Class 12 RD Sharma Solutions – Chapter 19 Indefinite Integrals – Exercise 19.20

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Question 1. Evaluate: $\int (x^2 + x + 1)/(x^2 - x)\, dx$

Solution:

Given that $I = \int (x^2 + x + 1)/(x^2 - x)\, dx$

$= \int [1 + (2x + 1)/(x^2 - x)] dx$

$= x + \int (2x + 1)/(x^2 - x)\, dx + c_1$

$= x + I_1 + c_1$ ....(i)

Now, $I_1 = \int (2x + 1)/(x^2 - x)\, dx$

Let $2x + 1 = \lambda\, d/dx\, (x^2 - x) + \mu = \lambda(2x - 1) + \mu$

$2x + 1 = (2\lambda)x - \lambda + \mu$

By comparing the coefficients of x, we get

$2 = 2\lambda \Rightarrow \lambda = 1$

$-\lambda + \mu = 1 \Rightarrow \mu = 2$

$I_1 = \int ((2x - 1) + 2)/(x^2 - x)\, dx)$

$= \int (2x - 1)/(x^2 - x)\, dx + 2\int 1/((x - 1/2)^2 - (1/2)^2)\, dx$

$= \log|x^2 - x| +$

$2 \times \frac{1}{2(\frac{1}{2})} \log\left|\frac{(x-\frac{1}{2}-\frac{1}{2})}{(x-\frac{1}{2}+\frac{1}{2})}\right|+c_1$

As we know that $\int 1/(x^2 - a^2)\, dx = 1/2a \log|(x - a)/(x + a)| + c$

So, $I_1 = \log|x^2 - x| + 2\log|(x - 1)/x| + c_2$ ......(ii)

Now put the value of I_1 in eq(i), we get

$I = x + \log|x^2 - x| + 2\log|(x - 1)/x| + c$

Question 2. $\int (x^2 + x - 1)/(x^2 + x - 6)\, dx$

Solution:

Given that $I = \int (x^2 + x - 1)/(x^2 + x - 6)\, dx$

$= \int [1 + 5/(x^2 + x - 6)]dx$

$I = x + \int 5/(x^2 + x - 6)\, dx + c_1$

Let us assume $I_1 = 5\int 1/(x^2 + x - 6)\, dx$

$I = x + I_1 + c_1$ .....(i)

$= 5\int 1/(x^2 + 2x(1/2) + (1/2)^2 - (1/2)^2 - 6)\, dx$

$= 5\int 1/((x + 1/2)^2 - (5/2)^2 dx$

$= 5\ 1/2(5/2) \log|(x + 1/2 - 5/2)/(x + 1/2 + 5/2)| + c_2$

As we know that $\int 1/(x^2 - a^2)\, dx = 1/2a \log|(x - a)/(x + a)| + c$

So, we get

$I_1 = \log|(x - 2)/(x + 3)| + c_2$ ....(ii)

$I = x + \log|(x - 2)/(x + 3)| + c$

Question 3. $\int (1 - x^2)/(x(1 - 2x))\, dx$

Solution:

Given that $I = \int (1 - x^2)/(x(1 - 2x))\, dx$

$= \int (1 - x^2)/(x - 2x^2)\, dx$

$= \int (x^2 - 1)/(2x^2 - x)\, dx$

$= \int [1/2 + (x/2 - 1)/(2x^2 - x)]dx$

$I = 1/2x + \int (x/2 - 1)/(2x^2 - x)\, dx + c_1$

Let us assume $I_1 = \int (x/2 - 1)/(2x^2 - x)\, dx$

So, $I = 1/2x + I_1 + c_1$ .....(i)

Now, let $x/2 - 1 = \lambda\, d/dx\, (2x^2 - x) + \mu = \lambda(4x - 1) + \mu$

$x/2 - 1 = (4\lambda)x - \lambda + \mu$

By comparing the coefficients of x, we get

$1/2 = 4\lambda \Rightarrow \lambda = 1/8$

$-\lambda + \mu = -1 \Rightarrow -(1/8) + \mu = -1$

$\mu = -7/8$

$I_1 = \int (1/8(4x - 1) - 7/8)/(2x^2 - x)\, dx$

$= 1/8 \int (4x - 1)/(2x^2 - x)\, dx - 7/8 \int 1/2(x^2 - x/2) dx$

$= 1/8 \int (4x - 1)/(2x^2 - x)\, dx - 7/16 \int 1/(x^2 - 2x(1/4) + (1/4)^2 - (1/4)^2)\, dx$

$= 1/8 \int (4x - 1)/(2x^2 - x)\, dx - 7/16\, [1/((x - 1/4)^2 - (1/4)^2)\, dx$

$= 1/8 \log|2x^2 - x| - 7/16 \times 1/2(1/4) \log|(x - 1/4 - 1/4)/(x - 1/4 + 1/4)| + c_2$

So, we get

$I_1 = 1/8 \log|x| + 1/8 \log|2x - 1| - 7/8 \log|1 - 2x| + 7/8 \log 2 + 7/8 \log|x| + c_2$

$I_1 = \log|x| - 3/4 \log|1 - 2x| + c_3$ [Here, $c_3 = c_2 + 7/8 \log 2$]

Now put the value of I_1 *in eq(i), we get*

$I = 1/2x + \log|x| - 3/4 \log|1 - 2x| + c$

Question 4. $\int (x^2 + 1)/(x^2 - 5x + 6)dx$

Solution:

Given that $I = \int (x^2 + 1)/(x^2 - 5x + 6)dx$

Now we convert I into proper rational function by dividing $x^2 + 1$ *by* $x^2 - 5x + 6$

So,

$(x^2 + 1)/(x^2 - 5x + 6) = 1 + (5x - 5)/(x^2 - 5x + 6) = 1 + (5x - 5)/((x - 2)(x - 3))$

Let

$(5x - 5)/((x - 2)(x - 3)) = A/(x - 2) + B/(x - 3)$

So, we get $A + B = 5$ *and* $3A + 2B = 5$

On solving both the equations we get $A = -5$ *and* $B = 10$

So, $\frac{(x^2 + 1)}{(x^2 - 5x + 6)} = 1 - \frac{5}{(x-2)} + \frac{10}{(x - 3)}$

Hence, $\int (x^2 + 1)/((x + 1)^2 (x + 3)) dx = \int dx - 5\int 1/(x - 2) dx + 10\int x/(x - 3)$

$I = x - 5\log|x - 2| + 10\log|x - 3| + c$

Question 5. $\int x^2/(x^2 + 7x + 10) dx$

Given that $I = \int x^2/(x^2 + 7x + 10)\, dx$

$= \int \{1 - (7x + 10)/(x^2 + 7x + 10)\} dx$

$I = x - \int (7x + 10)/(x^2 + 7x + 10)\, dx + c_1$

Let us assume $I_1 = \int (7x + 10)/(x^2 + 7x + 10)\, dx$

So, $I = x - I_1 + c_1$ .....(i)

Now let us assume $7x + 10 = \lambda\, d/dx\, (x^2 + 7x + 10) + \mu = \lambda(2x + 7) + \mu$

$7x + 10 = (2\lambda)x + 7\lambda + \mu$

By comparing the coefficients of x, we get

$7 = 2\lambda \Rightarrow \lambda = 7/2$

$7\lambda + \mu = 10 \Rightarrow 7(7/2) + \mu = 10\mu = -29/2$

So, $I_1 = \int (7/2(2x + 7) - 29/2)/(x^2 + 7x + 10)\, dx$

$= 7/2 \int ((2x + 7))/(x^2 + 7x + 10)\, dx - 29/2 \int 1/(x^2 + 2x(7/2) + (7/2)^2 - (7/2)^2 + 10)\, dx$

$= 7/2 \int (2x + 7)/(x^2 + 7x + 10)\, dx - 29/2 \{1/((x + 7/2)^2 - (3/2)^2)\, dx$

$= 7/2 \log|x^2 + 7x + 10| - 29/2 \times 1/2(3/2) \log|(x + 7/2 - 3/2)/(x + 7/2 + 3/2)| + c_2$

As we know that $\int 1/(x^2 - a^2)\, dx = 1/2a \log|(x - a)/(x + a)| + c$

So, we get

$I_1 = 7/2 \log|x^2 + 7x + 10| - 29/6 \log|(x + 2)/(x + 5)| + c_2$

Now put the value of I_1 in eq(i), we get

$I = x - 7/2 \log|x^2 + 7x + 10| + 29/6 \log|(x + 2)/(x + 5)| + c$

Question 6. $\int (x^2 + x + 1)/(x^2 - x + 1)\, dx$

Solution:

$= \int [1 + 2x/(x^2 - x + 1)]dx$

$= x + \int 2x/(x^2 - x + 1)\ dx + c_1$

Let us assume $I_1 = \int 2x/(x^2 - x + 1)\ dx$

So, $I = x + I_1 + c_1$ .....(i)

Now let $2x = \lambda\ d/dx\ (x^2 - x + 1) + \mu = \lambda(2x - 1) + \mu$

$2x = (2\lambda) \times -\lambda + \mu$

By comparing the coefficients of x, we get

$2 = 2\lambda \Rightarrow \lambda = 1$

$-\lambda + \mu = 0 \Rightarrow -1 + \mu = 0$

$\mu = 1$

So, $I_1 = \int ((2x - 1) + 1)/(x^2 - x + 1)\ dx$

$= \int ((2x - 1))/(x^2 - x + 1)\ dx + \int 1/(x^2 - 2x(1/2) + (1/2)^2 - (1/2)^2 + 1)\ dx$

$= \int (2x - 1)/(x^2 - x + 1)\ dx + \int 1/((x - 1/2)^2 + (\sqrt{3}/2)^2)\ dx$

$= \log|x^2 - x + 1| + 2/\sqrt{3} \tan^{-1}((x - 1/2)/(\sqrt{3}/2)) + c_2$

As we know that, $\int 1/(x^2 + a^2)\ dx = 1/a \tan^{-1}(x/a) + c$

So, we get

$I_1 = \log|x^2 - x + 1| + 2/\sqrt{3} \tan^{-1}((2x - 1)/\sqrt{3}) + c_2$

Now put the value of I_1 in eq (i), we get

$I = x + \log|x^2 - x + 1| + 2/\sqrt{3} \tan^{-1}((2x - 1)/\sqrt{3}) + c$

Question 7. $\int (x - 1)^2/(x^2 + 2x + 2)\ dx$

Solution:

$= \int (x^2 - 2x + 1)/(x^2 + 2x + 2)\ dx$

$= \int [1 - (4x + 1)/(x^2 + 2x + 2)]dx$

$= x - \int (4x + 1)/(x^2 + 2x + 2)\ dx + c_1$

Let us assume $l_1 = \int (4x + 1)/(x^2 + 2x + 2)\ dx$

So, $I = x - I_1 + c_1$ .....(i)

Now, let $4x + 1 = \lambda\ d/dx\ (x^2 + 2x + 2) + \mu$

$= \lambda(2x + 2) + \mu = (2k)x + (2\lambda + \mu)$

By comparing the coefficients of x, we get

$4 = 2\lambda \Rightarrow \lambda = 2$

$2\lambda + \mu = 1 \Rightarrow 2(2) + \mu = 1$

$\mu = -3$

$l_1 = \int\ (2(2x + 2) - 3)/(x^2 + 2x + 2)\ dx$

$= 2\int\ ((2x + 2))/(x^2 + 2x + 2)\ dx - 3\int 1/(x^2 - 2x + (1)^2 - (1)^2 + 2)\ dx$

$= 2\int\ (2x + 2)/(x^2 + 2x + 2)\ dx - 3\int 1/((x + 1)^2 + (1)^2)\ dx$

As we know that, $\int 1/(x^2 + 1)\ dx = \tan^{-1} x + c$

So, we get

$l_1 = 2\log|x^2 + 2x + 2| - 3\tan^{-1}(x + 1) + c_2$

Now put the value of I_1 in eq(i), we get

$I = x - 2\log|x^2 + 2x + 2| + 3\tan^{-1}(x + 1) + c$

Question 8. $\int (x^3 + x^2 + 2x + 1)/(x^2 - x + 1)\ dx$

Solution:

Given that, $I = \int (x^3 + x^2 + 2x + 1)/(x^2 - x + 1)\ dx$

Let us assume $I_1 = \int (3x - 1)/(x^2 - x + 1)\, dx$

So, $I = x^2/2 + 2x + I_1 + c_1$ .....(i)

Now, let $3x - 1 = \lambda\, d/dx\, (x^2 - x + 1) + \mu = \lambda(2x - 1) + \mu$

$3x - 1 = (2\lambda)x - \lambda + \mu$

By comparing the coefficients of x, we get

$3 = 2\lambda \Rightarrow \lambda = 3/2$

$-\lambda + \mu = -1 \Rightarrow -(3/2) + \mu = -1$

$\mu = 1/2$

So, $I_1 = \int (3/2(2x - 1) + 1/2)/(x^2 - x + 1)\, dx$

$= 3/2 \int ((2x - 1))/(x^2 - x + 1)\, dx + 1/2 \int 1/(x^2 - 2x(1/2) + (1/2)^2 - (1/2)^2 + 1)\, dx$

$= 3/2 \int (2x - 1)/(x^2 - x + 1)\, dx + 1/2 \int 1/((x + 1/2)^2 + (\sqrt{3}/2)^2)\, dx$

$= 3/2 \log|x^2 - x + 1| + 1/2 \times 2/\sqrt{3} \tan^{-1}((x + 1/2)/(\sqrt{3}/2)) + c_2$

As we know that, $\int 1/(x^2 + a^2)\, dx = 1/a \tan^{-1}(x/a) + c$

So, we get

$I_1 = 3/2 \log|x^2 - x + 1| + 1/\sqrt{3} \tan^{-1}((2x + 1)/\sqrt{3}) + c_2$

Now put the value of I_1 in eq(i), we get

$I = x^2/2 + 2x + 3/2 \log|x^2 - x + 1| + 1/\sqrt{3} \tan^{-1}((2x + 1)/\sqrt{3}) + c$

Question 9. $\int (x^2 (x^4 + 4))/(x^2 + 4)\, dx$

Solution:

Given that, $I = \int (x^2 (x^4 + 4))/((x^2 + 4))\, dx$

$= \int (x^6 + 4x^2)/((x^2 + 4))\, dx$

$= x^5/5 - (4x^3)/3 + 20x - 80\int 1/(x^2 + 4)\, dx + c_1$

Let us assume $I_1 = \int 1/(x^2 + 4)\, dx$

So, $I = x^5/5 - (4x^3)/3 + 20x - 80I_1 + c_1$ .....(i)

Now, $I_1 = \int 1/(x^2 + (2)^2)\, dx$

As we know that, $\int 1/(x^2 + a^2)\, dx = 1/a \tan^{-1}(x/a) + c$

So, we get

$I_1 = 1/2 \tan^{-1}(x/2) + c_2$

Now put the value of I_1 in eq(i), we get

$I = x^5/5 - (4x^3)/3 + 20x - 80/2 \tan^{-1}(x/2) + c$

$I = x^5/5 - (4x^3)/3 + 20x - 40\tan^{-1}(x/2) + c$

Question 10. $\int x^2/(x^2 + 6x + 12)\, dx$

Solution:

Given that, $I = \int x^2/(x^2 + 6x + 12)\, dx$

$= \int [1 - (6x + 12)/(x^2 + 6x + 12)]dx$

$= x - \int (6x + 12)/(x^2 + 6x + 12)\, dx + c_1$

Let us assume $I_1 = \int (6x + 12)/(x^2 + 6x + 12)\, dx$

So, $I = x - I_1 + c_1$ .....(i)

Now, let $6x + 12 = \lambda\, d/dx\, (x^2 + 6x + 12) + \mu = \lambda(2x + 6) + \mu$

$6x + 12 = (2\lambda)x + 6\lambda + \mu$

By comparing the coefficients of the power of x, we get

$6 = 2\lambda \Rightarrow \lambda = 3$

$6\lambda + \mu = 12$

So, $l_1 = \int (3(2x + 6) - 6)/(x^2 + 6x + 12)\ dx$

$=3\int ((2x + 6))/(x^2 + 6x + 12)\ dx - 6\int 1/(x^2 + 2x(3) + (3)^2 - (3)^2 + 12)\ dx$

$= 3\int (2x + 6)/(x^2 + 6x + 12)\ dx + 6\int 1/((x + 3)^2 + (\sqrt{3})^2)\ dx$

As we know that, $\int 1/(x^2 + a^2)\ dx = 1/2 \tan^{-1}(x/a) + c$

So, we get

$I_1 = 3\log|x^2 + 6x + 12| + 6/\sqrt{3} \tan^{-1}((x + 3)/\sqrt{3}) + c_2$

$I_1 = 3\log|x^2 + 6x + 12| + 2\sqrt{3} \tan^{-1}((x + 3)/\sqrt{3}) + c_2$

Now put the value of I_1 in eq(i), we get

$I = x - 3\log|x^2 + 6x + 12| + 2\sqrt{3} \tan^{-1}((x + 3)/\sqrt{3}) + c$