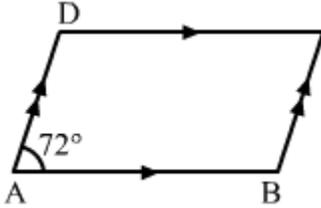


EXERCISE 10B

Answer 1:



Given, ABCD is parallelogram and $\angle A = 72^\circ$.

Then, as we know that opposite angles are equals.

$$\therefore \angle A = \angle C \text{ and } \angle B = \angle D$$

$$\therefore \angle C = 72^\circ$$

$\angle A$ and $\angle B$ are the adjacent angles.

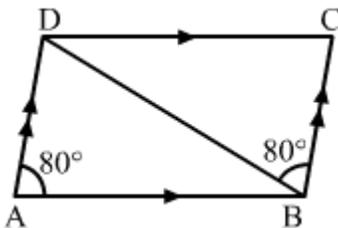
$$\text{as, } \angle A + \angle B = 180^\circ$$

$$\Rightarrow \angle B = 180^\circ - \angle A = 180^\circ - 72^\circ = 108^\circ$$

As above, $\angle B = \angle D = 108^\circ$

Hence, $\angle B = \angle D = 108^\circ$ and $\angle C = 72^\circ$

Answer 2:



Given: ABCD is parallelogram and $\angle DAB = 80^\circ$ and $\angle DBC = 60^\circ$

To find: Measure of $\angle CDB$ and $\angle ADB$

In parallelogram ABCD, $AD \parallel BC$

$$\therefore \angle DBC = \angle ADB = 60^\circ \text{ (Alternate interior angles) } \dots(i)$$

As $\angle DAB$ and $\angle ADC$ are the adjacent angles,

$$\angle DAB + \angle ADC = 180^\circ$$

$$\therefore \angle ADC = 180^\circ - \angle DAB = 180^\circ - 80^\circ = 100^\circ$$

$$\text{Also, } \angle ADC = \angle ADB + \angle CDB$$

$$\therefore \angle ADC = 100^\circ$$

Then,

$$\Rightarrow \angle ADB + \angle CDB = 100 \quad \dots(\text{ii})$$

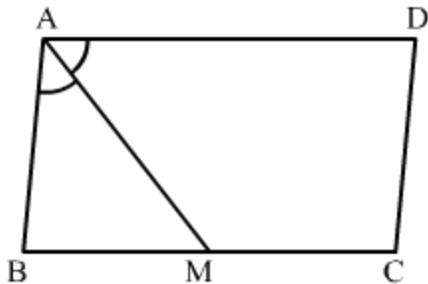
From (i) and (ii),

$$60^\circ + \angle CDB = 100^\circ$$

$$\Rightarrow \angle CDB = 100^\circ - 60^\circ = 40$$

Hence, $\angle CDB = 40^\circ$ and $\angle ADB = 60^\circ$

Answer 3:



Given: parallelogram ABCD, M is the midpoint of side BC and $\angle BAM = \angle DAM$.

To prove: $AD = 2CD$

Proof:

Since, $AD \parallel BC$ and AM is the transversal.

So, $\angle DAM = \angle AMB$ (Alternate interior angles)

But, $\angle DAM = \angle BAM$ (Given)

Thus, $\angle AMB = \angle BAM$

$$\Rightarrow AB = BM$$

As we know angles opposite to equal sides are equal and opposite sides of parallelogram are equal

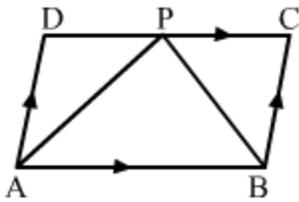
Now, $AB = CD$

$$\Rightarrow 2AB = 2CD$$

$$\begin{aligned} \text{So, } &\Rightarrow (AB + AB) = 2CD \\ &\Rightarrow BM + MC = 2CD \quad (AB = BM \text{ and } MC = BM) \\ &\Rightarrow BC = 2CD \end{aligned}$$

$\therefore AD = 2CD$ (AD=BC) hence proved

Answer 4:



ABCD is a parallelogram.

$$\begin{aligned} \therefore \angle A &= \angle C \text{ and } \angle B = \angle D \text{ (Opposite angles)} \\ \text{And } \angle A + \angle B &= 180^\circ \quad (\text{Adjacent angles are supplementary}) \\ \therefore \angle B &= 180^\circ - \angle A \\ &\Rightarrow 180^\circ - 60^\circ = 120^\circ \quad (\angle A = 60^\circ) \\ \therefore \angle A = \angle C &= 60^\circ \text{ and } \angle B = \angle D = 120^\circ \end{aligned}$$

(i) In ΔAPB , $\angle PAB = \frac{60}{2} = 30^\circ$

and $\angle PBA = \frac{120}{2} = 60^\circ$
 $\therefore \angle APB = 180^\circ - (30^\circ + 60^\circ) = 90^\circ$

(ii) In ΔADP , $\angle PAD = 30^\circ$ and $\angle ADP = 120^\circ$

$$\therefore \angle APD = 180^\circ - (30^\circ + 120^\circ) = 30^\circ$$

Thus, $\angle PAD = \angle APD = 30^\circ$

Hence, ΔADP is an isosceles triangle and $AD = DP$.

In ΔPBC , $\angle PBC = 60^\circ$, $\angle BPC = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$ and $\angle BCP = 60^\circ$

(Opposite angle of $\angle A$)

$$\therefore \angle PBC = \angle BPC = \angle BCP$$

Hence, ΔPBC is an equilateral triangle and, therefore, $PB = PC = BC$.

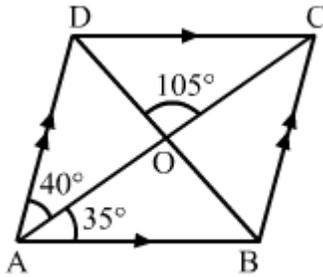
(iii) $DC = DP + PC$

From (ii), as ,

$$\begin{aligned}
 DC &= AD + BC \\
 \Rightarrow DC &= AD + AD \\
 \Rightarrow DC &= 2AD
 \end{aligned}$$

[AD = BC, opposite sides of a parallelogram]

Answer 5:



ABCD is a parallelogram.

$\therefore AB \parallel DC$ and $BC \parallel AD$

(i) In $\triangle AOB$, $\angle BAO = 35^\circ$,

As we know that, vertically opposite angles are equals

$$\angle AOB = \angle COD = 105^\circ$$

$$\therefore \angle ABO = 180^\circ - (35^\circ + 105^\circ) = 40^\circ$$

(ii) As we know that these angles are $\angle ODC$ and $\angle ABO$ are alternate interior angles.

$$\therefore \angle ODC = \angle ABO = 40^\circ$$

(iii) These are Alternate interior angles

$$\angle ACB = \angle CAD = 40^\circ$$

(iv) In $\triangle ABC$, we get

$$\angle CBD = \angle ABC - \angle ABD \quad \dots(i)$$

$$\angle ABC = 180^\circ - \angle BAD \quad (\text{Adjacent angles are supplementary})$$

$$\Rightarrow \angle ABC = 180^\circ - 75^\circ = 105^\circ$$

In $\triangle CBD$, we have

Then, $\angle CBD = \angle ABC - \angle ABD$

$$\Rightarrow \angle CBD = 105^\circ - \angle ABD \quad (\angle ABD = \angle ABO)$$

$$\Rightarrow \angle CBD = 105^\circ - 40^\circ = 65^\circ$$

Answer 6:

ABCD is a parallelogram.

i.e., $\angle A = \angle C$ and $\angle B = \angle D$ (Opposite angles)

Also, $\angle A + \angle B = 180^\circ$ (Adjacent angles are supplementary)

$$\therefore (2x + 25)^\circ + (3x - 5)^\circ = 180^\circ$$

$$\Rightarrow 5x + 20 = 180^\circ$$

$$\Rightarrow 5x = 180 - 20$$

$$\Rightarrow 5x = 160^\circ$$

$$\Rightarrow x = \frac{160}{5} = 32^\circ$$

$$\therefore \angle A = 2 \times 32 + 25 = 89^\circ \text{ and } \angle B = 3 \times 32 - 5 = 91^\circ$$

Hence, $x = 32^\circ$, $\angle A = \angle C = 89^\circ$ and $\angle B = \angle D = 91^\circ$

Answer 7:

Let PQRS be a parallelogram.

$\therefore \angle P = \angle R$ and $\angle Q = \angle S$

Let $\angle P = y^\circ$ and $\angle B = \left(\frac{4y}{5}\right)^\circ$

Now, $\angle P + \angle Q = 180^\circ$

$$\Rightarrow y + \left(\frac{4y}{5}\right)^\circ = 180^\circ \Rightarrow \left(\frac{9y}{5}\right)^\circ = 180^\circ \Rightarrow y = 100^\circ$$

Now, $\angle P = 100^\circ$ and $\angle B = \left(\frac{4}{5}\right) \times 100^\circ = 80^\circ$

Hence, $\angle P = \angle R = 100^\circ$; $\angle B = \angle S = 80^\circ$

Answer 8:

Let PQRS be a parallelogram.

$$\therefore \angle P = \angle R \text{ and } \angle Q = \angle S \quad (\text{Opposite angles})$$

Let $\angle P$ be the smallest angle whose measure is y° .

$$\therefore \angle Q = (2y - 30)^\circ$$

$$\text{Now, } \angle P + \angle Q = 180^\circ \quad (\text{Adjacent angles are supplementary})$$

$$\Rightarrow y + 2y - 30^\circ = 180^\circ$$

$$\Rightarrow 3y = 210^\circ$$

$$\Rightarrow y = \frac{210}{3} = 70$$

$$\Rightarrow y = 70^\circ$$

$$\therefore \angle Q = 2 \times 70^\circ - 30^\circ = 110^\circ$$

$$\text{Hence, } \angle P = \angle R = 70^\circ; \angle Q = \angle S = 110^\circ$$

Answer 9:

ABCD is a parallelogram.

The opposite sides of a parallelogram are parallel and equal.

$$\therefore AB = DC = 9.5 \text{ cm}$$

$$\text{Let } BC = AD = y$$

$$\therefore \text{Perimeter of ABCD} = AB + BC + CD + DA = 30 \text{ cm}$$

$$\Rightarrow 9.5 + y + 9.5 + y = 30$$

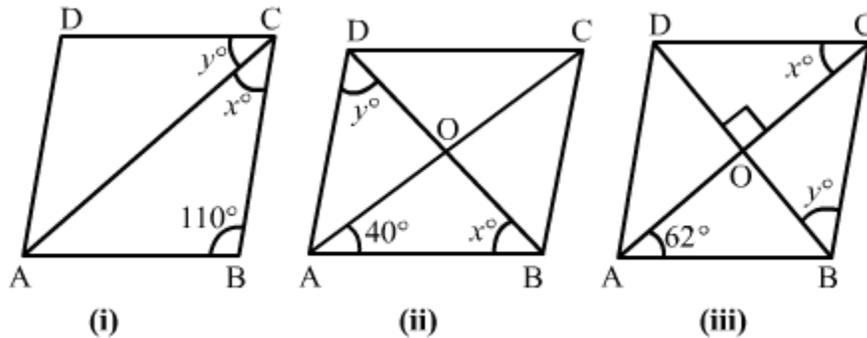
$$\Rightarrow 19 + 2y = 30$$

$$\Rightarrow 2y = 11$$

$$\Rightarrow y = \frac{11}{2} = 5.5 \text{ cm}$$

$$\text{Hence, } AB = DC = 9.5 \text{ cm and } BC = DA = 5.5 \text{ cm}$$

Answer 10:



ABCD is a rhombus and a rhombus is also a parallelogram. A rhombus has four equal sides.

(i) In $\triangle ABC$,

$$\angle BAC = \angle BCA = \frac{1}{2}(180 - 110)^\circ = 35^\circ$$

i.e., $x = 35^\circ$

Now by Adjacent angles are supplementary we get,

$$\angle B + \angle C = 180^\circ$$

As, $\angle C = x + y = 70^\circ$

$$\Rightarrow y = 70^\circ - x$$

$$\Rightarrow y = 70^\circ - 35^\circ = 35^\circ$$

Hence, $x = 35^\circ$; $y = 35^\circ$

(ii) The diagonals of a rhombus are perpendicular bisectors of each other.

So, in $\triangle AOB$, $\angle OAB = 40^\circ$, $\angle AOB = 90^\circ$ and

$$\angle ABO + \angle BOA + \angle OAB = 180$$

$$\angle ABO = 180^\circ - (40^\circ + 90^\circ) = 50^\circ$$

$$\therefore x = 50^\circ$$

In $\triangle ABD$, $AB = AD$

So, $\angle ABD = \angle ADB = 50^\circ$

Hence, $x = 50^\circ$; $y = 50^\circ$

(iii) $\angle BAC = \angle DCA$ (Alternate interior angles)

i.e., $x = 62^\circ$

In $\triangle BOC$, $\angle BCO = 62^\circ$

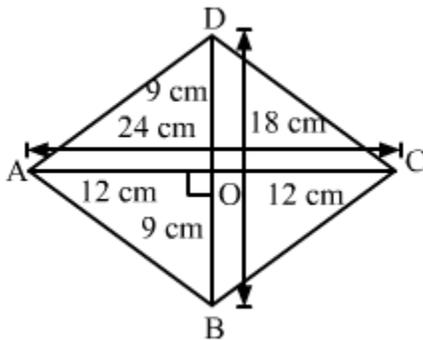
Also, $\angle BOC = 90^\circ$

$\angle BCO + \angle BOC + \angle OBC = 180$

$$\therefore \angle OBC = 180^\circ - (90^\circ + 62^\circ) = 28^\circ$$

Hence, $x = 62^\circ$; $y = 28^\circ$

Answer 11:



Let PQRS be a rhombus.

$\therefore PQ = QR = RS = SP$

Here, PR and QS are the diagonals of PQRS, where PR = 24 cm and QS = 18 cm.

Let the diagonals intersect each other at M.

$\therefore \triangle PMQ$ is a right angle triangle in which $MP = \frac{AC}{2} = \frac{24}{2} = 12$ cm and $MQ =$

$$\frac{QS}{2} = \frac{18}{2} = 9 \text{ cm.}$$

Now, $PQ^2 = MP^2 + MQ^2$ [Pythagoras theorem]

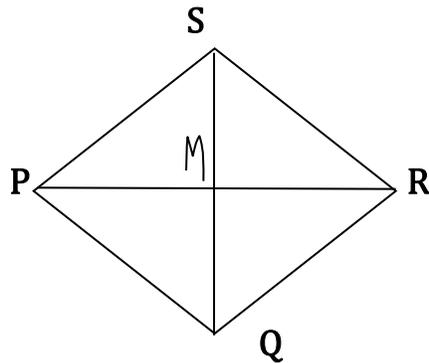
$$\Rightarrow PQ^2 = (12)^2 + (9)^2$$

$$\Rightarrow PQ^2 = 144 + 81 = 225$$

$$\Rightarrow PQ = 15 \text{ cm}$$

Hence, the side of the rhombus is 15 cm.

Answer 12:



Let PQRS be a rhombus.

$\therefore PQ = QR = RS = SP = 10 \text{ cm}$

Let PR and QS are the diagonals of PQRS. Let $PR = y$ and $QS = 16 \text{ cm}$ and M be the intersection point of the diagonals.

$\therefore \Delta PMQ$ is a right angle triangle, in which

$$MP = \frac{PR}{2} = \frac{y}{2} \text{ and } MQ = \frac{QS}{2} = \frac{16}{2} = 8 \text{ cm.}$$

Now, $PQ^2 = MP^2 + MQ^2$ [Pythagoras theorem]

$$\Rightarrow 10^2 = \left(\frac{y}{2}\right)^2 + 8^2 \Rightarrow 100 - 64 = \frac{y^2}{4} \Rightarrow 36 \times 4 = y^2$$

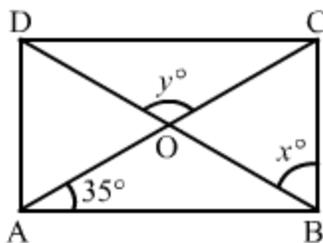
$$\Rightarrow y^2 = 144$$

$$\therefore y = 12 \text{ cm}$$

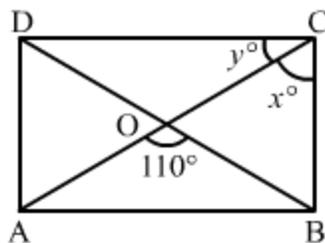
Hence, the other diagonal of the rhombus is 12 cm.

$$\therefore \text{Area of the rhombus} = 12 \times (12 \times 16) = 96 \text{ cm}^2$$

Answer 13:



(i)



(ii)

(i) ABCD is a rectangle.

The diagonals of a rectangle are congruent and bisect each other. Therefore, in ΔAOB , as ,

$$OA = OB$$

$$\therefore \angle OAB = \angle OBA = 35^\circ$$

$$\therefore x = 90^\circ - 35^\circ = 55^\circ$$

In ΔAOB

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$\text{And } \angle AOB = 180^\circ - (35^\circ + 35^\circ) = 110^\circ$$

$$\therefore y = \angle AOB = 110^\circ \quad [\text{Vertically opposite angles}]$$

Hence, $x = 55^\circ$ and $y = 110^\circ$

(ii) In ΔAOB , as ,

$$\text{Given, } \angle AOB = 100^\circ$$

$$OA = OB$$

$$\text{As, } \angle OAB = \angle OBA$$

$$\text{Then, } \angle AOB + \angle OBA + \angle OAB = 180$$

$$\Rightarrow 2\angle AOB = 180 - \angle AOB \dots\dots\dots(\angle OAB = \angle OBA)$$

$$\Rightarrow 2\angle AOB = 180 - 110 = 70^\circ$$

$$\Rightarrow \angle AOB = \frac{1}{2} \times 70 = 35^\circ$$

$$\text{so, } \therefore y = \angle BAC = 35^\circ \quad [\text{Interior alternate angles}]$$

Here at $\angle C$ is at right angle Δ by fig,

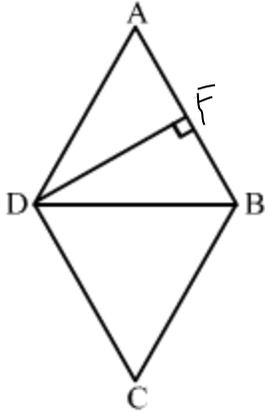
$$\Rightarrow 90^\circ = x + y$$

$$\Rightarrow x = 90^\circ - y$$

$$\Rightarrow x = 90^\circ - 35^\circ = 55^\circ$$

Thus, $x = 55^\circ$ and $y = 35^\circ$

Answer 14:



Given: ABCD is a rhombus, DF is altitude which bisects AB i.e. $AF = FB$

In $\triangle AFD$ and $\triangle BFD$,

$DF = DF$ (Common side)

$\angle DFA = \angle DFB = 90^\circ$ (Given)

$AF = FB$ (Given)

$\therefore \triangle AFD \cong \triangle BFD$ (By SAS congruence Criteria)

$\Rightarrow AD = BD$ (CPCT)

Also, $AD = AB$ (Sides of rhombus are equal)

$\Rightarrow AD = AB = BD$

Thus, $\triangle ABD$ is an equilateral triangle.

Therefore, $\angle A = 60^\circ$

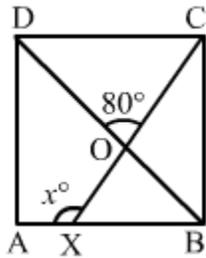
$\Rightarrow \angle C = \angle A = 60^\circ$ (Opposite angles of rhombus are equal)

And, $\angle ABC + \angle BCD = 180^\circ$ (Adjacent angles of rhombus are supplementary.)

$\Rightarrow \angle ABC + 60^\circ = 180^\circ \Rightarrow \angle ABC = 180^\circ - 60^\circ \Rightarrow \angle ABC = 120^\circ \Rightarrow \angle ADC = \angle ABC = 120^\circ$

Hence, the angles of rhombus are $60^\circ, 120^\circ, 60^\circ$ and 120°

Answer 15:



The angles of a square are bisected by the diagonals.

$$\angle OBX = \frac{1}{2} \times \angle CBA = \frac{1}{2} \times 90 = 45^\circ$$

$$\therefore \angle OBX = 45^\circ$$

$$\text{Given, } \angle COD = 80^\circ$$

$$\text{And } \angle BOX = \angle COD = 80^\circ \quad [\text{Vertically opposite angles}]$$

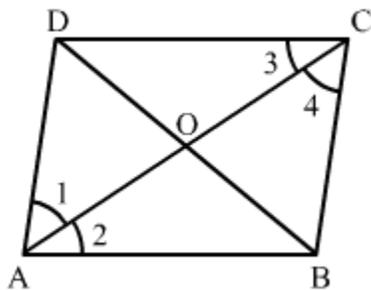
\therefore In $\triangle BOX$, as we know that exterior angle is sum of both interior angles.

$$\angle AXO = \angle OBX + \angle BOX$$

$$\Rightarrow \angle AXO = 45^\circ + 80^\circ = 125^\circ$$

$$\therefore x = 125^\circ$$

Answer 16:



Given: A rhombus ABCD.

To prove: Diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.

Proof:

In $\triangle ABC$,

$AB = BC$ (Sides of rhombus are equal.)

$\angle ACB = \angle CAB$ (Angles opposite to equal sides are equal.) ... (1)

$AD \parallel BC$ (Opposite sides of rhombus are parallel.)

AC is transversal.

$\angle DAC = \angle ACB$ (Alternate interior angles) ... (2)

From (1) and (2),

$\angle DAC = \angle CAB$

Thus, AC bisects $\angle A$.

As, $AB \parallel DC$ and AC is transversal.

$\angle CAB = \angle DCA$ (Alternate interior angles) ... (3)

From (1) and (3),

$\angle ACB = \angle DCA$

Thus, AC bisects $\angle C$.

Thus, AC bisects $\angle C$ and $\angle A$

In $\triangle DAB$,

$AD = AB$ (Sides of rhombus are equal.)

$\angle ADB = \angle ABD$ (Angles opposite to equal sides are equal.) ... (4)

Also,

$DC \parallel AB$ (Opposite sides of rhombus are parallel.)

BD is transversal.

$\angle CDB = \angle DBA$ (Alternate interior angles) ... (5)

From (4) and (5),

$\angle ADB = \angle CDB$

Therefore, DB bisects $\angle D$.

As, $AD \parallel BC$ and BD is transversal.

$\angle CBD = \angle ADB$ (Alternate interior angles) ... (6)

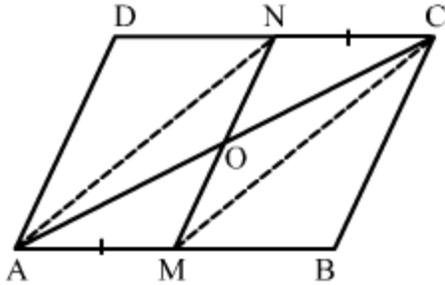
From (4) and (6)

$\angle CBD = \angle ABD$

Therefore, BD bisects $\angle B$.

Thus, BD bisects $\angle D$ and $\angle B$.

Answer 17:



Given: In a parallelogram ABCD, $AM = CN$.

To prove: AC and MN bisect each other.

Construction: Join AN and MC.

Proof:

As, ABCD is a parallelogram.

$\Rightarrow AB \parallel DC \Rightarrow AM \parallel NC$

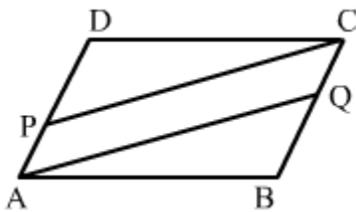
And, $AM = CN$ (Given)

Therefore, AMCN is a parallelogram.

As, the diagonals of a parallelogram bisect each other.

Thus, AC and MN also bisect each other.

Answer 18:



As, per by given fig,

$\angle B = \angle D$ [Opposite angles of parallelogram ABCD]

$AD = BC$ and $AB = DC$ [Opposite sides of parallelogram ABCD]

Also, $AD \parallel BC$ and $AB \parallel DC$

Given, $AP = \frac{1}{3}AD$ and $CQ = \frac{1}{3}BC$

So, we get

$$\therefore AP = CQ \quad [AD = BC]$$

In $\triangle DPC$ and $\triangle BQA$,

$$AB = CD, \angle B = \angle D \text{ and } DP = QB \quad [DP = \frac{2}{3}AD \text{ and } QB = \frac{2}{3}BC]$$

i.e., $\triangle DPC \cong \triangle BQA$

$$\therefore PC = QA$$

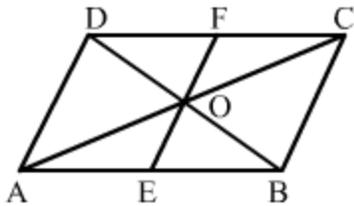
Thus, in quadrilateral AQCP,

$$AP = CQ \quad \dots(i)$$

$$PC = QA \quad \dots(ii)$$

\therefore AQCP is a parallelogram.

Answer 19:



Given, ABCD is a parallelogram whose diagonals intersect each other at O. A line segment EOF is drawn to meet AB at E and DC at F.

So in $\triangle ODF$ and $\triangle OBE$,

$$OD = OB \quad (\text{Diagonals bisect each other})$$

$$\angle DOF = \angle BOE \quad (\text{Vertically opposite angles})$$

$$\angle FDO = \angle OBE \quad (\text{Alternate interior angles})$$

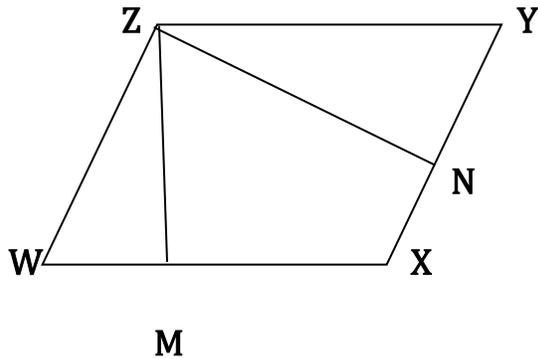
By parallelogram theorem

$$\triangle ODF \cong \triangle OBE$$

$$\therefore OF = OE$$

Hence, proved.

Answer 20:



Given: \square parallelogram WXYZ, $ZM \perp WX$, $WN \perp XY$ and $\angle MZN = 60^\circ$

In quadrilateral ZMXN, by angle sum property,

$$\angle MZN + \angle ZMX + \angle X + \angle XNZ = 360^\circ$$

$$\Rightarrow 60^\circ + 90^\circ + \angle X + 90^\circ = 360^\circ$$

$$\Rightarrow \angle X = 360^\circ - 240^\circ \Rightarrow \angle X = 120^\circ \Rightarrow \angle X = 120^\circ$$

Also,

$$\angle X = \angle Z = 120^\circ \quad (\text{Opposite angles of a parallelogram are equal.})$$

$$\angle W + \angle X = 180^\circ \quad (\text{Adjacent angles of a parallelogram are supplementary.})$$

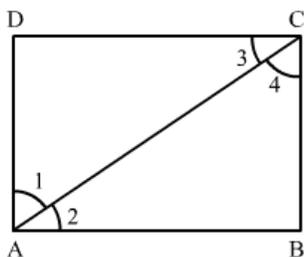
$$\Rightarrow \angle W + 120^\circ = 180^\circ \Rightarrow \angle W = 180^\circ - 120^\circ \Rightarrow \angle W = 60^\circ$$

Also,

$$\angle W = \angle Y = 60^\circ \quad (\text{Opposite angles of a parallelogram are equal.})$$

Thus, the angles of a parallelogram are 60° , 120° , 60° and 120° .

Answer 21:



Given: In

rectangle ABCD, AC bisects $\angle A$, i.e. $\angle DAC = \angle CAB$ and AC bisects $\angle C$, i.e. $\angle DCA = \angle ACB$.

To prove:

(i) ABCD is a square,

(ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

Proof:

(i) Since, $AD \parallel BC$ (Opposite sides of a rectangle are parallel.)

So, $\angle DAC = \angle ACB$ (Alternate interior angles)

But, $\angle DAC = \angle CAB$ (Given)

So, $\angle CAB = \angle ACB$

In $\triangle ABC$,

Since, $\angle CAB = \angle ACB$

So, $BC = AB$ (Sides opposite to equal angles are equal.)

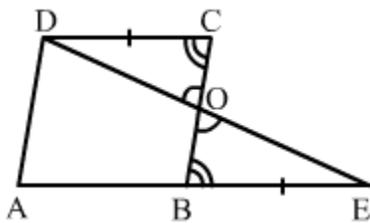
But these are adjacent sides of the rectangle ABCD.

Hence, ABCD is a square.

(ii) Since, the diagonals of a square bisect its angles.

So, diagonals BD bisect $\angle B$ as well as $\angle D$.

Answer 22:



Given, ABCD is parallelogram in which AB is produced to E.

$BE = AB$ (given)

So in $\triangle ODC$ and $\triangle OEB$, as ,

$DC = BE$ (DC = AB)

$$\angle OCD = \angle OBE \quad (\text{Alternate interior angles})$$

$$\angle COD = \angle BOE \quad (\text{Vertically opposite angles})$$

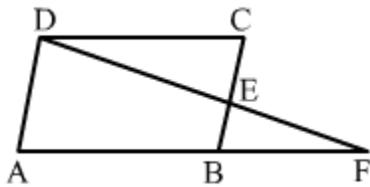
by parallelogram theorem we get,

$$\therefore \triangle ODC \cong \triangle OEB$$

$$\Rightarrow OC = OB$$

Hence, ED bisects BC.

Answer 23:



Given: ABCD is a parallelogram.

$$BE = CE$$

DE and AB when produced meet at F.

To prove: $AF = 2AB$

Proof: In parallelogram ABCD, as,

$$AB \parallel DC$$

$$\angle DCE = \angle EBF \quad (\text{Alternate interior angles})$$

In $\triangle DCE$ and $\triangle BFE$,

$$\angle DCE = \angle EBF \quad (\text{Proved above})$$

$$\angle DEC = \angle BEF \quad (\text{Vertically opposite angles})$$

And, $BE = CE$ (Given)

By parallelogram theorem

$$\therefore \triangle DCE \cong \triangle BFE$$

$$\text{hence } \therefore DC = BF$$

But $DC = AB$, as ABCD is a parallelogram.

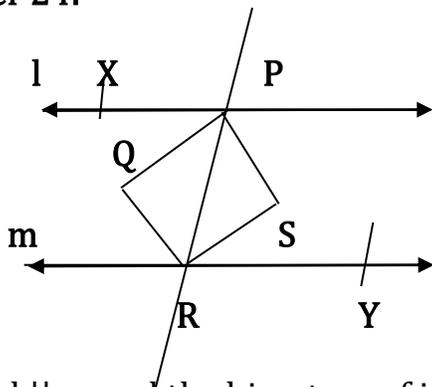
$$\therefore DC = AB = BF \quad \dots(i)$$

can also be written as, $AF = AB + BF \quad \dots(ii)$

$$AF = AB + AB = 2AB \quad \dots\text{from (i)}$$

Hence, proved. $AF = 2AB$.

Answer 24:



Given: $l \parallel m$ and the bisectors of interior angles intersect at X and Y.
 To prove: PQRS is a rectangle.

Proof:

Since, $l \parallel m$ (Given)

So, $\angle XPR = \angle PRY$ (Alternate interior angles)

$$\Rightarrow \frac{1}{2} \angle XPR = \frac{1}{2} \angle PRY$$

$\Rightarrow \angle QPR = \angle PRS$ but, these are a pair of alternate interior angles for PQ and RS.

$\Rightarrow PQ \parallel SR$

Similarly, $PR \parallel QS$

So, PQRS is a parallelogram.

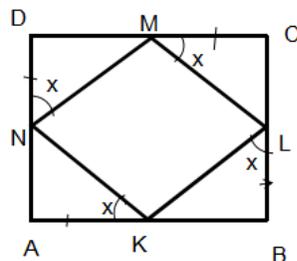
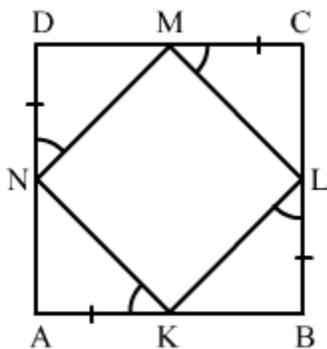
Also,

$$\angle XPR + \angle RPZ = 180^\circ \quad (\text{Linear pair})$$

$$\Rightarrow \frac{1}{2} \angle XPR + \frac{1}{2} \angle PRY = 90^\circ \Rightarrow \angle QPR + \angle RPS = 90^\circ \Rightarrow \angle QPS = 90^\circ$$

But, this an angle of the parallelogram PQRS
 Hence, PQRS is a rectangle.

Answer 25:



Given: In square ABCD, $AK = BL = CM = DN$.

To prove: KLMN is a square.

Proof:

In square ABCD,

$AB = BC = CD = DA$ (All sides of a square are equal.)

And, $AK = BL = CM = DN$ (Given)

So, $AB - AK = BC - BL = CD - CM = DA - DN$

$\Rightarrow KB = CL = DM = AN$...(1)

In $\triangle NAK$ and $\triangle KBL$,

$\angle NAK = \angle KBL = 90^\circ$ (Each angle of a square is a right angle.)

$AK = BL$ (Given)

$AN = KB$ [From (1)]

So, by parallelogram theorem,

$\triangle NAK \cong \triangle KBL$

$\Rightarrow NK = KL$ (CPCT) ...(2)

Similarly,

$\triangle MDN \cong \triangle NAK$ $\triangle DNM \cong \triangle CML$ $\triangle MCL \cong \triangle LBN$

$\Rightarrow MN = NK$ and $\angle DNM = \angle KNA$ (CPCT) ...(3)

$MN = JM$ and $\angle DNM = \angle CML$ (CPCT) ...(4)

$ML = LK$ and $\angle CML = \angle BLK$ (CPCT) ...(5)

From (2), (3), (4) and (5),

$NK = KL = MN = ML$...(6)

And, $\angle DNM = \angle AKN = \angle KLB = \angle LMC$

Now,

In $\triangle NAK$,

$\angle NAK = 90^\circ$

Let $\angle AKN = y^\circ$

So, $\angle DNK = 90^\circ + y^\circ$

$\Rightarrow \angle DNM + \angle MNK = 90^\circ + y^\circ \Rightarrow y^\circ + \angle MNK = 90^\circ + y^\circ \Rightarrow \angle MNK = 90^\circ$

Similarly,

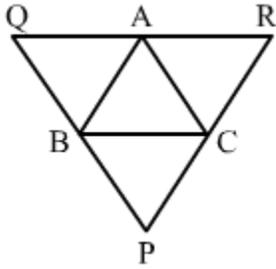
$\angle NKL = \angle KLM = \angle LMN = 90^\circ$...(7)

Using (6) and (7),

All sides of quadrilateral KLMN are equal and all angles are 90°

So, KLMN is a square.

Answer 26:



ΔABC , if lines are drawn through A, B, C parallel respectively to the sides BC, CA and AB. So, we get, $BC \parallel QA$ and $CA \parallel QB$
i.e., BCQA is a parallelogram.

$$\therefore BC = QA \quad \dots(i)$$

Similarly, $BC \parallel AR$ and $AB \parallel CR$.

i.e., BCRA is a parallelogram.

$$\therefore BC = AR \quad \dots(ii)$$

As $QR = QA + AR$

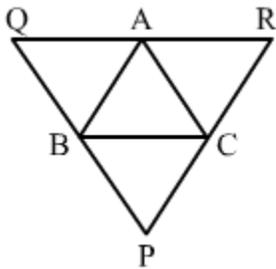
From (i) and (ii),

$$QR = BC + BC$$

$$\Rightarrow QR = 2BC$$

$$\therefore BC = \frac{1}{2}QR$$

Answer 27:



In ΔABC A, B, C lines drawn, parallel respectively to BC, CA and AB intersecting at P, Q and R. Acc to question,

$$\text{Perimeter of } \Delta ABC = AB + BC + CA \quad \dots(i)$$

$$\text{Perimeter of } \Delta PQR = PQ + QR + PR \quad \dots(ii)$$

By given figure,

BC || QA and CA || QB

i.e., BCQA is a parallelogram.

$$\therefore BC = QA \quad \dots(\text{iii})$$

Similarly, BC || AR and AB || CR

i.e., BCRA is a parallelogram.

$$\therefore BC = AR \quad \dots(\text{iv})$$

But, $QR = QA + AR$

From (iii) and (iv),

$$\Rightarrow QR = BC + BC$$

$$\Rightarrow QR = 2BC$$

$$\therefore BC = \frac{1}{2}QR$$

$$\text{Similarly, } CA = \frac{1}{2}PQ \text{ and } AB = \frac{1}{2}PR$$

From (i) and (ii),

$$\begin{aligned} \text{Perimeter of } \Delta ABC &= \frac{1}{2}QR + \frac{1}{2}PQ + \frac{1}{2}PR \\ &= \frac{1}{2}(PR + QR + PQ) \end{aligned}$$

$$\text{i.e., Perimeter of } \Delta ABC = \frac{1}{2}(\text{Perimeter of } \Delta PQR)$$

$$\therefore \text{Perimeter of } \Delta PQR = 2 \times \text{Perimeter of } \Delta ABC$$