# QUADRILATERALS - CHAPTER 10

## EXERCISE 10A

**Answer 1:**

Given: Three angles of a quadrilateral are 75°, 90° and 75°.
Let the fourth angle be y.
Using angle sum property of quadrilateral,
$75°+90°+75°+y=360°$

$\Rightarrow 240°+y=360°$

$\Rightarrow y=360°-240°$

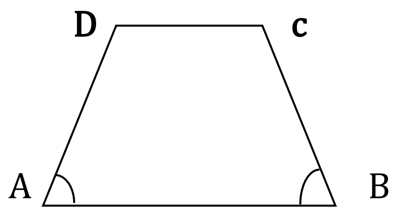
$\Rightarrow y=120°$
So, the measure of the fourth angle is 120°

.

**Answer 2:**

Let $\angle A = 2y^o$.
Then $\angle B = (4y)^o$; $\angle C = (5y)^o$ and $\angle D = (7y)^o$
Since the sum of the angles of a quadrilateral is $360^o$, as ,
$2y + 4y + 5y + 7y = 360^o$
$\Rightarrow 18\ y = 360^o$
$\Rightarrow y = 20^o$
$\therefore\ \angle A = 40^o; \angle B = 80^o; \angle C = 100^o; \angle D = 140^o$

**Answer 3:**




Given , AB || DC. As we know that the interior angles on the same side of transversal line, then $\angle A = 55° \ and \ \angle B = 70°$

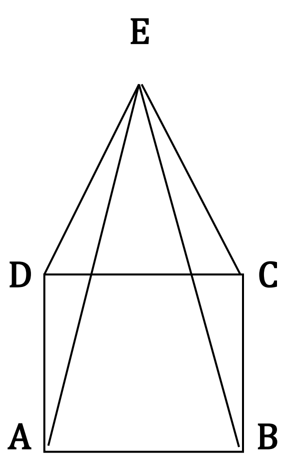
$\angle A + \angle D = 180°$

$\Rightarrow \angle D = 180° - \angle A = 180° - 55° = 125°$

Also , $\angle B + \angle C = 180°$

$\Rightarrow \angle C = 180° - \angle B = 180° - 70° = 110°$

**Answer 4:**



Given: ABCD is a square in which AB = BC = CD = DA. ΔEDC is an equilateral triangle in which ED = EC = DC and ∠EDC = ∠ DEC = ∠DCE = $60^o$.

To prove: AE = BE and ∠DAE = $15^o$

Proof: In ΔADE and ΔBCE, as ,

AD = BC [Sides of a square]

DE = EC [Sides of an equilateral triangle]

∠ADE = ∠BCE = $90^o + 60^o = 150^o$

∴ ΔADE ≅ ΔBCE

i.e., AE = BE

Now, ∠ADE = $150^o$

DA = DC [Sides of a square]

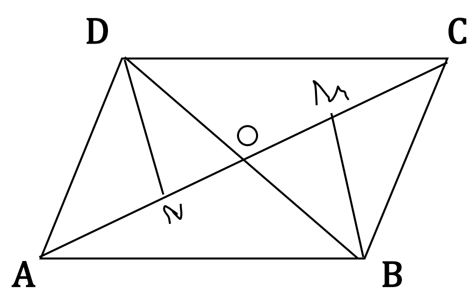
DC = DE [Sides of an equilateral triangle]

So, DA = DE

ΔADE and ΔBCE are isosceles triangles.

i.e., $\angle DAE = \angle DEA = \frac{1}{2}(180^\circ - 150^\circ) = \frac{30}{2} = 15^\circ$

**Answer 5:**




Given: by fig , both the diagonals intersect at O and BM ⊥ AC then

Let the diagonals intersect each other at O

Now, in ΔOND and ΔOMB,

∠OND = ∠OMB ($90^o$ each)

∠DON = ∠ BOM (Vertically opposite angles)

Also, DN = BM (Given)

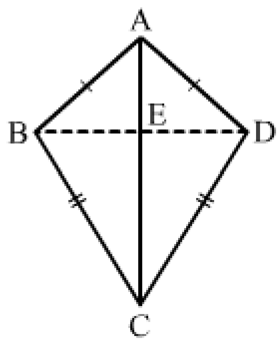
As we know that by parallelogram

$\Delta OND \cong \Delta OMB$

$\therefore OD = OB$ HENCE PROVED

Hence, AC bisects BD.

**Answer 6:**




Given: ABCD is a quadrilateral in which AB = AD and BC = DC

(i) To prove : AC bisects ∠A and ∠C

In ΔABC and ΔADC,

AB = AD

BC = DC

AC is common in both the traiangles.

i.e., ΔABC ≅ ΔADC (SSS congruence rule)

∴ ∠BAC = ∠DAC and ∠BCA = ∠DCA (By CPCT)

Hence proved, AC bisects both the angles, ∠A and ∠ C.

(ii) To prove BE = DE

In ΔABE and ΔADE,

AB = AD

S∠BAE = ∠DAE

AE is common.

∴ ΔABE ≅ ΔADE (SAS congruence rule)

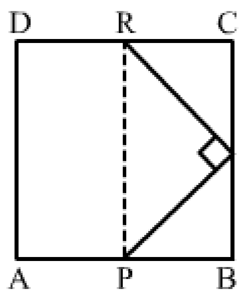
⇒ hence proved BE = DE

(iii) To prove : ∠ABC = ∠ADC

$\Delta ABC \cong \Delta ADC$ (Given)

Hence proved, $\angle ABC = \angle ADC$

**Answer 7:**




Given: ABCD is a square and $\angle PQR = 90°$.

$PB = QC = DR$

(i) To prove : $QB = DR$

$\therefore BC = CD$ (Sides of square)

and $CQ = DR$ (Given)

so , by fig $BC = BQ + CQ$

$\Rightarrow CQ = BC - BQ$

$\therefore DR = BC - BQ$ ...(i)

Also, $CD = RC + DR$

$\therefore DR = CD - RC = BC - RC$ ...(ii)

From (i) and (ii), we get

$BC - BQ = BC - RC$

$\therefore BQ = RC$

(ii)To prove, $PQ = QR$

In $\Delta RCQ$ and $\Delta QBP$,

$PB = QC$ (Given)

$BQ = RC$ (Given)

$\angle RCQ = \angle QBP$ ($90^{o}$ each)

By parallelogram theorem

$\Delta RCQ \cong \Delta QBP$ (SAS congruence rule)

$\therefore QR = PQ$ hence proved

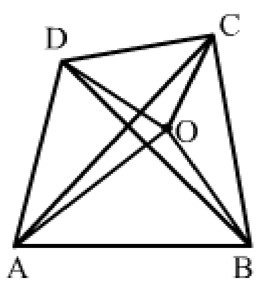
(iii) To prove, $\angle QPR = 45°$

$\Delta RCQ \cong \Delta QBP$ and $QR = PQ$

$\therefore$ In $\Delta RPQ$, $\angle QPR = \angle QRP = \frac{1}{2}(180° - 90°) = \frac{90}{2} = 45°$

Hence proved, $\angle QPR = 45°$

**Answer 8:**

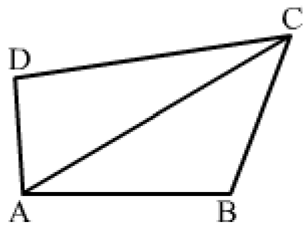



Let ABCD be a quadrilateral with diagonals AC and BD and O is a point within the quadrilateral.

Suppose
In $\Delta AOC$, $OA + OC > AC$ …………..(1)

And, in $\Delta BOD$, $OB + OD > BD$ ………………(2)
Adding these ,
$(OA + OC) + (OB + OD) > (AC + BD)$
$\Rightarrow OA + OB + OC + OD > AC + BD$

**Answer 9:**




Given: ABCD is a quadrilateral and AC is its diagonal.

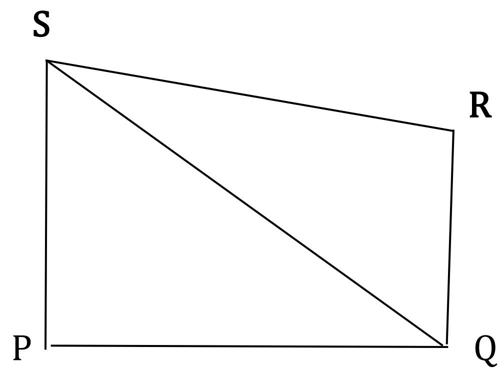
(i) As sum of any two sides of any triangle is greater than the third side.
In ΔABC, $AB + BC > AC$ ...(1)

In ΔACD, $CD + DA > AC$ ...(2)
Adding (1) and (2),
$AB + BC + CD + DA > 2AC$ ……………….hence proved

(ii) In ΔABC,
$AB + BC > AC$ ...(1)
In ΔACD,
$AC > |DA - CD|$ ...(2)
From (1) and (2),
$AB + BC > |DA - CD|$
$\Rightarrow AB + BC + CD > DA$………………………..hence proved

(iii) In ΔABC ,we know that $AB + BC > AC$
Same as, In ΔACD, $CD + DA > AC$
And
In Δ BCD,
$BC + CD > BD$
In Δ ABD,
$DA + AB > BD$
Adding these ,
$2(AB + BC + CD + DA) > 2(AC + BD)$
$\Rightarrow (AB + BC + CD + DA) > (AC + BD)$

**Answer 10:**




Let PQRS be a quadrilateral and $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$ are its four angles .

Join QR which divides PQRS in two triangles, $\Delta PQR$ and $\Delta QRS$.

In $\Delta PQR$,

$\angle 1 + \angle 2 + \angle P = 180°$ ...(i)

In $\Delta QRS$,

$\angle 3 + \angle 4 + \angle R = 180°$ ...(ii)

On adding (i) and (ii),

$(\angle 1 + \angle 3) + \angle P + \angle R + (\angle 4 + \angle 2) = 360°$

$\Rightarrow \angle P + \angle R + \angle Q + \angle S = 360°$ $\therefore \angle 1 + \angle 3 = \angle Q$ ; $\angle 4 + \angle 2 = \angle S$

Hence proved

$\therefore \angle P + \angle R + \angle Q + \angle S = 360°$