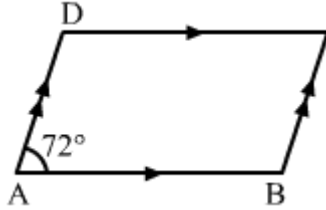


## EXERCISE 10B

Answer 1:



Given, ABCD is parallelogram and  $\angle A = 72^\circ$ .

Then, as we know that opposite angles are equals.

$$\therefore \angle A = \angle C \text{ and } \angle B = \angle D$$

$$\therefore \angle C = 72^\circ$$

$\angle A$  and  $\angle B$  are the adjacent angles.

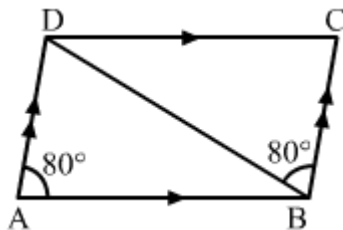
$$\text{as, } \angle A + \angle B = 180^\circ$$

$$\Rightarrow \angle B = 180^\circ - \angle A = 180^\circ - 72^\circ = 108^\circ$$

$$\text{As above, } \angle B = \angle D = 108^\circ$$

$$\text{Hence, } \angle B = \angle D = 108^\circ \text{ and } \angle C = 72^\circ$$

Answer 2:



Given: ABCD is parallelogram and  $\angle DAB = 80^\circ$  and  $\angle DBC = 60^\circ$

To find: Measure of  $\angle CDB$  and  $\angle ADB$

In parallelogram ABCD,  $AD \parallel BC$

$$\therefore \angle DBC = \angle ADB = 60^\circ \text{ (Alternate interior angles) } \dots(i)$$

As  $\angle DAB$  and  $\angle ADC$  are the adjacent angles,

$$\angle DAB + \angle ADC = 180^\circ$$

$$\therefore \angle ADC = 180^\circ - \angle DAB = 180^\circ - 80^\circ = 100^\circ$$

$$\text{Also, } \angle ADC = \angle ADB + \angle CDB$$

$$\therefore \angle ADC = 100^\circ$$

Then,

$$\Rightarrow \angle ADB + \angle CDB = 100 \quad \dots(ii)$$

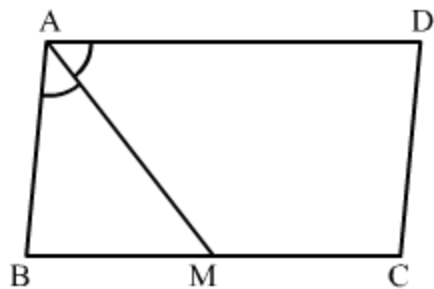
From (i) and (ii),

$$60^\circ + \angle CDB = 100^\circ$$

$$\Rightarrow \angle CDB = 100^\circ - 60^\circ = 40$$

Hence,  $\angle CDB = 40^\circ$  and  $\angle ADB = 60^\circ$

**Answer 3:**



Given: parallelogram ABCD, M is the midpoint of side BC and  $\angle BAM = \angle DAM$ .

To prove:  $AD = 2CD$

Proof:

Since,  $AD \parallel BC$  and AM is the transversal.

So,  $\angle DAM = \angle AMB$  (Alternate interior angles)

But,  $\angle DAM = \angle BAM$  (Given)

Thus,  $\angle AMB = \angle BAM$

$$\Rightarrow AB = BM$$

As we know angles opposite to equal sides are equal and opposite sides of parallelogram are equal

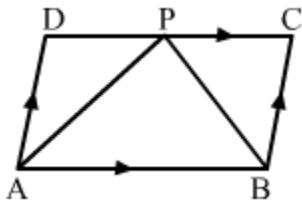
Now,  $AB = CD$

$$\Rightarrow 2AB = 2CD$$

$$\begin{aligned}\text{So, } &\Rightarrow (AB + AB) = 2CD \\ &\Rightarrow BM + MC = 2CD \quad (AB = BM \text{ and } MC = BM) \\ &\Rightarrow BC = 2CD\end{aligned}$$

$$\therefore AD = 2CD \quad (AD=BC)\text{hence proved}$$

**Answer 4:**



ABCD is a parallelogram.

$$\therefore \angle A = \angle C \text{ and } \angle B = \angle D \text{ (Opposite angles)}$$

$$\text{And } \angle A + \angle B = 180^\circ \quad (\text{Adjacent angles are supplementary})$$

$$\therefore \angle B = 180^\circ - \angle A$$

$$\Rightarrow 180^\circ - 60^\circ = 120^\circ \quad (\angle A = 60^\circ)$$

$$\therefore \angle A = \angle C = 60^\circ \text{ and } \angle B = \angle D = 120^\circ$$

$$(i) \text{ In } \triangle APB, \angle PAB = \frac{60}{2} = 30^\circ$$

$$\text{and } \angle PBA = \frac{120}{2} = 60^\circ$$

$$\therefore \angle APB = 180^\circ - (30^\circ + 60^\circ) = 90^\circ$$

$$(ii) \text{ In } \triangle ADP, \angle PAD = 30^\circ \text{ and } \angle ADP = 120^\circ$$

$$\therefore \angle APD = 180^\circ - (30^\circ + 120^\circ) = 30^\circ$$

$$\text{Thus, } \angle PAD = \angle APD = 30^\circ$$

Hence,  $\triangle ADP$  is an isosceles triangle and  $AD = DP$ .

$$\text{In } \triangle PBC, \angle PBC = 60^\circ, \angle BPC = 180^\circ - (90^\circ + 30^\circ) = 60^\circ \text{ and } \angle BCP = 60^\circ$$

(Opposite angle of  $\angle A$ )

$$\therefore \angle PBC = \angle BPC = \angle BCP$$

Hence,  $\triangle PBC$  is an equilateral triangle and, therefore,  $PB = PC = BC$ .

$$(iii) DC = DP + PC$$

From (ii), as ,

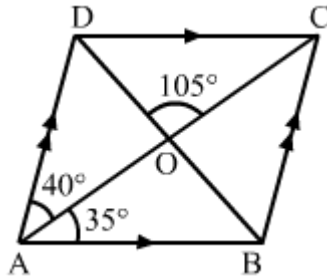
$$DC = AD + BC$$

$$\Rightarrow DC = AD + AD$$

$$\Rightarrow DC = 2 AD$$

[AD = BC, opposite sides of a parallelogram]

**Answer 5:**



ABCD is a parallelogram.

$\therefore AB \parallel DC$  and  $BC \parallel AD$

(i) In  $\triangle AOB$ ,  $\angle BAO = 35^\circ$ ,

As we know that, vertically opposite angles are equals

$$\angle AOB = \angle COD = 105^\circ$$

$$\therefore \angle ABO = 180^\circ - (35^\circ + 105^\circ) = 40^\circ$$

(ii) As we know that these angles are  $\angle ODC$  and  $\angle ABO$  are alternate interior angles.

$$\therefore \angle ODC = \angle ABO = 40^\circ$$

(iii) These are Alternate interior angles

$$\angle ACB = \angle CAD = 40^\circ$$

(iv) In  $\triangle ABC$ , we get

$$\angle CBD = \angle ABC - \angle ABD \quad \dots(i)$$

$$\angle ABC = 180^\circ - \angle BAD \quad (\text{Adjacent angles are supplementary})$$

$$\Rightarrow \angle ABC = 180^\circ - 75^\circ = 105^\circ$$

In  $\triangle CBD$ , we have

Then,  $\angle CBD = \angle ABC - \angle ABD$

$$\Rightarrow \angle CBD = 105^\circ - \angle ABD \quad (\angle ABD = \angle ABO)$$

$$\Rightarrow \angle CBD = 105^\circ - 40^\circ = 65^\circ$$

**Answer 6:**

ABCD is a parallelogram.

i.e.,  $\angle A = \angle C$  and  $\angle B = \angle D$  (Opposite angles)

Also,  $\angle A + \angle B = 180^\circ$  (Adjacent angles are supplementary)

$$\therefore (2x + 25)^\circ + (3x - 5)^\circ = 180^\circ$$

$$\Rightarrow 5x + 20 = 180^\circ$$

$$\Rightarrow 5x = 180 - 20$$

$$\Rightarrow 5x = 160^\circ$$

$$\Rightarrow x = \frac{160}{5} = 32^\circ$$

$$\therefore \angle A = 2 \times 32 + 25 = 89^\circ \text{ and } \angle B = 3 \times 32 - 5 = 91^\circ$$

Hence,  $x = 32^\circ$ ,  $\angle A = \angle C = 89^\circ$  and  $\angle B = \angle D = 91^\circ$

**Answer 7:**

Let PQRS be a parallelogram.

$\therefore \angle P = \angle R$  and  $\angle Q = \angle S$

Let  $\angle P = y^\circ$  and  $\angle B = \left(\frac{4y}{5}\right)^\circ$

Now,  $\angle P + \angle Q = 180^\circ$

$$\Rightarrow y + \left(\frac{4y}{5}\right)^\circ = 180^\circ \Rightarrow \left(\frac{9y}{5}\right)^\circ = 180^\circ \Rightarrow y = 100^\circ$$

Now,  $\angle P = 100^\circ$  and  $\angle B = \left(\frac{4}{5}\right) \times 100^\circ = 80^\circ$

Hence,  $\angle P = \angle R = 100^\circ$ ;  $\angle B = \angle S = 80^\circ$

**Answer 8:**

Let PQRS be a parallelogram.

$$\therefore \angle P = \angle R \text{ and } \angle Q = \angle S \quad (\text{Opposite angles})$$

Let  $\angle P$  be the smallest angle whose measure is  $y^\circ$ .

$$\therefore \angle Q = (2y - 30)^\circ$$

$$\text{Now, } \angle P + \angle Q = 180^\circ \quad (\text{Adjacent angles are supplementary})$$

$$\Rightarrow y + 2y - 30^\circ = 180^\circ$$

$$\Rightarrow 3y = 210^\circ$$

$$\Rightarrow y = \frac{210}{3} = 70$$

$$\Rightarrow y = 70^\circ$$

$$\therefore \angle Q = 2 \times 70^\circ - 30^\circ = 110^\circ$$

$$\text{Hence, } \angle P = \angle R = 70^\circ; \angle Q = \angle S = 110^\circ$$

**Answer 9:**

ABCD is a parallelogram.

The opposite sides of a parallelogram are parallel and equal.

$$\therefore AB = DC = 9.5 \text{ cm}$$

$$\text{Let } BC = AD = y$$

$$\therefore \text{Perimeter of ABCD} = AB + BC + CD + DA = 30 \text{ cm}$$

$$\Rightarrow 9.5 + y + 9.5 + y = 30$$

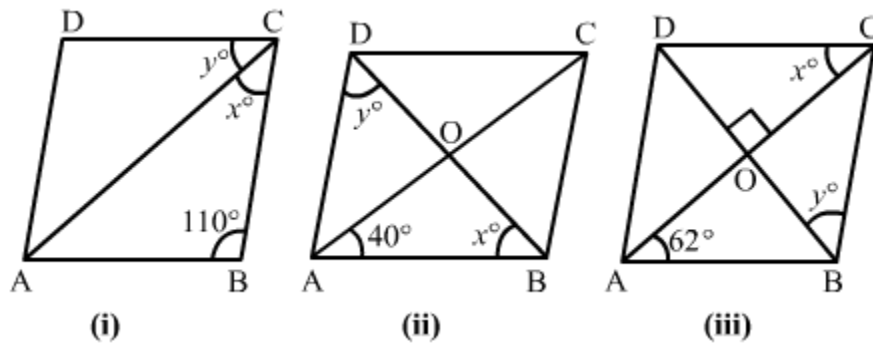
$$\Rightarrow 19 + 2y = 30$$

$$\Rightarrow 2y = 11$$

$$\Rightarrow y = \frac{11}{2} = 5.5 \text{ cm}$$

$$\text{Hence, } AB = DC = 9.5 \text{ cm and } BC = DA = 5.5 \text{ cm}$$

**Answer 10:**



ABCD is a rhombus and a rhombus is also a parallelogram. A rhombus has four equal sides.

(i) In  $\triangle ABC$ ,

$$\angle BAC = \angle BCA = \frac{1}{2}(180 - 110)^\circ = 35^\circ$$

i.e.,  $x = 35^\circ$

Now by Adjacent angles are supplementary we get,

$$\angle B + \angle C = 180^\circ$$

$$\text{As, } \angle C = x + y = 70^\circ$$

$$\Rightarrow y = 70^\circ - x$$

$$\Rightarrow y = 70^\circ - 35^\circ = 35^\circ$$

Hence,  $x = 35^\circ$ ;  $y = 35^\circ$

(ii) The diagonals of a rhombus are perpendicular bisectors of each other.

So, in  $\triangle AOB$ ,  $\angle OAB = 40^\circ$ ,  $\angle AOB = 90^\circ$  and

$$\angle ABO + \angle BOA + \angle OAB = 180$$

$$\angle ABO = 180^\circ - (40^\circ + 90^\circ) = 50^\circ$$

$$\therefore x = 50^\circ$$

In  $\triangle ABD$ ,  $AB = AD$

So,  $\angle ABD = \angle ADB = 50^\circ$

Hence,  $x = 50^\circ$ ;  $y = 50^\circ$

(iii)  $\angle BAC = \angle DCA$  (Alternate interior angles)

i.e.,  $x = 62^\circ$

In  $\triangle BOC$ ,  $\angle BCO = 62^\circ$

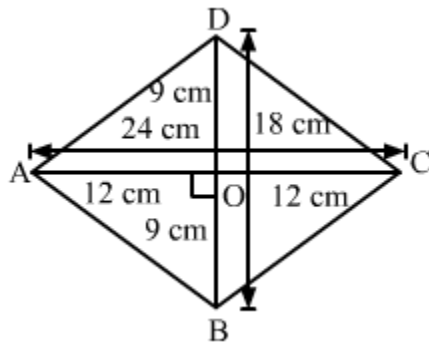
Also,  $\angle BOC = 90^\circ$

$$\angle BCO + \angle BOC + \angle OBC = 180$$

$$\therefore \angle OBC = 180^\circ - (90^\circ + 62^\circ) = 28^\circ$$

Hence,  $x = 62^\circ$ ;  $y = 28^\circ$

**Answer 11:**



Let PQRS be a rhombus.

$$\therefore PQ = QR = RS = SP$$

Here, PR and QS are the diagonals of PQRS, where PR = 24 cm and QS = 18 cm.

Let the diagonals intersect each other at M.

$$\therefore \triangle PMQ \text{ is a right angle triangle in which } MP = \frac{AC}{2} = \frac{24}{2} = 12 \text{ cm and } MQ =$$

$$\frac{QS}{2} = \frac{18}{2} = 9 \text{ cm.}$$

$$\text{Now, } PQ^2 = MP^2 + MQ^2 \quad [\text{Pythagoras theorem}]$$

$$\Rightarrow PQ^2 = (12)^2 + (9)^2$$

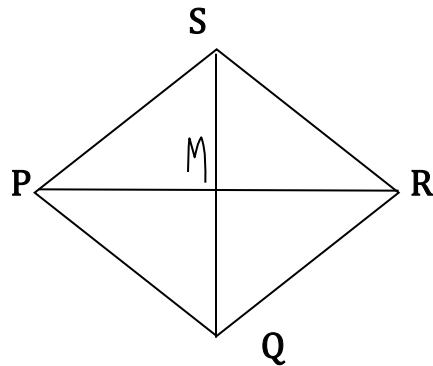
$$\Rightarrow PQ^2 = 144 + 81 = 225$$

$$\Rightarrow PQ = 15 \text{ cm}$$

Hence, the side of the rhombus is 15 cm.



**Answer 12:**



Let PQRS be a rhombus.

$\therefore PQ = QR = RS = SP = 10 \text{ cm}$

Let PR and QS are the diagonals of PQRS. Let  $PR = y$  and  $QS = 16 \text{ cm}$  and M be the intersection point of the diagonals.

$\therefore \triangle PMQ$  is a right angle triangle, in which

$$MP = \frac{PR}{2} = \frac{y}{2} \text{ and } MQ = \frac{QS}{2} = \frac{16}{2} = 8 \text{ cm.}$$

Now,  $PQ^2 = MP^2 + MQ^2$  [Pythagoras theorem]

$$\Rightarrow 10^2 = \left(\frac{y}{2}\right)^2 + 8^2 \Rightarrow 100 - 64 = \frac{y^2}{4} \Rightarrow 36 \times 4 = y^2$$

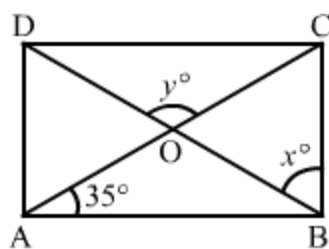
$$\Rightarrow y^2 = 144$$

$$\therefore y = 12 \text{ cm}$$

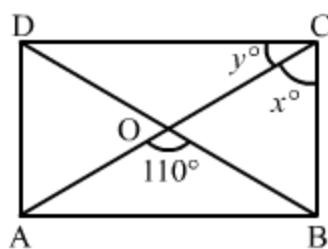
Hence, the other diagonal of the rhombus is 12 cm.

$$\therefore \text{Area of the rhombus} = 12 \times (12 \times 16) = 96 \text{ cm}^2$$

**Answer 13:**



(i)



(ii)

(i) ABCD is a rectangle.

The diagonals of a rectangle are congruent and bisect each other. Therefore, in  $\Delta AOB$ , as ,

$$OA = OB$$

$$\therefore \angle OAB = \angle OBA = 35^\circ$$

$$\therefore x = 90^\circ - 35^\circ = 55^\circ$$

In  $\Delta AOB$

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$\text{And } \angle AOB = 180^\circ - (35^\circ + 35^\circ) = 110^\circ$$

$$\therefore y = \angle AOB = 110^\circ \quad [\text{Vertically opposite angles}]$$

Hence,  $x = 55^\circ$  and  $y = 110^\circ$

(ii) In  $\Delta AOB$ , as ,

$$\text{Given, } \angle AOB = 100^\circ$$

$$OA = OB$$

$$\text{As, } \angle OAB = \angle OBA$$

$$\text{Then, } \angle AOB + \angle OBA + \angle OAB = 180$$

$$\Rightarrow 2\angle OAB = 180 - \angle AOB \dots\dots\dots(\angle OAB = \angle OBA)$$

$$\Rightarrow 2\angle OAB = 180 - 110 = 70^\circ$$

$$\Rightarrow \angle OAB = \frac{1}{2} \times 70 = 35^\circ$$

$$\text{so, } \therefore y = \angle BAC = 35^\circ \quad [\text{Interior alternate angles}]$$

Here at  $\angle C$  is at right angle  $\Delta$  by fig,

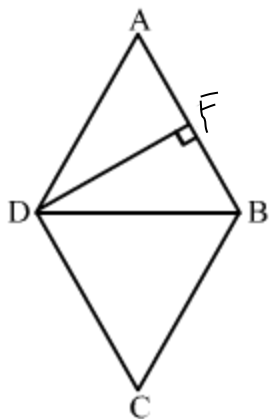
$$\Rightarrow 90^\circ = x + y$$

$$\Rightarrow x = 90^\circ - y$$

$$\Rightarrow x = 90^\circ - 35^\circ = 55^\circ$$

Thus,  $x = 55^\circ$  and  $y = 35^\circ$

Answer 14:



Given: ABCD is a rhombus, DF is altitude which bisects AB i.e.  $AF = FB$

In  $\triangle AFD$  and  $\triangle BFD$ ,

$DF = DF$  (Common side)

$\angle DFA = \angle DFB = 90^\circ$  (Given)

$AF = FB$  (Given)

$\therefore \triangle AFD \cong \triangle BFD$  (By SAS congruence Criteria)

$\Rightarrow AD = BD$  (CPCT)

Also,  $AD = AB$  (Sides of rhombus are equal)

$\Rightarrow AD = AB = BD$

Thus,  $\triangle ABD$  is an equilateral triangle.

Therefore,  $\angle A = 60^\circ$

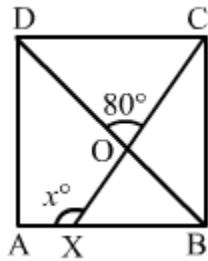
$\Rightarrow \angle C = \angle A = 60^\circ$  (Opposite angles of rhombus are equal)

And,  $\angle ABC + \angle BCD = 180^\circ$  (Adjacent angles of rhombus are supplementary.)

$\Rightarrow \angle ABC + 60^\circ = 180^\circ \Rightarrow \angle ABC = 180^\circ - 60^\circ \Rightarrow \angle ABC = 120^\circ \Rightarrow \angle ADC = \angle ABC = 120^\circ$

Hence, the angles of rhombus are  $60^\circ$ ,  $120^\circ$ ,  $60^\circ$  and  $120^\circ$

**Answer 15:**



The angles of a square are bisected by the diagonals.

$$\angle OBX = \frac{1}{2} \times \angle CBA = \frac{1}{2} \times 90 = 45^\circ$$

$$\therefore \angle OBX = 45^\circ$$

$$\text{Given, } \angle COD = 80^\circ$$

$$\text{And } \angle BOX = \angle COD = 80^\circ \quad [\text{Vertically opposite angles}]$$

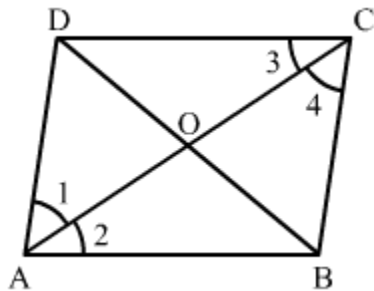
$\therefore$  In  $\triangle BOX$ , as we know that exterior angle is sum of both interior angles.

$$\angle AXO = \angle OBX + \angle BOX$$

$$\Rightarrow \angle AXO = 45^\circ + 80^\circ = 125^\circ$$

$$\therefore x = 125^\circ$$

**Answer 16:**



Given: A rhombus ABCD.

To prove: Diagonal AC bisects  $\angle A$  as well as  $\angle C$  and diagonal BD bisects  $\angle B$  as well as  $\angle D$ .

Proof:

In  $\triangle ABC$ ,

$AB = BC$  (Sides of rhombus are equal.)

$\angle ACB = \angle CAB$  (Angles opposite to equal sides are equal.) ... (1)

$AD \parallel BC$  (Opposite sides of rhombus are parallel.)

AC is transversal.

$\angle DAC = \angle ACB$  (Alternate interior angles) ... (2)

From (1) and (2),

$\angle DAC = \angle CAB$

Thus, AC bisects  $\angle A$ .

As,  $AB \parallel DC$  and AC is transversal.

$\angle CAB = \angle DCA$  (Alternate interior angles) ... (3)

From (1) and (3),

$\angle ACB = \angle DCA$

Thus, AC bisects  $\angle C$ .

Thus, AC bisects  $\angle C$  and  $\angle A$

In  $\triangle DAB$ ,

$AD = AB$  (Sides of rhombus are equal.)

$\angle ADB = \angle ABD$  (Angles opposite to equal sides are equal.) ... (4)

Also,

$DC \parallel AB$  (Opposite sides of rhombus are parallel.)

BD is transversal.

$\angle CDB = \angle DBA$  (Alternate interior angles) ... (5)

From (4) and (5),

$\angle ADB = \angle CDB$

Therefore, DB bisects  $\angle D$ .

As,  $AD \parallel BC$  and BD is transversal.

$\angle CBD = \angle ADB$  (Alternate interior angles) ... (6)

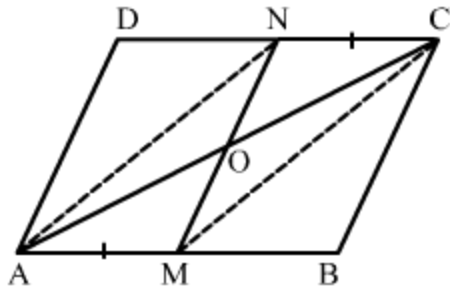
From (4) and (6)

$\angle CBD = \angle ABD$

Therefore, BD bisects  $\angle B$ .

Thus, BD bisects  $\angle D$  and  $\angle B$ .

**Answer 17:**



Given: In a parallelogram ABCD,  $AM = CN$ .

To prove: AC and MN bisect each other.

Construction: Join AN and MC.

Proof:

As, ABCD is a parallelogram.

$\Rightarrow AB \parallel DC \Rightarrow AM \parallel NC$

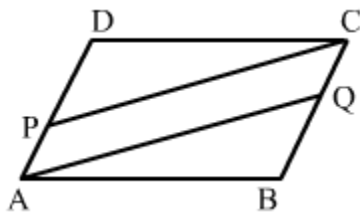
And,  $AM = CN$  (Given)

Therefore, AMCN is a parallelogram.

As, the diagonals of a parallelogram bisect each other.

Thus, AC and MN also bisect each other.

**Answer 18:**



As, per by given fig,

$\angle B = \angle D$  [Opposite angles of parallelogram ABCD]

$AD = BC$  and  $AB = DC$  [Opposite sides of parallelogram ABCD]

Also,  $AD \parallel BC$  and  $AB \parallel DC$

Given,  $AP = \frac{1}{3}AD$  and  $CQ = \frac{1}{3}BC$

So, we get

$$\therefore AP = CQ \quad [AD = BC]$$

In  $\triangle DPC$  and  $\triangle BQA$ ,

$$AB = CD, \angle B = \angle D \text{ and } DP = QB \quad [DP = \frac{2}{3}AD \text{ and } QB = \frac{2}{3}BC]$$

i.e.,  $\triangle DPC \cong \triangle BQA$

$$\therefore PC = QA$$

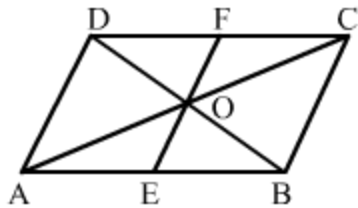
Thus, in quadrilateral AQCP,

$$AP = CQ \quad \dots(i)$$

$$PC = QA \quad \dots(ii)$$

$\therefore$  AQCP is a parallelogram.

**Answer 19:**



**Given,** ABCD is a parallelogram whose diagonals intersect each other at O. A line segment EOF is drawn to meet AB at E and DC at F.

So in  $\triangle ODF$  and  $\triangle OBE$ ,

$$OD = OB \quad (\text{Diagonals bisect each other})$$

$$\angle DOF = \angle BOE \quad (\text{Vertically opposite angles})$$

$$\angle FDO = \angle OBE \quad (\text{Alternate interior angles})$$

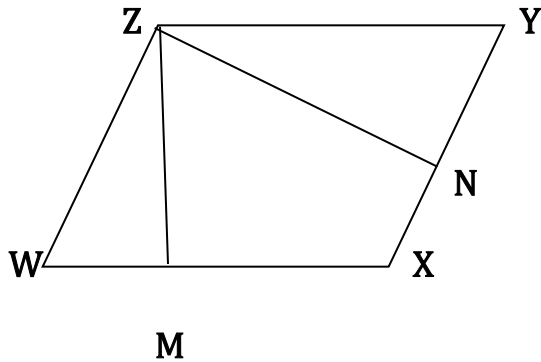
By parallelogram theorem

$$\triangle ODF \cong \triangle OBE$$

$$\therefore OF = OE$$

Hence, proved.

**Answer 20:**



Given: ▭ parallelogram WXYZ,  $ZM \perp WX$ ,  $WN \perp XY$  and  $\angle MZN = 60^\circ$

In quadrilateral ZMXN, by angle sum property,

$$\angle MZN + \angle ZMX + \angle X + \angle XNZ = 360^\circ$$

$$\Rightarrow 60^\circ + 90^\circ + \angle X + 90^\circ = 360^\circ$$

$$\Rightarrow \angle X = 360^\circ - 240^\circ \Rightarrow \angle X = 120^\circ \Rightarrow \angle X = 120^\circ$$

Also,

$$\angle X = \angle Z = 120^\circ \quad (\text{Opposite angles of a parallelogram are equal.})$$

$$\angle W + \angle X = 180^\circ \quad (\text{Adjacent angles of a parallelogram are supplementary.})$$

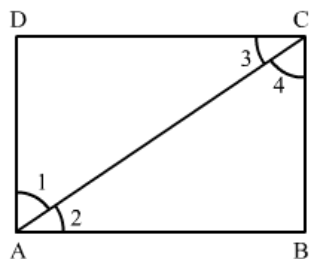
$$\Rightarrow \angle W + 120^\circ = 180^\circ \Rightarrow \angle W = 180^\circ - 120^\circ \Rightarrow \angle W = 60^\circ$$

Also,

$$\angle W = \angle Y = 60^\circ \quad (\text{Opposite angles of a parallelogram are equal.})$$

Thus, the angles of a parallelogram are  $60^\circ$ ,  $120^\circ$ ,  $60^\circ$  and  $120^\circ$ .

**Answer 21:**





Given: In

rectangle ABCD, AC bisects  $\angle A$ , i.e.  $\angle DAC = \angle CAB$  and AC bisects  $\angle C$ , i.e.  $\angle DCA = \angle ACB$ .

To prove:

- (i) ABCD is a square,
- (ii) diagonal BD bisects  $\angle B$  as well as  $\angle D$ .

Proof:

(i) Since,  $AD \parallel BC$  (Opposite sides of a rectangle are parallel.)

So,  $\angle DAC = \angle ACB$  (Alternate interior angles)

But,  $\angle DAC = \angle CAB$  (Given)

So,  $\angle CAB = \angle ACB$

In  $\triangle ABC$ ,

Since,  $\angle CAB = \angle ACB$

So,  $BC = AB$  (Sides opposite to equal angles are equal.)

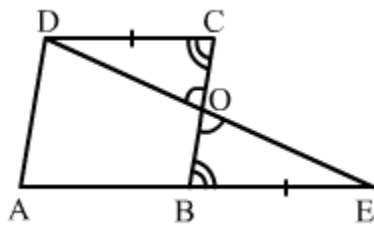
But these are adjacent sides of the rectangle ABCD.

Hence, ABCD is a square.

(ii) Since, the diagonals of a square bisect its angles.

So, diagonals BD bisect  $\angle B$  as well as  $\angle D$ .

**Answer 22:**



**Given,** ABCD is parallelogram in which AB is produced to E.

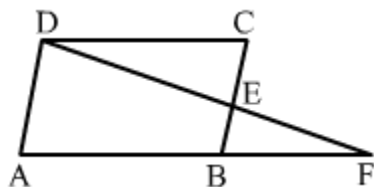
$BE = AB$  (given)

So in  $\triangle ODC$  and  $\triangle OEB$ , as ,

$DC = BE$  (DC = AB)

$\angle OCD = \angle OBE$  ( Alternate interior angles)  
 $\angle COD = \angle BOE$  (Vertically opposite angles)  
 by parallelogram theorem we get,  
 $\therefore \triangle ODC \cong \triangle OEB$   
 $\Rightarrow OC = OB$   
 Hence , ED bisects BC.

**Answer 23:**



**Given:** ABCD is a parallelogram.

$BE = CE$

DE and AB when produced meet at F.

**To prove:**  $AF = 2AB$

**Proof:** In parallelogram ABCD, as ,

$AB \parallel DC$

$\angle DCE = \angle EBF$  (Alternate interior angles)

In  $\triangle DCE$  and  $\triangle BFE$ ,

$\angle DCE = \angle EBF$  (Proved above)

$\angle DEC = \angle BEF$  (Vertically opposite angles)

And,  $BE = CE$  (Given)

By parallelogram theorem

$\therefore \triangle DCE \cong \triangle BFE$

hence  $\therefore DC = BF$

But  $DC = AB$ , as ABCD is a parallelogram.

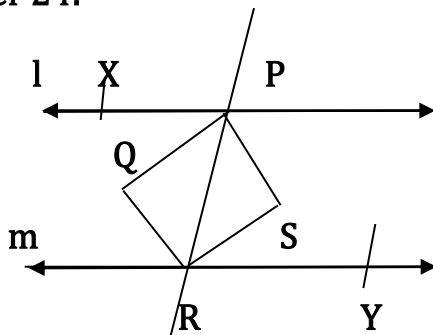
$\therefore DC = AB = BF$  ... (i)

can also be written as ,  $AF = AB + BF$  ... (ii)

$AF = AB + AB = 2AB$  .....from (i)

Hence, proved.  $AF = 2AB$ .

**Answer 24:**



Given:  $l \parallel m$  and the bisectors of interior angles intersect at X and Y.

To prove: PQRS is a rectangle.

Proof:

Since,  $l \parallel m$  (Given)

So,  $\angle XPR = \angle PRY$  (Alternate interior angles)

$$\Rightarrow \frac{1}{2} \angle XPR = \frac{1}{2} \angle PRY$$

$\Rightarrow \angle QPR = \angle PRS$  but, these are a pair of alternate interior angles for PQ and RS.

$\Rightarrow PQ \parallel SR$

Similarly,  $PR \parallel QS$

So, PQRS is a parallelogram.

Also,

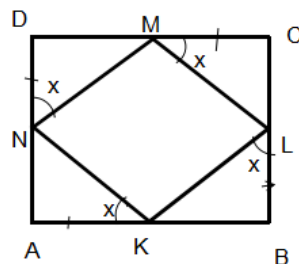
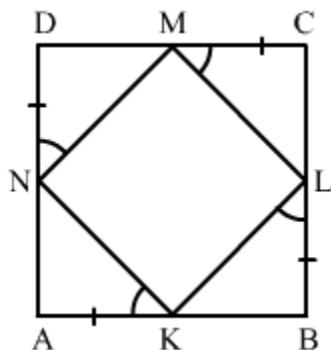
$$\angle XPR + \angle RPZ = 180^\circ \quad (\text{Linear pair})$$

$$\Rightarrow \frac{1}{2} \angle XPR + \frac{1}{2} \angle PRY = 90^\circ \Rightarrow \angle QPR + \angle RPS = 90^\circ \Rightarrow \angle QPS = 90^\circ$$

But, this is an angle of the parallelogram PQRS

Hence, PQRS is a rectangle.

**Answer 25:**



Given: In square ABCD,  $AK = BL = CM = DN$ .

To prove: KLMN is a square.

Proof:

In square ABCD,

$AB = BC = CD = DA$  (All sides of a square are equal.)

And,  $AK = BL = CM = DN$  (Given)

So,  $AB - AK = BC - BL = CD - CM = DA - DN$

$\Rightarrow KB = CL = DM = AN$  ...(1)

In  $\triangle NAK$  and  $\triangle KBL$ ,

$\angle NAK = \angle KBL = 90^\circ$  (Each angle of a square is a right angle.)

$AK = BL$  (Given)

$AN = KB$  [From (1)]

So, by parallelogram theorem,

$\triangle NAK \cong \triangle KBL$

$\Rightarrow NK = KL$  (CPCT) ...(2)

Similarly,

$\triangle MDN \cong \triangle NAK$   $\triangle DNM \cong \triangle CML$   $\triangle MCL \cong \triangle LBN$

$\Rightarrow MN = NK$  and  $\angle DNM = \angle KNA$  (CPCT) ...(3)

$MN = JM$  and  $\angle DNM = \angle CML$  (CPCT) ...(4)

$ML = LK$  and  $\angle CML = \angle BLK$  (CPCT) ...(5)

From (2), (3), (4) and (5),

$NK = KL = MN = ML$  ...(6)

And,  $\angle DNM = \angle AKN = \angle KLB = \angle LMC$

Now,

In  $\triangle NAK$ ,

$\angle NAK = 90^\circ$

Let  $\angle AKN = y^\circ$

So,  $\angle DNK = 90^\circ + y^\circ$

$\Rightarrow \angle DNM + \angle MNK = 90^\circ + y^\circ \Rightarrow y^\circ + \angle MNK = 90^\circ + y^\circ \Rightarrow \angle MNK = 90^\circ$

Similarly,

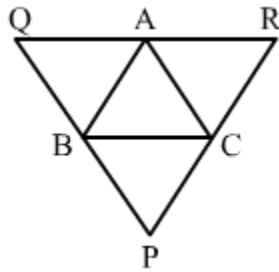
$\angle NKL = \angle KLM = \angle LMN = 90^\circ$  ...(7)

Using (6) and (7),

All sides of quadrilateral KLMN are equal and all angles are  $90^\circ$

So, KLMN is a square.

**Answer 26:**



$\Delta ABC$ , if lines are drawn through A, B, C parallel respectively to the sides BC, CA and AB. So, we get,  $BC \parallel QA$  and  $CA \parallel QB$

i.e., BCQA is a parallelogram.

$$\therefore BC = QA \quad \dots(i)$$

Similarly,  $BC \parallel AR$  and  $AB \parallel CR$ .

i.e., BCRA is a parallelogram.

$$\therefore BC = AR \quad \dots(ii)$$

$$\text{As } QR = QA + AR$$

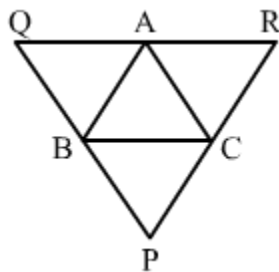
From (i) and (ii),

$$QR = BC + BC$$

$$\Rightarrow QR = 2BC$$

$$\therefore BC = \frac{1}{2}QR$$

**Answer 27:**



In  $\Delta ABC$  A, B, C lines drawn, parallel respectively to BC, CA and AB intersecting at P, Q and R. Acc to question,

$$\text{Perimeter of } \Delta ABC = AB + BC + CA \quad \dots(i)$$

$$\text{Perimeter of } \Delta PQR = PQ + QR + PR \quad \dots(ii)$$

By given figure,

BC || QA and CA || QB

i.e., BCQA is a parallelogram.

$$\therefore BC = QA \quad \dots(\text{iii})$$

Similarly, BC || AR and AB || CR

i.e., BCRA is a parallelogram.

$$\therefore BC = AR \quad \dots(\text{iv})$$

$$\text{But, } QR = QA + AR$$

From (iii) and (iv),

$$\Rightarrow QR = BC + BC$$

$$\Rightarrow QR = 2BC$$

$$\therefore BC = \frac{1}{2}QR$$

$$\text{Similarly, } CA = \frac{1}{2}PQ \text{ and } AB = \frac{1}{2}PR$$

From (i) and (ii),

$$\begin{aligned} \text{Perimeter of } \triangle ABC &= \frac{1}{2}QR + \frac{1}{2}PQ + \frac{1}{2}PR \\ &= \frac{1}{2}(PR + QR + PQ) \end{aligned}$$

$$\text{i.e., Perimeter of } \triangle ABC = \frac{1}{2}(\text{Perimeter of } \triangle PQR)$$

$$\therefore \text{Perimeter of } \triangle PQR = 2 \times \text{Perimeter of } \triangle ABC$$