - k^2 \sin^2 A$

$= k^2(\sin^2 C - \sin^2 A)$

$= k^2 \sin(C + A).\sin(C - A)$

$= k^2 \sin(\pi - B).\sin(C - A)$

$= k^2 \sin B.\sin(C - A)$

$= k\sin B.k\sin(C - A)$

$= bk\sin(C - A)$

$= bk(\sin C.\cos A - \sin A.\cos C)$

$= b(k\sin C.\cos A - k\sin A.\cos C)$

$= b(c\cos A - a\cos C) = LHS$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q6**

$c(a\cos B - b\cos A)$

$= ac.\cos B - bc\cos A$

$= ac.\dfrac{a^2 + c^2 - b^2}{2ac} - bc.\dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{a^2 + c^2 - b^2}{2} - \dfrac{b^2 + c^2 - a^2}{2}$

$= \dfrac{a^2 + c^2 - b^2 - b^2 - c^2 + a^2}{2}$

$= \dfrac{2a^2 - 2b^2}{2} = \left(a^2 - b^2\right) = RHS$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q7**

$2(bc\cos A + ca\cos B + ab\cos C) = a^2 + b^2 + c^2$

$LHS$

$= 2bc\cos A + 2ca\cos B + 2ab\cos C$

$= 2bc\dfrac{b^2 + c^2 - a^2}{2bc} + 2ca\dfrac{a^2 + c^2 - b^2}{2ca} + 2ab\dfrac{a^2 + b^2 - c^2}{2ab}$

$= b^2 + c^2 - a^2 + a^2 + c^2 - b^2 + a^2 + b^2 - c^2$

$= a^2 + b^2 + c^2 = RHS$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q8**

For any $\triangle ABC$, we have

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

therefore,

$\left(c^2 + b^2 - a^2\right)\tan A = \left(c^2 + b^2 - a^2\right)\dfrac{\sin A}{\cos A}$

$= \left(c^2 + b^2 - a^2\right)\dfrac{ka}{\frac{b^2 + c^2 - a^2}{2bc}}$

$= 2kabc$

Also,

$\left(a^2 + c^2 - b^2\right)\tan B = \left(a^2 + c^2 - b^2\right)\dfrac{\sin B}{\cos B}$

$= \left(a^2 + c^2 - b^2\right)\dfrac{kb}{\frac{a^2 + c^2 - b^2}{2ac}}$

$= 2kabc$

Now,

$\left(a^2 + b^2 - c^2\right)\tan C = \left(a^2 + b^2 - c^2\right)\dfrac{\sin C}{\cos C}$

$= \left(a^2 + b^2 - c^2\right)\dfrac{kc}{\frac{a^2 + b^2 - c^2}{2ab}}$

$= 2kabc$

Hence proved.

**Sine and Cosine Formulae and their Applications Ex-10.2 Q9**

$$\frac{c-b\cos A}{b-c\cos A}=\frac{\cos B}{\cos C}$$

$LHS$

$$=\frac{c-b\cos A}{b-c\cos A}$$

$$=\frac{k\sin C-k\sin B\cos A}{k\sin B-k\sin C\cos A}$$

$$=\frac{\sin(\pi-(A+B))-\sin B\cos A}{\sin(\pi-(A+C))-\sin C\cos A}$$

$$=\frac{\sin(A+B)-\sin B\cos A}{\sin(A+C)-\sin C\cos A}$$

$$=\frac{\sin A\cos B+\cos A\sin B-\sin B\cos A}{\sin A\cos C+\cos A\sin C-\sin C\cos A}$$

$$=\frac{\sin A\cos B}{\sin A\cos C}$$

$$=\frac{\cos B}{\cos C}=RHS$$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q10**

In any $\triangle ABC$, we have

$a=b\cos C+c\cos B$

$b=c\cos A+a\cos C$

$c=a\cos B+b\cos A$

Therefore,

$$L.H.S=a(\cos B+\cos C-1)+b(\cos C+\cos A-1)+c(\cos A+\cos B-1)$$

$$=a\cos B+a\cos C-a+b\cos C+b\cos A-b+c\cos A+c\cos B-c$$

$$=c-b\cos A+a\cos C-a+a-c\cos B+b\cos A-b+b-a\cos C+c\cos B-c$$

$$=0$$

$$=R.H.S$$

Hence proved.

**Sine and Cosine Formulae and their Applications Ex-10.2 Q11**

By sine rule we have

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=k$$

$k\sin A=a, k\sin B=b, k\sin C=c$

$$a\cos A+b\cos B+c\cos C=k\sin A\cos A+k\sin B\cos B+k\sin C\cos C$$

$$=\left(\frac{1}{2}\right)k[2\sin A\cos A+2\sin B\cos B+2\sin C\cos C]$$

$$=\left(\frac{1}{2}\right)k[\sin 2A+\sin 2B+\sin 2C]$$

$$=k[\sin(A+B)\cos(A-B)+\sin C\cos C]$$

$$=k[\sin(\pi-C)\cos(A-B)+\sin C\cos(\pi-(A+B))]$$

$$=k[\sin C\cos(A-B)-\sin C\cos(A+B)]$$

$$=k[\sin C(\cos(A-B)-\cos(A+B))]$$

$$=k\sin C[2\sin A\sin B]$$

$$=2\sin C(k\sin A)\sin B$$

$$=2a\sin B\sin C$$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q12**

We know that by cosine rule

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

$$\Rightarrow 2bc\cos A=b^2+c^2-a^2$$

$$\Rightarrow a^2=b^2+c^2-2bc\cos A$$

$$\Rightarrow a^2 = b^2 + c^2 - 2bc\left(2\cos^2\frac{A}{2} - 1\right)$$

$$\Rightarrow a^2 = b^2 + c^2 + 2bc - 4bc\cos^2\frac{A}{2}$$

$$\Rightarrow a^2 = (b+c)^2 - 4bc\cos^2\frac{A}{2}$$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q13**

$$4\left(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2}\right) = (a+b+c)^2$$

$LHS,$

$$4\left(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2}\right)$$

$$= 2\left(bc.2\cos^2\frac{A}{2} + ca.2\cos^2\frac{B}{2} + ab.2\cos^2\frac{C}{2}\right)$$

$$= 2(bc.(1-\cos A) + ca.(1-\cos B) + ab.(1-\cos C))$$

$$= 2bc - 2bc\cos A + 2ca - 2ca\cos B + 2ab - 2ab\cos C$$

$$= 2bc - 2bc\frac{b^2+c^2-a^2}{2bc} + 2ca - 2ca\frac{a^2+c^2-b^2}{2ca} + 2ab$$

$$-2ab\frac{b^2+a^2-c^2}{2ab}[\cos \text{ rule}]$$

$$= 2bc - b^2 - c^2 + a^2 + 2ca - a^2 - c^2 + b^2 + 2ab - b^2 - a^2 + c^2$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$= (a+b+c)^2 = RHS$$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q14**

$$\sin^3 A\cos(B-C) + \sin^3 B.\cos(C-A) + \sin^3 C.\cos(A-B)$$

$$= \sin^2 A\sin A\cos(B-C) + \sin^2 B.\sin B\cos(C-A) + \sin^2 C.\sin C.\cos(A-B)$$

$$= \sin^2 A\sin(\pi-(B+C))\cos(B-C) + \sin^2 B.\sin(\pi-(A+C)).\cos(C-A)$$

$$+\sin^2 C.\sin(\pi-(A+B)).\cos(A-B)$$

$$= \sin^2 A\sin(B+C)\cos(B-C) + \sin^2 B.\sin(C+A).\cos(C-A)$$

$$+\sin^2 C.\sin(A+B).\cos(A-B)$$

$$= \sin^2 A.(\sin 2B + \sin 2C) + \sin^2 B.(\sin 2C + \sin 2A) + \sin^2 C.(\sin 2A + \sin 2B)$$

$$= \sin^2 A.(2\sin B.\cos B + 2\sin C\cos C) + \sin^2 B.(2\sin C\cos C + 2\sin A\cos A)$$

$$+\sin^2 C.(2\sin A\cos A + 2\sin B\cos B)$$

$$= \sin^2 A.(2\sin B\cos B + 2\sin C\cos C) + \sin^2 B.(2\sin C\cos C + 2\sin A\cos A)$$

$$+\sin^2 C.(2\sin A\cos A + 2\sin B\cos B)$$

$$= \sin^2 A.2\sin B\cos B + \sin^2 A.2\sin C\cos C + \sin^2 B.2\sin C\cos C$$

$$+\sin^2 B.2\sin A\cos A + \sin^2 C.2\sin A\cos A + \sin^2 C.2\sin B\cos B$$

$$= k^2a^2 2kb\cos B + k^2a^2.2kc\cos C + k^2b^2.2ka\cos C$$

$$+k^2b^2.2ka\cos A + k^2c^2.2ka\cos A + k^2c^2.2kb\cos B$$

$$= k^3ab(a\cos B + b\cos A) + k^3ac(a\cos C + c\cos A) + k^3bc(c\cos B + b\cos C)$$

$$= k^3abc + k^3acb + k^3bca$$

$$= k^3 3abc$$

$$= 3(k\sin A.k\sin B.k\sin C)$$

$$= 3abc = RHS$$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q15**

$$\text{Let } \frac{b+c}{12} = \frac{c+a}{13} = \frac{a+b}{15} = \lambda(\text{say})$$

$$b+c = 12\lambda, c+a = 13\lambda, a+b = 15\lambda$$

$$(b+c+c+a+a+b) = 12\lambda + 13\lambda + 15\lambda$$

$$2(a+b+c) = 40\lambda$$

$$a+b+c = 20\lambda$$

$$b+c = 12\lambda \text{ and } a+b+c = 20\lambda \Rightarrow a = 8\lambda$$

$$c+a = 13\lambda \text{ and } a+b+c = 20\lambda \Rightarrow b = 7\lambda$$

$$a+b = 15\lambda \text{ and } a+b+c = 20\lambda \Rightarrow c = 5\lambda$$

$$\cos A = \frac{b^2+c^2-a^2}{\ldots} = \frac{49\lambda^2 + 25\lambda^2 - 64\lambda^2}{\ldots} = \frac{1}{\ldots}$$

$\cos A = \dfrac{\quad}{2bc} = \dfrac{\quad}{70\lambda^2} = \dfrac{\quad}{7}$

$\cos B = \dfrac{a^2+c^2-b^2}{2ac} = \dfrac{64\lambda^2+25\lambda^2-49\lambda^2}{80\lambda^2} = \dfrac{1}{2}$

$\cos C = \dfrac{a^2+b^2-c^2}{2ab} = \dfrac{64\lambda^2+49\lambda^2-25\lambda^2}{112\lambda^2} = \dfrac{11}{14}$

$\cos A : \cos B : \cos C = \dfrac{1}{7} : \dfrac{1}{2} : \dfrac{11}{14} = 2:7:11$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q16**

We have, $\angle B = 60^\circ$

$\cos B = \dfrac{1}{2} \Rightarrow \dfrac{a^2+c^2-b^2}{2ac} = \dfrac{1}{2}$

$\Rightarrow a^2+c^2-b^2 = ac$

$\Rightarrow a^2+c^2-ac = b^2 \qquad ......(i)$

$(a+b+c)(a-b+c) = 3ca$

$a^2-ab+ac+ab-b^2+bc+ac-bc+c^2 = 3ac$

$a^2+c^2-b^2+2ac-3ac = 0$

$a^2+c^2-ac = b^2$

which is given.

**Sine and Cosine Formulae and their Applications Ex-10.2 Q17**

Consider the given equation:

$\cos^2 A + \cos^2 B + \cos^2 C = 1$

$\Rightarrow 1 - \sin^2 A + 1 - \sin^2 B + 1 - \sin^2 C = 1$

$\Rightarrow 3 - \sin^2 A + 1 - \sin^2 B + 1 - \sin^2 C = 1$

**Sine and Cosine Formulae and their Applications Ex-10.2 Q18**

Let $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = k$. Then, $\sin A = ka, \sin B = kb, \sin C = kc$

Now, $\cos C = \dfrac{\sin A}{2\sin B}$

$2\sin B\cos C = \sin A$

$2\left(\dfrac{a^2+b^2-c^2}{2ab}\right)kb = ka$

$a^2+b^2-c^2 = a^2$

$b^2 = c^2$

$b = c$

$\triangle ABC$ is isosceles.

**Sine and Cosine Formulae and their Applications Ex-10.2 Q19**

Let $P$ and $Q$ be the position of two ships at the end of 3 hours.

Then,

$OP = 3\times 24 = 72\,km$ and $OQ = 3\times 32 = 96\,km$

Using cosine formula in $\Delta OPQ$, we get

$PQ^2 = OP^2 + OQ^2 - 2OP\times OQ\cos 90^\circ$

$PQ^2 = 72^2 + 96^2 - 2\times 72\times 96\cos 90^\circ$

$PQ^2 = 14400$

$PQ = 120\,km$