Exercise – 16B

1. Find the coordinates of the point which divides the join of $A(-1,7)$ *and* $B(4,-3)$. in the ratio $2:3$

Sol:

The end points of AB are $A(-1,7)$ *and* $B(4,-3)$.

Therefore, $(x_1=-1, y_1=7)$ *and* $(x_2=4, y_2=-3)$

Also, $m=2$ *and* $n=3$

Let the required point be $P(x,y)$.

By section formula, we get

$$x=\frac{(mx_2+nx_1)}{(m+n)}, y=\frac{(my_2+ny_1)}{(m+n)}$$

$$\Rightarrow x=\frac{\{2\times4+3\times(-1)\}}{2+3}, y=\frac{\{2\times(-3)+3\times7\}}{2+3}$$

$$\Rightarrow x=\frac{8-3}{5}, y=\frac{-6+21}{5}$$

$$\Rightarrow x=\frac{5}{5}, y=\frac{15}{5}$$

Therefore, $x=1$ *and* $y=3$

Hence, the coordinates of the required point are $(1,3)$.

2. Find the co-ordinates of the point which divides the join of A(-5, 11) and B(4,-7) in the ratio 7 : 2

Sol:

The end points of *AB* are $A(-5,11)$ *and* $B(4,-7)$.

Therefore, $(x_1=-5, y_1=11)$ *and* $(x_2=4, y_2=-7)$

Also, $m=7$ *and* $n=2$

Let the required point be $P(x,y)$.

By section formula, we get

$$x=\frac{(mx_2+nx_1)}{(m+n)}, y=\frac{(my_2+ny_1)}{(m+n)}$$

$$\Rightarrow x=\frac{\{7\times4+2\times(-5)\}}{7+2}, y=\frac{\{7\times(-7)+2\times11\}}{7+2}$$

$$\Rightarrow x=\frac{28-10}{9}, y=\frac{-49+22}{9}$$

$$\Rightarrow x = \frac{18}{9}, y = -\frac{27}{9}$$

Therefore, $x = 2$ *and* $y = -3$

Hence, the required point are $P(2, -3)$.

3. If the coordinates of points A and B are (-2, -2) and (2, -4) respectively. Find the coordinates of the point P such that AP=$\frac{3}{7}$ AB, where P lies on the segment AB.

Sol:

The coordinates of the points A and $B$are $(-2, -2)$ and $(2, -4)$ respectively, where

$AP = \frac{3}{7} AB$ *and* P lies on the line segment AB. So

$$AP + BP = AB$$

$$\Rightarrow AP + BP = \frac{7AP}{3} \qquad \because AP = \frac{3}{7} AB$$

$$\Rightarrow BP = \frac{7AP}{3} - AP$$

$$\Rightarrow \frac{AP}{BP} = \frac{3}{4}$$

Let (x, y) be the coordinates of P which divides AB in the ratio 3 : 4 internally Then

$$x = \frac{3 \times 2 + 4 \times (-2)}{3 + 4} = \frac{6 - 8}{7} = -\frac{2}{7}$$

$$y = \frac{3 \times (-4) + 4 \times (-2)}{3 + 4} = \frac{-12 - 8}{7} = -\frac{20}{7}$$

Hence, the coordinates of point $P$are $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

4. Point A lies on the line segment PQ joining P(6, -6) and Q(-4, -1) in such a way that $\frac{PA}{PQ} = \frac{2}{5}$. If that point A also lies on the line 3x + k(y + 1) = 0, find the value of k.

Sol:

Let the coordinates of A be (x, y). Here $\frac{PA}{PQ} = \frac{2}{5}$. So,

$$PA + AQ = PQ$$

$$\Rightarrow PA + AQ = \frac{5PA}{2} \qquad \left[\because PA = \frac{2}{5} PQ\right]$$

$\Rightarrow AQ = \frac{5PA}{2} - PA$

$\Rightarrow \frac{AQ}{PA} = \frac{3}{2}$

$\Rightarrow \frac{PA}{AQ} = \frac{2}{3}$

Let (x, y) be the coordinates of A, which dives PQ in the ratio 2 : 3 internally Then using section formula, we get

$$x = \frac{2\times(-4)+3\times(6)}{2+3} = \frac{-8+18}{5} = \frac{10}{5} = 2$$

$$y = \frac{2\times(-1)+3\times(-6)}{2+3} = \frac{-2-18}{5} = \frac{-20}{5} = -4$$

Now, the point $(2,-4)$ lies on the line $3x + k(y+1) = 0$, therefore

$3\times 2 + k(-4+1) = 0$

$\Rightarrow 3k = 6$

$\Rightarrow k = \frac{6}{3} = 2$

Hence, $k = 2$.

5. Points P, Q, R and S divide the line segment joining the points A(1,2) and B(6,7) in five equal parts. Find the coordinates of the points P,Q and R

Sol:

Since, the points *P, Q, R* and *S* divide the line segment joining the points $A(1,2)$ *and* $B(6,7)$ in five equal parts, so

$AP = PQ = QR = R = SB$

Here, point P divides AB in the ratio of 1 : 4 internally So using section formula, we get

Coordinates of $P = \left(\frac{1\times(6)+4\times(1)}{1+4}, \frac{1\times(7)+4\times(2)}{1+4}\right)$

$= \left(\frac{6+4}{5}, \frac{7+8}{5}\right) = (2,3)$

The point Q divides AB in the ratio of 2 : 3 internally. So using section formula, we get

Coordinates of $Q = \left(\frac{2\times(6)+3\times(1)}{2+3}, \frac{2\times(7)+3\times(2)}{2+3}\right)$

$= \left(\frac{12+3}{5}, \frac{14+6}{5}\right) = (3,4)$

The point R divides AB in the ratio of 3 : 2 internally So using section formula, we get

Coordinates of $R = \left(\frac{3\times(6)+2\times(1)}{3+2}, \frac{3\times(7)+2\times(2)}{3+2}\right)$

$= \left(\frac{18+2}{5}, \frac{21+4}{5}\right) = (4,5)$

Hence, the coordinates of the points *P, Q* and R are $(2,3),(3,4)$ and $(4,5)$ respectively

6. Points P, Q, and R in that order are dividing line segment joining A (1,6) and B(5, -2) in four equal parts. Find the coordinates of P, Q and R.

Sol:

The given points are $A(1,6)$ and $B(5,-2)$.

Then, $P(x,y)$ is a point that devices the line AB in the ratio $1:3$

By the section formula:

$x = \frac{(mx_2 + nx_1)}{(m+n)}, y = \frac{(my_2 + ny_1)}{(m+n)}$

$\Rightarrow x = \frac{(1\times 5 + 3\times 1)}{1+3}, y = \frac{(1\times(-2)+3\times 6)}{1+3}$

$\Rightarrow x = \frac{5+3}{4}, y = \frac{-2+18}{4}$

$\Rightarrow x = \frac{8}{4}, y = \frac{16}{4}$

$\Rightarrow x = 2 \text{ and } y = 4$

Therefore, the coordinates of point P are $(2,4)$

Let Q be the mid-point of AB

Then, $Q(x,y)$

$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$

$\Rightarrow x = \frac{1+5}{2}, y = \frac{6+(-2)}{2}$

$\Rightarrow x = \frac{6}{2}, y = \frac{4}{2}$

$\Rightarrow x = 3, y = 2$

Therefore, the coordinates of Q are $(3,2)$

Let $R(x,y)$ be a point that divides AB in the ratio $3:1$

Then, by the section formula:

$$x=\frac{(mx_2+nx_1)}{(m+n)}, y=\frac{(my_2+ny_1)}{(m+n)}$$

$$\Rightarrow x=\frac{(3\times5+1\times1)}{3+1}, y=\frac{(3\times(-2)+1\times6)}{3+1}$$

$$\Rightarrow x=\frac{15+1}{4}, y=\frac{-6+6}{4}$$

$$\Rightarrow x=\frac{16}{4}, y=\frac{0}{4}$$

$$\Rightarrow x=4 \text{ and } y=0$$

Therefore, the coordinates of R are $(4,0)$.

Hence, the coordinates of point P, Q and R are $(2,4),(3,2)$ *and* $(4,0)$ respectively.

7. The line segment joining the points A(3,-4) and B(1,2) is trisected at the points P(p, -2) and $Q\left(\frac{5}{3},q\right)$. Find the values of p and q.

Sol:

Let P and Q be the points of trisection of AB.

Then, P divides AB in the radio 1:2

So, the coordinates of P are

$$x=\frac{(mx_2+nx_1)}{(m+n)}, y=\frac{(my_2+ny_1)}{(m+n)}$$

$$\Rightarrow x=\frac{\{1\times1+2\times(3)\}}{1+2}, y=\frac{\{1\times2+2\times(-4)\}}{1+2}$$

$$\Rightarrow x=\frac{1+6}{3}, y=\frac{2-8}{3}$$

$$\Rightarrow x=\frac{7}{3}, y=-\frac{6}{3}$$

$$\Rightarrow x=\frac{7}{3}, y=-2$$

Hence, the coordinates of P are $\left(\frac{7}{3},-2\right)$

But $(p,-2)$ are the coordinates of P.

So, $p=\frac{7}{3}$

Also, Q divides the line AB in the ratio 2:1

So, the coordinates of Q are

$$x=\frac{(mx_2+mx_1)}{(m+n)}, y=\frac{(my_2+my_1)}{(m+n)}$$

$$\Rightarrow x=\frac{(2\times1+1\times3)}{2+1}, y=\frac{\{2\times2+1\times(-4)\}}{2+1}$$

$$\Rightarrow x=\frac{2+3}{3}, y=\frac{4-4}{3}$$

$$\Rightarrow x=\frac{5}{3}, y=0$$

Hence, coordinates of Q are $\left(\frac{5}{3},0\right)$.

But the given coordinates of Q are $\left(\frac{5}{3},q\right)$.

So, $q=0$

Thus, $p=\frac{7}{3}$ *and* $q=0$

8. Find the coordinates of the midpoints of the line segment joining
(i) A(3,0) and B(-5, 4) (ii) P(-11,-8) and Q(8,-2)

Sol:

(i) The given points are $A(3,0)$ *and* $B(-5,4)$.

Let (x,y) be the midpoint of AB. Then:

$$x=\frac{x_1+x_2}{2}, y=\frac{y_1+y_2}{2}$$

$$\Rightarrow x=\frac{3+(-5)}{2}, y=\frac{0+4}{2}$$

$$\Rightarrow x=\frac{-2}{2}, y=\frac{4}{2}$$

$$\Rightarrow x=-1, y=2$$

Therefore, $(-1,2)$. are the coordinates of midpoint of AB.

(ii) The given points are $P(-11,-8)$ *and* $Q(8,-2)$.

Let (x,y) be the midpoint of PQ. Then:

$$x=\frac{x_1+x_2}{2}, y=\frac{y_1+y_2}{2}$$

$$\Rightarrow x=\frac{-11+8}{2}, y=\frac{-8--2}{2}$$

$\Rightarrow x=-\frac{3}{2}, y=-\frac{10}{2}$

$\Rightarrow x=-\frac{3}{2}, y=-5$

Therefore, $\left(-\frac{3}{2},-5\right)$ are the coordinates of midpoint of PQ.

9. If (2, p) is the midpoint of the line segment joining the points A(6, -5) and B(-2,11) find the value of p.

Sol:

The given points are $A(6,-5)$ and $B(-2,11)$.

Let (x, y) be the midpoint of AB. Then,

$x=\frac{x_1+x_2}{2}, y=\frac{y_1+y_2}{2}$

$\Rightarrow x=\frac{6+(-2)}{2}, y=\frac{-5+11}{2}$

$\Rightarrow x=\frac{6-2}{2}, y=\frac{-5+11}{2}$

$\Rightarrow x=\frac{4}{2}, y=\frac{6}{2}$

$\Rightarrow x=2, y=3$

So, the midpoint of AB is $(2,3)$.

But it is given that midpoint of AB is $(2, p)$.

Therefore, the value of $p=3$.

10. The midpoint of the line segment joining A (2a, 4) and B (-2, 3b) is C (1, 2a+1). Find the values of a and b.

Sol:

The points are $A(2a,4)$ and $B(-2,3b)$.

Let $C(1,2a+1)$ be the mid-point of AB. Then:

$x=\frac{x_1+x_2}{2}, y=\frac{y_1+y_2}{2}$

$\Rightarrow 1=\frac{2a(-2)}{2}, 2a+1=\frac{4+3b}{2}$

$\Rightarrow 2=2a-2, 4a+2=4+3b$

$\Rightarrow 2a=2+2, 4a-3b=4-2$

$\Rightarrow a = \frac{4}{2}, 4a - 3b = 2$
$\Rightarrow a = 2, 4a - 3b = 2$
Putting the value of a in the equation $4a + 3b = 2$, we get:
$4(2) - 3b = 2$
$\Rightarrow -3b = 2 - 8 = -6$
$\Rightarrow b = \frac{6}{3} = 2$
Therefore, $a = 2$ and $b = 2$.

11. The line segment joining $A(-2,9)$ and $B(6,3)$ is a diameter of a circle with center C. Find the coordinates of C.
Sol:
The given points are $A(-2,9)$ *and* $B(6,3)$
Then, $C(x, y)$ is the midpoint of *AB*.
$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$
$\Rightarrow x = \frac{-2+6}{2}, y = \frac{9+3}{2}$
$\Rightarrow x = \frac{4}{2}, y = \frac{12}{2}$
$\Rightarrow x = 2, y = 6$
Therefore, the coordinates of point *C* are $(2,6)$.

12. Find the coordinates of a point *A*, where *AB* is a diameter of a circle with center $C(2,-3)$ and the other end of the diameter is $B(1,4)$.
Sol:
$C(2,-3)$ is the center of the given circle. Let $A(a,b)$ *and* $B(1,4)$ be the two end-points of the given diameter *AB*. Then, the coordinates of C are
$x = \frac{a+1}{2}, y = \frac{b+4}{2}$
It is given that $x = 2$ and $y = -3$.
$\Rightarrow 2 = \frac{a+1}{2}, -3 = \frac{b+4}{2}$
$\Rightarrow 4 = a + 1, -6 = b + 4$
$\Rightarrow a = 4 - 1, b = -6 - 4$

$\Rightarrow a = 3, b = -10$

Therefore, the coordinates of point A are $(3, -10)$.

13. In what ratio does the point P(2,5) divide the join of A (8,2) and B(-6, 9)?

Sol:

Let the point $P(2,5)$ divide AB in the ratio $k:1$.

Then, by section formula, the coordinates of P are

$$x = \frac{-6k+8}{k+1}, y = \frac{9k+2}{k+1}$$

It is given that the coordinates of P are $(2,5)$.

$$\Rightarrow 2 = \frac{-6k+8}{k+1}, 5 = \frac{9k+2}{k+1}$$

$$\Rightarrow 2k+2 = -6k+8, 5k+5 = 9k+2$$

$$\Rightarrow 2k+6k = 8-2, 5-2 = 9k-5k$$

$$\Rightarrow 8k = 6, 4k = 3$$

$$\Rightarrow k = \frac{6}{8}, k = \frac{3}{4}$$

$\Rightarrow k = \frac{3}{4}$ in each case..

Therefore, the point $P(2,5)$ divides AB in the ratio $3:4$

14. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B(2, -5).

Sol:

Let $k:1$ be the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ *and* $(2, -5)$. Then

$$\left(\frac{3}{4}, \frac{5}{12}\right) = \left(\frac{k(2)+\frac{1}{2}}{k+1}, \frac{k(-5)+\frac{2}{2}}{k+1}\right)$$

$$\Rightarrow \frac{k(2)+\frac{1}{2}}{k+1} = \frac{3}{4} \ and \ \frac{k(-5)+\frac{3}{2}}{k+1} = \frac{5}{12}$$

$\Rightarrow 8k+2=3k+3 \ and \ -60k+18=5k+5$

$\Rightarrow k=\frac{1}{5} \ and \ k=\frac{1}{5}$

Hence, the required ratio is $1:5$.

15. Find the ratio in which the point P(m, 6) divides the join of A(-4, 3) and B(2, 8) Also, find the value of m.

Sol:

Let the point $P(m,6)$ divide the line AB in the ratio $k:1$.

Then, by the section formula:

$x=\frac{mx_2+nx_1}{m+n}, y=\frac{my_2+ny_1}{m+n}$

The coordinates of P are $(m,6)$.

$m=\frac{2k-4}{k+1}, 6=\frac{8k+3}{k+1}$

$\Rightarrow m(k+1)=2k-4, 6k+6=8k+3$

$\Rightarrow m(k+1)=2k-4, 6-3=8k-6k$

$\Rightarrow m(k+1)=2k-4, 2k=3$

$\Rightarrow m(k+1)=2k-4, k=\frac{3}{2}$

Therefore, the point P divides the line AB in the ratio $3:2$

Now, putting the value of k in the equation $m(k+1)=2k-4,$ we get:

$m\left(\frac{3}{2}+1\right)=2\left(\frac{3}{2}\right)-4$

$\Rightarrow m\left(\frac{3+2}{2}\right)=3-4$

$\Rightarrow \frac{5m}{2}=-1 \Rightarrow 5m=-2 \Rightarrow m=-\frac{2}{5}$

Therefore, the value of $m=-\frac{2}{5}$

So, the coordinates of P are $\left(-\frac{2}{5},6\right)$.

16. Find the ratio in which the pint (-3, k) divide the join of A(-5, -4) and B(-2, 3),Also, find the value of k.

Sol:

Let the point $P(-3, k)$ divide the line AB in the ratio $s : 1$

Then, by the section formula:

$$x = \frac{mx_1 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

The coordinates of P are $(-3, k)$.

$$-3 = \frac{-2s-5}{s+1}, k = \frac{3s-4}{s+1}$$

$$\Rightarrow -3s - 3 = -2s - 5, k(s+1) = 3s-4$$

$$\Rightarrow -3s + 2s = -5 + 3, k(s+1) = 3s-4$$

$$\Rightarrow -s = -2, k(s+1) = 3s-4$$

$$\Rightarrow s = 2, k(s+1) = 3s-4$$

Therefore, the point P divides the line AB in the ratio 2 : 1.

Now, putting the value of s in the equation $k(s+1) = 3s-4$, we get:

$$k(2+1) = 3(2) - 4$$

$$\Rightarrow 3k = 6 - 4$$

$$\Rightarrow 3k = 2 \Rightarrow k = \frac{2}{3}$$

Therefore, the value of $k = \frac{2}{3}$

That is, the coordinates of P are $\left(-3, \frac{2}{3}\right)$.

17. In what ratio is the line segment joining A(2, -3) and B(5, 6) divide by the x-axis? Also, find the coordinates of the pint of division.

Sol:

Let AB be divided by the x-axis in the ratio $k:1$ at the point P.

Then, by section formula the coordination of P are

$$P = \left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1}\right)$$

But P lies on the x-axis; so, its ordinate is 0.

Therefore, $\frac{6k-3}{k+1} = 0$

$$\Rightarrow 6k - 3 = 0 \Rightarrow 6k = 3 \Rightarrow k = \frac{3}{6} \Rightarrow k = \frac{1}{2}$$

Therefore, the required ratio is $\frac{1}{2}:1$, which is same as 1 : 2

Thus, the x-axis divides the line AB li the ratio 1 : 2 at the point P.

Applying $k=\frac{1}{2}$, we get the coordinates of point.

$$P\left(\frac{5k+1}{k+1},0\right)$$

$$=P\left(\frac{5\times\frac{1}{2}+2}{\frac{1}{2}+1},0\right)$$

$$=p\left(\frac{\frac{5+4}{2}}{\frac{5+2}{2}},0\right)$$

$$=P\left(\frac{9}{3},0\right)$$

$$=P(3,0)$$

Hence, the point of intersection of AB and the x-axis is $P(3,0)$

18. In what ratio is the line segment joining the points A(-2, -3) and B(3,7) divided by the y-axis? Also, find the coordinates of the point of division.

Sol:

Let AB be divided by the x-axis in the ratio $k:1$ at the point P.

Then, by section formula the coordination of P are

$$P=\left(\frac{3k-2}{k+1},\frac{7k-3}{k+1}\right)$$

But P lies on the y-axis; so, its abscissa is 0.

Therefore, $\frac{3k-2}{k+1}=0$

$$\Rightarrow 3k-2=0\Rightarrow 3k=2\Rightarrow k=\frac{2}{3}\Rightarrow k=\frac{2}{3}$$

Therefore, the required ratio is $\frac{2}{3}:1$, which is same as 2 : 3

Thus, the x-axis divides the line AB in the ratio 2 : 3 at the point P.

Applying $k=\frac{2}{3}$, we get the coordinates of point.

$$P\left(0,\frac{7k-3}{k+1}\right)$$

$= P\left(0, \dfrac{7 \times \frac{2}{3} - 3}{\frac{2}{3} + 1}\right)$

$= p\left(0, \dfrac{\frac{14-9}{3}}{\frac{2+3}{3}}\right)$

$= P\left(0, \dfrac{5}{5}\right)$

$= P(0,1)$

Hence, the point of intersection of AB and the x-axis is $P(0,1)$.

19. In what ratio does the line $x - y - 2 = 0$ divide the line segment joining the points $A(3,-1)$ *and* $B(8,9)$?

Sol:

Let the line $x - y - 2 = 0$ divide the line segment joining the points $A(3,-1)$ and $B(8,9)$ in the ratio $k:1$ at P.

Then, the coordinates of P are

$P\left(\dfrac{8k+3}{k+1}, \dfrac{9k-1}{k+1}\right)$

Since, P lies on the line $x - y - 2 = 0$, we have:

$\left(\dfrac{8k+3}{k+1}\right) - \left(\dfrac{9k-1}{k+1}\right) - 2 = 0$

$\Rightarrow 8k + 3 - 9k + 1 - 2k - 2 = 0$

$\Rightarrow 8k - 9k - 2k + 3 + 1 - 2 = 0$

$\Rightarrow -3k + 2 = 0$

$\Rightarrow -3k = -2$

$\Rightarrow k = \dfrac{2}{3}$

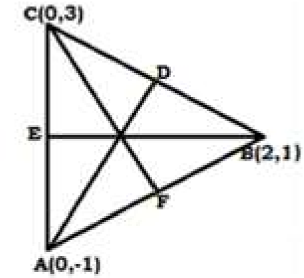
So, the required ratio is $\dfrac{2}{3}:1$, which is equal to $2:3$.

20. Find the lengths of the medians of a ΔABC whose vertices are $A(0,-1), B(2,1)$ and $C(0,3)$.

Sol:

The vertices of ΔABC are $A(0,-1), B(2,1)$ and $C(0,3)$.

Let AD, BE and CF be the medians of ΔABC.



Let D be the midpoint of BC. So, the coordinates of D are

$D\left(\frac{2+0}{2},\frac{1+3}{2}\right)$ i.e. $D\left(\frac{2}{2},\frac{4}{2}\right)$ i.e. $D(1,2)$

Let E be the midpoint of AC. So the coordinate of E are

$E\left(\frac{0+0}{2},\frac{-1+3}{2}\right)$ i.e. $E\left(\frac{0}{2},\frac{0}{2}\right)$ i.e. $E(0,1)$

Let F be the midpoint of AB. So, the coordinates of F are

$F\left(\frac{0+2}{2},\frac{-1+1}{2}\right)$ i.e. $F\left(\frac{2}{2},\frac{0}{2}\right)$ i.e. $F(1,0)$

$AD=\sqrt{(1-0)^2+(2-(-1))^2}=\sqrt{(1)^2+(3)^2}=\sqrt{1+9}=\sqrt{10}$ *units.*

$BE=\sqrt{(0-2)^2+(1-1)^2}=\sqrt{(-2)^2+(0)^2}=\sqrt{4+0}=\sqrt{4}=2$ *units.*

$CF=\sqrt{(1-0)^2+(0-3)^2}=\sqrt{(1)^2+(-3)^2}=\sqrt{1+9}=\sqrt{10}$ *units.*

Therefore, the lengths of the medians: $AD=\sqrt{10}$ *units*, $BE=2$ *units and* $CF=\sqrt{10}$ *units*.

21. Find the centroid of ΔABC whose vertices are A(-1, 0) B(5, -2) and C(8,2)

Sol:

Here, $(x_1=-1, y_1=0),(x_2=5, y_2=-2)$ *and* $(x_3=8, y_3=2)$

Let $G(x,y)$ be the centroid of the ΔABC. Then,

$x=\frac{1}{3}(x_1+x_2+x_3)=\frac{1}{3}(-1+5+8)=\frac{1}{3}(12)=4$

$y=\frac{1}{3}(y_1+y_2+y_3)=\frac{1}{3}(0-2+2)=\frac{1}{3}(0)=0$

Hence, the centroid of ΔABC is $G(4,0)$.

22. If G(-2, 1) is the centroid of a ΔABC and two of its vertices are A(1, -6) and B(-5, 2) , find the third vertex of the triangle.

Sol:

Two vertices of ΔABC are $A(1,-6)$ and $B(-5,2)$. Let the third vertex be C(a, b).

Then the coordinates of its centroid are

$$C\left(\frac{1-5+a}{3},\frac{-6+2+b}{3}\right)$$

$$C\left(\frac{-4+a}{3},\frac{-4+b}{3}\right)$$

But it is given that $G(-2,1)$ is the centroid. Therefore,

$$-2=\frac{-4+a}{3},1=\frac{-4+b}{3}$$

$$\Rightarrow -6=-4+a,3=-4+b$$

$$\Rightarrow -6+4=a,3+4=b$$

$$\Rightarrow a=-2,b=7$$

Therefore, the third vertex of ΔABC is $C(-2,7)$.

23. Find the third vertex of a ΔABC if two of its vertices are $B(-3,1)$ and $C(0,-2)$, and its centroid is at the origin

Sol:

Two vertices of ΔABC are $B(-3,1)$ and $C(0,-2)$. Let the third vertex be $A(a, b)$.

Then, the coordinates of its centroid are

$$\left(\frac{-3+0+a}{3},\frac{1-2+b}{3}\right)$$

i.e., $\left(\frac{-3+a}{3},\frac{-1+b}{3}\right)$

But it is given that the centroid is at the origin, that is $G(0,0)$. Therefore

$$0=\frac{-3+a}{3},0=\frac{-1+b}{3}$$

$$\Rightarrow 0=-3+a,0=-1+b$$

$$\Rightarrow 3=a,1=b$$

$$\Rightarrow a=3,b=1$$

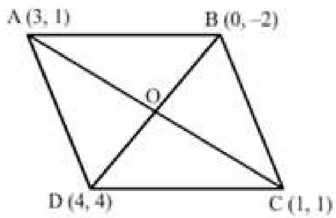
Therefore, the third vertex of ΔABC is $A(3,1)$.

24. Show that the points $A(3,1), B(0,-2), C(1,1)$ and $D(4,4)$ are the vertices of parallelogram ABCD.

Sol:

The points are $A(3,1), B(0,-2), C(1,1)$ and $D(4,4)$

Join *AC* and *BD*, intersecting at *O*.



We know that the diagonals of a parallelogram bisect each other.

Midpoint of $AC = \left(\frac{3+1}{2}, \frac{1+1}{2}\right) = \left(\frac{4}{2}, \frac{2}{2}\right) = (2,1)$

Midpoint of $BD = \left(\frac{0+4}{2}, \frac{-2+4}{2}\right) = \left(\frac{4}{2}, \frac{2}{2}\right) = (2,1)$

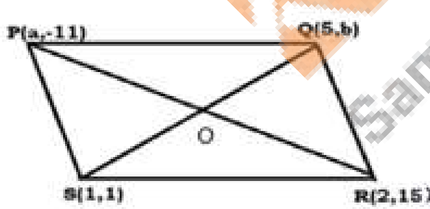
Thus, the diagonals *AC* and *BD* have the same midpoint

Therefore, *ABCD* is a parallelogram.

25. If the points $P(a,-11), Q(5,b), R(2,15)$ *and* $S(1,1)$ are the vertices of a parallelogram PQRS, find the values of a and b.

Sol:

The points are $P(a,-11), Q(5,b), R(2,15)$ *and* $S(1,1)$.



Join *PR* and Q*S*, intersecting at *O*.

We know that the diagonals of a parallelogram bisect each other

Therefore, *O* is the midpoint of *PR* as well as *QS*.

Midpoint of $PR = \left(\frac{a+2}{2}, \frac{-11+15}{2}\right) = \left(\frac{a+2}{2}, \frac{4}{2}\right) = \left(\frac{a+2}{2}, 2\right)$

Midpoint of $QS = \left(\frac{5+1}{2}, \frac{b+1}{2}\right) = \left(\frac{6}{2}, \frac{b+1}{2}\right) = \left(3, \frac{b+1}{2}\right)$

Therefore, $\frac{a+2}{2} = 3, \frac{b+1}{2} = 2$

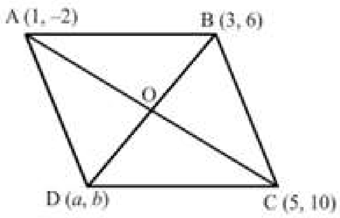
$\Rightarrow a+2=6, b+1=4$
$\Rightarrow a=6-2, b=4-1$
$\Rightarrow a=4 \ and \ b=3$

26. If three consecutive vertices of a parallelogram $ABCD$ are $A(1,-2), B(3,6)$ *and* $C(5,10)$, find its fourth vertex *D*.

Sol:

Let $A(1,-2), B(3,6)$ *and* $C(5,10)$ be the three vertices of a parallelogram *ABCD* and the fourth vertex be $D(a,b)$.

Join *AC* and *BD* intersecting at *O*.



We know that the diagonals of a parallelogram bisect each other

Therefore, *O* is the midpoint of *AC* as well as *BD*.

Midpoint of $AC = \left(\frac{1+5}{2}, \frac{-2+10}{2}\right) = \left(\frac{6}{2}, \frac{8}{2}\right) = (3,4)$

Midpoint of $BD = \left(\frac{3+a}{2}, \frac{6+b}{2}\right)$

Therefore, $\frac{3+a}{2} = 3$ and $\frac{6+b}{2} = 4$

$\Rightarrow 3+a=6 \ and \ 6+b=8$
$\Rightarrow a=6-3 \ and \ b=8-6$
$\Rightarrow a=3 \ and \ b=2$

Therefore, the fourth vertex is $D(3,2)$.

27. In what ratio does y-axis divide the line segment joining the points (-4, 7) and (3, -7)?

Sol:

Let *y*-axis divides the e segment pining the points $(-4,7)$ and $(3,-7)$ in the ratio $k:1$. Then

$0 = \frac{3k-4}{k+1}$
$\Rightarrow 3k = 4$
$\Rightarrow k = \frac{4}{3}$

Hence, the required ratio is $4:3$

28. If the point $P\left(\frac{1}{2}, y\right)$ lies on the line segment joining the points A(3, -5) and B(-7, 9) then find the ratio in which P divides AB. Also, find the value of y.

Sol:

Let the point $P\left(\frac{1}{2}, y\right)$ divides the line segment joining the points $A(3,-5)$ *and* $B(-7,9)$ in the ratio $k:1$. Then

$$\left(\frac{1}{2}, y\right) = \left(\frac{k(-7)+3}{k+1}, \frac{k(9)-3}{k+1}\right)$$

$$\Rightarrow \frac{-7k+3}{k+1} = \frac{1}{2} \text{ and } \frac{9k-5}{k+1} = y$$

$$\Rightarrow k+1 = -14k+6 \Rightarrow k = \frac{1}{3}$$

Now, substituting $k = \frac{1}{3}$ in $\frac{9k-5}{k+1} = y$, we get

$$\frac{\frac{9}{3}-5}{\frac{1}{3}+1} = y \Rightarrow y = \frac{9-15}{1+3} = -\frac{3}{2}$$

Hence, required ratio is $1:3$ and $y = -\frac{3}{2}$.

29. Find the ratio which the line segment joining the pints A(3, -3) and B(-2,7) is divided by x-axis Also, find the point of division.

Sol:

The line segment joining the points $A(3,-3)$ *and* $B(-2,7)$ is divided by *x*-axis. Let the required ratio be $k:1$. So,

$$0 = \frac{k(7)-3}{k+1} \Rightarrow k = \frac{3}{7}$$

Now,

$$\text{Point of division} = \left(\frac{k(-2)+3}{k+1}, \frac{k(7)-3}{k+1}\right)$$

$$= \left(\frac{\frac{3}{7}\times(-2)+3}{\frac{3}{7}+1}, \frac{\frac{3}{7}\times(7)-3}{\frac{3}{7}+1}\right) \qquad \left(\because k = \frac{3}{7}\right)$$

$$= \left(\frac{-6+21}{3+7}, \frac{21-21}{3+7} \right)$$

$$= \left(\frac{3}{2}, 0 \right)$$

Hence, the required ratio is $3:7$ and the point of division is $\left(\frac{3}{2}, 0 \right)$

30. The base QR of a n equilateral triangle PQR lies on x-axis. The coordinates of the point Q are (-4, 0) and origin is the midpoint of the base. Find the coordinates of the points P and R.

Sol:

Let $(x, 0)$ be the coordinates of R. Then

$$0 = \frac{-4+x}{2} \Rightarrow x = 4$$

Thus, the coordinates of R are $(4, 0)$.

Here, $PQ = QR = PR$ and the coordinates of P lies on $y-axis$. Let the coordinates of P be $(0, y)$. Then,

$$PQ = QR \Rightarrow PQ^2 = QR^2$$

$$\Rightarrow (0+4)^2 + (y-0)^2 = 8^2$$

$$\Rightarrow y^2 = 64 - 16 = 48$$

$$\Rightarrow y = \pm 4\sqrt{3}$$

Hence, the required coordinates are $R(4,0)$ *and* $P(0, 4\sqrt{3})$ *or* $P(0, -4\sqrt{3})$.

31. The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point C are (0, -3). The origin is the midpoint of the base. Find the coordinates of the points A and B. Also, find the coordinates of another point D such that ABCD is a rhombus.

Sol:

Let $(0, y)$ be the coordinates of B. Then

$$0 = \frac{-3+y}{2} \Rightarrow y = 3$$

Thus, the coordinates of B are $(0, 3)$

Here. $AB = BC = AC$ and by symmetry the coordinates of A lies on x-axis Let the coordinates of A be $(x, 0)$. Then

$$AB = BC \Rightarrow AB^2 = BC^2$$

$$\Rightarrow (x-0)^2 + (0-3)^2 = 6^2$$

$\Rightarrow x^2 = 36 - 9 = 27$

$\Rightarrow x = \pm 3\sqrt{3}$

If the coordinates of point A are $\left(3,\sqrt{3},0\right)$. then the coordinates of D are $\left(-3\sqrt{3},0\right)$.

If the coordinates of point A are $\left(-3\sqrt{3},0\right)$. then the coordinates of D are $\left(-3\sqrt{3},0\right)$.

Hence the required coordinates are $A\left(3\sqrt{3},0\right), B\left(0,3\right)$ and $D\left(-3\sqrt{3},0\right)$ or

$A\left(-3\sqrt{3},0\right), B\left(0,3\right)$ *and* $D\left(3\sqrt{3},0\right)$.

32. Find the ratio in which the point (-1, y) lying on the line segment joining points A(-3, 10) and (6, -8) divides it. Also, find the value of y.

Sol:

Let k be the ratio in which $P\left(-1, y\right)$ divides the line segment joining the points $A\left(-3,10\right)$ *and* $B\left(6,-8\right)$

Then,

$$\left(-1, y\right) = \left(\frac{k\left(6\right)-3}{k+1}, \frac{k\left(-8\right)+10}{k+1}\right)$$

$$\Rightarrow \frac{k\left(6\right)-3}{k+1} = -1 \text{ and } y = \frac{k\left(-8\right)+10}{k+1}$$

$$\Rightarrow k = \frac{2}{7}$$

Substituting $k = \frac{2}{7}$ in $y = \frac{k\left(-8\right)+10}{k+1}$, we get

$$y = \frac{\frac{-8\times 2}{7}+10}{\frac{2}{7}+1} = \frac{-16+70}{9} = 6$$

Hence, the required ratio is $2:7$ *and* $y = 6$.

33. ABCD is rectangle formed by the points A(-1, -1), B(-1, 4), C(5, 4) and D(5, -1). If P,Q,R and S be the midpoints of AB, BC, CD and DA respectively, Show that PQRS is a rhombus.

Sol:

Here, the points P, Q, R and S are the midpoint of AB, BC, CD *and* DA respectively. Then

Coordinates of $P = \left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$

Coordinates of $Q = \left(\frac{-1+5}{2}, \frac{4+4}{2}\right) = (2,4)$

Coordinates of $R = \left(\frac{5+5}{2}, \frac{4-1}{2}\right) = \left(5, \frac{3}{2}\right)$

Coordinates of $S = \left(\frac{-1+5}{2}, \frac{-1-1}{2}\right) = (2,-1)$

Now,

$$PQ = \sqrt{(2+1)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$QR = \sqrt{(5-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$RS = \sqrt{(5-2)^2 + \left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$SP = \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$PR = \sqrt{(5-1)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = \sqrt{36} = 6$$

$$QS = \sqrt{(2-2)^2 + (-1-4)^2} = \sqrt{25} = 5$$

Thus, $PQ = QR = RS = SP$ and $PR \neq QS$ therefore $PQRS$ is a rhombus.

34. The midpoint P of the line segment joining points A(-10, 4) and B(-2, 0) lies on the line segment joining the points C(-9, -4) and D(-4, y). Find the ratio in which P divides CD. Also, find the value of y.

Sol:

The midpoint of AB is $\left(\frac{-10-2}{2}, \frac{4+10}{2}\right) = P(-6,2)$.

Let k be the ratio in which P divides CD. So

$$(-6,2) = \left(\frac{k(-4)-9}{k+1}, \frac{k(y)-4}{k+1}\right)$$

$$\Rightarrow \frac{k(-4)-9}{k+1} = -6 \text{ and } \frac{k(y)-4}{k+1} = 2$$

$$\Rightarrow k = \frac{3}{2}$$

Now, substituting $k = \frac{3}{2}$ in $\frac{k(y)-4}{k+1} = 2,$ we get

$$\frac{y \times \frac{3}{2} - 4}{\frac{3}{2} + 1} = 2$$

$$\Rightarrow \frac{3y-8}{5} = 2$$

$$\Rightarrow y = \frac{10+8}{3} = 6$$

Hence, the required ratio is $3:2$ and $y = 6.$