Exercise – 8B

1. If $a\cos\theta+b\sin\theta=m$ and $a\sin\theta-b\cos\theta=n$, prove that, $\left(m^2+n^2\right)=\left(a^2+b^2\right)$

Sol:

We have $m^2+n^2=[(a\cos\theta+b\sin\theta)^2+(a\sin\theta-b\cos\theta)^2]$

$=(a^2\cos^2\theta+b^2\sin^2\theta+2ab\cos\theta\sin\theta)$

$+(a^2\sin^2\theta+b^2\cos^2\theta-2abcos\theta\sin\theta)$

$=a^2\cos^2\theta+b^2\sin^2\theta+a^2\sin^2\theta+b^2\cos^2\theta$

$=(a^2\cos^2\theta+b^2\sin^2\theta)+(b^2\cos^2\theta+b^2\sin^2\theta)$

$=a^2(\cos^2\theta+\sin^2\theta)+b^2(\cos^2\theta+\sin^2\theta)$

$=a^2+b^2 \quad [\because \sin^2+\cos^2=1]$

Hence, $m^2+n^2=a^2+b^2$

2. If $x=a\ sec\ \theta+b\tan\theta$ and $y=a\tan\ \theta+b\ sec\theta$, prove that $\left(x^2-y^2\right)=\left(a^2-b^2\right)$.

Sol:

We have $x^2-y^2=[(asec\theta+btan\ \theta)^2-(atan\theta+bsec\ \theta)^2]$

$=(a^2\sec^2\theta+b^2\tan^2\theta+2absec\theta\tan\theta)$

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$$-(a^2\tan^2\theta + b^2\sec^2\theta + 2abtan\theta\sec\theta)$$
$$= a^2\sec^2\theta + b^2\tan^2\theta - a^2\tan^2\theta - b^2\sec^2\theta$$
$$= (a^2\sec^2\theta - a^2\tan^2\theta) - (b^2\sec^2\theta - b^2\tan^2\theta)$$
$$= a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$$
$$= a^2 - b^2 \qquad [\because \sec^2\theta - \tan^2\theta = 1]$$

Hence, $x^2 - y^2 = a^2 - b^2$

3. If $\left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right) = 1$ and $\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right) = 1$, prove that $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = 2$.

Sol:

We have $(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta) = 1$

Squaring both side, we have:

$(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta)^2 = (1)^2$

$\Rightarrow (\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\frac{x}{a}\times\frac{y}{b}\sin\theta\cos\theta) = 1 \quad \ldots(i)$

Again, $(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta) = 1$

Squaring both side, we get:

$(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta)^2 = (1)^2$

$\Rightarrow (\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{x}{a}\times\frac{y}{a}\sin\theta\cos\theta) = \ldots(ii)$

Now, adding (i) and (ii), we get:

$(\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\frac{x}{a}\times\frac{y}{b}\sin\theta\cos\theta) + (\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{x}{a}\times \frac{y}{b}\sin\theta\cos\theta)$

$\Rightarrow \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta + \frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta = 2$

$\Rightarrow (\frac{x^2}{a^2}\sin^2\theta + \frac{x^2}{a^2}\cos^2\theta) + (\frac{y^2}{b^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta) = 2$

$\Rightarrow \frac{x^2}{a^2}(\sin^2\theta + \cos^2\theta) + \frac{y^2}{b^2}(\cos^2\theta + \sin^2\theta) = 2$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2 \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

4. If $(\sec\theta + \tan\theta) = m$ and $(\sec\theta - \tan\theta) = n$, show that $mn = 1$.

Sol:

We have $(\sec\theta + \tan\theta) = m \quad \ldots(i)$

Again, $(\sec\theta - \tan\theta) = n \quad \ldots(ii)$

Now, multiplying (i) and (ii), we get:

$(\sec\theta + \tan\theta)\times(\sec\theta - \tan\theta) = mn$

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$\Rightarrow \sec^2\theta - \tan^2\theta = mn$

$\Rightarrow 1 = mn \quad [\because \sec^2\theta - \tan^2\theta = 1]$

$\therefore mn = 1$

5. If $(\text{cosec}\,\theta + \cot\theta) = m$ and $(\text{cosec}\,\theta - \cot\theta) = n$, show that $mn = 1$.

Sol:

We have $(\text{cosec}\,\theta + \cot\theta) = m \quad \ldots.(i)$

$Again, (\text{cosec}\,\theta - \cot\theta) = n \quad \ldots(ii)$

$Now, multiplying\ (i) and\ (ii), we\ get:$

$(\text{cosec}\,\theta + \cot\theta) \times (\text{cosec}\,\theta - \cot\theta) = mn$

$\Rightarrow \text{cosec}^2\theta - \cot^2\theta = mn$

$\Rightarrow 1 = mn \quad [\because \text{cosec}^2\theta - \cot^2\theta = 1]$

$\therefore mn = 1$

6. If $x = a\cos^3\theta$ and $y = b\sin^3\theta$, prove that $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1.$

Sol:

$We\ have\ x = a\cos^3\theta$

$\Rightarrow \frac{x}{a} = \cos^3\theta \quad \ldots(i)$

$Again, y = b\sin^3\theta$

$\Rightarrow \frac{y}{b} = \sin^3\theta \quad \ldots(ii)$

$Now, LHS = \left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}}$

$= (\cos^3\theta)^{\frac{2}{3}} + (\sin^3\theta)^{\frac{2}{3}} \quad [From\ (i) and\ (ii)]$

$= \cos^2\theta + \sin^2\theta$

$= 1$

$Hence, LHS = RHS$

7. If $(\tan\theta + \sin\theta) = m$ and $(\tan\theta - \sin\theta) = n$, prove that $\left(m^2 - n^2\right)^2 = 16\ mn.$

Sol:

$We\ have\ (\tan\theta + \sin\theta) = m\ and\ (\tan\theta - \sin\theta) = n$

$Now, LHS = (m^2 - n^2)^2$

$= [(\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2]^2$

$= [(\tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta) - (\tan^2\theta + \sin^2\theta - 2\tan\theta\sin\theta)]^2$

$= [(\tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta)]^2$

$= (4\tan\theta\sin\theta)^2$

$= 16\tan^2\theta\sin^2\theta$

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$= 16\frac{\sin^2\theta}{\cos^2\theta}\sin^2\theta$

$= 16\frac{(1-\cos^2\theta)\sin^2\theta}{\cos^2\theta}$

$= 16[\tan^2\theta(1-\cos^2\theta)]$

$= 16(\tan^2\theta - \tan^2\theta\cos^2\theta)$

$= 16(\tan^2\theta - \frac{\sin^2\theta}{\cos^2\theta}\times\cos^2\theta)$s

$= 16(\tan^2\theta - \sin^2\theta)$

$= 16(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)$

$= 16\,mn \qquad [(\tan\theta+\sin\theta)(\tan\theta-\sin\theta) = mn]$

$\therefore (m^2-n^2)(m^2-n^2)^2 = 16mn$

8. If $(\cot\theta+\tan\theta)=m$ and $(\sec\theta-\cos\theta)=n$ prove that $(m^2n)^{2/3}-(mn^2)^{2/3}=1$

Sol:

We have $(\cot\theta+\tan\theta)=m$ *and* $(\sec\theta-\cos\theta)=n$

Now, $m^2n = [(\cot\theta+\tan\theta)^2(\sec\theta-\cos\theta)]$

$= [(\frac{1}{\tan\theta}+\tan\theta)^2(\frac{1}{\cos\theta}-\cos\theta)]$

$= \frac{(1+\tan^2\theta)^2}{\tan^2\theta}\times\frac{(1-\cos^2\theta)}{\cos\theta}$

$= \frac{\sec^4\theta}{\tan^2\theta}\times\frac{\sin^2\theta}{\cos\theta}$

$= \frac{\sec^4\theta}{\frac{\sin^2\theta}{\cos^2\theta}}\times\frac{\sin^2\theta}{\cos\theta}$

$= \frac{\cos^2\theta\times\sec^4\theta}{\cos\theta}$

$= \cos\theta\sec^4\theta$

$= \frac{1}{\sec\theta}\times\sec^4\theta = \sec^3\theta$

$\therefore (m^2n)^{\frac{2}{3}} = (\sec^3\theta)^{\frac{2}{3}} = \sec^2\theta$

Again, $mn^2 = [(\cot\theta+\tan\theta)(\sec\theta-\cos\theta)^2]$

$= [(\frac{1}{\tan\theta}+\tan\theta).(\frac{1}{\cos\theta}-\cos\theta)^2]$

$= \frac{(1+\tan^2\theta)}{\tan\theta}\times\frac{(1-\cos^2\theta)^2}{\cos^2\theta}$

$= \frac{\sec^2\theta}{\tan\theta}\times\frac{\sin^4\theta}{\cos^2\theta}$

$= \frac{\sec^2\theta}{\frac{\sin\theta}{\cos\theta}}\times\frac{\sin^4\theta}{\cos^2\theta}$

$= \frac{\sec^2\theta\times\sin^3\theta}{\cos\theta}$

$= \frac{1}{\cos^2\theta}\times\frac{\sec^3\theta}{\cos\theta} = \tan^3\theta$

$\therefore (mn^2)^{\frac{2}{3}} = (\tan^3\theta)^{\frac{2}{3}} = \tan^2\theta$

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$Now, (m^2n)^{\frac{2}{3}} - (mn^2)^{\frac{2}{3}}$

$= \sec^2\theta - \tan^2\theta = 1$

$= \text{RHS}$

Hence proved.

9. If $(\text{cosec}\,\theta - \sin\theta) = a^3 and (\sec\theta - \cos\theta) = b^3$, prove that $a^2b^2(a^2 + b^2) = 1$

Sol:

$We\ have\ (cosec\ \theta - \sin\theta) = a^3$

$=> a^3 = (\frac{1}{\sin\theta} - \sin\theta)$

$=> a^3 = \frac{(1-\sin^2\theta)}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$

$\therefore\ a = \frac{\cos^{\frac{2}{3}}\theta}{\sin^{\frac{1}{3}}\theta}$

$Again, (\sec\theta - \cos\theta) = b^3$

$=> b^3 = (\frac{1}{\cos\theta} - \cos\theta)$

$= \frac{(1-\cos^2\theta)}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$\therefore b = \frac{\sin^{\frac{2}{3}}\theta}{\cos^{\frac{1}{3}}\theta}$

$Now, LHS = a^2b^2(a^2 + b^2)$

$= a^4b^2 + a^2b^4$

$= a^3(ab^2) + (a^2b^2)b^3$

$= \frac{\cos^2\theta}{\sin\theta} \times \left[\frac{\cos^{\frac{2}{3}}\theta}{\sin^{\frac{1}{3}}\theta} \times \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}\right] + \left[\frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta} \times \frac{\sin^{\frac{2}{3}}\theta}{\cos^{\frac{1}{3}}\theta}\right] \times \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\cos^2\theta}{\sin\theta} \times \sin\theta + \cos\theta \times \frac{\sin^2\theta}{\cos\theta}$

$= \cos^2\theta + \sin^2\theta = 1$

$= \text{RHS}$

Hence, proved.

10. If $\left(2\sin\theta + 3\cos\theta\right) = 2$, prove that $\left(3\sin\theta - 2\cos\theta\right) = \pm 3$.

Sol:

$Given, (2\sin\theta + 3\cos\theta) = 2 \quad \ldots(i)$

$We\ have\ (2\sin\theta + 3\cos\theta)^2 + (3\sin\theta - 2\cos\theta)^2$

$= 4\sin^2\theta + 9\cos^2\theta + 12\sin\theta\cos\theta + 9\sin^2\theta + 4\cos^2\theta - 12\sin\theta\cos\theta$

$= 4(\sin^2\theta + \cos^2\theta) + 9(\sin^2\theta + \cos^2\theta)$

$= 4+9$

$= 13$

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i.e., $(2\sin\theta + 3\cos\theta)^2 + (3\sin\theta - 2\cos\theta)^2 = 13$

$=> 2^2 + (3\sin\theta - 2\cos\theta)^2 = 13$

$=> (3\sin\theta - 2\cos\theta)^2 = 13 - 4$

$=> (3\sin\theta - 2\cos\theta)^2 = 9$

$=> (3\sin\theta - 2\cos\theta) = \pm 3$

11. If $(\sin\theta + \cos\theta) = \sqrt{2}$, prove that $\cot\theta = \left(\sqrt{2}+1\right)$.

Sol:

We have, $(\sin\theta + \cos\theta) = \sqrt{2}\cos\theta$

Dividing both sides by $\sin\theta$ *, we get*

$\frac{\text{Sin}\,\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{2}\cos\theta}{\sin\theta}$

$\Rightarrow 1 + \cot\theta = \sqrt{2}\cot\theta$

$\Rightarrow \sqrt{2}\cot\theta - \cot\theta = 1$

$\Rightarrow (\sqrt{2}-1)\cot\theta = 1$

$\Rightarrow \cot\theta = \frac{1}{(\sqrt{2}-1)}$

$\Rightarrow \cot\theta = \frac{1}{(\sqrt{2}-1} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}$

$\Rightarrow \cot\theta = \frac{(\sqrt{2}+1)}{2-1}$

$\Rightarrow \cot\theta = \frac{(\sqrt{2}+1)}{1}$

$\therefore \cot\theta = (\sqrt{2}+1)$

12. If $(\cos\theta + \sin\theta) = \sqrt{2}\sin\theta$, prove that $(\sin\theta - \cos\theta) = \sqrt{2}\cos\theta$.

Sol:

Given: $\cos\theta + \sin\theta = \sqrt{2}\sin\theta$

We have $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2(\sin^2\theta + \cos^2\theta)$

$=> \left(\sqrt{2}\sin\theta\right)^2 + (\sin\theta - \cos\theta)^2 = 2$

$=> 2\sin^2\theta + (\sin\theta - \cos\theta)^2 = 2$

$=> (\sin\theta - \cos\theta)^2 = 2 - 2\sin^2\theta$

$=> (\sin\theta - \cos\theta)^2 = 2(1 - \sin^2\theta)$

$=> (\sin\theta - \cos\theta)^2 = 2\cos^2\theta$

$=> (\sin\theta - \cos\theta) = \sqrt{2}\cos\theta$

Hence proved.

13. If $\sec\theta + \tan\theta = p$, prove that

(i) $\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$ (ii) $\tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$ (iii) $s\,\text{in}\,\theta = \frac{p^2-1}{p^2+1}$

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Sol:

(i) We have, $\sec\theta+\tan\theta=p$ ……..(1)

$$\Rightarrow \frac{\sec\theta+\tan\theta}{1}\times\frac{\sec\theta-\tan\theta}{\sec\theta-\tan\theta}=p$$

$$\Rightarrow \frac{\sec^2\theta-\tan^2\theta}{\sec\theta-\tan\theta}=p$$

$$\Rightarrow \frac{1}{\sec\theta-\tan\theta}=p$$

$$\Rightarrow \sec\theta-\tan\theta=\frac{1}{p} \quad \text{.....(2)}$$

Adding (1) and (2), we get

$$2\sec\theta=p+\frac{1}{p}$$

$$\Rightarrow \sec\theta=\frac{1}{2}\left(p+\frac{1}{p}\right)$$

(ii) Subtracting (2) from (1), we get

$$2\tan\theta=\left(p-\frac{1}{p}\right)$$

$$\Rightarrow \tan\theta=\frac{1}{2}\left(p-\frac{1}{p}\right)$$

(iii) Using (i) and (ii), we get

$$\sin\theta=\frac{\tan\theta}{\sec\theta}$$

$$=\frac{\frac{1}{2}\left(p-\frac{1}{p}\right)}{\frac{1}{2}\left(p+\frac{1}{p}\right)}$$

$$=\frac{\left(\frac{p^2-1}{p}\right)}{\frac{\left(p^2+1\right)}{p}}$$

$$\therefore \sin\theta=\frac{p^2-1}{p^2+1}$$

14. If tan A = n tan B and sin A =m sin B, prove that $\cos^2 A=\frac{\left(m^2-1\right)}{\left(n^2-1\right)}$.

Sol:

We have $\tan A=n\tan B$

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$$\Rightarrow \cot B = \frac{n}{\tan A} \quad .....(i)$$

Again, sin A = m sin B

$$\Rightarrow \cos ecB = \frac{m}{\sin A} \quad .....(ii)$$

Squaring (i) and (ii) and subtracting (ii) form (i), we get

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = \cos ec^2 B - \cot^2 B$$

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2 \cos}{\sin^2 A} = 1$$

$$\Rightarrow m^2 - n^2 \cos^2 A = \sin^2 A$$

$$\Rightarrow m^2 - n^2 \cos^2 A = 1 - \cos^2 A$$

$$\Rightarrow n^2 \cos^2 A - \cos^2 A = m^2 - 1$$

$$\Rightarrow \cos^2 A\left(n^2 - 1\right) = \left(m^2 - 1\right)$$

$$\Rightarrow \cos^2 A = \frac{\left(m^2 - 1\right)}{\left(n^2 - 1\right)}$$

$$\therefore \cos^2 A = \frac{\left(m^2 - 1\right)}{\left(n^2 - 1\right)}$$

15. 15. if $m = (\cos\theta - \sin\theta)$ and $n = (\cos\theta + \sin\theta)$ then show that $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \frac{2}{\sqrt{1 - \tan^2\theta}}$.

Sol:

$$LHS = \sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}}$$

$$= \frac{\sqrt{m}}{\sqrt{n}} + \frac{\sqrt{n}}{\sqrt{m}}$$

$$= \frac{m+n}{\sqrt{mn}}$$

$$= \frac{(\cos\theta - \sin\theta) + (\cos\theta + \sin\theta)}{\sqrt{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}}$$

$$= \frac{2\cos\theta}{\sqrt{\cos^2\theta - \sin^2\theta}}$$

$$= \frac{2\cos\theta}{\sqrt{\cos^2\theta - \sin^2\theta}}$$

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$$= \frac{\left(\frac{2\cos\theta}{\cos\theta}\right)}{\left(\frac{\sqrt{\cos^2\theta - \sin^2\theta}}{\cos\theta}\right)}$$

$$= \frac{2}{\sqrt{\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}}}$$

$$= \frac{2}{\sqrt{1 - \tan^2\theta}}$$

$$= RHS$$

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