Exercise – 8A

1. (i) $\left(1-\cos^2\theta\right)\operatorname{cosec}^2\theta=1$

(ii) $\left(1+\cot^2\theta\right)\sin^2\theta=1$

Sol:

(i) $LHS=(1-\cos^2\theta)\ cosec^2\theta$

$=\sin^2\theta\ \ cosec^2\theta \qquad (\because \cos^2\theta+\sin^2\theta=1)$

$=\frac{1}{cosec^2\theta}\times cosec^2\theta$

$=1$

Hence, LHS = RHS

(ii) $LHS=(1+\cot^2\theta)\sin^2\theta$

$=cosec^2\theta\sin^2\theta \qquad (\because cosec^2\theta-\cot^2\theta=1)$

$=\frac{1}{\sin^2\theta}\times\sin^2\theta$

$=1$

Hence, LHS = RHS

2. (i) $\left(\sec^2\theta-1\right)\cot^2\theta=1$

(ii) $\left(sec^2\theta-1\right)\left(\operatorname{cosec}^2\theta-1\right)=1$

(iii) $\left(1-\cos^2\theta\right)sec^2\theta=\tan^2\theta$

Sol:

(i) $LHS=(\sec^2\theta-1)\cot^2\theta$

$=\tan^2\theta\times\cot^2\theta \qquad (\because sec^2\theta-\tan^2\theta=1)$

$=\frac{1}{\cot^2\theta}\times\cot^2\theta$

$=1$

= RHS

(ii) $LHS=(\sec^2\theta-1)(cosec^2\theta-1)$

$=\tan^2\theta\times\cot^2\theta \qquad (\because \sec^2\theta-\tan^2\theta=1 and\ cosec^2\theta-\cot^2\theta=1)$

$=\tan^2\theta\times\frac{1}{\tan^2\theta}$

$=1$

=RHS

(iii) $LHS=(1-\cos^2\theta)\sec^2\theta$

$=\sin^2\theta\times\sec^2\theta \qquad (\because \sin^2\theta+\cos^2\theta=1)$

$=\sin^2\theta\times\frac{1}{\cos^2\theta}$

$=\frac{\sin^2\theta}{\cos^2\theta}$

$=\tan^2\theta$

= RHS

3. (i) $\sin^2\theta + \frac{1}{(1+\tan^2\theta)} = 1$

(ii) $\frac{1}{(1+\tan^2\theta)} + \frac{1}{(1+\cot^2\theta)} = 1$

Sol:

(i) $LHS = \sin^2\theta + \frac{1}{(1+\tan^2\theta)}$

$= \sin^2\theta + \frac{1}{\sec^2\theta}$ $\quad (\because \sec^2\theta - \tan^2\theta = 1)$

$= \sin^2\theta + \cos^2\theta$

$= 1$

= RHS

(ii)$LHS = \frac{1}{(1+\tan^2\theta)} + \frac{1}{(1+\cot^2\theta)}$

$= \frac{1}{\sec^2\theta} + \frac{1}{cosec^2\theta}$

$= \cos^2\theta + \sin^2\theta$

$= 1$

= RHS

4. (i) $(1+\cos\theta)(1-\cos\theta)(1+\cos^2\theta) = 1$

(ii) $\cos ec\theta(1+\cos\theta)(cosec\theta - \cot\theta) = 1$

Sol:

(i) $LHS = (1+\cos\theta)\,(1-\cos\theta)\,\,(1+\cot^2\theta)$

$= (1-\cos^2\theta)cosec^2\theta$

$= \sin^2\theta \times cosec^2\theta$

$= \sin^2\theta \times \frac{1}{\sin^2\theta}$

$= 1$

= RHS

(ii) $LHS = cosec\theta(1+\cos\theta)(cosec\,\theta - \cot\theta)$

$= (cosec\,\theta + cosec\,\theta \times \cos\theta)\,(cosec\,\theta - \cot\theta)$

$= \left(cosec\theta + \frac{1}{\sin\theta} \times \cos\theta\right)(cosec\,\theta - \cot\theta)$

$= (cosec\,\theta + \cot\theta)\,(cosec\theta - \cot\theta)$

$= cosec^2\theta - \cot^2\theta$ $\quad (\because cosec^2\theta - \cot^2\theta = 1)$

$= 1$

= RHS

5. (i) $\cot^2\theta - \frac{1}{\sin^2\theta} = -1$

(ii) $\tan^2\theta - \frac{1}{\cos^2\theta} = -1$

(iii) $\cos^2\theta + \frac{1}{(1+\cot^2\theta)} = 1$

Sol:

(i) $LHS = \cot^2\theta - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta - 1}{\sin^2\theta}$

$= \frac{-\sin^2\theta}{\sin^2\theta}$

= -1

=RHS

(ii) $LHS = \tan^2\theta - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta - 1}{\cos^2\theta}$

$= \frac{-\cos^2\theta}{\cos^2\theta}$

= -1

= RHS

(iii) $LHS = \cos^2\theta + \frac{1}{(1+\cot^2\theta)}$

$= \cos^2\theta + \frac{1}{cosec^2\theta}$

$= \cos^2\theta + \sin^2\theta$

=1

= RHS

6. $\frac{1}{(1+\sin\theta)} + \frac{1}{(1-\sin\theta)} = 2\sec^2\theta$

Sol:

$LHS = \frac{1}{(1+\sin\theta)} + \frac{1}{(1-\sin\theta)}$

$= \frac{(1-\sin\theta)+(1+\sin\theta)}{(1+\sin\theta)\,(1-\sin\theta)}$

$= \frac{2}{1-\sin^2\theta}$

$= \frac{2}{\cos^2\theta}$

$= 2\sec^2\theta$

= RHS

7. (i) $\sec\theta(1-\sin\theta)(\sec\theta+\tan\theta)=1$

(ii) $\sin\theta(1+\tan\theta)+\cos\theta(1+\cot\theta)=(\sec\theta+\operatorname{cosec}\theta)$

Sol:

(i) $LHS = \sec\theta(1-\sin\theta)\ (\sec\theta+\tan\theta)$

$= (\sec\theta - \sec\theta\sin\theta)\ (\sec\theta+\tan\theta)$

$=(\sec\theta - \frac{1}{\cos\theta}\times\sin\theta)\ (\sec\theta+\tan\theta)$

$= (\sec\theta-\tan\theta)\ (\sec\theta+\tan\theta)$

$= \sec^2\theta - \tan^2\theta$

$= 1$

$=$ RHS

(ii) $LHS = \sin\theta(1+\tan\theta)+\cos\theta(1+\cot\theta)$

$=\sin\theta + \sin\theta\times\frac{\sin\theta}{\cos\theta} + \cos\theta + \cos\theta\times\frac{\cos\theta}{\sin\theta}$

$= \frac{\cos\theta\sin^2\theta+\sin^3\theta+\cos^2\theta\sin\theta+\cos^3\theta}{\cos\theta\sin\theta}$

$= \frac{(\sin^3\theta+\cos^3\theta)+(\cos\theta\sin^2\theta+\cos^2\theta\sin\theta)}{\cos\theta\sin\theta}$

$= \frac{(\sin\theta+\cos\theta)(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta)+\sin\theta\cos\theta(\sin\theta+\cos\theta)}{\cos\theta\sin\theta}$

$= \frac{(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta+\sin\theta\cos\theta)}{\cos\theta\sin\theta}$

$= \frac{(\sin\theta+\cos\theta)\ (1)}{\cos\theta\sin\theta}$

$=\frac{\sin\theta}{\cos\theta\sin\theta} + \frac{\cos\theta}{\cos\theta\sin\theta}$

$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$

$= \sec\theta + cosec\ \theta$

$=$RHS

8. (i) $1+\frac{\cot^2\theta}{(1+\operatorname{cosec}\theta)}=\operatorname{cosec}\theta$

(ii) $1+\frac{\tan^2\theta}{(1+sec\theta)}=sec\theta$

Sol:

(i) $LHS = 1 + \frac{\cot^2\theta}{(1+cosec\theta)}$

$= 1 + \frac{(cosec^2\theta-1)}{(cosec\theta+1)}$ $(\because cosec^2\theta - \cot^2\theta = 1)$

$= 1 + \frac{(cosec\theta+1)(cosec\ \theta-1)}{(cosec\ \theta+1)}$

$=1+(cosec\ \theta - 1)$

$= cosec\ \theta$

= RHS

(ii) $LHS = 1 + \frac{\tan^2\theta}{(1+\sec\theta)}$

$= 1 + \frac{(\sec^2\theta - 1)}{(\sec\theta + 1)}$

$= 1 + \frac{(\sec\theta+1)(\sec\theta-1)}{(\sec\theta+1)}$

$= 1 + (\sec\theta - 1)$

$= \sec\theta$

= RHS

9. $1 + \frac{(\tan^2\theta)\cot\theta}{\cos ec^2\theta} = \tan\theta$

Sol:

$LHS = \frac{(1+\tan^2\theta)\cot\theta}{cosec^2\theta}$

$= \frac{\sec^2\theta\cot\theta}{cosec^2\theta}$

$= \frac{\frac{1}{\cos^2\theta}\times\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin^2\theta}}$

$= \frac{1}{cos\,\theta sin\theta} \times sin^2\,\theta$

$= \frac{sin\,\theta}{cos\,\theta}$

$= tan\,\theta$

=RHS

Hence, LHS = RHS

10. $\frac{\tan^2\theta}{(1+\tan^2\theta)} + \frac{\cot^2\theta}{(1+\cot^2\theta)} = 1$

Sol:

LHS = $\frac{\tan^2\theta}{(1+\tan^2\theta)} + \frac{\cot^2\theta}{(1+\cot^2\theta)}$

$= \frac{\tan^2\theta}{\sec^2\theta} + \frac{\cot^2\theta}{cosec^2\theta}$ $(\because \sec^2\theta - \tan^2\theta = 1 and\ cosec^2\theta - \cot^2\theta = 1)$

$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}}$

$= \sin^2\theta + \cos^2\theta$

= 1

= RHS

Hence, LHS = RHS

11. $\dfrac{\sin\theta}{(1+\cos\theta)}+\dfrac{(1+\cos\theta)}{\sin\theta}=2\operatorname{cosec}\theta$

Sol:

LHS $=\dfrac{\sin\theta}{(1+\cos\theta)}+\dfrac{(1+\cos\theta)}{\sin\theta}$

$=\dfrac{\sin^2\theta+(1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$

$=\dfrac{\sin^2\theta+1+\cos^2\theta+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$=\dfrac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$=\dfrac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$=\dfrac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

$=\dfrac{2}{\sin\theta}$

$=2\operatorname{cosec}\theta$

= RHS

Hence, L.H.S = R.H.S.

12. $\dfrac{\tan\theta}{(1-\cot\theta)}+\dfrac{\cot\theta}{(1-\tan\theta)}=(1+\sec\theta\operatorname{cosec}\theta)$

Sol:

LHS $=\dfrac{\tan\theta}{(1-\cot\theta)}+\dfrac{\cot\theta}{(1-\tan\theta)}$

$=\dfrac{\tan\theta}{\left(1-\frac{\cos\theta}{\sin\theta}\right)}+\dfrac{\cot\theta}{\left(1-\frac{\sin\theta}{\cos\theta}\right)}$

$=\dfrac{\sin\theta\tan\theta}{(\sin\theta-\cos\theta)}+\dfrac{\cos\theta\cot\theta}{(\cos\theta-\sin\theta)}$

$=\dfrac{\sin\theta\times\frac{\sin\theta}{\cos\theta}-\cos\theta\times\frac{\cos\theta}{\sin\theta}}{(\sin\theta-\cos\theta)}$

$=\dfrac{\frac{\sin^2\theta}{\cos\theta}-\frac{\cos^2\theta}{\sin\theta}}{(\sin\theta-\cos\theta)}$

$=\dfrac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$

$=\dfrac{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$

$=\dfrac{1+\sin\theta\cos\theta}{\cos\theta\sin\theta}$

$=\dfrac{1}{\cos\theta\sin\theta}+\dfrac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$

$=\dfrac{1}{\cos\theta\sin\theta}+\dfrac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$

$=\sec\theta cosec\ \theta+1$

$=1+\sec\theta cosec\ \theta$

= RHS

13. $\frac{\cos^2\theta}{(1-\tan\theta)}+\frac{\sin^3\theta}{(\sin\theta-\cos\theta)}=(1+\sin\theta\cos\theta)$

Sol:

$\frac{\cos^2\theta}{(1-\tan\theta)}+\frac{\sin^3\theta}{(\sin\theta-\cos\theta)}=(1+\sin\theta\cos\theta)$

$LHS=\frac{\cos^2\theta}{(1-\tan\theta)}+\frac{\sin^3\theta}{(\sin\theta-\cos\theta)}$

$=\frac{\cos^2\theta}{(1-\frac{\sin\theta}{\cos\theta})}+\frac{\sin^3\theta}{(\sin\theta-\cos\theta)}$

$=\frac{\cos^3\theta}{(\cos\theta-\sin\theta)}+\frac{\sin^3\theta}{(\sin\theta-\cos\theta)}$

$=\frac{\cos^3\theta-\sin^3\theta}{(\cos\theta-\sin\theta)}$

$=\frac{(\cos\theta-\sin\theta)(\cos^2\theta+\cos\theta sin+\sin^2\theta)}{(\cos\theta-\sin\theta)}$

$=(\sin^2\theta+\cos^2\theta+\cos\theta\sin\theta)$

$=(1+\sin\theta\cos\theta)$

= RHS

Hence, L.H.S = R.H.S.

14. $\frac{\cos\theta}{(1-\tan\theta)}+\frac{\sin^2\theta}{(\cos\theta-\sin\theta)}=(\cos\theta+\sin\theta)$

Sol:

LHS $=\frac{\cos\theta}{(1-\tan\theta)}-\frac{\sin^2\theta}{(\cos\theta-\sin\theta)}$

$=\frac{\cos\theta}{\left(1-\frac{\sin\theta}{\cos\theta}\right)}-\frac{\sin^2\theta}{(\cos\theta-\sin\theta)}$

$=\frac{\cos^2\theta}{(\cos\theta-\sin\theta)}-\frac{\sin^2\theta}{(\cos\theta-\sin\theta)}$

$=\frac{\cos^2\theta-\sin^2\theta}{(\cos\theta-\sin\theta)}$

$=\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{(\cos\theta-\sin\theta)}$

$=(\cos\theta+\sin\theta)$

= RHS

Hence, LHS = RHS

15. $(1+\tan^2\theta)(1+\cot^2\theta)=\frac{1}{(\sin^2\theta-\sin^4\theta)}$

Sol:

$LHS=(1+\tan^2\theta)\,(1+\cot^2\theta)$

$=\sec^2\theta.cosec^2\theta \qquad (\because \sec^2\theta-\tan^2\theta=1 \ and \ cosec^2\ -\cot^2\theta=1)$

$= \frac{1}{\cos^2\theta . \sin^2\theta}$

$= \frac{1}{(1-\sin^2\theta)\sin^2\theta}$

$= \frac{1}{\sin^2\theta - \sin^4\theta}$

=RHS

Hence, LHS = RHS

16. $\frac{\tan\theta}{\left(1+\tan^2\theta\right)^2} + \frac{\cot\theta}{\left(1+\cot^2\theta\right)^2} = \sin\theta\cos\theta$

Sol:

$LHS = \frac{\tan\theta}{(1+\tan^2\theta)^2} + \frac{\cot\theta}{(1+\cot^2\theta)^2}$

$= \frac{\tan\theta}{(\sec^2\theta)^2} + \frac{\cot\theta}{(cosec^2\theta)^2}$

$= \frac{\tan\theta}{\sec^4\theta} + \frac{\cot\theta}{cosec^4\theta}$

$= \frac{\sin\theta}{\cos\theta} \times \cos^4\theta + \frac{\cos\theta}{\sin\theta} \times \sin^4\theta$

$= \sin\theta\cos^3\theta + \cos\theta\sin^3\theta$

$= \sin\theta\cos\theta(\cos^2\theta + \sin^2\theta)$

$= \sin\theta\cos\theta$

= RHS

17. (i) $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

(ii) $\sin^2\theta + \cos^4\theta = \cos^2\theta + \sin^4\theta$

(iii) $\cos ec^4\theta + \cos ec^2\theta = \cot^4\theta + \cot^2\theta$

Sol:

(i) $LHS = \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta)$

$= 1 \times \{(\sin^2\theta)^2 + 2\sin^2\theta\cos^2\theta + (\cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta\}$

$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$

$= (1)^2 - 3\sin^2\theta\cos^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta$

= RHS

Hence, LHS = RHS

(ii) $LHS = \sin^2\theta + \cos^4\theta$

$= \sin^2\theta + (\cos^2\theta)^2$

$= \sin^2\theta + (1 - \sin^2\theta)^2$

$= \sin^2\theta + 1 - 2\sin^2\theta + \sin^4\theta$

$= 1 - \sin^2\theta + \sin^4\theta$

$= \cos^2\theta + \sin^4\theta$

$=$ RHS

Hence, LHS = RHS

(iii) $LHS = cosec^4\theta - cosec^2\theta$

$= cosec^2\theta(cosec^2\theta - 1)$

$= cosec^2\theta \times \cot^2\theta \qquad (\because cosec^2\theta - \cot^2\theta = 1)$

$= (1 + \cot^2\theta) \times \cot^2\theta$

$= \cot^2\theta + \cot^4\theta$

$=$ RHS

Hence, LHS = RHS

18. (i) $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \left(\cos^2\theta - \sin^2\theta\right)$

(ii) $\frac{1-\tan^2\theta}{\cot^2 - 1} = \tan^2\theta$

Sol:

(i) $LHS = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{1}$

$= \cos^2\theta - \sin^2\theta$

$=$ RHS

(ii) $LHS = \frac{1-\tan^2\theta}{\cot^2\theta - 1}$

$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta} - 1}$

$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}}$

$= \frac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta$

$=$ RHS

19. (i) $\frac{\tan\theta}{(sec\ \theta-1)}+\frac{\tan\theta}{(sec\ \theta+1)}=2\cos ec\theta$

(ii) $\frac{\cot\theta}{(\cos ec\theta+1)}+\frac{(\cos ec\theta+1)}{\cot\theta}=2\sec\theta$

Sol:

(i) $LHS=\frac{\tan\theta}{(\sec\theta-1)}+\frac{\tan\theta}{(\sec\theta+1)}$

$=\tan\theta\{\frac{\sec\theta+1+\sec\theta-1}{(\sec\theta-1)(\sec\theta+1)}\}$

$=\tan\theta\{\frac{2\sec\theta}{(\sec^2\theta-1)}\}$

$=\tan\theta\times\frac{2\sec\theta}{\tan^2\theta}$

$=2\frac{\sec\theta}{\tan\theta}$

$=2\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$

$=2\frac{1}{\sin\theta}$

$=2cosec\ \theta$

= RHS

Hence, LHS = RHS

(ii) $LHS=\frac{\cot\theta}{(cosec\ \theta+1)}+\frac{(cosec\theta+1)}{\cot\theta}$

$=\frac{\cot^2\theta+(cosec\ \theta+1)^2}{(cosec\ \theta+1)\cot\theta}$

$=\frac{\cot^2\theta+cosec^2\theta+2cosec\ \theta+1}{(cosec\ \theta+1)\cot\theta}$

$=\frac{\cot^2\theta+cosec^2\theta+2cosec\ \theta+cosec^2\theta-\cot^2\theta}{(cosec\theta+1)\cot\theta}$

$=\frac{2cosec^2\theta+2cosec\ \theta}{(cosec\ \theta+1)\cot\theta}$

$=\frac{2cosec\theta(cosec\theta+1)}{(cosec\theta+1)\cot\theta}$

$=\frac{2cosec\theta}{\cot\theta}$

$=2\times\frac{1}{\sin\theta}\times\frac{\sin\theta}{\cos\theta}$

$=2\sec\theta$

= RHS

Hence, LHS = RHS

20. (i) $\frac{\sec\theta - 1}{\sec\theta + 1} = \frac{\sin^2\theta}{(1+\cos\theta)^2}$

(ii) $\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = \frac{\cos^2\theta}{(1+\sin\theta)^2}$

Sol:

(i)$LHS = \frac{\sec\theta - 1}{\sec\theta + 1}$

$= \frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}$

$= \frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}$

$= \frac{1-\cos\theta}{1+\cos\theta}$

$= \frac{(1-\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1+\cos\theta)}$ $\left\{\begin{array}{l}\text{Dividing the numerator and} \\ deno\min ator\, by (1+\cos\theta)\end{array}\right\}$

$= \frac{1-\cos^2\theta}{(1+\cos\theta)^2}$

$= \frac{\sin^2\theta}{(1+\cos\theta)^2}$

= RHS

(ii) $LHS = \frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}$

$= \frac{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$

$= \frac{\frac{1-\sin\theta}{\cos\theta}}{\frac{1+\sin\theta}{\cos\theta}}$

$= \frac{1-\sin\theta}{1+\sin\theta}$

$= \frac{(1-\sin\theta)(1+\sin\theta)}{(1+\sin\theta)(1+\sin\theta)}$ $\left\{\begin{array}{l}\text{Dividing the numerator and} \\ deno\min ator\, by (1+\cos\theta)\end{array}\right\}$

$= \frac{(1-\sin^2\theta)}{(1+\sin\theta)^2}$

$= \frac{\cos^2\theta}{(1+\sin\theta)^2}$

= RHS

21. (i) $\sqrt{\frac{1+sin\theta}{1-sin\theta}} = (\sec\theta + \tan\theta)$ (ii) $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = (\cos ec\theta - \cot\theta)$

(iii) $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = 2\cos ec\theta$

Sol:

(i) $LHS = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

$= \sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

$= \frac{1+\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= (\sec\theta + \tan\theta)$

= RHS

(ii) $LHS = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)} \times \frac{(1-\cos\theta)}{(1-\cos\theta)}}$

$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

$= \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

$= \frac{1-\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$= (cosec\ \theta - \cot\theta)$

= RHS

(iii) $LHS = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}} + \sqrt{\frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos^2\theta)}} + \sqrt{\frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

$= \frac{(1+\cos\theta)}{\sin\theta} + \frac{(1-\cos\theta)}{\sin\theta}$

$= \frac{1+\cos\theta+1-\cos\theta}{\sin\theta}$

$= \frac{2}{\sin\theta}$

$= 2\cos ec\theta$

= RHS

22. $\frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}+\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}=2$

Sol:

$LHS = \frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}+\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$

$= \frac{(\cos\theta+\sin\theta)(\cos^2\theta-\cos\theta\sin\theta+\sin^2\theta)}{(\cos\theta+\sin\theta)}+\frac{(\cos\theta-\sin\theta)(\cos^2\theta+\cos\theta\sin\theta+\sin^2\theta)}{(\cos\theta-\sin\theta)}$

$= (\cos^2\theta+\sin^2\theta-\cos\theta\sin\theta)+(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta)$

$= (1-\cos\theta\sin\theta)+(1+\cos\theta\sin\theta)$

$= 2$

= RHS

Hence, LHS = RHS

23. $\frac{\sin\theta}{(\cot\theta+\operatorname{cosec}\theta)}-\frac{\sin\theta}{(\cot\theta-\operatorname{cosec}\theta)}=2$

Sol:

$LHS = \frac{\sin\theta}{(\cot\theta+cosec\,\theta)}-\frac{\sin\theta}{(\cot\theta-cosec\,\theta)}$

$= \sin\theta\left\{\frac{(\cot\theta-cosec\,\theta)-(\cot\theta+cosec\,\theta)}{(\cot\theta+cosec\,\theta)(\cot\theta-cosec\,\theta)}\right\}$

$= \sin\theta\left\{\frac{-2cosec\theta}{-1}\right)$ $\quad (\because cosec^2\theta-\cot^2\theta=1)$

$= \sin\theta.2\operatorname{cosec}\theta$

$= \sin\theta\times2\times\frac{1}{\sin\theta}$

$= 2$

= RHS

24. (i) $\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}+\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=\frac{2}{(2\sin^2\theta-1)}$

(ii) $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\frac{2}{(1-2\cos^2\theta)}$

Sol:

(i) $LHS = \frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}+\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$

$= \frac{(\sin\theta-\cos\theta)^2+(\sin\theta+\cos\theta)^2}{(\sin\theta+cos\,\theta)(\sin\theta-\cos\theta)}$

$= \frac{\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$

$= \frac{1+1}{\sin^2\theta-(1-\sin^2\theta)}$ $\quad (\because \sin^2\theta+\cos^2\theta=1)$

$= \frac{2}{\sin^2\theta-1+\sin^2\theta}$

$= \frac{2}{\sin^2\theta-1}$

= RHS

(ii) $LHS = \frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} + \frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

$= \frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$

$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{(\sin^2\theta-\cos^2\theta)}$

$= \frac{1+1}{(1-\cos^2\theta)-\cos^2\theta}$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{2}{1-2\cos^2\theta}$

= RHS

25. $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \cot\theta$

Sol:

$LHS = \frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{(1+\cos\theta)-(1-\cos^2\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

= RHS

Hence, L.H.S. = R.H.S.

26. (i) $\frac{\cos ec\theta+\cot\theta}{\cos ec\theta-\cot\theta} = (\cos ec\theta+\cot\theta)^2 = 1+2\cot^2\theta+2\cos ec\theta\cot\theta$

(ii) $\frac{sec\theta+\tan\theta}{\sec\theta-\tan\theta} = (\sec\theta+\tan\theta)^2 = 1+2\tan^2\theta+2sec\theta\tan\theta$

Sol:

(i) Here, $\frac{cosec\theta+\cot\theta}{cosec\theta-\cot\theta}$

$= \frac{(cosec\theta+\cot\theta)(cosec\theta+\cot\theta)}{(cosec\theta-\cot\theta)(cosec\theta+\cot\theta)}$

$= \frac{(cosec\theta+\cot\theta)^2}{(cosec^2\theta-\cot^2\theta)}$

$= \frac{(cosec\theta+\cot\theta)^2}{1}$

$= (cosec\theta + \cot\theta)^2$

Again, $(cosec\theta + \cot\theta)^2$

$= cosec^2\theta + \cot^2\theta + 2cosec\theta\cot\theta$

$= 1 + \cot^2\theta + \cot^2\theta + 2cosec\theta\cot\theta \quad (\because cosec^2\theta - \cot^2\theta = 1)$

$= 1 + 2\cot^2\theta + 2cosec\theta\cot\theta$

(ii) Here, $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}$

$= \frac{(\sec\theta+\tan\theta)(\sec\theta+\tan\theta)}{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}$

$= \frac{(\sec\theta+\tan\theta)^2}{\sec^2\theta-\tan^2\theta}$

$= \frac{(\sec\theta+\tan\theta)^2}{1}$

$= (\sec\theta + \tan\theta)^2$

Again, $(\sec\theta + \tan\theta)^2$

$= \sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$

$= 1 + \tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$

$= 1 + 2\tan^2\theta + 2\sec\theta\tan\theta$

27. (i) $\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$

(ii) $\frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$

Sol:

(i) LHS $= \frac{1+\cos\theta+\sin\theta}{1+\cos\theta-sin\theta}$

$= \frac{\{(1+\cos\theta)+\sin\theta\}\{(1+\cos\theta)+\sin\theta\}}{\{(1+\cos\theta)-\sin\theta\}\{(1+\cos\theta)+\sin\theta\}}$ $\left\{\begin{matrix}\text{Multiplying the numerator and} \\ \text{denominator by } (1+\cos\theta+\sin\theta)\end{matrix}\right\}$

$= \frac{\{(1+\cos\theta)+\sin\theta\}^2}{\{(1+\cos\theta)^2-\sin^2\theta\}}$

$= \frac{1+\cos^2\theta+2\cos\theta+\sin^2\theta+2\sin\theta(1+\cos\theta)}{1+\cos^2\theta+2\cos\theta-\sin^2\theta}$

$= \frac{2+2\cos\theta+2\sin\theta(1+\cos\theta)}{1+\cos^2\theta+2\cos\theta-(1-\cos^2\theta)}$

$= \frac{2(1+\cos\theta)+2\sin\theta(1+\cos\theta)}{2\cos^2\theta+2\cos\theta}$

$= \frac{2(1+\cos\theta)(1+\sin\theta)}{2\cos\theta(1+\cos\theta)}$

$= \frac{1+\sin\theta}{\cos\theta}$

= RHS

(ii) $LHS = \frac{\sin\theta+1\cos\theta}{\cos\theta-1+\sin\theta}$

$= \frac{(\sin\theta+1-\cos\theta)(\sin\theta+\cos\theta+1)}{(\cos\theta-1+\sin\theta)(\sin\theta+\cos\theta+1)}$ $\left\{\begin{matrix}\text{Multiplying the numerator and} \\ \text{denominator by } (1+\cos\theta+\sin\theta)\end{matrix}\right\}$

$= \frac{(\sin\theta+1)^2-\cos^2\theta}{(\sin\theta+\cos\theta)^2-1^2}$

$= \frac{\sin^2\theta+1+2\sin\theta-\cos^2\theta}{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}$

$= \frac{\sin^2\theta + \sin^2\theta + \cos^2\theta + 2\sin\theta - \cos^2\theta}{2\sin\theta\cos\theta}$

$= \frac{2\sin^2\theta + 2sin\,\theta}{2\sin\theta\cos\theta}$

$= \frac{2\sin\theta(1+\sin\theta)}{2\sin\theta\cos\theta}$

$= \frac{1+\sin\theta}{\cos\theta}$

= RHS

28. $\frac{\sin\theta}{(\sec\theta + \tan\theta - 1)} + \frac{\cos\theta}{(\cos ec\theta + \cot\theta - 1)} = 1$

Sol:

$LHS = \frac{\sin\theta}{(\sec\theta + \tan\theta - 1)} + \frac{\cos\theta}{(cosec\,\theta + \cot\theta - 1)}$

$= \frac{\sin\theta\cos\theta}{1+\sin\theta - \cos\theta} + \frac{\cos\theta\sin\theta}{1+\cos\theta - \sin\theta}$

$= \sin\theta\cos\theta\,[\frac{1}{1+(\sin\theta-\cos\theta)} + \frac{1}{1-(\sin\theta-\cos\theta)}]$

$= \sin\theta\cos\theta\left[\frac{1-(\sin\theta-\cos\theta)+1+(\sin\theta-\cos\theta)}{\{1+(\sin\theta-\cos\theta)\}\{1-(\sin\theta-\cos\theta)\}}\right]$

$= \sin\theta\cos\theta[\frac{1-\sin\theta+\cos\theta+1+\sin\theta-\cos\theta}{1-(\sin\theta-\cos\theta)^2}$

$= \frac{2\sin\theta cos\,\theta}{1-(\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta)}$

$= \frac{2\sin\theta\cos\theta}{2\sin\theta\cos\theta}$

= 1

= RHS

Hence, LHS = RHS

29. $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} + \frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta} = \frac{2}{(\sin^2\theta-\cos^2\theta)} = \frac{2}{(2\sin^2\theta-1)}$

Sol:

We have $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} + \frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

$= \frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$

$= \frac{\sin^2\theta+\cos^2\theta+2sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$

$= \frac{1+1}{\sin^2\theta-\cos^2\theta}$

$= \frac{2}{\sin^2\theta-\cos^2\theta}$

Again, $\frac{2}{\sin^2\theta-\cos^2\theta}$

$= \frac{2}{\sin^2\theta-(1-\sin^2\theta)}$

$= \frac{2}{2\sin^2\theta-1}$

30. $\dfrac{\cos\theta \cos ec\theta - \sin\theta \sec\theta}{\cos\theta + \sin\theta} = \cos ec\theta - \sec\theta$

Sol:

$LHS = \frac{\cos\theta cosec\theta - \sin\theta sec\theta}{\cos\theta + \sin\theta}$

$= \frac{\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}}{\cos\theta + \sin\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta \sin\theta(\cos\theta + \sin\theta)}$

$= \frac{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}{\cos\theta \sin\theta(\cos\theta + \sin\theta)}$

$= \frac{(\cos\theta - \sin\theta)}{\cos\theta \sin\theta}$

$= \frac{1}{\sin\theta} - \frac{1}{\cos\theta}$

$= cosec\ \theta - \sec\theta$

= RHS

Hence, LHS = RHS

31. $(1 + \tan\theta + \cot\theta)(\sin\theta - \cos\theta) = \left(\dfrac{sec\theta}{\cos ec^2\theta} - \dfrac{\cos ec\theta}{sec^2\theta}\right)$

Sol:

$LHS = (1 + \tan\theta + \cot\theta)(\sin\theta - \cos\theta)$

$= \sin\theta + tan\theta \sin\theta + \cot\theta \sin\theta - \cos\theta - \tan\theta \cos\theta - \cot\theta \cos\theta$

$= \sin\theta + \tan\theta \sin\theta + \frac{\cos\theta}{\sin\theta} \times \sin\theta - \cos\theta - \frac{\sin\theta}{\cos\theta} \times \cos\theta - \cot\theta \cos\theta$

$= \sin\theta + \tan\theta \sin\theta + \cos\theta - \cos\theta - \sin\theta - \cot\theta \cos\theta$

$= \tan\theta \sin\theta - \cot\theta \cos\theta$

$= \frac{\sin\theta}{\cos\theta} \times \frac{1}{cosec\theta} - \frac{\cos\theta}{\sin\theta} \times \frac{1}{\sec\theta}$

$= \frac{1}{cosec\theta} \times \frac{1}{cosec\theta} \times \sec\theta - \frac{1}{\sec\theta} \times \frac{1}{\sec\theta} \times cosec\theta$

$= \frac{\sec\theta}{cosec^2\theta} - \frac{cosec\theta}{\sec^2\theta}$

= RHS

Hence, LHS = RHS

32. $\dfrac{\cot^2\theta(sec\theta - 1)}{(1 + \sin\theta)} + \dfrac{\sec^2\theta(\sin\theta - 1)}{(1 + \sec\theta)} = 0$

Sol:

$LHS = \frac{\cot^2\theta(\sec\theta - 1)}{(1 + \sin\theta)} + \frac{\sec^2\theta(\sin\theta - 1)}{(1 + \sec\theta)}$

$= \frac{\frac{\cos^2\theta}{\sin^2\theta}(\frac{1}{\cos\theta} - 1)}{(1 + \sin\theta)} + \frac{\frac{1}{\cos^2\theta}(\sin\theta - 1)}{(1 + \frac{1}{\cos\theta})}$

$= \frac{\frac{\cos^2\theta}{\sin^2\theta}(\frac{1-\cos\theta}{\cos\theta})}{(1+\sin\theta)} + \frac{\frac{(\sin\theta-1)}{\cos^2\theta}}{(\frac{\cos\theta+1}{\cos\theta})}$

$= \frac{\cos^2\theta(1-\cos\theta)}{\sin^2\theta\cos\theta(1+\sin\theta)} + \frac{(\sin\theta-1)\cos\ \theta}{(\cos\theta+1)\cos^2\theta}$

$= \frac{\cos\theta(1-\cos\theta)}{(1-\cos^2\theta)(1+\sin\theta)} + \frac{(\sin\theta-1)\cos\ \theta}{(\cos\theta+1)(1-\sin^2\theta)}$

$= \frac{\cos\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)(1+\sin\theta)} + \frac{-(1\sin\theta)\cos\theta}{(\cos\theta+1)(1-\sin\theta)(1+\sin\theta)}$

$= \frac{\cos\ \theta}{(1+\cos\theta)(1+\sin\theta)} - \frac{\cos\theta}{(\cos\theta+1)(1+\sin\theta)}$

$= \theta$

= RHS

33. $\left\{\frac{1}{\left(sec^2\theta-\cos^2\theta\right)}+\frac{1}{\left(cosec^2\theta-\sin^2\theta\right)}\right\}\left(\sin^2\theta\cos^2\theta\right)=\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$

Sol:

$LHS = \{\frac{1}{\sec^2\theta-\cos^2\theta} + \frac{1}{cosec^2\theta-\sin^2\theta}\}(\sin^2\theta\cos^2\theta)$

$= \{\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\}(\sin^2\theta\cos^2\theta)$

$= \left\{\frac{\cos^2\theta}{(1-\cos^2\theta)(1+\cos^2\theta)} + \frac{\sin^2\theta}{(1-\sin^2\theta)(1+\sin^2\theta)}\right\}(\sin^2\theta\cos^2\theta)$

$= \left[\frac{\cot^2\theta}{1+\cos^2\theta} + \frac{\tan^2\theta}{1+\sin^2\theta}\right]\sin^2\theta\cos^2\theta$

$= \frac{\cos^4\theta}{1+\cos^2\theta} + \frac{\sin^4\theta}{1+\sin^2\theta}$

$= \frac{(\cos^2\theta)^2}{1+\cos^2\theta} + \frac{(\sin^2\theta)^2}{1+\sin^2\theta}$

$= \frac{(1-\sin^2\theta)}{1+\cos^2\theta} + \frac{(1-\cos^2\theta)^2}{1+\sin^2\theta}$

$= \frac{(1-\sin^2\theta)^2(1+\sin^2)+(1-\cos^2\theta)^2(1+\cos^2\theta)}{(1+\sin^2\theta)(1+\cos^2\theta)}$

$= \frac{\cos^4\theta(1+\sin^2\theta)+\sin^4\theta(1+\cos^2\theta)}{1+\sin^2\theta+\cos^2\theta+\sin^2\theta\cos^2\theta}$

$= \frac{\cos^4\theta\cos^4\theta\sin^2\theta+\sin^4\theta+\sin^4\theta\cos^2\theta}{1+1\sin^2\theta\cos^2\theta}$

$= \frac{\cos^4\theta+\sin^4\theta+\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)}{2+\sin^2\theta\cos^2\theta}$

$= \frac{(\cos^2\theta)^2+(\sin^2\theta)^2+\sin^2\theta\cos^2\theta(1)}{2+\sin^2\theta\cos^2\theta}$

$= \frac{(\cos^2\theta+\sin^2\theta)^2-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta(1)}{2+\sin^2\theta\cos^2\theta}$

$= \frac{1^2+\cos^2\theta\sin^2\theta-2\cos^2\theta\sin^2\theta}{2+\sin^2\theta\cos^2\theta}$

$= \frac{1-\cos^2\theta\sin^2\theta}{2+\sin^2\theta\cos^2\theta}$

= RHS

34. $\frac{(\sin A - \sin B)}{(\cos A + \cos B)} + \frac{(\cos A - \cos B)}{(\sin A + \sin B)} = 0$

Sol:

$LHS = \frac{(sinA - sinB)}{(cosA + cosB)} + \frac{(cosA - cosB}{(sinA + SinB)}$

$= \frac{(sinA - sinB)(sinA + sinB) + (cosA - cosB)(cosA - cosB)}{(cosA + cosB)(sinA + sinB)}$

$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(cosA + cosB)(sinA + sinB)}$

$= \frac{0}{(cosA + cosB)(sinA + sinB)}$

$= 0$

= RHS

35. $\frac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B$

Sol:

$LHS = \frac{tanA + tanB}{cotA + cotB}$

$= \frac{tanA + tanB}{\frac{1}{tanA} + \frac{1}{tanB}}$

$= \frac{tanA + tanB}{\frac{tanA + tanB}{tanA\ tanB}}$

$= \frac{tanA\ tanB(tanA + tanB)}{(\tan A + \tan B)}$

$= tanA\ tanB$

= RHS

Hence, LHS = RHS

36. Show that none of the following is an identity:

(i) $\cos^2\theta + \cos\theta = 1$

(ii) $\sin^2\theta + \sin\theta = 2$

(iii) $tan^2\theta + \sin\theta = \cos^2\theta$

Sol:

$(i)\ \cos^2\theta + \cos\theta = 1$

$LHS = \cos^2\theta + \cos\theta$

$= 1 - \sin^2\theta + \cos\theta$

$= 1 - (\sin^2\theta - \cos\theta)$

Since LHS ≠ RHS, this not an identity.

(ii) $\sin^2\theta + \sin\theta = 1$

$LHS = \sin^2\theta + \sin\theta$

$= 1 - \cos^2\theta + \sin\theta$

$= 1 - (\cos^2\theta - \sin\theta)$

Since LHS ≠ RHS, this is not an identity.

(iii) $\tan^2\theta + \sin\theta = \cos^2\theta$

$LHS = \tan^2\theta + \sin\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} + \sin\theta$

$= \frac{1-\cos^2\theta}{\cos^2\theta} + \sin\theta$

$= \sec^2\theta - 1 + \sin\theta$

Since LHS ≠ RHS, this is not an identity.

37. Prove that $\left(\sin\theta - 2\sin^3\theta\right) = \left(2\cos^3\theta - \cos\theta\right)\tan\theta$

Sol:

$RHS = (2\cos^3\theta - \cos\theta)\tan\theta$

$= (2\cos^2\theta - 1)\cos\ \theta \times \frac{\sin\theta}{\cos\theta}$

$= [2(1 - \sin^2\theta) - 1]\sin\ \theta$

$= (2 - 2\sin^2\theta - 1)\sin\ \theta$

$= (1 - 2\sin^2\theta)\sin\theta$

$= (\sin\theta - 2\sin^3\theta)$

= LHS