Exercise – 11D

1. Find the sum of each of the following Aps: (i) 2, 7, 12, 17, to 19 terms. (ii) 9, 7, 5, 3 ... to 14 terms (iii) -37, -33, -29, ... to 12 terms. (iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$ to 11 terms. (v) 0.6, 1.7, 2.8, to 100 terms Sol: The given AP is 2, 7, 12, 17,..... (i) Here, a = 2 and d = 7 - 2 = 5Using the formula, $S_n = \frac{n}{2} [2a + (n-1)d]$, we have $S_{19} = \frac{19}{2} \left[2 \times 2 + (19 - 1) \times 5 \right]$ $=\frac{19}{2} \times (4+90)$ $=\frac{19}{2} \times 94$ = 893The given AP is 9, 7, 5, 3,.... (ii) Here, a = 9 and d = 7 - 9 = -2Using the formula, $S_n = \frac{n}{2} [2a + (n-1)d]$, we have $S_{14} = \frac{14}{2} \Big[2 \times 9 + (14 - 1) \times (-2) \Big]$ $=7 \times (18 - 26)$ $=7\times(-8)$ = -56(iii) The given AP is -37, -33, -29,..... Here, a = -37 and d = -33 - (-37) = -33 + 37 = 4Using the formula, $S_n = \frac{n}{2} [2a + (n-1)d]$, we have $S_{12} = \frac{12}{2} \left[2 \times (-37) + (12 - 1) \times 4 \right]$ $=6 \times (-74 + 44)$ $=6 \times (-30)$

= -180(iv) The given AP is $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$ Here, $a = \frac{1}{15}$ and $d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$ Using the formula, $S_n = \frac{n}{2} [2a + (n-1)d]$, we have $S_{11} = \frac{11}{2} \left[2 \times \left(\frac{1}{15} \right) + (11 - 1) \times \frac{1}{60} \right]$ $=\frac{11}{2} \times \left(\frac{2}{15} + \frac{10}{60}\right)$ $=\frac{11}{2}\times\left(\frac{18}{60}\right)$... is 0.6,1.7,2.8,..... riere, a = 0.6 and d = 1.7 - 0.6 = 1.1Using formula, $S_n = \frac{n}{2} [2a + (n-1)d]$, we have $S_{100} = \frac{100}{2} [2 \times 0.6 + (100 - 1) \times 1.1]$ $= 50 \times (1.2 + 108.9)$ $= 50 \times 110.1$ 5505sum of each $c^{0.1}$ (v)

Find the sum of each of the following arithmetic series: 2.

(i)
$$7+10\frac{1}{2}+14+...+84$$

(ii) $34+32+30+...+10$
(iii) $(-5)+(-8)+(-11)+...+(-230)$
Sol:

(i) The given arithmetic series is
$$7+10\frac{1}{2}+14+\dots+84$$
.

Here,
$$a = 7, d = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{21-4}{2} = \frac{7}{2}$$
 and $l = 84$

Let the given series contains n terms. Then,

	$a_n = 84$
	$\Rightarrow 7 + (n-1) \times \frac{7}{2} = 84 \qquad \left[a_n = a + (n-1)d\right]$
	$\Rightarrow \frac{7}{2}n + \frac{7}{2} = 84$
	$\Rightarrow \frac{7}{2}n = 84 - \frac{7}{2} = \frac{161}{2}$
	$\Rightarrow n = \frac{161}{7} = 23$
	$\therefore \text{Required sum} = \frac{23}{2} \times (7+84) \qquad \left[S_n = \frac{n}{2}(a+l)\right]$
	$=\frac{23}{2} \times 91$
	$=\frac{2030}{2}$
	$1046\frac{1}{2}$
(ii)	The given arithmetic series is $34 + 32 + 30 + \dots + 10$.
	Here, $a = 34, d = 32 - 34 = -2$ and $l = 10$.
	Let the given series contain <i>n</i> terms. Then,
	$a_n = 10$
	$\Rightarrow 34 + (n-1) \times (-2) = 10 \qquad \qquad \boxed{a_n = a + (n-1)d}$
	$\Rightarrow -2n + 36 = 10$
	$\Rightarrow -2n = 10 - 36 = -26$
	$\Rightarrow n = 13$
	$\therefore \text{Required sum} = \frac{13}{2} \times (34+10) \qquad \left[S_n = \frac{n}{2}(a+l)\right]$
	$=\frac{13}{2}\times44$
	= 286
(iii)	The given arithmetic series is $(-5)+(-8)+(-11)++(-230)$.

(iii) The given arithmetic series is $(-5)+(-8)+(-11)+\dots+(-230)$ Here, a = -5, d = -8-(-5) = -8+5 = -3 and l = 230. Let the given series contain *n* terms. Then,

$$a_{n} = -230$$

$$\Rightarrow -5 + (n-1) \times (-3) = -230 \qquad [a_{n} = a + (n-1)d]$$

$$\Rightarrow -3n - 2 = -230$$

$$\Rightarrow -3n = -230 + 2 = -228$$

$$\Rightarrow n = 76$$

$$\therefore \text{ Required sum} = \frac{76}{2} \times [(-5) + (-230)] \qquad [S_{n} = \frac{n}{2}(a+l)]$$

$$= \frac{76}{2} \times (-235)$$

$$= -8930$$

Find the sum of first n terms of an AP whose nth term is (5 - 6n). Hence, find the sum of its 3. first 20 terms.

$$\therefore a_n = 5 - 6n$$

$$a_2 = 5 - 6 \times 2 = -$$

Find the sum of first in terms of an AP whose function is (3 - 6n). Hence, find
first 20 terms.
Sol:
Let
$$a_n$$
 be the nth term of the AP.
 $\therefore a_n = 5 - 6n$
Putting $n = 1$, we get
First term, $a = a_1 = 5 - 6 \times 1 = -1$
Putting $n = 2$, we get
 $a_2 = 5 - 6 \times 2 = -7$
Let d be the common difference of the AP.
 $\therefore d = a_2 - a_1 = -7 - (-1) = -7 + 1 = -6$
Sum of first n term of the AP, S_n
 $= \frac{n}{2} [2 \times (-1) + (n-1) \times (-6)]$ $\{S_n = \frac{n}{2} [2a + (n-1)d]\}$
 $= \frac{n}{2} (-2 - 6n + 6)$
 $= n(2 - 3n)$
 $= 2n - 3n^2$
Putting $n = 20$, we get
 $S_{20} = 2 \times 20 - 3 \times 20^2 = 40 - 1200 = -1160$

The sum of the first n terms of an AP is $(3n^2 + 6n)$. Find the nth term and the 15th term of this 4. AP. Sol:

Let S_n denotes the sum of first *n* terms of the AP.

$$\therefore S_n = 3n^2 + 6n$$

$$\Rightarrow S_{n-1} = 3(n-1)^2 + 6(n-1)$$

$$= 3(n^2 - 2n + 1) + 6(n-1)$$

$$= 3n^2 - 3$$

$$\therefore n^{th} \text{ term of the AP, } a_n$$

$$= S_n - S_{n-1}$$

$$= (3n^2 + 6n) - (3n^2 - 3)$$

$$= 6n + 3$$

Putting $n = 15$, we get
 $a_{15} = 6 \times 15 + 3 = 90 + 3 = 93$
Hence, the n^{th} term is $(6n + 3)$ and 15^{th} term is 93

5.

Hence, the
$$n^{th}$$
 term is $(6n+3)$ and 15^{th} term is 93.
The sum of the first n terms of an AP is given by $S_n = (3n^2 - n)$. Find its
(i) nth term,
(ii) first term and
(iii) common difference.
Sol:
Given: $S_n = (3n^2 - n)$ (*i*)
Replacing *n* by $(n-1)$ in (i), we get:
 $S_{n-1} = 3(n-1)^2 - (n-1)$
 $= 3(n^2 - 2n + 1) - n + 1$
 $= 3n^2 - 7n + 4$
(i) Now, $T_n = (S_n - S_{n-1})$
 $= (3n^2 - n) - (3n^2 - 7n + 4) = 6n - 4$
 $\therefore n^{th}$ term, $T_n = (6n - 4)$ (*ii*)
(ii) Putting $n = 1$ in (ii), we get:
 $T_1 = (6 \times 1) - 4 = 2$
(iii) Putting $n = 2$ in (ii), we get:
 $T_2 = (6 \times 2) - 4 = 8$
 \therefore Common difference, $d = T_2 - T_1 = 8 - 2 = 6$

The sum of the first n terms of an AP is $\left(\frac{5n^2}{2} + \frac{3n}{2}\right)$. Find its nth term and the 20th term of 6.

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this AP. Sol:

 $S_n = \left(\frac{5n^2}{2} + \frac{3n}{2}\right) = \frac{1}{2}(5n^2 + 3n) \qquad \dots \dots (i)$

Replacing *n* by (n-1) in (i), we get:

$$S_{n-1} = \frac{1}{2} \times \left[5(n-1)^2 + 3(n-1) \right]$$

= $\frac{1}{2} \times \left[5n^2 - 10n + 5 + 3n - 3 \right] = \frac{1}{2} \times \left[5n^2 - 7n + 2 \right]$
 $\therefore T_n = S_n - S_{n-1}$
= $\frac{1}{2} (5n^2 + 3n) - \frac{1}{2} \times \left[5n^2 - 7n + 2 \right]$
= $\frac{1}{2} (10n - 2) = 5n - 1$ (*ii*)
Putting $n = 20$ in (ii), we get
 $T_{20} = (5 \times 20) - 1 = 99$
Hence, the 20th term is 99.

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Hence, the 20th term is 99.

 $3n^2$ $\left(\frac{5n}{2}\right)$. Find its nth term and the 25th term The sum of the first n term sofa an AP is 7.

Sol:

Let S_n denotes the sum of first *n* terms of the AP.

$$\therefore S_n = \frac{3n^2}{2} + \frac{5n}{2}$$

$$\Rightarrow S_{n-1} = \frac{3(n-1)^2}{2} + \frac{5(n-1)^2}{2}$$

$$= \frac{3(n^2 - 2n + 1)}{2} + \frac{5(n-1)}{2}$$

$$= \frac{3n^2 - n - 2}{2}$$

$$\therefore n^{th} \text{ term of the AP, } a_n$$

$$= S_n - S_{n-1}$$

$$= \left(\frac{3n^2 + 5n}{2}\right) - \left(\frac{3n^2 - n - 2}{2}\right)$$
$$= \frac{6n + 2}{2}$$
$$= 3n + 2$$
Putting $n = 25$, we get
$$a_{25} = 3 \times 25 + 1 = 75 + 1 = 76$$

Hence, the nth term is (3n+1) and 25^{th} term is 76.

8. How many terms of the AP 21, 18, 15, ... must be added to get the sum 0? Sol:

The given AP is 21, 18, 15,.... Here, a = 21 and d = 18 - 21 = -3Let the required number of terms be n. Then, $S_n = 0$ $\Rightarrow \frac{n}{2} [2 \times 21 + (n-1) \times (-3)] = 0$ $\left\{ S_n = \frac{n}{2} [2a + (n-1)d] \right\}$ $\Rightarrow \frac{n}{2} (42 - 3n + 3) = 0$ $\Rightarrow n (45 - 3n) = 0$ $\Rightarrow n = 0 \text{ or } 45 - 3n = 0$ $\Rightarrow n = 0 \text{ or } n = 15$ $\therefore n = 15$ (Number of terms cannot be zero) Hence, the required number of terms is 15.

9. How many terms of the AP 9, 17, 25, ... must be taken so that their sum is 636? Sol:

The given AP is 9, 17, 25,..... Here, a = 9 and d = 17 - 9 = 8Let the required number of terms be n. Then, $S_n = 636$ $\Rightarrow \frac{n}{2} [2 \times 9 + (n-1) \times 8] = 636$ $\begin{cases} S_n = \frac{n}{2} [2a + (n-1)d] \\ \\ \\ \end{cases}$ $\Rightarrow \frac{n}{2} (18 + 8n - 8) = 636$ $\Rightarrow \frac{n}{2} (10 + 8n) = 636$

$$\Rightarrow n(5+4n) = 636$$

$$\Rightarrow 4n^{2} + 5n - 636 = 0$$

$$\Rightarrow 4n^{2} - 48n + 53n - 636 = 0$$

$$\Rightarrow 4n(n-12) + 53(n-12) = 0$$

$$\Rightarrow (n-12)(4n + 53) = 0$$

$$\Rightarrow n - 12 = 0 \text{ or } 4n + 53 = 0$$

$$\Rightarrow n = 12 \text{ or } n = -\frac{53}{4}$$

$$\therefore n = 12$$
 (Number of terms cannot negative)
Hence, the required number of terms is 12.
10. How many terms of the AP 63, 60, 57, 54, must be taken so that their sum is 693?
Explain the double answer.
Sol:
The given AP is 63, 60, 57, 54, must be taken so that their sum is 693?
Explain the double answer.
Sol:
The given AP is 63, 60, 57, 54,
Here, $a = 63$ and $d = 60 - 63 = -3$
Let the required number of terms be *n*. Then,
 $S_{n} = 693$

$$\Rightarrow \frac{n}{2} [2 \times 63 + (n-1) \times (-3)] = 693$$

$$\Rightarrow n(129 - 3n) = 1386$$

$$\Rightarrow 3n^{2} - 129n + 1386 = 0$$

$$\Rightarrow 3n(2-2) - 63(n-22) = 0$$

$$\Rightarrow (n-22)(3n-63) = 0$$

$$\Rightarrow n - 22 = 0 \text{ or } 3n - 63 = 0$$

$$\Rightarrow n = 22 \text{ or } n = 21$$

So, the sum of 21 terms as well as that of 22 terms is 693. This is because the 22nd term of the AP is 0.

$$a_{22} = 63 + (22 - 1) \times (-3) = 63 - 63 = 0$$

Hence, the required number of terms is 21 or 22.

11. How many terms of the AP $20,19\frac{1}{3},18\frac{2}{3},\dots$ must be taken so that their sum is 300? Explain

the double answer.

Sol:

The given AP is 20,
$$19\frac{1}{3}, 18\frac{2}{3}, \dots$$

Here, $a = 20$ and $d = 19\frac{1}{3} - 20 = \frac{58}{3} - 20 = \frac{58 - 60}{3} = -\frac{2}{3}$

Let the required number of terms be n. Then,

$$S_{n} = 300$$

$$\Rightarrow \frac{n}{2} \left[2 \times 20 + (n-1) \times \left(-\frac{2}{3} \right) \right] = 300 \qquad \left\{ S_{n} = \frac{n}{2} \left[2a + (n-1)d \right] \right\}$$

$$\Rightarrow \frac{n}{2} \left(40 - \frac{2}{3}n + \frac{2}{3} \right) = 300$$

$$\Rightarrow \frac{n}{2} \times \frac{(122 - 2n)}{3} = 300$$

$$\Rightarrow 122n - 2n^{2} = 1800$$

$$\Rightarrow 2n^{2} - 122n + 1800 = 0$$

$$\Rightarrow 2n^{2} - 50n - 72n + 1800 = 0$$

$$\Rightarrow 2n(n-25) - 72(n-25) = 0$$

$$\Rightarrow (n-25)(2n-72) = 0$$

$$\Rightarrow n-25 = 0 \text{ or } 2n - 72 = 0$$

$$\Rightarrow n = 25 \text{ or } n = 36$$

So, the sum of first 25 terms as well as that of first 36 terms is 300. This is because the sum of all terms from 26^{th} to 36^{th} is 0.

12. Find the sum of all odd numbers between 0 and 50.Sol:

 $\therefore \text{Required sum} = \frac{n}{2}(a+l)$ $= \frac{25}{2}[1+49] = 25 \times 25 = 625$

$$=\frac{-1}{2}[1+49] = 25 \times 25 = 625$$

Hence, the required sum is 625.

13. Find the sum of all natural numbers between 200 and 400 which are divisible by 7.Sol:

Natural numbers between 200 and 400 which are divisible by 7 are 203, 210 ,.... 399. This is an AP with a = 203, d = 7 and l = 399.

Suppose there are n terms in the AP. Then,

$$a_n = 399$$

$$\Rightarrow 203 + (n-1) \times 7 = 399 \qquad [a_n = a + (n-1)d]$$

$$\Rightarrow 7n + 196 = 399$$

$$\Rightarrow 7n = 399 - 196 = 203$$

$$\Rightarrow n = 29$$

$$\therefore \text{ Required sum } = \frac{29}{2}(203 + 399) \qquad [S_n = \frac{n}{2}(a+1)]$$

$$= \frac{29}{2} \times 602$$

$$= 8729$$
Hence, the required sum is 8729.
Find the sum of first forty positive integers divisible by 6.
Sol:
The positive integers divisible by 6 are 6, 12, 18,.....

This is an AP with a = 6 and d = 6. Also, n = 40 (Given)

Using the formula, $S_n = \frac{n}{2} [2a + (n-1)d]$, we get

$$S_{40} = \frac{40}{2} [2 \times 6 + (40 - 1) \times 6]$$

= 20(12 + 234)
= 20 \times 246
= 4920

Hence, the required sum is 4920.

14.

Find the sum of first 15 multiples of 8. 15. Sol: The first 15 multiples of 8 are 8, 16, 24, 32,..... This is an AP in which a = 8, d = (16-8) = 8 and n = 15. Thus, we have: l = a + (n-1)d=8+(15-1)8=120 \therefore Required sum $=\frac{n}{2}(a+l)$ $=\frac{15}{2}[8+120]=15\times 64=960$ Hence, the required sum is 960. Find the sum of all multiples of 9 lying between 300 and 700. 16. Sol: The multiples of 9 lying between 300 and 700 are 306, 315,... ., 693. This is an AP with a = 306, d = 9 and l = 693. Suppose these are *n* terms in the AP. Then, $a_n = 693$ $\int a_n = a + (n)$ \Rightarrow 306 + (n-1) × 9 = 693 $\Rightarrow 9n + 297 = 693$ $\Rightarrow 9n = 693 - 297 = 396$ $\Rightarrow n = 44$ $\left\lceil S_n = \frac{n}{2} \left(a + l \right) \right\rceil$ \therefore Required sum = $\frac{44}{2}(306 = 693)$ $= 22 \times 999$ = 21978Hence, the required sum is 21978. 17. Find the sum of all three-digits natural numbers which are divisible by 13.

Sol: All three-digit numbers which are divisible by 13 are 104, 117, 130, 143,...... 938. This is an AP in which a = 104, d = (117 - 104) = 13 and l = 938Let the number of terms be n Then $T_n = 938$ $\Rightarrow a + (n-1)d = 988$ $\Rightarrow 104 + (n-1) \times 13 = 988$ $\Rightarrow 13n = 897$ $\Rightarrow n = 69$ $\therefore \text{ Required sum} = \frac{n}{2}(a+l)$ $= \frac{69}{2}[104 + 988] = 69 \times 546 = 37674$ Hence, the required sum is 37674.

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18. Find the sum of first 100 even number which are divisible by 5.

Sol:

The first few even natural numbers which are divisible by 5 are 10, 20, 30, 40, ... This is an AP in which a = 10, d = (20 - 10) = 10 and n = 100The sum of n terms of an AP is given by

$$S_{n} = \frac{n}{2} \Big[2a + (n-1)d \Big]$$

= $\Big(\frac{100}{2}\Big) \times \Big[2 \times 10 + (100-1) \times 10 \Big]$ [:: $a = 10, d = 10$ and $n = 100$]
= $50 \times \Big[20 + 990 \Big] = 50 \times 1010 = 50500$

Hence, the sum of the first hundred even natural numbers which are divisible by 5 is 50500.

19. Find the sum of the following.

$$\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\dots$$
 up to n terms.

Sol:

On simplifying the given series, we get:

$$\left(1 - \frac{1}{n}\right) + \left(1 - \frac{2}{n}\right) + \left(1 - \frac{3}{n}\right) + \dots n \text{ terms}$$

= $\left(1 + 1 + 1 + \dots n \text{ terms}\right) - \left(\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{n}{n}\right)$
= $n - \left(\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{n}{n}\right)$

Here, $\left(\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{n}{n}\right)$ is an AP whose first term is $\frac{1}{n}$ and the common difference

is $\left(\frac{2}{n} - \frac{1}{n}\right) = \frac{1}{n}$.

The sum of terms of an AP is given by

$$S_{n} = \frac{n}{2} \Big[2a + (n-1)d \Big]$$
$$= n - \left[\frac{n}{2} \Big\{ 2 \times \Big(\frac{1}{n} \Big) + (n-1) \times \Big(\frac{1}{n} \Big) \Big\} \Big]$$
$$= n - \left[\frac{n}{2} \Big[\Big(\frac{2}{n} \Big) + \Big(\frac{n-1}{n} \Big) \Big] \Big] = n - \left\{ \frac{n}{2} \Big(\frac{n+1}{n} \Big) \right\}$$
$$= n - \Big(\frac{n+1}{2} \Big) = \frac{n-1}{2}$$

20. In an AP. It is given that $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of its first n terms.

Sol:

Let a be the first term and d be the common difference of thee AP. Then,

$$S_{5} + S_{7} = 167$$

$$\Rightarrow \frac{5}{2}(2a + 4d) + \frac{7}{2}(2a + 6d) = 167$$

$$\Rightarrow 5a + 10d + 7a + 21d = 167$$

$$\Rightarrow 12a + 31d = 167$$
(1)
Also,
$$S_{10} = 235$$

$$\Rightarrow \frac{10}{2}(2a + 9d) = 235$$

$$\Rightarrow 5(2a + 9d) = 235$$

$$\Rightarrow 5(2a + 9d) = 235$$

$$\Rightarrow 2a + 9d = 47$$
Multiplying both sides by 6, we get
$$12a + 54d = 282$$
(2)
Subtracting (1) from (2), we get
$$12a + 54d - 12a - 31d = 282 - 167$$

$$\Rightarrow 23d = 115$$

$$\Rightarrow d = 5$$
Putting $d = 5$ in (1), we get

 $12a+31\times 5 = 167$ $\Rightarrow 12a+155 = 167$ $\Rightarrow 12a = 167 - 155 = 12$ $\Rightarrow a = 1$ Hence, the AP is 1, 6, 11, 16,.....

21. In an AP, the first term is 2, the last term is 29 and the sum of all the terms is 155. Find the common difference.

Sol:

Here, a = 2, l = 29 and $S_n = 155$

Let d be the common difference of the given AP and n be the total number of terms.

Then,
$$T_n = 29$$

$$\Rightarrow a + (n-1)d = 29$$
$$\Rightarrow 2 + (n-1)d = 29 \qquad \dots$$

$$\Rightarrow 2 + (n-1)d = 29 \qquad \dots \dots (i)$$

The sum of *n* terms of an AP is given by

$$S_n = \frac{n}{2} [a+l] = 155$$
$$\Rightarrow \frac{n}{2} [2+29] = \left(\frac{n}{2}\right) \times 31 = 155$$

 $\Rightarrow n = 10$

Putting the value of n in (i), we get:

$$\Rightarrow 2 + 9d = 29$$

$$\Rightarrow 9d = 27$$

$$\Rightarrow d = 3$$

Thus, the common difference of the given AP is 3.

22. In an AP, the first term is -4, the last term is 29 and the sum of all its terms is 150. Find its common difference.

Sol:

Suppose there are *n* terms in the AP. Here a = -4 l = 29 and S = 150

$$S_{n} = 150$$

$$\Rightarrow \frac{n}{2}(-4+29) = 150$$

$$S_{n} = \frac{150 \times 2}{25} = 12$$

$$S_{n} = \frac{150 \times 2}{25} = 12$$

Thus, the AP contains 12 terms. Let d be the common difference of the AP.

 $\therefore a_{12} = 29$ $\Rightarrow -4 + (12 - 1) \times d = 29 \qquad [a_n = a + (n - 1)d]$ $\Rightarrow 11d = 29 + 4 = 33$ $\Rightarrow d = 3$ Hence, the common difference of the AP is 3.

23. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Suppose there are n terms in the AP. Here, a = 17, d = 9 and l = 350 $\therefore a_n = 350$ $\Rightarrow 17 + (n-1) \times 9 = 350$ $\left[a_n = a + (n-1)d\right]$ $\Rightarrow 9n + 8 = 350$ $\Rightarrow 9n = 350 - 8 = 342$ $\Rightarrow n = 38$ Thus, there are 38 terms in the AP. $\therefore S_{38} = \frac{28}{2}(17 + 350)$ $\left[S_n = \frac{n}{2}(a+l)\right]$ $= 19 \times 367$ = 6973Hence, the required sum is 6973.

24. The first and last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find the common difference and the number of terms.

Sol:

Suppose there are n term in the AP.

Here,
$$a = 5, l = 45$$
 and $S_n = 400$

$$S_n = 400$$

$$\Rightarrow \frac{n}{2}(5+45) = 400 \qquad \left[S_n = \frac{n}{2}(a+l)\right]$$

$$\Rightarrow \frac{n}{2} \times 50 = 400$$

$$\Rightarrow n = \frac{400 \times 2}{50} = 16$$

Thus, there are 16 terms in the AP. Let d be the common difference of the AP. $\therefore a_{16} = 45$ $\Rightarrow 5 + (16 - 1) \times d = 45 \qquad [a_n = a + (n - 1)d]$ $\Rightarrow 15d = 45 - 5 = 40$ $\Rightarrow d = \frac{40}{15} = \frac{8}{3}$

Hence, the common difference of the AP is $\frac{8}{3}$.

25. In an AP, the first term is 22, nth terms is -11 and sum of first n terms is 66. Find the n and hence find the4 common difference.

Sol:

Here, $a = 22, T_n = -11$ and $S_n = 66$ Let d be the common difference of the given AP. Then, $T_n = -11$ $\Rightarrow a + (n-1)dd = 22 + (n-1)d = -11$ $\Rightarrow (n-1)d = -33$ (i) The sum of *n* terms of an AP is given by $S_n = \frac{n}{2} [2a + (n-1)d] = 66$ [Substituting the value of (n-1)d from (i)] $\Rightarrow \frac{n}{2} [2 \times 22 + (-33)] = (\frac{n}{2}) \times 11 = 66$ $\Rightarrow n = 12$ Putting the value of *n* in (i), we get: 11d = -33 $\Rightarrow d = -3$ Thus, n = 12 and d = -3

26. The 12th term of an AP is -13 and the sum of its first four terms is 24. Find the sum of its first 10 terms.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

 $a_{12} = -13$ $\Rightarrow a + 11d = -13 \qquad \dots \dots (1) \qquad \left[a_n = a + (n-1)d\right]$ Also, $S_4 = 24$

$$\Rightarrow \frac{4}{2}(2a+3d) = 24 \qquad \left\{ S_n = \frac{n}{2} \left[2a + (n-1)d \right] \right\}$$

$$\Rightarrow 2a+3d = 12 \qquad \dots \dots (2)$$

Solving (1) and (2), we get

$$2(-13-11d)+3d = 12$$

$$\Rightarrow -26-22d+3d = 12$$

$$\Rightarrow -19d = 12+26 = 38$$

$$\Rightarrow d = -2$$

Putting $d = -2$ in (1), we get
 $a+11\times(-2) = -13$

$$\Rightarrow a = -13+22 = 9$$

$$\therefore$$
 Sum of its first 10 terms, S_{10}

$$= \frac{10}{2} \left[2 \times 9 + (10-1) \times (-2) \right]$$

$$= 5 \times (18-18)$$

$$= 5 \times 0$$

$$= 0$$

Hence, the required sum is 0.

27. The sum of the first 7 terms of an AP is 182. If its 4th and 17th terms are in the ratio 1:5, find the AP.

Sol:

Let a be the first term and d be the common difference of the AP.

$$\therefore S_7 = 182$$

$$\Rightarrow \frac{7}{2}(2a+6d) = 182$$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow a+3d = 26$$
.....(1)

Also,

 $a_4: a_{17} = 1:5$

$$\Rightarrow \frac{a+3d}{a+16d} = \frac{1}{5} \qquad \left[a_n = a + (n-1)d \right]$$
$$\Rightarrow 5a+15d = a+16d$$
$$\Rightarrow d = 4a \qquad \dots \dots \dots (2)$$

Solving (1) and (2), we get

 $a+3 \times 4a = 26$ $\Rightarrow 13a = 26$ $\Rightarrow a = 2$ Putting a = 2 in (2), we get $d = 4 \times 2 = 8$ Hence, the required AP is 2, 10, 18, 26,.....

28. The sum of the first 9 terms of an AP is 81 and that of its first 20 terms is 400. Find the first term and common difference of the AP.

Sol: Here, a = 4, d = 7 and l = 81Let the nth term be 81. Then $T_n = 81$ $\Rightarrow a + (n-1)d = 4 + (n-1)7 = 81$ $\Rightarrow (n-1)7 = 77$ $\Rightarrow (n-1) = 11$ $\Rightarrow n = 12$ Thus, there are 12 terms in the AP. The sum of *n* terms of an AP is given by $S_n = \frac{n}{2}[a+l]$ $\therefore S_{12} = \frac{12}{2}[4+81] = 6 \times 85 = 510$ Thus, the required sum is 510.

29. The sum of the first 7 terms of an AP is 49 and the sum of its first 17 term is 289. Find the sum of its first n terms.

Sol:

Let a be the first term and d be the common difference of the given AP. Then, we have:

$$S_{n} = \frac{n}{2} [2a + (n-1)d]$$

$$S_{7} = \frac{7}{2} [2a + 6d] = 7[a + 3d]$$

$$S_{17} = \frac{17}{2} [2a + 16d] = 17[a + 8d]$$
However, $S_{7} = 49$ and $S_{17} = 289$
Now, $7(a + 3d) = 49$

 $\Rightarrow a + 3d = 7$ (*i*) Also, 17[a+8d] = 289 $\Rightarrow a + 8d = 17$(*ii*) Subtracting (i) from (ii), we get: 5d = 10 $\Rightarrow d = 2$ Putting d = 2 in (i), we get a + 6 = 7 $\Rightarrow a = 1$ Thus, a = 1 and d = 2 $\therefore \text{ Sum of n terms of AP} = \frac{n}{2} \left[2 \times 1 + (n-1) \times 2 \right] = n \left[1 + (n-1) \right] = n^2$

Two Aps have the same common difference. If the fist terms of these Aps be 3 and 8 30. respectively. Find the difference between the sums of their first 50 terms.

Sol:

Let a_1 and a_2 be the first terms of the two *APs*.

Here, $a_1 = 8$ and $a_2 = 3$

Suppose d be the common difference of the two Aps

Let
$$S_{50}$$
 and $S'_{50} = \frac{50}{2} [2a_1 + (50 - 1)d] - \frac{50}{2} [2a_2 + (50 - 1)d]$
= $25(2 \times 8 \times 49d) - 25(2 \times 3 + 49d)$
= $25 \times (16 - 6)$
= 250

Hence, the required difference between the two sums is 250.

The sum first 10 terms of an AP is -150 and the sum of its next 10 terms is -550. Find the 31. AP.

Sol:

a

Let a be the first term and d be the common difference of the AP. Then, 150 (\mathbf{C})

$$S_{10} = -150 \qquad (Given)$$

$$\Rightarrow \frac{10}{2}(2a+9d) = -150 \qquad \left\{ S_n = \frac{n}{2} \left[2a + (n-1)d \right] \right\}$$

$$\Rightarrow 5(2a+9d) = -150$$

$$\Rightarrow 2a+9d = -30 \qquad \dots \dots (1)$$

It is given that the sum of its next 10 terms is -550.

Now,

$$S_{20}$$
 = Sum of first 20 terms = Sum of first 10 terms + Sum of the next 10 terms =

$$-150 + (-550) = -700$$

$$\Rightarrow \frac{20}{2} (2a + 19d) = -700$$

$$\Rightarrow 10 (2a + 19d) = -700$$

$$\Rightarrow 2a + 19d = -70$$
(2)
Subtracting (1) from (2), we get
 $(2a + 19d) - (2a + 9d) = -70 - (-30)$

$$\Rightarrow 10d = -40$$

$$\Rightarrow d = -4$$

Putting $d = -4$ in (1), we get
 $2a + 9 \times (-4) = -30$

$$\Rightarrow 2a = -30 + 36 = 6$$

$$\Rightarrow a = 3$$

Hence, the required AP is 3, -1, -5, -9,

32. The 13th terms of an AP is 4 times its 3rd term. If its 5th term is 16, Find the sum of its first 10 terms.

 c_{2}

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$$a_{13} = 4 \times a_{3}$$
(Given)

$$\Rightarrow a + 12d = 4(a + 2d)$$

$$a_{n} = a + (n - 1)d$$

$$\Rightarrow a + 12d = 4a + 8d$$

$$\Rightarrow 3a = 4d$$
(Given)

$$\Rightarrow 3a = 4d$$
(Also,

$$a_{5} = 16$$
(Given)

$$\Rightarrow a + 4d = 16$$
(Given)

$$\Rightarrow d = 3$$

Using the formula, $S_4 = \frac{n}{2} [2a + (n-1)d]$, we get $S_{10} = \frac{10}{2} [2 \times 4 + (10-1) \times 3]$ $= 5 \times (8+27)$ $= 5 \times 35$ = 175Hence, the required sum is 175.

33. The 16th term of an AP is 5 times its 3rd term. If its 10th term is 41, find the sum of its first 15 terms.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$$a_{16} = 5 \times a_{3} \qquad (Given)$$

$$\Rightarrow a + 15d = 5(a + 2d) \qquad \left[a_{n} = a + (n + 1)d\right]$$

$$\Rightarrow a + 15d = 5a + 10d$$

$$\Rightarrow 4a = 5d$$
Also,

$$a_{10} = 41 \qquad (Given)$$

$$\Rightarrow a + 9d - 41 \qquad \dots \dots (2)$$
Solving (1) and (2), we get

$$a + 9 \times \frac{4a}{5} = 41$$

$$\Rightarrow \frac{5a + 36a}{5} = 41$$

$$\Rightarrow \frac{41a}{5} = 41$$

$$\Rightarrow a = 5$$
Putting $a = 5$ in (1), we get

$$5d = 4 \times 5 = 20$$

$$\Rightarrow d = 4$$
Using the formula, $S_{n} = \frac{n}{2} [2a + (n - 1)d]$, we get

 $S_{15} = \frac{15}{2} [2 \times 5 + (15 - 1) \times 4]$ = $\frac{15}{2} \times (10 + 56)$ = $\frac{15}{2} \times 66$ = 495

Hence, the required sum is 495.

34. An AP 5, 12, 19, has 50 term. Find its last term. Hence, find the sum of its last 15 terms.Sol:

The given AP is 5,12,19,.....

Here, a = 5, d = 12 - 5 = 7 and n = 50.

Since there are 50 terms in the AP, so the last term of the AP is a_{50} .

$$l = a_{50} = 5 + (50 - 1) \times 7 \qquad [a_n = a + (n - 1)d]$$

= 5 + 343
= 348
Thus, the last term of the AP is 348.
Now,
Sum of the last 15 terms of the AP
= $S_{50} - S_{35}$
= $\frac{50}{2} [2 \times 5 + (50 - 1) \times 7] - \frac{35}{2} [2 \times 5 + (35 - 1) \times 7]$
 $\left\{ S_n = \frac{n}{2} [2a + (n - 1)d] \right\}$
= $\frac{50}{2} \times (10 + 343) - \frac{35}{2} \times (10 + 238)$
= $\frac{50}{2} \times 353 - \frac{35}{2} \times 248$
= $\frac{17650 - 8680}{2}$
= $\frac{8970}{2}$
= 4485
Hence, the require sum is 4485.

35. An AP 8, 10, 12, ... has 60 terms. Find its last term. Hence, find the sum of its last 10 terms.
Sol: The given AP is 8, 10, 12,.....

36.

Here, a = 8, d = 10 - 8 = 2 and n = 60Since there are 60 terms in the AP, so the last term of the AP is a_{60} . $l = a_{60} = 8 + (60 - 1) \times 2$ $\left\lceil a_n = a + (n-1)d \right\rceil$ =8+118=126Thus, the last term of the AP is 126. Now. Sum of the last 10 terms of the AP $=S_{60}-S_{50}$ $=\frac{60}{2} \Big[2 \times 8 + (60-1) \times 2 \Big] - \frac{50}{2} \Big[2 \times 8 + (50-1) \times 2 \Big]$ S. March. away $\left\{S_n = \frac{n}{2} \left[2a + (n-1)d\right]\right\}$ $= 30 \times (16 + 118) - 25 \times (16 + 98)$ $= 30 \times 134 - 25 \times 114$ =4020-2850=1170Hence, the required sum is 1170. The sum of the 4th and 8th terms of an AP is 24 and the sum of its 6th and 10th terms is 44. Find the sum of its first 10 terms. Sol: Let a be the first and d be the common difference of the AP. $\therefore a_4 + a_8 = 24$ (Given) $\left[a_n = a + (n-1)d\right]$ $\Rightarrow (a+3d)+(a+7d)=24$ $\Rightarrow 2a + 10d = 24$(1) $\Rightarrow a + 5d = 12$ Also,

$$\therefore a_{6} + a_{10} = 44 \qquad \text{(Given)}$$

$$\Rightarrow (a+5d) + (a+9d) = 44 \qquad \left[a_{n} = a + (n-1)d\right]$$

$$\Rightarrow 2a+14d = 44$$

$$\Rightarrow a+7d = 22 \qquad \dots \dots (2)$$
Subtracting (1) from (2), we get
$$(a+7d) - (a+5d) = 22 - 12$$

$$\Rightarrow 2d = 10$$

$$\Rightarrow d = 5$$

Putting d = 5 in (1), we get $a + 5 \times 5 = 12$ $\Rightarrow a = 12 - 25 = -13$ Using the formula, $S_n = \frac{n}{2} [2a + (n-1)d]$, we get $S_{10} = \frac{10}{2} \left[2 \times (-13) + (10 - 1) \times 5 \right]$ $= 5 \times (-26 + 45)$ $= 5 \times 19$ =95Hence, the required sum is 95.

The sum of fist m terms of an AP is $(4m^2 - m)$. If its nth term is 107, find the value of n. Also, 37. Find the 21st term of this AP.

Find the 21st term of this AP.
Sol:
Let
$$S_m$$
 denotes the sum of the first m terms of the AP. Then,
 $S_m = 4m^2 - m$
 $\Rightarrow S_{m-1} = 4(m-1)^2 - (m-1)$
 $= 4(m^2 - 2m + 1) - (m-1)$
 $= 4m^2 - 9m + 5$
Suppose a_m denote the m^{th} term of the AP.
 $\therefore a_m = S_m - S_{m-1}$
 $= (4m^2 - m) - (4m^2 - 9m + 5)$
 $= 8m - 5$ (1)
Now,
 $a_n = 107$ (Given)
 $\Rightarrow 8n - 5 = 107$ [From (1)]

$$\Rightarrow 8n-5=107$$
 [From (1)]

$$\Rightarrow 8n = 107 + 5 = 112$$

$$\Rightarrow n = 14$$

Thus, the value of n is 14.

Putting m = 21 in (1), we get

 $a_{21} = 8 \times 21 - 5 = 168 - 5 = 163$

Hence, the 21^{st} term of the AP is 163.

The sum of first q terms of an AP is $(63q-3q^2)$. If its pth term is -60, find the value of p. Also, 38. find the 11th term of its AP.

Sol:

Let S_q denote the sum of the first q terms of the AP. Then,

$$S_q = 63q - 3q^2$$

$$\Rightarrow S_{q-1} = 63(q-1) - 3(q-1)^2$$

$$= 63q - 63 - 3(q^2 - 2q + 1)$$

$$= -3q^2 + 69q - 66$$

Suppose a_q denote the q^{th} term of the AP.

$$\therefore a_q = S_q - S_{q-1}$$

= $(63q - 3q^2) - (-3q^2 + 69q - 66)$
= $-6q + 66$ (1)

Now,

Suppose
$$a_q$$
 denote the q^- term of the AT.
 $\therefore a_q = S_q - S_{q-1}$
 $= (63q - 3q^2) - (-3q^2 + 69q - 66)$
 $= -6q + 66$ (1)
Now,
 $a_p = -60$ (Given)
 $\Rightarrow -6p + 66 = -60$ [From (1)]
 $\Rightarrow -6p = -60 - 66 = -126$
 $\Rightarrow p = 21$
Thus, the value of p is 21.
Putting $q = 11$ in (1), we get
 $a_{11} = -6 \times 11 + 66 = -66 + 66 = 0$
Hence, the 11th term of the AP is 0.

39. Find the number of terms of the AP -12, -9, -6, ..., 21. If 1 is added to each term of this AP then the sum of all terms of the AP thus obtained. Sol: The given AP is -12, -9, -6,, 21. Here, a = -12, d = -9 - (-12) = -9 + 12 = 3 and l = 2lSuppose there are n terms in the AP. $\therefore l = a_n = 21$ $\left\lceil a_n = a + (n-1)d \right\rceil$ $\Rightarrow -12 + (n-1) \times 3 = 21$ \Rightarrow 3*n*-15 = 21

$$\Rightarrow 3n = 21 + 15 = 36$$

$$\Rightarrow n = 12$$

Thus, there are 12 terms in the AP.

If 1 is added to each term of the AP, then the new AP so obtained is $-11, -8, -5, \dots, 22$. Here, first term, A = -11; last term, L = 22 and n = 12 \therefore Sum of the terms of this AP $= \frac{12}{2}(-11+22)$ $\left[S_n = \frac{n}{2}(a+l)\right]$

40. Sum of the first 14 terms of and AP is 1505 and its first term is 10. Find its 25th term.Sol:

Let d be the common difference of the AP. Here, a = 10 and n = 14Now, $S_{14} = 1505$ (Given) $\Rightarrow \frac{14}{2} [2 \times 10 + (14 - 1) \times d] = 1505$ $\Rightarrow 7(20 + 13d) = 1505$ $\Rightarrow 20 + 13d = 215$ $\Rightarrow 13d = 215 - 20 = 195$ $\Rightarrow d = 15$ $\therefore 25^{\text{th}}$ term of the AP, a_{25} $= 10 + (25 - 1) \times 15$ $[a_n = a + (n - 1)d]$ = 10 + 360 = 370Hence, the required term is 370.

41. Find the sum of fist 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

 $d = a_3 - a_2 = 18 - 14 = 4$ Now, $a_2 = 14$ (Given) $\Rightarrow a + d = 14$ $\begin{bmatrix} a_n = a + (n-1)d \end{bmatrix}$ $\Rightarrow a + 4 = 14$ $\Rightarrow a = 14 - 4 = 10$ Using the formula, $S_n = \frac{n}{2} [2a + (n-1)d]$, we get $S_{51} = \frac{51}{2} [2 \times 10 + (51-1) \times 4]$ $= \frac{51}{2} (20 + 200)$ $= \frac{51}{2} \times 220$ = 5610Hence, the required sum is 5610.

42. In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees that each section of each class will plant will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two section, find how many trees were planted by student. Which value is shown in the question?

Sol:

Number of trees planted by the students of each section of class 1 = 2

There are two sections of class 1.

: Number of trees planted by the students of class $1 = 2 \times 2 = 4$

Number of trees planted by the students of each section of class 2 = 4

There are two sections of class 2.

: Number of trees planted by the students of class $2 = 2 \times 4 = 8$ Similarly,

Number of trees planted by the students of class $3 = 2 \times 6 = 12$

So, the number of trees planted by the students of different classes are 4, 8, 12,

 \therefore Total number of trees planted by the students = 4+8+12+..... up to 12 terms This series is an arithmetic series.

Here, a = 4, d = 8 - 4 = 4 and n = 12

Using the formula,
$$S_n = \frac{n}{2} [2a + (n-1)d]$$
, we get
 $S_{12} = \frac{12}{2} [2 \times 4 + (12-1) \times 4]$
 $= 6 \times (8+44)$
 $= 6 \times 52$
 $= 312$

Hence, the total number of trees planted by the students is 312.

The values shown in the question are social responsibility and awareness for conserving nature.

43. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3m apart in a straight line. There are 10 potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and he continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

Sol:

Distance covered by the competitor to pick and drop the first potato $= 2 \times 5 m = 10 m$ Distance covered by the competitor to pick and drop the second potato

$$=2\times(5+3)m=2\times8m=16m$$

Distance covered by the competitor to pick and drop the third potato

 $=2\times(5+3+3)m=2\times11m=22$ m and so on.

... Total distance covered by the competitor =10m+16m+22m+.... up to 10 terms This is an arithmetic series.

Here,
$$a = 10, d = 16 - 10 = 6$$
 and $n = 10$

Using the formula, $S_n = \frac{n}{2} [2a + (n-1)d]$, we get

$$S_{10} = \frac{10}{2} [2 \times 10 + (10 - 1) \times 6]$$

= 5 \times (20 + 54)
= 5 \times 74
= 370

Hence, the total distance the competitor has to run is 370 m.

44. There are 25 trees at equal distance of 5 m in a line with a water tank, the distance of the water tank from the nearest tree being 10 m. A gardener waters all the trees separately, starting from the water tank and returning back to the water tank after watering each tree to get water for the next. Find the total distance covered by the gardener in order to water all the trees.



Sol:

Distance covered by the gardener to water the first tree and return to the water tank =10 m + 10 m = 20 m

Distance covered by the gardener to water the second tree and return to the water tank =15m+15m=30m

Distance covered by the gardener to water the third tree and return to the water tank

= 20m + 20m = 40m and so on.

 \therefore Total distance covered by the gardener to water all the trees = 20m + 30m + 40m + up CK BW to 25 terms

This series is an arithmetic series.

Here, a = 20, d = 30 - 20 = 10 and n = 25

a textbooks, hi Using the formula, $S_n = \frac{n}{2} [2a + (n-1)d]$, we get

$$S_{25} = \frac{25}{2} [2 \times 20 + (25 - 1) \times 10]$$

= $\frac{25}{2} (40 + 240)$

$$=\frac{23}{2}=280$$

=3500

Hence, the total distance covered by the gardener to water all the trees 3500 m.

45. A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each prize.

Sol:

Let the value of the first prize be *a*.

Since the value of each prize is 20 less than its preceding prize, so the values of the prizes are in AP with common difference - ₹20.

$$\Rightarrow \frac{40}{2} [2a + (40 - 1)d] = 36000$$
$$\therefore d = - \textcircled{3} \Rightarrow 20(2a + 39d) = 36000$$
$$\Rightarrow 2a + 39d = 1800 \qquad \dots \dots \dots (2)$$

Number of cash prizes to be given to the students, n = 7Total sum of the prizes, $S_7 = \gtrless 700$

Using the formula,
$$S_n = \frac{n}{2} [2a + (n-1)d]$$
, we get

$$S_{7} = \frac{7}{2} \Big[2a + (7-1) \times (-20) \Big] = 700$$

$$\Rightarrow \frac{7}{2} (2a - 120) = 700$$

$$\Rightarrow 7a - 420 = 700$$

$$\Rightarrow 7a = 700 + 420 = 1120$$

$$\Rightarrow a = 160$$

Thus, the value of the first prize is $\gtrless 160$.

Hence, the value of each prize is ₹160, ₹140, ₹120, ₹100, ₹80, ₹60 and ₹40.

46. A man saved ₹33000 in 10 months. In each month after the first, he saved ₹100 more than he did in the preceding month. How much did he save in the first month?

Sol:

Let the money saved by the man in the first month be $\exists a$

It is given that in each month after the first, he saved ₹100 more than he did in the preceding month. So, the money saved by the man every month is in AP with common difference ₹100.

Number of months, n = 10

Sum of money saved in 10 months, $S_{10} = ₹ 33,000$

Using the formula,
$$S_n = \frac{n}{2} [2a + (n-1)d]$$
, we get

$$S_{10} = \frac{10}{2} [2a + (10 - 1) \times 100] = 33000$$

$$\Rightarrow 5(2a + 900) = 33000$$

$$\Rightarrow 2a + 900 = 6600$$

$$\Rightarrow 2a = 6600 - 900 = 5700$$

$$\Rightarrow a = 2850$$

Hence, the money saved by the man in the first month is ₹2,850.

47. A man arranges to pay off debt of ₹36000 by 40 monthly instalments which form an arithmetic series. When 30 of the installments are paid, he dies leaving on-third of the debt unpaid. Find the value of the first instalment.
Sol:

Let the value of the first installment be $\mathbf{E} a$.

Since the monthly installments form an arithmetic series, so let us suppose the man increases the value of each installment by R d every month.

: Common difference of the arithmetic series = $\mathbf{E}d$

Amount paid in 30 installments = ₹36,000 - $\frac{1}{3}$ × ₹ 36,000 = ₹ 36,000 - ₹ 12,000 = ₹ 24,000

Let S_n denote the total amount of money paid in the *n* installments. Then,

$$S_{30} = 24,000$$

$$\Rightarrow \frac{30}{2} [2a + (30 - 1)d] = 24000$$

$$\Rightarrow 15(2a + 29d) = 24000$$

$$\Rightarrow 2a + 29d = 1600$$
(1)
Also,
$$S_{40} = ₹36,000$$

$$\Rightarrow \frac{40}{2} [2a + (40 - 1)d] = 36000$$

$$\Rightarrow 20(2a + 39d) = 36000$$

$$\Rightarrow 2a + 39d = 1800$$
(2)
Subtracting (1) from (2), we get
$$(2a + 39d) - (2a + 29d) = 1800 - 1600$$

$$\Rightarrow 10d = 200$$

$$\Rightarrow d = 20$$
Putting $d = 20$ in (1), we get
$$2a + 29 \times 20 = 1600$$

$$\Rightarrow 2a + 580 = 1600$$

$$\Rightarrow 2a = 1600 - 580 = 1020$$

$$\Rightarrow a = 510$$
Thus, the value of the first installment is ₹510.

48. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day, ₹ 250 for the second day, ₹300 for the third day, etc. the penalty for each succeeding day being ₹50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Sol:

It is given that the penalty for each succeeding day is 50 more than for the preceding day, so the amount of penalties are in AP with common difference ₹50 Number of days in the delay of the work = 30The amount of penalties are ₹200, ₹250, ₹300,... up to 30 terms. .: Total amount of money paid by the contractor as penalty, $S_{30} = ₹ 200 + ₹ 250 + ₹ 300 + up to 30 terms$ Here, a = ₹ 200, d = ₹ 50 and n = 30Using the formula, $S_n = \frac{n}{2} [2a + (n-1)d]$, we get $S_{30} = \frac{30}{2} \left[2 \times 200 + (30 - 1) \times 50 \right]$ =15(400-1450) $=15 \times 1850$ =27750Hence, the contractor has to pay ₹27,750 as penalty