VOLUME AND SURFACE AREA OF SOLIDS - CHAPTER15

EXERCISE – 15A

Answer1.

(i) Given, l = 12cm, b = 8cm, h = 4.5cm.

Volume of cuboid= (l × b × h) cubic units

$= 12cm \times 8cm \times 4.5cm$

$= 432cm^3$

Lateral surface area of cuboid

$= [2(l + b) \times h]$ surface unit

$= [2(12+8) \times 4.5]\ cm^2$

$= [2 \times 20 \times 4.5]\ cm^2$

$= 180\ cm^2$

Total surface area of cuboid

$= 2(lb + bh + hl)$ square units

$= 2(12cm \times 8cm + 8cm \times 4.5cm + 4.5cm \times 12cm)$

$= 2(96cm^2 + 36cm^2 + 54cm^2)$

$= 2 \times 186cm^2$

$= 372\ cm^2$

(ii) Given, l = 26m, b= 14m, h= 6.5m

Volume of cuboid = (l × b × h) cubic unit

$= 26m \times 14m \times 6.5m$

$= 2366m^3$

Lateral surface Area of cuboid

$= [2(l + b) \times h]$ surface unit

$= [2\ (26+14) \times 6.5]\ m^2$

$= [2 \times 40 \times 6.5]\ m^2$

$= 520\ m^2.$

Total surface Area of cuboid

$= 2(lb + bh + hl)$ square units

$= 2(26 \times 14 + 14 \times 6.5 + 6.5 \times 26)$

$= 2(364+91+169)$

$= 2 \times 624$

$= 1248 m^2.$

(iii)

Given, l= 15m, b= 6m,

h= 5dm

h= $5 \times 1/10$m [11Equation Section (Next) ∵ 1dm = 1/10m]

h= 0.5m

Volume of cuboid = (l×b×h)cubic unit

$= (15 \times 6 \times 0.5)\ m^3$

$= 45 m^3$

Lateral surface Area of cuboid

$= [2(l + b) \times h]$ square unit

$= [2(15+6) \times 0.5]\ m^2$

$= [2 \times 21 \times 0.5]\ m^2$

$= 21\ m^2.$

Total surface Area of cuboid

$=2(lb + bh + hl)$ square units

$=2(15\times6 + 6\times0.5 + 15\times0.5)\ m^2$

$=2(90+3+7.5)\ m^2$

$=2\times100.5\ m^2$

$=201m^2$.

(iv) Given,

$l=24m$

$b=25cm = 0.25m$ [$\because$ $1cm = 1/100$ m]

$h=6m$,

Volume of cuboid = ($l\times b\times h$)cubic unit

$= (24\times.25\times6)\ m^3$

$= 36m^3$

Lateral surface area of cuboid

$= [2(l + b) \times h]$ square unit

$= [2(24+0.25) \times 6]\ m^2$

$= [2\times24.25\times6]$

$= 291\ m^2$.

Total surface area of cuboid

$=2(lb + bh + hl)$ square units

$=2(24\times0.25 + 0.25\times6 + 24\times6)\ m^2$

$=2(6+1.5+144)\ m^2$

$= 2\times 151.2$

$= 303\ m^2$

Answer2. Given,

A match box measure = 4cm × 2.5cm × 1.5cm

Volume of 1 match box = 4cm × 2.5cm × 1.5cm

$= 15cm^3$

∵ volume of one matchbox = $15cm^3$

∴ volume of 12 matchbox = 15 × 12 cm^3

$= 180\ cm^3$.

Answer 3. Given,

Cuboid water tank

Length (l) =6m

Width(b)= 5m

Height(h)=4.5m

Volume of cuboid water tank = (l×b×h) $= (6\times5\times4.5)\ m^3 = 135\ m^3$

Given, $1m^3 = 1000$litres

So, $135m^3 = 135 \times 1000$litres

= 135000 litre

Litre of water hold by tank = 135000 litre.

Answer 4. Given,

Capacity of a cuboid tank = 50000 litre

Length(l) = 10m

Depth(h)=2.5m

Width(b)=?

Volume of tank = length × depth × width

Capacity = 50000 litre

∵ 1000 litre = $1m^3$ (given)

∴ 1 litre = $\frac{1}{1000}m^3$

$\therefore 50000 \text{ litre} = \frac{50000}{1000}\text{m}^3 = 50\text{m}^3$

$50 = 10 \times b \times 2.5$

$50 = 25 \times b$

$b = \frac{50}{25} = 2\text{m}$

width of tank $= 2\text{m}$

Answer5. Given,

Go down measures $= 40\text{m} \times 25\text{m} \times 15\text{m}$

Each wooden crates measures$= 1.5\text{m} \times 1.25\text{m} \times 0.5\text{m}$

$$\text{Maximum no. of wooden crates} = \frac{volume of godown}{volume of one wooden crates}$$

$$= \frac{40m \times 25m \times 15m}{1.5m \times 1.25m \times 0.5m}$$

$$= \frac{15000m^3}{0.9375m^3}$$

Maximum no. of wooden crates $= 16000$.

Answer 6. Given,

Dimensions of plank $= 5\text{m} \times 25\text{m} \times 10\text{cm}$ $(5\text{m} \times 0.25\text{m} \times 0.1\text{m})$

Length of pit(l) $= 20\text{m}$

Width of pit(b) $= 6\text{m}$

Deep of pit(h) $= 80\text{cm} = 0.8\text{m}$

$$\text{Total no. of planks stored in pit} = \frac{volume of pit}{volume of one plank}$$

Volume of pit = l×b×h

$= 20\times6\times0.8 = 96m^3$

Volume of plank = l×b×h

$= 5\times0.25\times0.1 = 0.125m^3$

Total no. of plank stored in pit = $96m^3/0.125m^3$

$= 768.$

Answer 7. Given,

Length of wall(l) = 8m = 800cm {∵ 1m = 100cm}

Height of wall(h) = 6m = 600cm

Thick of wall(b) = 22.5cm

Volume of wall = l×b×h

$= 800\times600\times22.5$

$= 10800000cm^3$

Dimension of each brick = 25cm×11.25cm×6cm

Volume of each brick = l×b×h

$= 1687.5cm^3$

Let total required bricks to construct wall = x

$$x = \frac{volume of wall}{volume of one brick}$$

$$= \frac{10800000}{1687.5}$$

$$x = 6400$$

Answer 8. Given,

Length of cistern = 8m(l)

Breadth of cistern = 6m(b)

Depth of cistern = 2.5m

Let , capacity of closed rectangular cistern = x

$x = l \times b \times h$

$= 8 \times 6 \times 2.5$

$= 120 m^3$

Area of the iron sheet require to make the cistern = surface area of cistern surface area of cistern = 2(lb+bh+hl)

$= 2(8 \times 6 + 6 \times 2.5 + 2.5 \times 8)$

$= 2(48 + 15.0 + 20)$

$= 2 \times 83$

$= 166\ m^2$

Answer9. Given,

Room dimensions = (9m×8m×6.5m)

Room has one door , two windows

dimension of door = 2m×1.5m

dimensions of windows = 1.5m×1m

cost of white wishing the walls = 25 per sq. meter

area of wall=lateral surface area of wall

lateral surface area of wall = [2(l+b)×h]

let l= 9, b=8,h=6.5(given)

$= [2(9+8) \times 6.5] m^3$

$= 2 \times 17 \times 6.5$

Area of wall = $221 m^3$

Let area of wall which will be white wishing = x

x = area of wall – [area of door + area of windows]

area of door = $2\times1.5 = 3m^2$

area of windows = $1.5\times1 = 1.5m^2$

but there are two windows then

$= 1.5 \times 2$

$= 3m^2$

x = $221 – (3+3) = 215m^2$

∵ per square meter cost = 25

∴ 216 square meter cost = 25×215

$= 5375$

Answer 10. Given,

Length of the wall = 15m

Width of wall = 30cm = 0.3m

Height of wall = 4m

Volume of wall = l×b×h

$= 15\times.3\times4 = 18.0m^3$

Brick dimension = 22cm×12.5cm×7.5cm

Volume of brick = $2062.5\ cm^3$

➔ 1/12 of the total volume of the wall consist mortar

So, volume of mortar = $1/12 \times 18 = 1.5m^3$

Volume of wall which is made of brick = $18.15 = 16.5m^3$

Let total brick require = x

$x = \frac{volume of wall}{volume of brick}$

volume of brick = $2062.5cm^3$

$$= \frac{2062.5}{100\times100\times100}\text{m}^3$$

$$x = 16.5/\frac{2062.5}{100\times100\times100} = 8000$$

Answer11. Given,

External dimension of box = 36cm×25×16.5cm

Total dimension of box = (36-3)×(25-3)×(16.5-1.5)

=33×22×15

Because box is 1.5cm throughout

Volume of external box = 36×25×16.5

$= 14850\text{cm}^3$

Volume of internal box = 33×22×15

$= 10890\text{cm}^3$

Volume of iron in box = 14850 – 10890

$= 3960\text{cm}^3$

∵ 1 cm^3 of iron weighs = 15gm

∴ 3960 cm^3 of iron weighs is = 15 × 3960

= 59400gm

∵ 1kg = 1000gm

∴ 59400gm = 59400/1000 = 59.4kg

Answer 12. Given

Sheet metal costs=6480

Per square meter cost = 120

Area of sheet metal = $\frac{totalcost}{costpersquaremeter}$

$$=\frac{6480}{120}\,sqmeter$$

Area of sheet metal = $54m^2$

Length(l) = 5m

Breadth(b)= 3m

Height(h)= ?

Area of sheet metal = 2(lb+bh+hl)

$54m^2$= 2(5×3+3×h+5×h)

54 = 2(15+3h+5h)

54=2(15+8h)

2×8h=24

$h = \frac{24}{16} = 1.5m$

Answer13. Given,

Volume of cuboid =$1596m^2$

Length = 16m

Ratio of breadth & height = 3:2

Let

Breadth = b

Height = h

$\Rightarrow \frac{b}{h} = \frac{3}{2}$

$b= \frac{3}{2}h$

volume = l×b×h

putting the values

$\Rightarrow 1536 = 16 \times \frac{3}{2} h \times h$

$\Rightarrow 1536 = 8 \times 3 \times h^2$

$\Rightarrow h^2 = \frac{1536}{8 \times 3} = 64$

$\Rightarrow h = \sqrt{64}$

$\Rightarrow h = 8m$

$\Rightarrow b = 3/2h$

$b = 1.5 \times 8$

$b = 12m$

breadth $= 12m$

height $= 8m$

Answer14. Given,

Dining hall of dimension $= 20m \times 16m \times 4.5m$

Volume of dining hall $= 1440m^2$

One person require $5m^3$ of air

Total no. of person accommodate in hall $= \frac{volume of hall}{volume of a person}$

$= \frac{1440}{5} = 288$ persons

Answer15. Given,

Length of classroom(l)=10m

Width of classroom (b)=6.4m

Height of classroom(h)=5m

Area of classroom floor = 10×6.4

$= 64m^2$

One student require area = $1.6m^2$

No. of students in classroom = $\frac{64}{1.6}$= 40

Volume of air = volume of classroom

= l×b×h

=10×6.4×5

$=320m^3$

Require cubic meters of air for each student

$$= \frac{volumeofair}{totalstudents}$$

$=\frac{320}{40}= 8m^3$

Answer16. Given,

Surface area of cuboid = $758cm^2$

Length of cuboid = 14cm

Breadth of cuboid = 11cm

Surface area of cuboid = 2(lb+bh+hl)

Let h be the height of cuboid

$\Rightarrow 758 = 2(14\times11+11\times h+14\times h)$

$\Rightarrow 758=2(154+25h)$

$\Rightarrow 154+25h=379$

$\Rightarrow 25h=379-154$

$\Rightarrow 25h=225$

$\Rightarrow h=\frac{225}{25} = 9cm$

Height of cuboid is 9cm

Answer17. Given,

Height of rain falls (h)= 5cm

Area of ground = 2 hectares

$\because$ 1 hectares = $10000m^2$

$\therefore$ 2 hectares = $20000m^2$

Volume of water falls on ground = $area \times depth$

$\Rightarrow 2 \times 10000 \times \frac{5}{100} = 2 \times 100 \times 5 = 1000$

Volume of water = $1000m^3$

Answer 18. Given,

Edge measure of cube (a) = 9m

Volume of cube = a^3

Volume of cube = $9\times9\times9 = 729m^3$

Lateral surface area of cube = $4a^2$

$\Rightarrow 4\times9\times9$

$\Rightarrow 4\times81$

$\Rightarrow 324m^2$

Total surface area of cube $= 6a^2$

$= 4\times9\times9$

$\Rightarrow 6\times81 \quad = 486m^2$

A diagonal of a cube $= \sqrt{3}\ a$

$= \sqrt{3}\times9$

$= 1.79\times9 = 15.57m.$

Answer 19. Given,

Total surface area of cube $= 1176cm^3$

Total surface area of cube $= 6a^2$

$6a^2=1176$

$a^2=\frac{1176}{6} = 196$

$\Rightarrow a = \sqrt{196} = 14$

$a = 14cm$

volume of cube $= a^3$

$= 14\times14\times14$

$= 2744cm^3$

Answer 20. Given,

Lateral surface area of cube $= 900cm^2$

Lateral surface area of cube $= 4a^2$

$\Rightarrow 4a^2=900$

$\Rightarrow a^2=\frac{900}{4}=225$

$\Rightarrow a=\sqrt{225}=15\text{cm}$

volume of cube $= a^3$

$\Rightarrow 15\times15\times15$

$\Rightarrow 3375\text{cm}^3$

Answer 21. Given,

Volume of cube $= 512\ \text{cm}^3$

$a^3 = 512$

$a = 8\ \text{cm}$

surface area of cube $= 6a^2$

$= 6\times8\times8$

$= 6\times64$

$= 384\text{cm}^2$

Answer 22. Given,

Size of cube $= 3\text{cm}\times4\text{cm}\times5\text{cm}$

Volume of cube which is form by these three $= (3^3\times4^3\times5^3)\text{cm}^3$

$= 27\times64\times125$

$= 216\text{cm}^3$

Let side of new cube $= a$

volume $= a^3$

$216 = a^3$

$a = \sqrt{216} = 6cm$

lateral surface area of new cube $= 4a^2$

$= 4 \times 6 \times 6$

$= 144cm^2$

Answer 23. Given,

Longest side in a cuboid = diagonal of cuboid

Diagonal of cuboid $= \sqrt{l^2+b^2+h^2}$

Given, $l=10$, $b= 10$, $h=5$

Length of longest pole in room $= \sqrt{100 + 100 + 25}$

$= \sqrt{225}$

$= 15$ m

Answer 24. Given,

$\Rightarrow l + b + h = 19cm$.....(1)

length of diagonal = 11cm

Diagonal of cuboid $= \sqrt{l^2+b^2+h^2}$

$\sqrt{l^2+b^2+h^2} = 11$

$l^2+b^2+h^2 = 121$ --------(2)

do square of equation of (1)

$\Rightarrow (l + b + h)2 = (19)2$

$l2 + b2 + h2 + 2(lb + bh + hl) = 361$ ---------- (3)

put the values in equation (3)

$\Rightarrow 121 + 2(lb + bh + hl) = 361$

$\Rightarrow 2(lb + bh + hl) = 361 - 121$

$\Rightarrow 2(lb + bh + hl) = 240$

Surface area of cuboid $= 240cm^2$

Answer 25. Given,

Let edge of cube = a

Surface area of cube (a) $= 6a^2$

edge is increased by 50% so,

new edge a' $= a + \frac{a \times 50}{100}$

$\Rightarrow a + \frac{a}{2} = \frac{3a}{2}$

Surface area of new cube $= 6 \times a^2$

$a' = 6 \times (\frac{3a}{2})^2$

$a' = \frac{27}{2} a^2$

percentage increase in surface are $= (\frac{a - a'}{a}) \times 100$

$= \frac{(\frac{27}{2})a \times a - 6 \times a \times a}{6 \times a \times a} \times 100$

$= \frac{27a \times a + 2a \times a}{6 \times 2 \times a \times a} \times 100$

$= 125\%$

Answer 26.

Volume of cuboid = V

Dimension of cuboid = a,b,c

Surface area = S

V=abc, S = 2(ab+bc+ca)

To be proven

$$\frac{1}{v} = \frac{2}{5}\left(\frac{1}{a} \times \frac{1}{b} \times \frac{1}{c}\right)$$

RHS $= \frac{2}{S}\left(\frac{1}{a} \times \frac{1}{b} \times \frac{1}{c}\right)$

$= \frac{2}{2(ab+bc+ca)} \times \left(\frac{1}{a} \times \frac{1}{b} \times \frac{1}{c}\right)$

$= \frac{bc+ab+ca}{(ab+bc+ca)abc}$

$= \frac{1}{abc}$

$= \frac{1}{V}$ LHS

Answer 27 Given, canal dimension 30 dm wide and 12 dm deep, velocity 20km/hr.

Distance covered by in 30 min = velocity of water × time

$$= \left(20000 \times \frac{30}{60}\right) m = 10000m$$

Volume of water flown in 30 min $= (l \times b \times h) = \left(10000 \times \frac{30}{10} \times \frac{12}{10}\right) m^3 = 36000m^3$

Let the area irrigated be $x\ m^2$

Hence, $x \times \frac{9}{100} = 36000$

$\Rightarrow x = \left(36000 \times \frac{100}{9}\right) = 400000\ m^2$

Answer 28. dimension of cuboid $= 9\text{m} \times 8\text{m} \times 2\text{m}$

$$\text{Volume} = 144\text{m}^3$$

Edge of cube $= a^3$

$$= (2)^3$$

$$= 8\text{m}^3$$

Total cube $= \frac{volume of cuboid}{volume of one cube}$

$$= \frac{144}{8} = 18 \text{ Cubes}$$