# RD Sharma Solutions Class 11 Maths Chapter 10 Ex 10.1

p>**Sine and Cosine Formulae and their Applications Ex-10.1 Q1**

$\angle A = 45^\circ, \angle B = 60^\circ \text{ and } \angle C = 75^\circ$

Using sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\frac{a}{\sin 45} = \frac{b}{\sin 60} = \frac{c}{\sin 75} = k$$

$$\frac{a}{\frac{1}{\sqrt{2}}} = \frac{b}{\frac{\sqrt{3}}{2}} = \frac{c}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = k$$

$$a : b : c = 2 : \sqrt{6} : (\sqrt{3}+1)$$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q2**

$\angle C = 105^\circ, \angle B = 45^\circ, a = 2$

From here we can calculate that

$\angle A = 30^\circ$

$$a \sin B = b \sin A$$

$$\Rightarrow 2 \sin 45 = b \sin 30$$

$$\Rightarrow 2 \times \frac{1}{\sqrt{2}} = b \times \frac{1}{2}$$

$$\Rightarrow \sqrt{2} = \frac{b}{2}$$

$$\Rightarrow b = 2\sqrt{2}$$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q3**

$a = 18, b = 24, c = 30, \angle C = 90^\circ$

let $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$$\frac{\sin A}{18} = \frac{\sin B}{24} = \frac{\sin 90}{30}$$

$$\frac{\sin A}{18} = \frac{\sin B}{24} = \frac{1}{30}$$

$$\frac{\sin A}{18} = \frac{1}{30} \Rightarrow \sin A = \frac{18}{30} = \frac{3}{5}$$

$$\frac{\sin B}{24} = \frac{1}{30} \Rightarrow \sin B = \frac{24}{30} = \frac{4}{5}$$

$$\therefore \sin A = \frac{3}{5}, \sin B = \frac{4}{5}, \sin C = 1$$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q4**

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$$

Let $a = k \sin A, b = k \sin B$ (Using sine rule)

$LHS$

$$= \frac{a-b}{a+b}$$

$$= \frac{k \sin A - k \sin B}{k \sin A + k \sin B}$$

$$= \frac{\sin A - \sin B}{\sin A + \sin B}$$

$$= \frac{2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})}{2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})}$$

$$= \frac{\tan(\frac{A-B}{2})}{\quad} = RHS$$

$\tan(\frac{A+B}{2})$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q5**

$$(a-b)\cos\frac{C}{2} = c\sin\left(\frac{A-B}{2}\right)$$

Let $a = k\sin A, b = k\sin B, c = k\sin C$

LHS

$$(a-b)\cos\frac{C}{2}$$

$$= k(\sin A - \sin B).\cos\frac{C}{2}$$

$$= 2k\cos(\frac{A+B}{2})\sin(\frac{A-B}{2}).\cos\frac{C}{2}$$

$$= 2k\cos(\frac{\pi-C}{2})\sin(\frac{A-B}{2}).\cos\frac{C}{2}$$

$$= 2k\sin(\frac{C}{2}).\cos\frac{C}{2}.\sin(\frac{A-B}{2}) \qquad [\cos(\frac{\pi}{2}-\theta) = \sin\theta]$$

$$= k\sin C.\sin(\frac{A-B}{2})$$

$$= c.\sin(\frac{A-B}{2}) = RHS$$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q6**

$$\frac{c}{a-b} = \frac{\tan\left(\frac{A}{2}\right)+\tan\left(\frac{B}{2}\right)}{\tan\left(\frac{A}{2}\right)-\tan\left(\frac{B}{2}\right)}$$

$LHS$

$$\frac{c}{a-b}$$

$$= \frac{k\sin C}{k\sin A - k\sin B}$$

$$= \frac{\sin C}{\sin A - \sin B}$$

$$= \frac{2\sin\frac{C}{2}\cos\frac{C}{2}}{\sin A - \sin B}$$

$$= \frac{2\sin\frac{C}{2}\cos\frac{C}{2}}{2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})}$$

$$= \frac{\sin\frac{C}{2}\cos\frac{(\pi-(A+B))}{2}}{\cos(\frac{\pi-C}{2})\sin(\frac{A-B}{2})}$$

$$= \frac{\sin\frac{C}{2}\sin\frac{(A+B)}{2}}{\sin\frac{C}{2}\sin(\frac{A-B}{2})}$$

$$= \frac{\sin\frac{(A+B)}{2}}{\sin(\frac{A-B}{2})}$$

$$= \frac{\sin(\frac{A}{2}).\cos(\frac{B}{2})+\sin(\frac{B}{2}).\cos(\frac{A}{2})}{\sin(\frac{A}{2}).\cos(\frac{B}{2})-\sin(\frac{B}{2}).\cos(\frac{A}{2})}$$

$$= \frac{\tan(\frac{A}{2})+\tan(\frac{B}{2})}{\tan(\frac{A}{2}).-\tan(\frac{B}{2}).}$$ [Dividing both Numerator and Denominator by $\cos(\frac{A}{2}).\cos(\frac{B}{2})$ ]

$= RHS$



**Sine and Cosine Formulae and their Applications Ex-10.1 Q7**

$$\frac{c}{a+b} = \frac{1-\tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)}{1+\tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)}$$

$LHS$

$$= \frac{c}{a+b}$$

$$= \frac{k \sin C}{k \sin A + k \sin B}$$

$$= \frac{2 \sin \frac{C}{2} \cos \frac{C}{2}}{2 \sin(\frac{A+B}{2}). \cos(\frac{A-B}{2})}$$

$$= \frac{\sin \frac{C}{2} \cos \frac{C}{2}}{\sin(\frac{\pi - C}{2}). \cos(\frac{A-B}{2})}$$

$$= \frac{\sin(\frac{\pi-(A+B)}{2}) \cos \frac{C}{2}}{\cos(\frac{C}{2}). \cos(\frac{A-B}{2})}$$

$$= \frac{\cos(\frac{A+B}{2})}{\cos(\frac{A-B}{2})}$$

$$= \frac{\cos \frac{A}{2} . \cos \frac{B}{2} - \sin \frac{A}{2} \sin \frac{B}{2}}{\cos \frac{A}{2} . \cos \frac{B}{2} + \sin \frac{A}{2} . \sin \frac{B}{2}}$$

$$= \frac{1 - \tan \frac{A}{2} \tan \frac{B}{2}}{1 + \tan \frac{A}{2} . \tan \frac{B}{2}}$$ [Dividing both Numerator and Denominator by $\cos(\frac{A}{2}). \cos(\frac{B}{2})$ ]

$= RHS$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q8**

$$\frac{a+b}{c} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin \frac{C}{2}}$$

Let $a = k \sin A, b = k \sin B, c = k \sin C$

$LHS$

$$\frac{k \sin A + k \sin B}{k \sin C}$$

$$= \frac{\sin A + \sin B}{\sin C}$$

$$= \frac{2 \sin \frac{A+B}{2} . \cos \frac{A-B}{2}}{2 \sin \frac{C}{2} . \cos \frac{C}{2}}$$

$$= \frac{\sin(\frac{\pi - C}{2}). \cos \frac{A-B}{2}}{\sin \frac{C}{2} . \cos \frac{C}{2}}$$

$$= \frac{\cos \frac{A-B}{2}}{\sin \frac{C}{2}} = RHS$$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q9**

$$\sin\left(\frac{B-C}{2}\right) = \frac{b-c}{a} \cos \frac{A}{2}$$

Let $a = k \sin A, b = k \sin B, c = k \sin C$

$RHS$

$$\frac{b-c}{a} \cos \frac{A}{2}$$

$$= \frac{k \sin B - k \sin C}{k \sin A} . \cos \frac{A}{2}$$

$\sin B - \sin C \quad A$

$$\frac{\sin B-\sin C}{\sin A}.\cos\frac{A}{2}$$

$$=\frac{2\cos\frac{B+C}{2}.\sin\frac{B-C}{2}}{2\sin\frac{A}{2}.\cos\frac{A}{2}}\cos\frac{A}{2}$$

$$=\frac{\cos\frac{\pi-A}{2}\sin\frac{B-C}{2}}{\sin\frac{A}{2}}$$

$$=\frac{\sin\frac{A}{2}\sin\frac{B-C}{2}}{\sin\frac{A}{2}}=\sin\frac{B-C}{2}=RHS$$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q10**

let $\frac{\text{a}}{\sin \text{A}}=\frac{\text{b}}{\sin \text{B}}=\frac{\text{c}}{\sin \text{C}}=k$

LHS,

$$\frac{a^2-c^2}{b^2}$$

$$=\frac{k^2\sin^2 A-k^2\sin^2 C}{k^2\sin^2 B}$$

$$=\frac{k^2(\sin^2 A-\sin^2 C)}{k^2\sin^2 B}$$

$$=\frac{(\sin^2 A-\sin^2 C)}{\sin^2(\pi-(A+C))}$$

$$=\frac{\sin(A+C)\sin(A-C)}{\sin^2(A+C)}$$

$$=\frac{\sin(A-C)}{\sin(A+C)}=RHS$$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q11**

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=k$$

RHS,

$$a\ \sin(B-C)$$

$$=a\sin B.\cos C-a\sin C.\cos B$$

$$=a(bk).\left(\frac{a^2+b^2-c^2}{2ab}\right)-a(ck).\left(\frac{a^2+c^2-b^2}{2ac}\right)$$

$$=k.\frac{(a^2+b^2-c^2)}{2}-k\frac{(a^2+c^2-b^2)}{2}$$

$$=2k.\frac{(b^2-c^2)}{2}$$

$$=b.(kb)-c(kc)$$

$$=b(\sin B)-c(\sin C)$$

LHS

**Sine and Cosine Formulae and their Applications Ex-10.1 Q12**

$$a^2\sin(B-C)=\ (b^2-c^2)\sin A$$

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=k$$

$LHS$,

$$a^2\sin(B-C)$$

$$=a^2\{\sin B.\cos C-\sin C.\cos B\}$$

$$=a^2kb.\frac{a^2+b^2-c^2}{2ab}-a^2ck.\frac{a^2+c^2-b^2}{2ac}\text{[Using cos rule and sine rule]}$$

$$=a^2k.\frac{a^2+b^2-c^2}{2a}-a^2k.\frac{a^2+c^2-b^2}{2a}$$

$$=a^2k.\left(\frac{a^2+b^2-c^2-a^2-c^2+b^2}{2a}\right)$$

$$= a^2k.\left(\frac{2b^2-2c^2}{2a}\right)$$
$$= ak.(b^2-c^2)$$
$$= \sin A(b^2-c^2) = RHS$$
Hence Proved

**Sine and Cosine Formulae and their Applications Ex-10.1 Q13**

$$\frac{\sqrt{\sin A}-\sqrt{\sin B}}{\sqrt{\sin A}+\sqrt{\sin B}} = \frac{a+b-2\sqrt{ab}}{a-b}$$
$RHS$
$$\frac{a+b-2\sqrt{ab}}{a-b}$$
$$= \frac{\left(\sqrt{a}\right)^2+\left(\sqrt{b}\right)^2-2\sqrt{ab}}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}$$
$$= \frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}$$
$$= \frac{\left(\sqrt{a}-\sqrt{b}\right)}{(\sqrt{a}+\sqrt{b})}$$
$$= \frac{\left(\sqrt{k\sin A}-\sqrt{k\sin B}\right)}{(\sqrt{k\sin A}+\sqrt{k\sin B})}$$
$$= \frac{\left(\sqrt{\sin A}-\sqrt{\sin B}\right)}{(\sqrt{\sin A}+\sqrt{\sin B})}\text{[taking } k \text{ common and cancelling them]}$$
$$= LHS$$
Hence Proved

**Sine and Cosine Formulae and their Applications Ex-10.1 Q14**

$LHS,$
$$a(\sin B-\sin C)+b(\sin C-\sin A)+c(\sin A-\sin B)$$
$$= a\sin B - a\sin C + b\sin C - b\sin A + c\sin A - c\sin B$$
$$= b\sin A - c\sin A + c\sin B - b\sin A + c\sin A - c\sin B[\because b\sin A = a\sin B, b\sin C = c\sin B, c\sin A = a\sin C]$$
$$= 0 = RHS$$
Hence Proved

**Sine and Cosine Formulae and their Applications Ex-10.1 Q15**

$$\frac{a^2\sin(B-C)}{\sin A}+\frac{b^2\sin(C-A)}{\sin B}+\frac{c^2\sin(A-B)}{\sin C} = 0$$
$$\frac{a}{\sin A} = \frac{b}{b\sin C} = \frac{c}{\sin C} = k$$
$LHS$
$$\frac{a^2\sin(B-C)}{\sin A}+\frac{b^2\sin(C-A)}{\sin B}+\frac{c^2\sin(A-B)}{\sin C}$$
$$= ak\sin(B-C)+bk\sin(C-A)+ck\sin(A-B)$$
$$= \sin A\sin(B-C)+\sin B\sin(C-A)+\sin C\sin(A-B)$$
$$= \sin(\pi-(B+C))\sin(B-C)+\sin(\pi-(C+A))\sin(C-A)$$
$$+\sin(\pi-(A+B))\sin(A-B)$$
$$= \sin(B+C)\sin(B-C)+\sin(C+A)\sin(C-A)$$
$$+\sin(A+B)\sin(A-B)$$
$$= \sin^2 B-\sin^2 C+\sin^2 C-\sin^2 A+\sin^2 A-\sin^2 B = 0 = RHS$$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q16**

$$a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2(\cos^2 A - \cos^2 B) = 0$$
LHS

$= a^2(1-\sin^2 B - 1 + \sin^2 C) + b^2(1-\sin^2 C - 1 + \sin^2 A)$
$+ c^2(1-\sin^2 A - 1 + \sin^2 B)$
$= a^2(\sin^2 C - \sin^2 B) + b^2(\sin^2 A - \sin^2 C) + c^2(\sin^2 B - \sin^2 A)$
$= a^2(k^2c^2 - k^2b^2) + b^2(k^2a^2 - k^2c^2) + c^2(k^2b^2 - k^2a^2)$
$= k^2(a^2c^2 - a^2b^2 + b^2a^2 - b^2c^2 + b^2c^2 - a^2c^2)$
$= k^2 \times 0 = 0 = RHS$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q17**

Let $a = k\sin A, b = k\sin B, c = k\sin C$

$LHS$

$b\cos B + c\cos C$

$= k\sin B\cos B + k\sin C\cos C$

$= \frac{k}{2}(2\sin B\cos B + 2\sin C\cos C)$

$= \frac{k}{2}(\sin 2B + \sin 2C)$

$= \frac{k}{2}2\sin(B+C)\cos(B-C)$

$= k\sin(\pi - A)\cos(B-C)$

$= k\sin A\cos(B-C)$

$= a\cos(B-C) = RHS$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q18**

$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \frac{1}{a^2} - \frac{1}{b^2}$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$

$LHS$

$= \frac{1-2\sin^2 A}{a^2} - \frac{1-2\sin^2 B}{b^2}$

$= \frac{1}{a^2} - \frac{1}{b^2} - 2\left(\frac{\sin^2 A}{a^2} - \frac{\sin^2 B}{b^2}\right)$

$= \frac{1}{a^2} - \frac{1}{b^2} - 2\left(k^2 - k^2\right)$[Using sine rule]

$= \frac{1}{a^2} - \frac{1}{b^2} = RHS$

hence Proved

**Sine and Cosine Formulae and their Applications Ex-10.1 Q19**

$\frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c+a} + \frac{\cos^2 A - \cos^2 B}{a+b} = 0$

$LHS$

$\frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c+a} + \frac{\cos^2 A - \cos^2 B}{a+b}$

$= \frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c+a} + \frac{\cos^2 A - \cos^2 B}{a+b}$

$= \frac{1-\sin^2 B - 1 + \sin^2 C}{b+c} + \frac{1-\sin^2 C - 1 + \sin^2 A}{c+a} + \frac{1-\sin^2 A - 1 + \sin^2 B}{a+b}$

$= \frac{\sin^2 C - \sin^2 B}{b+c} + \frac{\sin^2 A - \sin^2 C}{c+a} + \frac{\sin^2 B - \sin^2 A}{a+b}$

$= \frac{k^2c^2 - k^2b^2}{b+c} + \frac{k^2a^2 - k^2c^2}{c+a} + \frac{k^2b^2 - k^2a^2}{a+b}$

$= k^2(\frac{c^2-b^2}{b+c} + \frac{a^2-c^2}{c+a} + \frac{b^2-a^2}{a+b})$

$= k^2(c - b + a - c + b - a)$[Using $b^2 - a^2 = (b-a)(b+a)$]

$= 0 = RHS$

Hence Proved

**Sine and Cosine Formulae and their Applications Ex-10.1 Q20**

We know, $a\sin B = b\sin A, c\sin B = b\sin C, a\sin C = c\sin B$

We know $a\sin B = b\sin A, c\sin B = b\sin C, a\sin C = c\sin B$

$$a\sin\frac{A}{2}\sin\left(\frac{B-C}{2}\right)+b\sin\frac{B}{2}\sin\left(\frac{C-A}{2}\right)+c\sin\frac{C}{2}\sin\left(\frac{A-B}{2}\right)=0$$

$LHS$

$$=a\sin\left(\frac{\pi-(B+C)}{2}\right)\sin\left(\frac{B-C}{2}\right)+b\sin\left(\frac{\pi-(C+A)}{2}\right)\sin\left(\frac{C-A}{2}\right)$$

$$+c\sin\left(\frac{\pi-(A+B)}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$=a\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)+b\cos\left(\frac{C+A}{2}\right)\sin\left(\frac{C-A}{2}\right)$$

$$+c\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$=a(\sin B-\sin C)+b(\sin C-\sin A)+c(\sin A-\sin B)$$

$$=a\sin B-a\sin C+b\sin C-b\sin A+c\sin A-c\sin B$$

$$=b\sin A-a\sin C+b\sin C-b\sin A+a\sin C-b\sin C$$

$$=0=RHS$$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q21**

$$\frac{b\sec B+c\sec C}{\tan B+\tan C}=\frac{c\sec C+a\sec A}{\tan C+\tan A}=\frac{a\sec A+b\sec B}{\tan A+\tan B}$$

$$\frac{b\sec B+c\sec C}{\tan B+\tan C}$$

$$=\frac{k\sin B\sec B+k\sin C\sec C}{\tan B+\tan C}$$

$$=\frac{k\sin B\frac{1}{\cos B}+k\sin C\frac{1}{\cos C}}{\tan B+\tan C}$$

$$=\frac{k\tan B+k\tan C}{\tan B+\tan C}=\frac{k(\tan B+\tan C)}{\tan B+\tan C}=k$$

$$Similarly, \frac{c\sec C+a\sec A}{\tan C+\tan A}=k$$

$$Similarly, \frac{a\sec A+b\sec B}{\tan A+\tan B}=k$$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q22**

$$a\cos A + b\cos B + c\cos C = 2b\sin A\ \sin C = 2c\sin A\ \sin B$$

LHS

$$a\cos A + b\cos B + c\cos C$$

$$=k\ \sin A\cos A+k\sin B\cos B+k\sin C\cos C$$

$$=\frac{k}{2}(\sin 2A+\sin 2B+\sin 2C)$$

$$=\frac{k}{2}(2\sin(A+B).\cos(A-B)+2\sin C.\cos C)$$

$$=\frac{2k}{2}(\sin(\pi-C).\cos(A-B)+\sin C.\cos C)$$

$$=k(\sin C.\cos(A-B)+\sin C.\cos C)$$

$$=k\sin C(\cos(A-B)+\cos C)$$

$$=k\sin C.2\cos(\frac{A-B+C}{2}).\cos(\frac{A-B-C}{2})$$

$$=k\sin C.2\cos(\frac{\pi-2B}{2}).\cos(\frac{A-\pi+A}{2})$$

$$=k\sin C.2\sin B.\cos(\frac{2A-\pi}{2})$$

$$=k\sin C.2\sin B.\cos(\frac{\pi-2A}{2})$$

$$=k\sin C.2\sin B.\sin A$$

$$=2\sin B\sin C(k\sin A)=2a\sin B\sin C$$

$$=RHS$$

Similarly, $a\cos A + b\cos B + c\cos C = 2c\sin A\ \sin B$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q23**

$a(\cos B\cos C+\cos A)=b(\cos A\cos C+\cos B)=c(\cos A\cos B+\cos C)$

$a(\cos B\cos C-\cos(\pi-(B+C)))$

$=a(\cos B\cos C-\cos(B+C))$

$=a(\cos B\cos C-\cos B.\cos C+\sin B\sin C)$

$=a\sin B\sin C$

$=k\sin A\sin B\sin C$

Similarly, $b(\cos A\cos C+\cos B)=k\sin A\sin B\sin C$

Similarly, $c(\cos A\cos B+\cos C)=k\sin A\sin B\sin C$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q24**

Let $a=k\sin A$

$a(\cos C-\cos B)=\ 2(b-c)\ \cos^2\frac{A}{2}$

$LHS$

$=a(\cos C-\cos B)$

$=a2.\sin\frac{C+B}{2}.\sin\frac{B-C}{2}$

$=2k\sin A\sin\frac{\pi-A}{2}.\sin\frac{B-C}{2}$

$=2k2\sin\frac{A}{2}.\cos\frac{A}{2}.\cos\frac{A}{2}.\sin\frac{B-C}{2}$

$=2k\cos^2\frac{A}{2}\left(2\sin\frac{B-C}{2}.\sin\frac{A}{2}\right)$

$=2k\cos^2\frac{A}{2}\left(2\sin\frac{B-C}{2}.\sin\frac{\pi-(B+C)}{2}\right)$

$=2k\cos^2\frac{A}{2}\left(2\sin\frac{B-C}{2}.\cos\frac{B+C}{2}\right)$

$=2k\cos^2\frac{A}{2}(\sin B-\sin C)$

$=2\cos^2\frac{A}{2}(k\sin B-k\sin C)$

$=2\cos^2\frac{A}{2}(b-c)=RHS$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q25**

$b\cos\theta=c\cos(A-\theta)\ +a\cos(C+\theta)$

Let $a\sin C=c\sin A$[Using sine rule]

$RHS$

$=c\cos(A-\theta)+a\cos(C+\theta)$

$=c\cos A\cos\theta+c\sin A\cos\theta+a\cos C.\cos\theta-a\sin C\sin\theta$

$=k\sin C\cos A.\cos\theta+k\sin C\sin A\cos\theta+k\sin A\cos C.\cos\theta$

$-k\sin A\sin C\sin\theta$

$=k\sin C\cos A.\cos\theta+k\sin A\cos C.\cos\theta$

$=k\cos\theta(\sin C\cos A+\sin A\cos C)$

$=k\cos\theta\sin(C+A)$

$=k\cos\theta\sin(\pi-B)$

$=k\cos\theta\sin B$

$=k\sin B.\cos\theta=b\cos\theta=\ LHS$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q26**

Let $\sin A=ak,\sin B=bk,\sin C=ck$

$\sin^2 A+\sin^2 B=\sin^2 C$

$\Rightarrow k^2a^2+k^2b^2=k^2c^2$[Using sine rule]

$\Rightarrow a^2+b^2=c^2$

Since the triangle satisfies the Pythagoras theorem, therefore it is right angled.

**Sine and Cosine Formulae and their Applications Ex-10.1 Q27**

$a^2, b^2, c^2$ are in A.P.

$\Rightarrow -2a^2, -2b^2, -2c^2$ are in A.P.

$\Rightarrow (a^2+b^2+c^2)-2a^2, (a^2+b^2+c^2)-2b^2, (a^2+b^2+c^2)-2c^2$ are in A.P.

$\Rightarrow (b^2+c^2-a^2), (c^2+a^2-b^2), (b^2+a^2-c^2)$ are in A.P.

$\Rightarrow \frac{(b^2+c^2-a^2)}{2abc}, \frac{(c^2+a^2-b^2)}{2abc}, \frac{(b^2+a^2-c^2)}{2abc}$ are in A.P.

$\Rightarrow \frac{1}{a}\frac{(b^2+c^2-a^2)}{2bc}, \frac{1}{b}\frac{(c^2+a^2-b^2)}{2ac}, \frac{1}{c}\frac{(b^2+a^2-c^2)}{2ab}$ are in A.P.

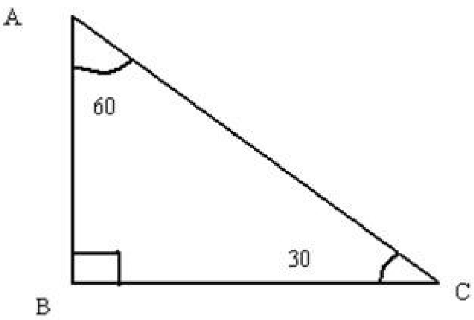
$\Rightarrow \frac{1}{a}\cos A, \frac{1}{b}\cos B, \frac{1}{c}\cos C$ are in A.P.

$\Rightarrow \frac{k}{a}\cos A, \frac{k}{b}\cos B, \frac{k}{c}\cos C$ are in A.P.

$\Rightarrow \frac{\cos A}{\sin A}, \frac{\cos B}{\sin B}, \frac{\cos C}{\sin C}$ are in A.P.

$\Rightarrow \cot A, \cot B, \cot C$ are in A.P.

**Sine and Cosine Formulae and their Applications Ex-10.1 Q28**




BC=15m, AB=h

From the diagram we can calculate, $\angle A = 60°$

Using sine rule,

$\frac{\sin A}{15} = \frac{\sin C}{h}$

$\Rightarrow \frac{\sin 60}{15} = \frac{\sin 30}{h}$

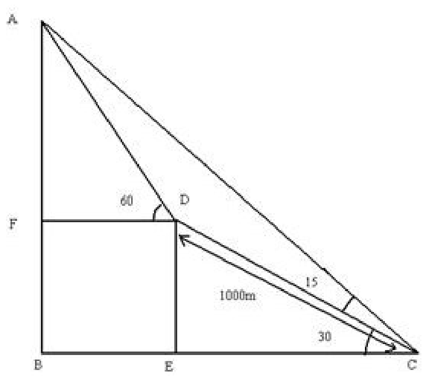
$\Rightarrow \frac{\sqrt{3}}{2\times 15} = \frac{1}{2\times h}$

$\Rightarrow \frac{\sqrt{3}}{15} = \frac{1}{h}$

$\Rightarrow h = \frac{15}{\sqrt{3}} \Rightarrow h = 5\sqrt{3}$



**Sine and Cosine Formulae and their Applications Ex-10.1 Q29**




$DE = 1000\sin 30 = 1000\times\frac{1}{2} = 500m = FB$

$EC = 1000\cos 30 = 1000\times\frac{\sqrt{3}}{2} = 500\sqrt{3}m$

$EC = 1000\cos 30 = 1000\times\frac{\sqrt{3}}{2} = 500\sqrt{3}m$

Let $AF = x\ m$

$DF = \frac{x}{\sqrt{3}}m = BE$

We know,

From $\Delta ABC$,

$\tan 45 = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{AF+FB}{BE+EC}$

$\Rightarrow 1 = \frac{x+500}{\frac{x}{\sqrt{3}}+500\sqrt{3}}$

$\Rightarrow \frac{x}{\sqrt{3}}+500\sqrt{3} = x+500$

$\Rightarrow x+1500 = x\sqrt{3}+500\sqrt{3}$

$\Rightarrow 1500-500\sqrt{3} = x\sqrt{3}-x$

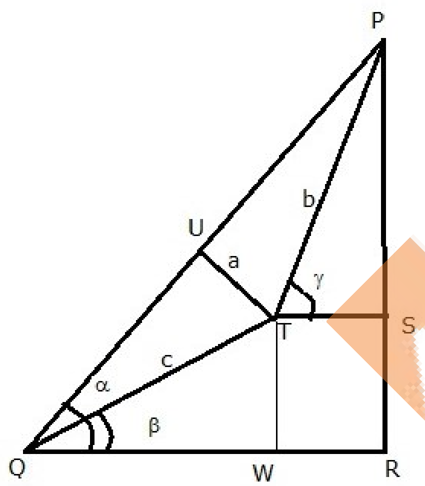
$\Rightarrow 500\sqrt{3}(\sqrt{3}-1) = x(\sqrt{3}-1)$

$\therefore x = 500\sqrt{3}m$

The height of the triangle is $AB = AF+FB = 500(\sqrt{3}+1)m$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q30**

*Consider the following figure.*






*The person is observing the peak P from the point Q.*

*The distance he travelled is $QT = c$ metres and the angle of inclination of QT is $\beta$.*

*He is observing the peak from the point and the angle of inclination is $\gamma$.*

*Now consider the triangle $\Delta QUT$.*

$\angle TQU = \beta - \alpha$

*Thus,* $\sin(\alpha-\beta) = \frac{a}{c}$

$\Rightarrow a = c\times\sin(\alpha-\beta)....(1)$

*Now consider the triangle $\Delta PQR$.*

*We know that* $\angle QPR = 90° - \alpha$

*In triangle $\Delta PTS$,* $\angle TPS = 90° - \gamma$

*Thus,* $\angle TPU = \angle QPR - \angle TPS$

$\Rightarrow \angle TPU = (90° - \alpha) - (90° - \gamma)$

$\Rightarrow \angle TPU = \gamma - \alpha$

*Now consider the $\Delta TPU$,*

*Thus,* $\sin(\gamma-\alpha) = \frac{a}{b}$

$\Rightarrow b = \dfrac{a}{\sin(\gamma - \alpha)}$

*Substituting the value of a from equation (1), we have,*

$b = \dfrac{c \times \sin(\alpha - \beta)}{\sin(\gamma - \alpha)}$ ...(2)

*We need to find the total height of the peak PR.*

*Here,* $PR = PS + SR$....(3)

*From the triangle PST,*

$\sin\gamma = \dfrac{PS}{PT} = \dfrac{PS}{b}$

$\Rightarrow PS = b\sin\gamma$....(4)

*From the triangle QTW,*

$\sin\beta = \dfrac{TW}{QT} = \dfrac{TW}{c}$

$\Rightarrow TW = SR = c\sin\beta$....(5)

*Substituting the values of PS and SR from equations (4) and (5) in equation (3), we have*

$PR = PS + SR$

$\Rightarrow PR = b\sin\gamma + c\sin\beta$

$\Rightarrow PR = \dfrac{c \times \sin(\alpha - \beta)}{\sin(\gamma - \alpha)}\sin\gamma + c\sin\beta$ [*from equation* (2)]

$\Rightarrow PR = \dfrac{c \times \sin(\alpha - \beta) \times \sin\gamma + c\sin\beta \times \sin(\gamma - \alpha)}{\sin(\gamma - \alpha)}$

$\Rightarrow PR = c\left[\dfrac{\sin\alpha \times \cos\beta \times \sin\gamma - \cos\alpha \times \sin\beta \times \sin\gamma + \sin\beta \times \sin\gamma \times \cos\alpha - \sin\beta \times \sin\alpha \times \cos\gamma}{\sin(\gamma - \alpha)}\right]$

$\Rightarrow PR = c\left[\dfrac{\sin\alpha \times \cos\beta \times \sin\gamma - \sin\beta \times \sin\alpha \times \cos\gamma}{\sin(\gamma - \alpha)}\right]$

$\Rightarrow PR = \dfrac{c\sin\alpha \times (\cos\beta \times \sin\gamma - \sin\beta \times \cos\gamma)}{\sin(\gamma - \alpha)}$

$\Rightarrow PR = \dfrac{c\sin\alpha \times \sin(\gamma - \beta)}{\sin(\gamma - \alpha)}$

**Sine and Cosine Formulae and their Applications Ex-10.1 Q31**

If the sides a, b, c of a $\Delta$ ABC are in H.P.

$\therefore \dfrac{1}{a}, \dfrac{1}{b}$ and $\dfrac{1}{c}$ are in AP

$\therefore \dfrac{1}{b} - \dfrac{1}{a} = \dfrac{1}{c} - \dfrac{1}{b}$

$\Rightarrow \dfrac{a - b}{ba} = \dfrac{b - c}{ca}$

$\Rightarrow \dfrac{\sin A - \sin B}{\sin B \sin A} = \dfrac{\sin B - \sin C}{\sin C \sin B}$........[Using sine rule]

$\Rightarrow \dfrac{2\sin\dfrac{A-B}{2}\cos\dfrac{A+B}{2}}{\sin A} = \dfrac{2\sin\dfrac{B-C}{2}\cos\dfrac{B+C}{2}}{\sin C}$

But A + B + C = $\pi$

A + B = $\pi - C$

$\cos\dfrac{A+B}{2} = \cos\left(\dfrac{\pi}{2} - \dfrac{C}{2}\right) = \sin\dfrac{C}{2}$

$\sin^2\dfrac{C}{2}\cos\dfrac{C}{2}\sin\dfrac{A-B}{2} = \sin\dfrac{B-C}{2}\cos\dfrac{A}{2}\sin^2\dfrac{A}{2}$

$\sin^2\dfrac{C}{2}\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2} = \sin\dfrac{B-C}{2}\cos\dfrac{B+C}{2}\sin^2\dfrac{A}{2}$

$\sin^2\dfrac{C}{2}\left[\sin^2\dfrac{A}{2} - \sin^2\dfrac{B}{2}\right] = \sin^2\dfrac{A}{2}\left[\sin^2\dfrac{B}{2} - \sin^2\dfrac{C}{2}\right]$

$\sin^2\dfrac{C}{2}\sin^2\dfrac{A}{2} - \sin^2\dfrac{C}{2}\sin^2\dfrac{B}{2} = \sin^2\dfrac{A}{2}\sin^2\dfrac{B}{2} - \sin^2\dfrac{A}{2}\sin^2\dfrac{C}{2}$

$\dfrac{1}{\sin^2\dfrac{B}{2}} - \dfrac{1}{\sin^2\dfrac{A}{2}} = \dfrac{1}{\sin^2\dfrac{C}{2}} - \dfrac{1}{\sin^2\dfrac{B}{2}}$

Hence 1 1 1 are in AP

Hence $\frac{}{\sin^2\frac{A}{2}}, \frac{}{\sin^2\frac{B}{2}}, \frac{}{\sin^2\frac{C}{2}}$ are in AP.

$\therefore \sin^2\frac{A}{2}, \sin^2\frac{B}{2}, \sin^2\frac{C}{2}$ are in HP.