Exercise – 3D

1. Show that the following system of equations has a unique solution:

$3x + 5y = 12,$

$5x + 3y = 4.$

Also, find the solution of the given system of equations.

Sol:

The given system of equations is:

3x + 5y = 12
5x + 3y = 4
These equations are of the forms:
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 3$, $b_1= 5$, $c_1 = -12$ and $a_2 = 5$, $b_2 = 3$, $c_2 = -4$
For a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, i.e., $\frac{3}{5} \neq \frac{5}{3}$
Hence, the given system of equations has a unique solution.
Again, the given equations are:
3x + 5y = 12 …..(i)
5x + 3y = 4 …..(ii)
On multiplying (i) by 3 and (ii) by 5, we get:
9x + 15y = 36 …….(iii)
25x + 15y = 20 ……(iv)
On subtracting (iii) from (iv), we get:
16x = -16
⇒x = -1
On substituting x = -1 in (i), we get:
3(-1) + 5y = 12
⇒5y = (12 + 3) = 15
⇒y = 3
Hence, x = -1 and y = 3 is the required solution.

2. Show that the following system of equations has a unique solution:
2x - 3y = 17,
4x + y = 13.
Also, find the solution of the given system of equations.
Sol:
The given system of equations is:
2x - 3y - 17 = 0 ….(i)
4x + y - 13 = 0 …..(ii)
The given equations are of the form
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= -3$, $c_1 = -17$ and $a_2 = 4$, $b_2 = 1$, $c_2 = -13$
Now,
$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{-3}{1} = -3$
Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, therefore the system of equations has unique solution.
Using cross multiplication method, we have

$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow \frac{x}{-3(-13)-1\times(-17)} = \frac{y}{-17\times4-(-13)\times2} = \frac{1}{2\times1-4\times(-3)}$

$\Rightarrow \frac{x}{39+17} = \frac{y}{-68+26} = \frac{1}{2+12}$

$\Rightarrow \frac{x}{56} = \frac{y}{-42} = \frac{1}{14}$

$\Rightarrow$ x = $\frac{56}{14}$, y = $\frac{-42}{14}$

$\Rightarrow$ x = 4, y = -3

Hence, x = 4 and y = -3.

3. Show that the following system of equations has a unique solution:

$\frac{x}{3} + \frac{y}{2} = 3$, x – 2y = 2.

Also, find the solution of the given system of equations.

Sol:

The given system of equations is:

$\frac{x}{3} + \frac{y}{2} = 3$

$\Rightarrow \frac{2x+3y}{6} = 3$

2x + 3y = 18

$\Rightarrow$2x + 3y – 18 = 0 ….(i)

and

x – 2y = 2

x – 2y – 2 = 0 …..(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1 = 3$, $c_1 = -18$ and $a_2 = 1$, $b_2 = -2$, $c_2 = -2$

For a unique solution, we must have:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, i.e., $\frac{2}{1} \neq \frac{3}{-2}$

Hence, the given system of equations has a unique solution.

Again, the given equations are:

2x + 3y – 18 = 0 …..(iii)

x – 2y – 2 = 0 …..(iv)

On multiplying (i) by 2 and (ii) by 3, we get:

4x + 6y – 36 = 0 …….(v)

3x - 6y – 6 = 0 ……(vi)

On adding (v) from (vi), we get:

7x = 42

$\Rightarrow$x = 6

On substituting x = 6 in (iii), we get:

2(6) + 3y = 18

⇒3y = (18 - 12) = 6

⇒y = 2

Hence, x = 6 and y = 2 is the required solution.

4. Find the value of k for which the system of equations has a unique solution:
2x + 3y = 5,
kx - 6y = 8.
Sol:
The given system of equations are
2x + 3y – 5 = 0
kx - 6y - 8 = 0
This system is of the form:
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -5$ and $a_2 = k$, $b_2 = -6$, $c_2 = -8$
Now, for the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{2}{k} \neq \frac{3}{-6}$
⇒ k ≠ -4
Hence, k ≠ -4

5. Find the value of k for which the system of equations has a unique solution:
x – ky = 2,
3x + 2y + 5=0.
Sol:
The given system of equations are
x - ky – 2 = 0
3x + 2y + 5 = 0
This system of equations is of the form:
$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$
where, $a_1 = 1$, $b_1= -k$, $c_1 = -2$ and $a_2 = 3$, $b_2 = 2$, $c_2 = 5$
Now, for the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{1}{3} \neq \frac{-k}{2}$
$\Rightarrow k \neq -\frac{2}{3}$
Hence, $k \neq -\frac{2}{3}$.

6. Find the value of k for which the system of equations has a unique solution:
$5x - 7y = 5$,
$2x + ky = 1$.
Sol:
The given system of equations are
$5x - 7y - 5 = 0$ ....(i)
$2x + ky - 1 = 0$ ...(ii)
This system is of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = 5$, $b_1= -7$, $c_1 = -5$ and $a_2 = 2$, $b_2 = k$, $c_2 = -1$
Now, for the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{5}{2} \neq \frac{-7}{k}$
$\Rightarrow k \neq -\frac{14}{5}$
Hence, $k \neq -\frac{14}{5}$.

7. Find the value of k for which the system of equations has a unique solution:
$4x + ky + 8=0$,
$x + y + 1 = 0$.
Sol:
The given system of equations are
$4x + ky + 8 = 0$
$x + y + 1 = 0$
This system is of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = 4$, $b_1= k$, $c_1 = 8$ and $a_2 = 1$, $b_2 = 1$, $c_2 = 1$
For the given system of equations to have a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
$\Rightarrow \frac{4}{1} \neq \frac{k}{1}$
$\Rightarrow k \neq 4$
Hence, $k \neq 4$.

8. Find the value of k for which the system of equations has a unique solution:
$4x - 5y = k$,
$2x - 3y = 12$.
Sol:

The given system of equations are
4x - 5y = k
$\Rightarrow$ 4x - 5y - k = 0 ….(i)
And, 2x - 3y = 12
$\Rightarrow$2x - 3y - 12 = 0 …(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
Here, $a_1 = 4$, $b_1= -5$, $c_1 = -k$ and $a_2 = 2$, $b_2 = -3$, $c_2 = -12$
For a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
i.e., $\frac{4}{2} \neq \frac{-5}{-3}$
$\Rightarrow 2 \neq \frac{5}{3} \Rightarrow 6 \neq 5$
Thus, for all real values of k, the given system of equations will have a unique solution.

9. Find the value of k for which the system of equations has a unique solution:
kx + 3y = (k – 3),
12x + ky = k
Sol:
The given system of equations:
kx + 3y = (k – 3)
$\Rightarrow$ kx + 3y – (k - 3) = 0 ….(i)
And, 12x + ky = k
$\Rightarrow$12x + ky - k = 0 …(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
Here, $a_1 = k$, $b_1= 3$, $c_1 = -(k – 3)$ and $a_2 = 12$, $b_2 = k$, $c_2 = -k$
For a unique solution, we must have:
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
i.e., $\frac{k}{12} \neq \frac{3}{k}$
$\Rightarrow k^2 \neq 36 \Rightarrow k \neq \pm 6$
Thus, for all real values of k, other than ± 6, the given system of equations will have a unique solution.

10. Show that the system equations
2x - 3y = 5,
6x - 9y = 15

has an infinite number of solutions

Sol:

The given system of equations:

2x - 3y = 5

$\Rightarrow$ 2x - 3y – 5 = 0 ….(i)

6x - 9y = 15

$\Rightarrow$6x - 9y - 15 = 0 …(ii)

These equations are of the following forms:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

Here, $a_1 = 2$, $b_1= -3$, $c_1 = -5$ and $a_2 = 6$, $b_2 = -9$, $c_2 = -15$

$\therefore \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{-5}{-15} = \frac{1}{3}$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given system of equations has an infinite number of solutions.

11. Show that the system of equations

6x + 5y = 11,

$9x + \frac{15}{2}y = 21$

has no solution.

Sol:

The given system of equations can be written as

6x + 5y – 11 = 0 ….(i)

$\Rightarrow 9x + \frac{15}{2}y - 21 = 0$ …(ii)

This system is of the form

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

Here, $a_1 = 6$, $b_1= 5$, $c_1 = -11$ and $a_2 = 9$, $b_2 = \frac{15}{2}$, $c_2 = -21$

Now,

$\frac{a_1}{a_2} = \frac{6}{9} = \frac{2}{3}$

$\frac{b_1}{b_2} = \frac{5}{\frac{15}{2}} = \frac{2}{3}$

$\frac{c_1}{c_2} = \frac{-11}{-21} = \frac{11}{21}$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, therefore the given system has no solution.

12. For what value of k, the system of equations

kx + 2y = 5,

3x - 4y = 10

has (i) a unique solution, (ii) no solution?

Sol:

The given system of equations:

kx + 2y = 5

$\Rightarrow$ kx + 2y - 5 = 0 ....(i)

3x - 4y = 10

$\Rightarrow$ 3x - 4y - 10 = 0 ...(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1= 2$, $c_1 = -5$ and $a_2 = 3$, $b_2 = -4$, $c_2 = -10$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{k}{3} \neq \frac{2}{-4} \Rightarrow k \neq \frac{-3}{2}$

Thus for all real values of k other than $\frac{-3}{2}$, the given system of equations will have a unique solution.

(ii) For the given system of equations to have no solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{3} = \frac{2}{-4} \neq \frac{-5}{-10}$

$\Rightarrow \frac{k}{3} = \frac{2}{-4}$ and $\frac{k}{3} \neq \frac{1}{2}$

$\Rightarrow k = \frac{-3}{2}, k \neq \frac{3}{2}$

Hence, the required value of k is $\frac{-3}{2}$.

13. For what value of k, the system of equations

x + 2y = 5,

3x + ky + 15 = 0

has (i) a unique solution, (ii) no solution?

Sol:

The given system of equations:

x + 2y = 5

$\Rightarrow$ x + 2y - 5 = 0 ....(i)

3x + ky + 15 = 0 ...(ii)

These equations are of the forms:

$a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

where, $a_1 = 1$, $b_1= 2$, $c_1 = -5$ and $a_2 = 3$, $b_2 = k$, $c_2 = 15$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{1}{3} \neq \frac{2}{k} \Rightarrow k \neq 6$

Thus for all real values of k other than 6, the given system of equations will have a unique solution.

(ii) For the given system of equations to have no solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15}$

$\Rightarrow \frac{1}{3} = \frac{2}{k}$ and $\frac{2}{k} \neq \frac{-5}{15}$

$\Rightarrow$k = 6, k ≠ -6

Hence, the required value of k is 6.

14. For what value of k, the system of equations

x + 2y = 3,

5x + ky + 7 = 0

Have (i) a unique solution, (ii) no solution?

Also, show that there is no value of k for which the given system of equation has infinitely namely solutions

Sol:

The given system of equations:

x + 2y = 3

$\Rightarrow$ x + 2y - 3 = 0 ….(i)

And, 5x + ky + 7 = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 1$, $b_1 = 2$, $c_1 = -3$ and $a_2 = 5$, $b_2 = k$, $c_2 = 7$

(i) For a unique solution, we must have:

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., $\frac{1}{5} \neq \frac{2}{k} \Rightarrow k \neq 10$

Thus for all real values of k other than 10, the given system of equations will have a unique solution.

(ii) In order that the given system of equations has no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{1}{5} \neq \frac{2}{k} \neq \frac{-3}{7}$

$\Rightarrow \frac{1}{5} \neq \frac{2}{k}$ and $\frac{2}{k} \neq \frac{-3}{7}$

$\Rightarrow$k = 10, $k \neq \frac{14}{-3}$

Hence, the required value of k is 10.

There is no value of k for which the given system of equations has an infinite number of solutions.

15. Find the value of k for which the system of linear equations has an infinite number of solutions:
$2x + 3y = 7$,
$(k - 1)x + (k + 2)y = 3k$.
Sol:
The given system of equations:
$2x + 3y = 7$,
$\Rightarrow 2x + 3y - 7 = 0$ ….(i)
And, $(k - 1)x + (k + 2)y = 3k$
$\Rightarrow (k - 1)x + (k + 2)y - 3k = 0$ …(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = (k - 1)$, $b_2 = (k + 2)$, $c_2 = -3k$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\frac{2}{(k-1)} = \frac{3}{(k+2)} = \frac{-7}{-3k}$
$\Rightarrow \frac{2}{(k-1)} = \frac{3}{(k+2)} = \frac{7}{3k}$
Now, we have the following three cases:
Case I:
$\frac{2}{(k-1)} = \frac{3}{k+2}$
$\Rightarrow 2(k + 2) = 3(k - 1) \Rightarrow 2k + 4 = 3k - 3 \Rightarrow k = 7$
Case II:
$\frac{3}{(k+2)} = \frac{7}{3k}$
$\Rightarrow 7(k + 2) = 9k \Rightarrow 7k + 14 = 9k \Rightarrow 2k = 14 \Rightarrow k = 7$
Case III:
$\frac{2}{(k-1)} = \frac{7}{3k}$
$\Rightarrow 7k - 7 = 6k \Rightarrow k = 7$
Hence, the given system of equations has an infinite number of solutions when k is equal to 7.

16. Find the value of k for which the system of linear equations has an infinite number of solutions:
$2x + (k - 2)y = k$,
$6x + (2k - 1)y = (2k + 5)$.
Sol:
The given system of equations:

$2x + (k - 2)y = k$

$\Rightarrow 2x + (k - 2)y - k = 0$ ….(i)

And, $6x + (2k - 1) y = (2k + 5)$

$\Rightarrow 6x + (2k - 1) y - (2k + 5) = 0$ …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= (k - 2)$, $c_1 = -k$ and $a_2 = 6$, $b_2 = (2k - 1)$, $c_2 = -(2k + 5)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{6} = \frac{(k-2)}{(2k-1)} = \frac{-k}{-(2k+5)}$

$\Rightarrow \frac{1}{3} = \frac{(k-2)}{(2k-1)} = \frac{k}{(2k+5)}$

Now, we have the following three cases:

Case I:

$\frac{1}{3} = \frac{(k-2)}{(2k-1)}$

$\Rightarrow (2k - 1) = 3(k - 2)$

$\Rightarrow 2k - 1 = 3k - 6 \Rightarrow k = 5$

Case II:

$\frac{(k-2)}{(2k-1)} = \frac{k}{(2k+5)}$

$\Rightarrow (k - 2) (2k + 5) = k(2k - 1)$

$\Rightarrow 2k^2 + 5k - 4k - 10 = 2k^2 - k$

$\Rightarrow k + k = 10 \Rightarrow 2k = 10 \Rightarrow k = 5$

Case III:

$\frac{1}{3} = \frac{k}{(2k+5)}$

$\Rightarrow 2k + 5 = 3k \Rightarrow k = 5$

Hence, the given system of equations has an infinite number of solutions when k is equal to 5.

17. Find the value of k for which the system of linear equations has an infinite number of solutions:

$kx + 3y = (2k + 1)$,

$2(k + 1)x + 9y = (7k + 1)$.

Sol:

The given system of equations:

$kx + 3y = (2k + 1)$

$\Rightarrow kx + 3y - (2k + 1) = 0$ ….(i)

And, 2(k + 1)x + 9y = (7k + 1)

⇒ 2(k + 1)x + 9y - (7k + 1) = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1= 3$, $c_1 = -(2k + 1)$ and $a_2 = 2(k + 1)$, $b_2 = 9$, $c_2 = -(7k + 1)$

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e., $\frac{k}{2(k+1)} = \frac{3}{9} = \frac{-(2k+1)}{-(7k+1)}$

$\Rightarrow \frac{k}{2(k+1)} = \frac{1}{3} = \frac{(2k+1)}{(7k+1)}$

Now, we have the following three cases:

Case I:

$\frac{k}{2(k+1)} = \frac{1}{3}$

⇒ 2(k + 1) = 3k

⇒ 2k + 2 = 3k

⇒ k = 2

Case II:

$\frac{1}{3} = \frac{(2k+1)}{(7k+1)}$

⇒ (7k + 1) = 6k + 3

⇒ k = 2

Case III:

$\frac{k}{2(k+1)} = \frac{(2k+1)}{(7k+1)}$

⇒ k(7k + 1) = (2k + 1) × 2(k + 1)

⇒ $7k^2 + k = (2k + 1)(2k + 2)$

⇒ $7k^2 + k = 4k^2 + 4k + 2k + 2$

⇒ $3k^2 – 5k – 2 = 0$

⇒ $3k^2 – 6k + k – 2 = 0$

⇒ 3k(k – 2) + 1(k – 2) = 0

⇒ (3k + 1) (k – 2) = 0

⇒ k = 2 or k = $\frac{-1}{3}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 2.

18. Find the value of k for which the system of linear equations has an infinite number of solutions:
$5x + 2y = 2k$,
$2(k + 1)x + ky = (3k + 4)$.
Sol:
The given system of equations:
$5x + 2y = 2k$
$\Rightarrow 5x + 2y - 2k = 0$ ….(i)
And, $2(k + 1)x + ky = (3k + 4)$
$\Rightarrow 2(k + 1)x + ky - (3k + 4) = 0$ …(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 5$, $b_1= 2$, $c_1 = -2k$ and $a_2 = 2(k + 1)$, $b_2 = k$, $c_2 = -(3k + 4)$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\frac{5}{2(k+1)} = \frac{2}{k} = \frac{-2k}{-(3k+4)}$
$\Rightarrow \frac{5}{2(k+1)} = \frac{2}{k} = \frac{2k}{(3k+4)}$
Now, we have the following three cases:
Case I:
$\frac{5}{2(k+1)} = \frac{2}{k}$
$\Rightarrow 2 \times 2(k + 1) = 5k$
$\Rightarrow 4(k + 1) = 5k$
$\Rightarrow 4k + 4 = 5k$
$\Rightarrow k = 4$
Case II:
$\frac{2}{k} = \frac{2k}{(3k+4)}$
$\Rightarrow 2k^2 = 2 \times (3k + 4)$
$\Rightarrow 2k^2 = 6k + 8 \Rightarrow 2k^2 - 6k - 8 = 0$
$\Rightarrow 2(k^2 - 3k - 4) = 0$
$\Rightarrow k^2 - 4k + k - 4 = 0$
$\Rightarrow k(k - 4) + 1(k - 4) = 0$
$\Rightarrow (k + 1) (k - 4) = 0$
$\Rightarrow (k + 1) = 0$ or $(k - 4) = 0$

$\Rightarrow$ k = -1 or k = 4

Case III:

$\frac{5}{2(k+1)} = \frac{2k}{(3k+4)}$

$\Rightarrow$ 15k + 20 = 4k^2 + 4k

$\Rightarrow$ 4k^2 – 11k – 20 = 0

$\Rightarrow$ 4k^2– 16k + 5k – 20 = 0

$\Rightarrow$ 4k(k – 4) + 5(k – 4) = 0

$\Rightarrow$ (k – 4) (4k + 5) = 0

$\Rightarrow$ k = 4 or k = $\frac{-5}{4}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 4.

19. Find the value of k for which the system of linear equations has an infinite number of solutions:

(k – 1)x – y = 5,

(k + 1)x + (1 – k)y = (3k + 1).

Sol:

The given system of equations:

(k – 1)x – y = 5

$\Rightarrow$ (k – 1)x – y – 5 = 0 ….(i)

And, (k + 1)x + (1 – k)y = (3k + 1)

$\Rightarrow$ (k + 1)x + (1 – k)y – (3k + 1) = 0 …(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, a_1 = (k – 1), b_1= -1, c_1 = -5 and a_2 = (k + 1), b_2 = (1 – k), c_2 = -(3k + 1)

For an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e., $\frac{(k-1)}{(k+1)} = \frac{-1}{-(k-1)} = \frac{-5}{-(3k+1)}$

$\Rightarrow \frac{(k-1)}{(k+1)} = \frac{1}{(k-1)} = \frac{5}{(3k+1)}$

Now, we have the following three cases:

Case I:

$\frac{(k-1)}{(k+1)} = \frac{1}{(k-1)}$

$\Rightarrow$ (k - 1)^2 = (k + 1)

$\Rightarrow$ k^2 + 1 – 2k = k + 1

$\Rightarrow k^2 - 3k = 0 \Rightarrow k(k - 3) = 0$

$\Rightarrow k = 0$ or $k = 3$

Case II:

$\frac{1}{(k-1)} = \frac{5}{(3k+1)}$

$\Rightarrow 3k + 1 = 5k - 5$

$\Rightarrow 2k = 6 \Rightarrow k = 3$

Case III:

$\frac{(k-1)}{(k+1)} = \frac{5}{(3k+1)}$

$\Rightarrow (3k + 1)(k - 1) = 5(k + 1)$

$\Rightarrow 3k^2 + k - 3k - 1 = 5k + 5$

$\Rightarrow 3k^2 - 2k - 5k - 1 - 5 = 0$

$\Rightarrow 3k^2 - 7k - 6 = 0$

$\Rightarrow 3k^2 - 9k + 2k - 6 = 0$

$\Rightarrow 3k(k - 3) + 2(k - 3) = 0$

$\Rightarrow (k - 3)(3k + 2) = 0$

$\Rightarrow (k - 3) = 0$ or $(3k + 2) = 0$

$\Rightarrow k = 3$ or $k = \frac{-2}{3}$

Hence, the given system of equations has an infinite number of solutions when k is equal to 3.

20. Find the value of k for which the system of linear equations has a unique solution:
$(k - 3)x + 3y - k, kx + ky - 12 = 0$.

Sol:

The given system of equations can be written as

$(k - 3)x + 3y - k = 0$

$kx + ky - 12 = 0$

This system is of the form:

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

where, $a_1 = k$, $b_1= 3$, $c_1 = -k$ and $a_2 = k$, $b_2 = k$, $c_2 = -12$

For the given system of equations to have a unique solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{k-3}{k} = \frac{3}{k} = \frac{-k}{-12}$

$\Rightarrow k - 3 = 3$ and $k^2 = 36$

$\Rightarrow$ k = 6 and k = ±6
$\Rightarrow$ k = 6
Hence, k = 6.

21. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
(a – 1) x + 3y = 2, 6x + (1 – 2b)y = 6
Sol:
The given system of equations can be written as
(a – 1) x + 3y = 2
$\Rightarrow$(a – 1) x + 3y – 2 = 0 ….(i)
and 6x + (1 – 2b)y = 6
$\Rightarrow$6x + (1 – 2b)y – 6 = 0 ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = (a – 1)$, $b_1= 3$, $c_1 = -2$ and $a_2 = 6$, $b_2 = (1 – 2b)$, $c_2 = -6$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\Rightarrow \frac{a-1}{6} = \frac{3}{(1-2b)} = \frac{-2}{-6}$
$\Rightarrow \frac{a-1}{6} = \frac{3}{(1-2b)} = \frac{1}{3}$
$\Rightarrow \frac{a-1}{6} = \frac{1}{3}$ and $\frac{3}{(1-2b)} = \frac{1}{3}$
$\Rightarrow$ 3a – 3 = 6 and 9 = 1 – 2b
$\Rightarrow$ 3a = 9 and 2b = -8
$\Rightarrow$ a = 3 and b = -4
$\therefore$ a = 3 and b = -4

22. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
(2a – 1) x + 3y = 5, 3x + (b – 1)y = 2.
Sol:
The given system of equations can be written as
(2a – 1) x + 3y = 5
$\Rightarrow$(2a – 1) x + 3y – 5 = 0 ….(i)
and 3x + (b – 1)y = 2
$\Rightarrow$3x + (b – 1)y – 2 = 0 ….(ii)
These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = (2a – 1)$, $b_1= 3$, $c_1 = -5$ and $a_2 = 3$, $b_2 = (b – 1)$, $c_2 = -2$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\Rightarrow \frac{(2a-1)}{3} = \frac{3}{(b-1)} = \frac{-5}{-2}$
$\Rightarrow \frac{(2a-1)}{6} = \frac{3}{(b-1)} = \frac{5}{2}$
$\Rightarrow \frac{(2a-1)}{6} = \frac{5}{2}$ and $\frac{3}{(b-1)} = \frac{5}{2}$
$\Rightarrow 2(2a – 1) = 15$ and $6 = 5(b – 1)$
$\Rightarrow 4a – 2 = 15$ and $6 = 5b – 5$
$\Rightarrow 4a = 17$ and $5b = 11$
$\therefore a = \frac{17}{4}$ and $b = \frac{11}{5}$

23. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
$2x - 3y = 7$, $(a + b)x - (a + b – 3)y = 4a + b$.
Sol:
The given system of equations can be written as
$2x - 3y = 7$
$\Rightarrow 2x - 3y - 7 = 0$ ….(i)
and $(a + b)x - (a + b – 3)y = 4a + b$
$\Rightarrow (a + b)x - (a + b – 3)y - 4a + b = 0$ ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
Here, $a_1 = 2$, $b_1= -3$, $c_1 = -7$ and $a_2 = (a + b)$, $b_2 = -(a + b - 3)$, $c_2 = -(4a + b)$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\frac{2}{a+b} = \frac{-3}{-(a+b-3)} = \frac{-7}{-(4a+b)}$
$\Rightarrow \frac{2}{a+b} = \frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$
$\Rightarrow \frac{2}{a+b} = \frac{7}{(4a+b)}$ and $\frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$
$\Rightarrow 2(4a + b) = 7(a + b)$ and $3(4a + b) = 7(a + b - 3)$
$\Rightarrow 8a + 2b = 7a + 7b$ and $12a + 3b = 7a + 7b - 21$
$\Rightarrow 4a = 17$ and $5b = 11$
$\therefore a = 5b$ …….(iii)
and $5a = 4b – 21$ ……(iv)
On substituting $a = 5b$ in (iv), we get:

25b = 4b – 21
⇒21b = -21
⇒b = -1
On substituting b = -1 in (iii), we get:
a = 5(-1) = -5
∴a = -5 and b = -1.

24. Find the values of a and b for which the system of linear equations has an infinite number of solutions:
2x + 3y = 7, (a + b + 1)x - (a + 2b + 2)y = 4(a + b) + 1.
Sol:
The given system of equations can be written as
2x + 3y = 7
⇒2x + 3y - 7 = 0 ….(i)
and (a + b + 1)x - (a + 2b + 2)y = 4(a + b) + 1
(a + b + 1)x - (a + 2b + 2)y – [4(a + b) + 1] = 0 ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = (a + b + 1)$, $b_2 = (a + 2b + 2)$, $c_2 = – [4(a + b) + 1]$
For an infinite number of solutions, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
$\frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)} = \frac{-7}{-[4(a+b)+1]}$
$\Rightarrow \frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)} = \frac{7}{[4(a+b)+1]}$
$\Rightarrow \frac{2}{(a+b+1)} = \frac{3}{(a+2b+2)}$ and $\frac{3}{(a+2b+2)} = \frac{7}{[4(a+b)+1]}$
⇒ 2(a + 2b + 2) = 3(a + b + 1) and 3[4(a + b) + 1] = 7(a + 2b + 2)
⇒ 2a + 4b + 4 = 3a + 3b + 3 and 3(4a + 4b + 1) = 7a + 14b + 14
⇒ a – b - 1=0 and 12a + 12b + 3 = 7a + 14b + 14
⇒ a - b = 1 and 5a – 2b = 11
a = (b + 1) …….(iii)
5a - 2b = 11 ……(iv)
On substituting a = (b + 1) in (iv), we get:
5(b + 1) – 2b = 11
⇒5b + 5 – 2b = 11
⇒ 3b = 6
⇒ b = 2
On substituting b = 2 in (iii), we get:
a = 3

∴a = 3 and b = 2.

25. Find the values of a and b for which the system of linear equations has an infinite number of solutions:

2x + 3y = 7, (a + b)x + (2a - b)y = 21.

Sol:

The given system of equations can be written as

2x + 3y - 7 = 0 ….(i)

(a + b)x + (2a - b)y – 21 = 0 ….(ii)

This system is of the form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = a + b$, $b_2 = 2a - b$, $c_2 = -21$

For the given system of linear equations to have an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{a+b} = \frac{3}{2a-b} = \frac{-7}{-21}$

$\Rightarrow \frac{2}{a+b} = \frac{-7}{-21} = \frac{1}{3}$ and $\frac{3}{2a-b} = \frac{-7}{-21} = \frac{1}{3}$

⇒ a + b = 6 and 2a –b = 9

Adding a + b = 6 and 2a – b = 9 ,we get

$3a = 15 \Rightarrow a = \frac{15}{3} = 3$

Now substituting a = 5 in a + b = 6, we have

5 + b = 6 ⇒ b = 6 – 5 = 1

Hence, a = 5 and b = 1.

26. Find the values of a and b for which the system of linear equations has an infinite number of solutions:

2x + 3y = 7, 2ax + (a + b)y = 28.

Sol:

The given system of equations can be written as

2x + 3y - 7 = 0 ….(i)

2ax + (a + b)y – 28 = 0 ….(ii)

This system is of the form:

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

where, $a_1 = 2$, $b_1= 3$, $c_1 = -7$ and $a_2 = 2a$, $b_2 = a + b$, $c_2 = -28$

For the given system of linear equations to have an infinite number of solutions, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$

$\Rightarrow \frac{2}{2a} = \frac{-7}{-28} = \frac{1}{4}$ and $\frac{3}{a+b} = \frac{-7}{-28} = \frac{1}{4}$

⇒ a = 4 and a + b = 12

Substituting a = 4 in a + b = 12, we get

4 + b = 12 ⇒ b = 12 – 4 = 8

Hence, a = 4 and b = 8.

27. Find the value of k for which the system of equations

8x + 5y = 9, kx + 10y = 15

has a non-zero solution.

Sol:

The given system of equations:

8x + 5y = 9

8x + 5y - 9 = 0 ….(i)

kx + 10y = 15

kx + 10y - 15 = 0 ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 8$, $b_1= 5$, $c_1 = -9$ and $a_2 = k$, $b_2 = 10$, $c_2 = – 15$

In order that the given system has no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e., $\frac{8}{k} = \frac{5}{10} \neq \frac{-9}{-15}$

i.e., $\frac{8}{k} = \frac{1}{2} \neq \frac{3}{5}$

$\frac{8}{k} = \frac{1}{2}$ and $\frac{8}{k} \neq \frac{3}{5}$

⇒ k = 16 and $k \neq \frac{40}{3}$

Hence, the given system of equations has no solutions when k is equal to 16.

28. Find the value of k for which the system of equations

kx + 3y = 3, 12x + ky = 6 has no solution.

Sol:

The given system of equations:

kx + 3y = 3

kx + 3y - 3 = 0 ….(i)

12x + ky = 6

12x + ky - 6 = 0 ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = k$, $b_1= 3$, $c_1 = -3$ and $a_2 = 12$, $b_2 = k$, $c_2 = –6$
In order that the given system has no solution, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
i.e., $\frac{k}{12} = \frac{3}{k} \neq \frac{-3}{-6}$
$\frac{k}{12} = \frac{3}{k}$ and $\frac{3}{k} \neq \frac{1}{2}$
$\Rightarrow k^2 = 36$ and $k \neq 6$
$\Rightarrow k = \pm 6$ and $k \neq 6$
Hence, the given system of equations has no solution when k is equal to -6.

29. Find the value of k for which the system of equations
3x - y = 5, 6x - 2y = k
has no solution.
Sol:
The given system of equations:
3x - y - 5 = 0 ….(i)
And, 6x - 2y + k = 0 ….(ii)
These equations are of the following form:
$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$
where, $a_1 = 3$, $b_1= -1$, $c_1 = -5$ and $a_2 = 6$, $b_2 = -2$, $c_2 = k$
In order that the given system has no solution, we must have:
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
i.e., $\frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{k}$
$\Rightarrow \frac{-1}{-2} \neq \frac{-5}{k} \Rightarrow k \neq -10$
Hence, equations (i) and (ii) will have no solution if $k \neq -10$.

30. Find the value of k for which the system of equations
kx + 3y + 3 - k = 0, 12x + ky - k = 0
has no solution.
Sol:
The given system of equations can be written as
kx + 3y + 3 - k = 0 ….(i)
12x + ky - k = 0 ….(ii)
This system of the form:
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
where, $a_1 = k$, $b_1= 3$, $c_1 = 3 - k$ and $a_2 = 12$, $b_2 = k$, $c_2 = –k$

For the given system of linear equations to have no solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{12} = \frac{3}{k} \neq \frac{3-k}{-k}$

$\Rightarrow \frac{k}{12} = \frac{3}{k}$ and $\frac{3}{k} \neq \frac{3-k}{-k}$

$\Rightarrow k^2 = 36$ and $-3 \neq 3 - k$

$\Rightarrow k = \pm 6$ and $k \neq 6$

$\Rightarrow k = -6$

Hence, k = -6.

31. Find the value of k for which the system of equations

5x - 3y = 0, 2x + ky = 0

has a non-zero solution.

Sol:

The given system of equations:

5x - 3y = 0 ….(i)

2x + ky = 0 ….(ii)

These equations are of the following form:

$a_1x+b_1y+c_1 = 0$, $a_2x+b_2y+c_2 = 0$

where, $a_1 = 5$, $b_1 = -3$, $c_1 = 0$ and $a_2 = 2$, $b_2 = k$, $c_2 = 0$

For a non-zero solution, we must have:

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow \frac{5}{2} = \frac{-3}{k}$

$\Rightarrow 5k = -6 \Rightarrow k = \frac{-6}{5}$

Hence, the required value of k is $\frac{-6}{5}$.