Binary Operations Ex 3.4 Q1

Given,

$a * b = a + b - 4$ for all $a, b \in Z$

(i)

Commutative: Let $a, b \in Z$, then

$\Rightarrow \quad a * b = a + b - 4 = b + a - 4 = b * a$

$\Rightarrow \quad a * b = b * a$

So, '$*$' is commutative on Z.

Associativity: Let $a, b, c \in Z$, then

$(a * b) * c = (a + b - 4) * c = a + b - 4 + c - 4$

$= a + b + c - 8 \quad ---(i)$

and, $a * (b * c) = a * (b + c - 4) = a + b + c - 8 \quad ---(ii)$

From (i) & (ii)

$(a * b) * c = a * (b * c)$

So, '$*$' is associative on Z.

(ii)

Let $e \in Z$ be the identity element with respect to $*$.

By identity property, we have

$a * e = e * a = a$ for all $a \in Z$

$\Rightarrow \quad a + e - 4 = a$

$\Rightarrow \quad e = 4$

So, $e = 4$ will be the identity element with respect to $*$

(iii)

Let $b \in Z$ be the inverse element of $a \in Z$

Then, $a * b = b * a = e$

$\Rightarrow \quad a + b - 4 = e$

$\Rightarrow \quad a + b - 4 = 4 \quad [\because e = 4]$

$\Rightarrow \quad b = 8 - a$

Thus, $b = 8 - a$ will be the inverse element of $a \in Z$.

Binary Operations Ex 3.4 Q2

We have,

$a * b = \frac{3ab}{5}$ for all $a, b \in Q_0$

(i)

Commutative: Let $a, b \in Q_0$, then

$$a * b = \frac{3ab}{5} = \frac{3ba}{5} = b * a$$

$\Rightarrow$ $a * b = b * a$

So, '*' is commutative on Q_0

Associativity: Let $a, b, c \in Q_0$, then

$$(a * b) * c = \frac{3ab}{5} * c$$

$$= \frac{9abc}{25} \qquad ---(i)$$

and, $$a * (b * c) = a * \frac{3bc}{5}$$

$$= \frac{9abc}{25} \qquad ---(ii)$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '*' is associative on Q_0

(ii)

Let $e \in Q_0$ be the identity element with respect to *, then

$a * e = e * a = a$ for all $a \in Q_0$

$\Rightarrow$ $\frac{3ae}{5} = a$

$\Rightarrow$ $e = \frac{5}{3}$

will be the identity element with respect to *.

(iii)

Let $b \in Q_0$ be the inverse element of $a \in Q_0$, then

$a * b = b * a = e$

$\Rightarrow$ $\frac{3}{5}ab = e$

$\Rightarrow$ $\frac{3}{5}ab = \frac{5}{3}$ $\qquad \left[\because e = \frac{5}{3}\right]$

$\Rightarrow$ $b = \frac{25}{9a}$

$\therefore$ $b = \frac{25}{9a}$ is the inverse of $a \in Q_0$.

Binary Operations Ex 3.4 Q3

We have,

$a * b = a + b + ab$ for all $a, b \in Q - \{-1\}$

(i)

Commutativity: Let $a, b \in Q - \{-1\}$

$\Rightarrow \quad a * b = a + b + ab = b + a + ba = b * a$

$\Rightarrow \quad a * b = b * a$

$\Rightarrow \quad$ '$*$' is commutative on $Q - \{-1\}$

Associativity: Let $a, b, c \in Q - \{-1\}$, then

$\Rightarrow \quad (a * b) * c = (a + b + ab) * c$

$= a + b + ab + c + ac + bc + abc \quad ---(i)$

and, $\quad a * (b * c) = a * (b + c + bc)$

$= a + b + c + bc + ab + ac + abc \quad ---(ii)$

From (i) & (ii)

$(a * b) * c = a * (b * c)$

$\Rightarrow \quad *$ is associative on $Q - \{-1\}$

(ii)

Let e be identity element with respect to $*$.

By identity property,

$a * e = a = e * a$ for all $a \in Q - \{-1\}$

$\Rightarrow \quad a + e + ae = a$

$\Rightarrow \quad e(1 + a) = 0 \quad \Rightarrow e = 0 \qquad [\because 1 + a \neq 0 \text{ as } a \neq -1]$

$\therefore \quad e = 0$ is the identity element with respect to $*$

(iii)

Let b be the inverse of $a \in Q - \{-1\}$

Then, $\quad a * b = b * a = e \qquad$ [e is the identity element]

$\Rightarrow \quad a + b + ab = e$

$\Rightarrow \quad a + b + ab = 0$

$\Rightarrow \quad b(1 + a) = -a$

$\Rightarrow \quad b = \frac{-a}{1 + a} \qquad \left[\because \frac{-a}{1+a} \neq -1, \text{ because if } \frac{-a}{1+a} = -1 \Rightarrow a = 1 + a \Rightarrow 1 = 0 \text{ Not possible}\right]$

$\therefore \quad b = \frac{-a}{1 + a}$ is the inverse of a with respect to $*$

Binary Operations Ex 3.4 Q4

We have,

$$(a,b)\odot(c,d)=(ac,bc+d) \text{ for all } (a,b),(c,d)\in R_0\times R$$

(i)

Commutativity: Let $(a,b),(c,d)\in R_0\times R$, then

$\Rightarrow \quad (a,b)\odot(c,d)=(ac,bc+d) \quad ---(i)$

and, $\quad (c,d)\odot(a,b)=(ca,da+b) \quad ---(ii)$

From (i) & (ii)

$$(a,b)\odot(c,d)\neq(c,d)\odot(a,b)$$

$\Rightarrow \quad$ '$\odot$' is not commutative on $R_0\times R$.

Associativity: Let $(a,b),(c,d),(e,f)\in R_0\times R$, then

$\Rightarrow \quad ((a,b)\odot(c,d))\odot(e,f)=(ac,bc+d)\odot(e,f)$

$\qquad =(ace,bce,de+f) \quad ---(i)$

and, $\quad (a,b)\odot(c,d\odot(e,f))=(a,b)\odot(ce,de+f)$

$\qquad =(ace,bce+de+f) \quad ---(ii)$

$\Rightarrow \quad ((a,b)\odot(c,d))\odot(e,f) = (a,b)\odot((c,d)\odot(e,f))$

$\Rightarrow \quad$ '$\odot$' is associative on $R_0 \times R$.

(ii)

Let $(x,y) \in R_0 \times R$ be the identity element with respect to $\odot$, then

$$(a,b)\odot(x,y) = (x,y)\odot(a,b) = (a,b) \text{ for all } (a,b) \in R_0 \times R$$

$\Rightarrow \quad (ax, bx+y) = (a,b)$

$\Rightarrow \quad ax = a$ and $bx + y = b$

$\Rightarrow \quad x = 1,$ and $y = 0$

$\therefore \quad (1,0)$ will be the identity element with respect to $\odot$.

(iii)

Let $(c,d) \in R_0 \times R$ be the inverse of $(a,b) \in R_0 \times R$, then

$$(a,b)\odot(c,d) = (c,d)\odot(a,b) = e$$

$\Rightarrow \quad (ac, bc+d) = (1,0) \qquad [\because e = (1,0)]$

$\Rightarrow \quad ac = 1$ and $bc + d = 0$

$\Rightarrow \quad c = \frac{1}{a}$ and $d = -\frac{b}{a}$

$\therefore \quad \left(\frac{1}{a}, -\frac{b}{a}\right)$ will be the inverse of (a,b).

Binary Operations Ex 3.4 Q5

We have,

$$a * b = \frac{ab}{2} \text{ for all } a, b \in Q_0$$

(i)

Commutativity: Let $a, b \in Q_0$, then

$$\Rightarrow \quad a * b = \frac{ab}{2} = \frac{ba}{2} = b * a$$

$$\Rightarrow \quad a * b = b * a$$

Hence, '$*$' is commutative on Q_0.

Associativity: Let $a, b, c \in Q_0$, then

$$\Rightarrow \quad (a * b) * c = \frac{ab}{2} * c = \frac{abc}{4} \qquad ---(i)$$

$$\text{and,} \quad a * (b * c) = a * \frac{bc}{2} = \frac{abc}{4} \qquad ---(ii)$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow$ $*$ is associative on Q_0.

(ii)

Let $e \in Q_0$ be the identity element with respect to $*$.

By identity property, we have,

$$a * e = e * a = a \text{ for all } a \in Q_0$$

$$\Rightarrow \quad \frac{ae}{2} = a \qquad \Rightarrow e = 2$$

Thus, the required identity element is 2.

(iii)

Let $b \in Q_0$ be the inverse of $a \in Q_0$ with respect to $*$, then,

$$a * b = b * a = e \text{ for all } a \in Q_0$$

$$\Rightarrow \quad \frac{ab}{2} = e \qquad \Rightarrow \frac{ab}{2} = 2$$

$$\Rightarrow b = \frac{4}{a}$$

Thus, $b = \frac{4}{a}$ is the inverse of a with respect to $*$.

Binary Operations Ex 3.4 Q6

We have,

$$a*b = a+b-ab \text{ for all } a,b \in R-\{+1\}$$

(i)

Commutative: Let $a,b \in R-\{+1\}$, then,

$\Rightarrow \quad a*b = a+b-ab = b+a-ba = b*a$

$\Rightarrow \quad a*b = b*a$

So, '$*$' is commutative on $R-\{+1\}$.

Associativity: Let $a,b,c \in R-\{+1\}$, then

$$\begin{aligned}(a*b)*c &= (a+b-ab)*c \\ &= a+b-ab+c-ac-bc+abc \\ &= a+b+c-ab-ac-bc+abc \qquad ---(i)\end{aligned}$$

and,

$$\begin{aligned}a*(b*c) &= a*(b+c-bc) \\ &= a+b+c-bc-ab-ac+abc \qquad ---(ii)\end{aligned}$$

From (i) & (ii)

$$(a*b)*c = a*(b*c)$$

So, '$*$' is associative on $R-\{+1\}$.

(ii)

Let $e \in R-\{+1\}$ be the identity element with respect to $*$, then

$$a*e = e*a = a \text{ for all } a \in R-\{+1\}$$

$\Rightarrow \quad a+e-ae = a$

$\Rightarrow \quad e(1-a) = 0$

$\Rightarrow \quad e = 0 \qquad [\because a \neq 1 \Rightarrow 1-a \neq 0]$

$\therefore \quad e = 0$ will be the identity element with respect to $*$.

(iii)

Let $b \in R-\{1\}$ be the inverse element of $a \in R-\{1\}$, then

$$a*b = b*a = e$$

$\Rightarrow \quad a+b-ab = 0 \qquad [\because e = 0]$

$\Rightarrow \quad b(1-a) = -a$

$\Rightarrow \quad b = \dfrac{-a}{1-a} \neq 1 \qquad \left[\begin{array}{l}\because \text{if } \dfrac{-a}{1-a} = 1 \\ \Rightarrow -a = 1-a \quad \Rightarrow 1 = 0 \\ \quad \text{Not possible}\end{array}\right]$

$\therefore \quad b = \dfrac{-a}{1-a}$ is the inverse of $a \in R-\{1\}$ with respect to $*$.

Binary Operations Ex 3.4 Q7

We have,

$$(a,b)*(c,d)=(ac,bd) \text{ for all } (a,b),(c,d)\in A$$

(i)

Let $(a,b),(c,d)\in A$, then

$$(a,b)*(c,d)=(ac,bd)$$
$$=(ca,db) \qquad [\because ac=ca \text{ and } bd=db]$$
$$=(c,d)*(a,b)$$

$\Rightarrow \quad (a,b)*(c,d)=(c,d)*(a,b)$

So, '$*$' is commutative on A

Associativity: Let $(a,b),(c,d),(e,f)\in A$, then

$\Rightarrow \quad ((a,b)*(c,d))*(e,f)=(ac,bd)*(e,f)$

$$=(ace,bdf) \qquad ---(i)$$

and, $\quad (a,b)*((c,d)*(e,f))=(a,b)*(ce,df)$

$$=(ace,bdf) \qquad ---(ii)$$

From (i) & (ii)

$\Rightarrow \quad ((a,b)*(c,d))*(e,f)=(a,b)*((c,d)*(e,f))$

So, '$*$' is associative on A.

(ii)

Let $(x,y)\in A$ be the identity element with respect to $*$.

$$(a,b)*(x,y)=(x,y)*(a,b)=(a,b) \text{ for all } (a,b)\in A$$

$\Rightarrow \quad (ax,by)=(a,b)$

$\Rightarrow \quad ax=a$ and $by=b$

$\Rightarrow \quad x=1$, and $y=1$

$\therefore \quad (1,1)$ will be the identity element

(iii)

Let $(c,d)\in A$ be the inverse of $(a,b)\in A$, then

$$(a,b)*(c,d)=(c,d)*(a,b)=e$$

$\Rightarrow \quad (ac,bd)=(1,1) \qquad [\because e=(1,1)]$

$\Rightarrow \quad ac=1$ and $bd=1$

$\Rightarrow \quad c=\frac{1}{a}$ and $d=\frac{1}{b}$

$\therefore \quad \left(\frac{1}{a}, \frac{1}{b}\right)$ will be the inverse of (a, b) with respect to $*$.

Binary Operations Ex 3.4 Q8

The binary operation * on **N** is defined as:

$a * b$ = H.C.F. of a and b

It is known that:

H.C.F. of a and b = H.C.F. of b and a, $a, b \in$ **N**.

Therefore, $a * b = b * a$

Thus, the operation * is commutative.

For $a, b, c \in$ **N**, we have:

$(a * b) * c$ = (H.C.F. of a and b) $* c$ = H.C.F. of a, b, and c

$a * (b * c) = a *$(H.C.F. of b and c) = H.C.F. of a, b, and c

Therefore, $(a * b) * c = a * (b * c)$

Thus, the operation * is associative.

Now, an element $e \in$ **N** will be the identity for the operation * if $a * e = a = e * a$, $\forall\ a \in$ **N**.

But this relation is not true for any $a \in$ **N**.

Thus, the operation * does not have any identity in **N**.

RD Sharma Solutions Class 12 Maths Chapter 6 Ex 6.5

Chapter 6 Determinants Ex 6.5 Q1

Here $D = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

$= 1(3) - 1(-3) - 2(3)$

$= 3 + 3 - 6$

$= 0$

Since $D = 0$, so the system has infinite solutions:

Now let $z = k$,

$x + y = 2k$

$2x + y = 3k$

Solving there equations by cramer's Rule

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} 2k & 1 \\ 3k & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-k}{-1} = k$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 1 & 2k \\ 2 & 3k \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-k}{-1} = k$$

thus, we have $x = k, y = k, z = k$

and there values satisfy eq.(3)

Hence $x = k, y = k, z = k$

Chapter 6 Determinants Ex 6.5 Q2

Here $D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix}$

$= 2(4) - 3(1) + 4(-3)$

$= 8 - 3 - 7$

$= -2$

$\neq 0$

So, the given system of equations has only the trivial solutions i.e $x = 0 = y = z$:

Hence $x = 0$

$y = 0$

$z = 0$

Chapter 6 Determinants Ex 6.5 Q3

$$\text{Here } D = \begin{vmatrix} 3 & 1 & 1 \\ 1 & -4 & 3 \\ 2 & 5 & -2 \end{vmatrix}$$

$$= 3(8-15) - 1(-2-6) + 1(13)$$

$$= -21 + 8 + 13$$

$$= 0$$

So, the system has infinite solutions:

Let $z = k$,

so, $3x + y = -k$

$x - 4y = -3k$

Now,

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} -k & 1 \\ -3k & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \frac{7k}{-13}$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 3 & -k \\ 1 & -3k \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \frac{-8k}{-13}$$

$$x = \frac{-7k}{13}, y = \frac{8k}{13}, z = k$$

and there values satisfy eq.(3)

Hence $x = -7k, y = 8k, z = 13k$

Chapter 6 Determinants Ex 6.5 Q4

$$D = \begin{vmatrix} 2\lambda & -2 & 3 \\ 1 & \lambda & 2 \\ 2 & 0 & \lambda \end{vmatrix}$$

$$= 3\lambda^3 + 2\lambda - 8 - 6\lambda$$
$$= 2\lambda^3 - 4\lambda - 8$$

which is satisfied by $\lambda = 2$ $\left[\because \quad \text{for non-trivial solutions } \lambda = 2\right]$

Now Let $z = k$,

$$4x - 2y = -3k$$
$$x + 2y = -3k$$

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} -3k & -2 \\ -2k & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix}} = \frac{-10k}{10} = -k$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 4 & -3k \\ 1 & -2k \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix}} = \frac{-5k}{10} = \frac{-k}{2}$$

Hence solution is $x = -k, y = \frac{-k}{2}, z = k$

Chapter 6 Determinants Ex 6.5 Q5

$$D = \begin{vmatrix} (a-1) & -1 & -1 \\ -1 & (b-1) & -1 \\ -1 & -1 & (c-1) \end{vmatrix}$$

Now for non-trivial solution, $D = 0$

$$0 = (a-1)\left[(b-1)(c-1) - 1\right] + 1\left[-c + \not{1} - \not{1}\right] - \left[\not{1} + b - \not{1}\right]$$
$$0 = (a-1)\left[bc - b - c + \not{1} - \not{1}\right] - c - b$$
$$0 = abc - ab - ac + \not{b} + \not{c} - \not{c} - \not{b}$$
$$ab + bc + ac = abc$$

Hence proved