RD Sharma Solutions Class 12 Maths Chapter 3 Ex 3.1

Binary Operations Ex 3.1 Q1(i)

We have,

$$a * b = a^b \text{ for all } a, b \in N$$

Let $a \in N$ and $b \in N$

$\Rightarrow \quad a^b \in N$

$\Rightarrow \quad a * b \in N$

The operation $*$ defines a binary operation on N

Binary Operations Ex 3.1 Q1(ii)

We have,

$a \circ b = a^b$ for all $a, b \in Z$

Let $a \in Z$ and $b \in Z$

$\Rightarrow \quad a^b \notin Z \qquad \Rightarrow a \circ b \notin Z$

For example, if $\quad a = 2, \; b = -2$

$\Rightarrow \quad a^b = 2^{-2} = \frac{1}{4} \notin Z$

$\therefore$ The operation '$\circ$' does not define a binary operation on Z.

Binary Operations Ex 3.1 Q1(iii)

We have,

$a * b = a + b - 2$ for all $a, b \in N$

Let $a \in N$ and $b \in N$

Then, $a + b - 2 \notin N$ for all $a, b \in N$

$\Rightarrow \quad a * b \notin N$

For example $\quad a = 1, \; b = 1$

$\Rightarrow \quad a + b - 2 = 0 \notin N$

$\therefore$ The operation $*$ does not define a binary operation on N

Binary Operations Ex 3.1 Q1(iv)

We have,

$S = \{1, 2, 3, 4, 5\}$

and, $\quad a \times_6 b$ = Remainder when ab is divided by 6

Let $a \in S$ and $b \in S$

$\Rightarrow \quad a \times_6 b \notin S$ for all $a, b \in S$

For example, $\; a = 2, \; b = 3$

$\Rightarrow \quad 2 \times_6 3$ = Remainder when 6 is divided by 6 = $0 \notin S$

$\therefore \quad \times_6$ does not define a binary opartion on S

Binary Operations Ex 3.1 Q1(v)

We have,

$$S = \{0, 1, 2, 3, 4, 5\}$$

and, $a +_6 b = \begin{cases} a+b; \text{ if } a+b < 6 \\ a+b-6; \text{ if } a+b \geq 6 \end{cases}$

Let $a \in S$ and $b \in S$ such that $a + b < 6$

Then $a +_6 b = a + b \in S$ $\quad [\because\ a + b < 6 = 0, 1, 2, 3, 4, 5]$

Let $a \in S$ and $b \in S$ such that $a + b > 6$

Then $a +_6 b = a + b - 6 \in S$ $\quad [\because \text{if } a + b \geq 6 \text{ then } a + b - 6 \geq 0 = 0, 1, 2, 3, 4, 5]$

$\therefore \quad a +_6 b \in S$ for $a, b \in S$

$\therefore \quad +_6$ defines a binary oparation on S

Binary Operations Ex 3.1 Q1(vi)

We have,

$$a \circ b = a^b + b^a \text{ for all } a, b \in N$$

Let $a \in N$ and $b \in N$

$\Rightarrow \quad a^b \in N$ and $b^a \in N$

$\Rightarrow \quad a^b + b^a \in N$

$\Rightarrow \quad a \circ b \in N$

Thus, the operation '$\circ$' defines a binary relation on N

Binary Operations Ex 3.1 Q1(vii)

We have,

$$a * b = \frac{a-1}{b+1} \text{ for all } a, b \in Q$$

Let $a \in Q$ and $b \in Q$

Then $\frac{a-1}{b+1} \notin Q$ for $b = -1$

$\Rightarrow \quad a * b \notin Q$ for all $a, b \in Q$

Thus, the operation $*$ does not define a binary operation on Q

Binary Operations Ex 3.1 Q2

(i) On $\mathbf{Z}^+$, * is defined by $a * b = a - b$.
It is not a binary operation as the image of (1, 2) under * is $1 * 2 = 1 - 2 = -1 \notin \mathbf{Z}^+$.

(ii) On $\mathbf{Z}^+$, * is defined by $a * b = ab$.
It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element ab in $\mathbf{Z}^+$.
This means that * carries each pair (a, b) to a unique element $a * b = ab$ in $\mathbf{Z}^+$.
Therefore, * is a binary operation.

(iii) On $\mathbf{R}$, * is defined by $a * b = ab^2$.
It is seen that for each $a, b \in \mathbf{R}$, there is a unique element ab^2 in $\mathbf{R}$.
This means that * carries each pair (a, b) to a unique element $a * b = ab^2$ in $\mathbf{R}$.
Therefore, * is a binary operation.

(iv) On $\mathbf{Z}^+$, * is defined by $a * b = |a - b|$.
It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element $|a - b|$ in $\mathbf{Z}^+$.
This means that * carries each pair (a, b) to a unique element $a * b = |a - b|$ in $\mathbf{Z}^+$.
Therefore, * is a binary operation.

(v) On $\mathbf{Z}^+$, * is defined by $a * b = a$.
* carries each pair (a, b) to a unique element $a * b = a$ in $\mathbf{Z}^+$.
Therefore, * is a binary operation.

(vi) on R, * is defined by $a * b = a + 4b^2$
it is seen that for each element $a, b \in R$, there is unique element $a + 4b^2$ in R
This means that * carries each pair (a, b) to a unique element $a * b = a + 4b^2$ in R.
Therefore, * is a binary operation.

Binary Operations Ex 3.1 Q3

It is given that, $a * b = 2a + b - 3$
Now

$$3 * 4 = 2 \times 3 + 4 - 3$$
$$= 10 - 3$$
$$= 7$$

Binary Operations Ex 3.1 Q4

The operation * on the set A = {1, 2, 3, 4, 5} is defined as

$a * b$ = L.C.M. of a and b.

2*3 = L.C.M of 2 and 3 = 6. But 6 does not belong to the given set.

Hence, the given operation * is not a binary operation.

Binary Operations Ex 3.1 Q5

We have,

$$S = \{a, b, c\}$$

We know that the total number of binary operation on a set S with n element is n^{n^2}

$\Rightarrow$ Total number of binary operation on $S = \{a, b, c\} = 3^{3^2} = 3^9$

Binary Operations Ex 3.1 Q6

We have,

$$S = \{a, b\}$$

The total number of binary operation on $S = \{a, b\}$ in $2^{2^2} = 2^4 = 16$

Binary Operations Ex 3.1 Q7

We have,

$$M = \left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : \ a, b \in R - \{0\}\right\} \text{ and}$$

$$A * B = AB \text{ for all } A, B \in M$$

Let $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \in M$ and $B = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} \in M$

Now, $AB = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$

$\therefore \quad a \in R, \ b \in R, \ c \in R, \ \& \ d \in R$

$\Rightarrow \quad ac \in R$ and $bd \in R$

$\Rightarrow \quad \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix} \in M$

$\Rightarrow \quad A * B \in M$

Thus, the operater $*$ difines a binary operation on M

Binary Operations Ex 3.1 Q8

S = set of rational numbers of the form $\frac{m}{n}$ where $m \in Z$ and $n = 1, 2, 3$

Also, $a * b = ab$

Let $a \in S$ and $b \in S$

$\Rightarrow \quad ab \notin S$

For example $a = \frac{7}{3}$ and $b = \frac{5}{2}$

$\Rightarrow \quad ab = \frac{35}{6} \notin S$

$\therefore \quad a * b \notin S$

Hence, the operater $*$ does not define a binary operation on S

Binary Operations Ex 3.1 Q9

It is given that, a*b = 2a + b

Now

$(2*3) = 2 \times 2 + 3$
$= 4 + 3$
$= 7$

$(2*3)*4 = 7*4 = 2 \times 7 + 4$
$= 14 + 4$
$= 18$

Binary Operations Ex 3.1 Q10

It is given that, a*b = LCM (a, b)

Now

5*7 = LCM (5, 7)
= 35

RD Sharma Solutions Class 12 Maths Chapter 3 Ex 3.2

Binary Operations Ex 3.2 Q1

We have,

$a * b = l.c.m.(a, b)$ for all $a, b \in N$

(1)

Now,

$2 * 4 = l.c.m\ (2,4) = 4$

$3 * 5 = l.c.m\ (3,5) = 15$

$1 * 6 = l.c.m\ (1,6) = 6$

(ii)

Commutativity:

Let $a, b \in N$ then,

$$\begin{aligned} a * b &= l.c.m(a, b) \\ &= l.c.m(b, a) \\ &= b * a \end{aligned}$$

$\Rightarrow \quad a * b = b * a$

$\therefore \quad *$ is commutative on N.

Associativity:

Let $a, b, c \in N$ then,

$$\begin{aligned} (a * b) * c &= l.c.m(a, b) * c \\ &= l.c.m(a, b, c) \quad \text{---(i)} \end{aligned}$$

and, $$\begin{aligned} a * (b * c) &= a * l.c.m(b, c) \\ &= l.c.m(a, b, c) \quad \text{---(ii)} \end{aligned}$$

From (i) and (ii)

$(a * b) * c = a * (b * c)$

$\therefore \quad *$ is associative on N.

Binary Operations Ex 3.2 Q2

(i) Clearly, by definition $a * b = 1 = b * a$, $\forall a, b \in \mathbf{N}$

Also, $(a * b) * c = (1 * c) = 1$

and $a * (b * c) = (a * 1) = 1 \qquad \forall a, b, c \in \mathbf{N}$

Hence, $\mathbf{N}$ is both associative and commutative.

(ii) $a * b = \dfrac{a+b}{2} = \dfrac{b+a}{2} = b * a$,

which shows $*$ is commutative.

Further, $(a * b) * c = \left(\dfrac{a+b}{2}\right) * c = \dfrac{\left(\dfrac{a+b}{2}\right) + c}{2} = \dfrac{a+b+2c}{4}$

$$a * (b * c) = a * \left(\frac{b+c}{2}\right) = \frac{a + \left(\dfrac{b+c}{2}\right)}{2} = \frac{2a+b+c}{2} \neq \frac{a+b+2c}{4}$$

Hence, $*$ is not associative.

Binary Operations Ex 3.2 Q3

We have, binary operator $*$ defined on A and is given by

$$a * b = b \text{ for all } a, b \in A$$

Commutativity: Let $a, b \in A$, then

$$a * b = b \neq a = b * a$$

$\Rightarrow$ $a * b \neq b * a$

$\therefore$ '$*$' is not commutative on A.

Associativity: Let $a, b, c \in A$, then

$$(a * b) * c = b * c = c \qquad ---(i)$$

and, $a * (b * c) = a * c = c \qquad ---(ii)$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow$ '$*$' is associative on A.

Binary Operations Ex 3.2 Q4(i)

'$*$' is a binary operator on Z defined by $a * b = a + b + ab$ for all $a, b \in Z$.

Commutativity of '$*$':

Let $a, b \in Z$, then

$$a * b = a + b + ab = b + a + ba = b * a$$

$\therefore$ $a * b = b * a$

Associative of '$*$':

Let $a, b \in Z$, then

$$(a * b) * c = (a + b + ab) * c = a + b + ab + c + ac + bc + abc$$
$$= a + b + c + ab + bc + ac + abc \qquad ---(i)$$

Again, $a * (b * c) = a * (b + c + bc)$

$$= a + b + c + bc + ab + ac + abc \qquad ---(ii)$$

From (i) & (ii), we get

$$(a * b) * c = a * (b * c)$$

$\therefore$ $*$ is commutative and associative on Z

Binary Operations Ex 3.2 Q4(ii)

Commutative:

Let $a, b \in N$, then

$$a * b = 2^{ab} = 2^{ba} = b * a$$

$\therefore \quad a * b = b * a$

$\therefore \quad *$ is commutative on N

Associative:

Let $a, b, c \in N$, then

$$(a * b) * c = 2^{ab} * c = 2^{2^{ab} \cdot c} \qquad ---(i)$$

and, $\quad a * (b * c) = a * 2^{bc} = 2^{a \cdot 2^{bc}} \qquad ---(ii)$

From (i) & (ii), we get

$$(a * b) * c \neq a * (b * c)$$

$\therefore \quad *$ is not associative on N

Binary Operations Ex 3.2 Q4(iii)

Commutativity:

Let $a, b \in Q$, then

$$a * b = a - b \neq b - a = b * a$$

$\therefore \quad a * b \neq b * a$

$\Rightarrow \quad *$ is not commutative on Q

Associative:

Let $a, b, c \in Q$, then

$$(a * b) * c = (a - b) * c = a - b - c \qquad ---(i)$$

and, $\quad a * (b * c) = a * (b - c) = a - b + c \qquad ---(ii)$

From (i) & (ii), we get

$$(a * b) * c \neq a * (b * c)$$

$\therefore \quad *$ is not associative on Q

Binary Operations Ex 3.2 Q4(iv)

Commutative:

Let $a, b \in Q$, then

$$a * b = a^2 + b^2 = b^2 + a^2 = b * a$$

$\Rightarrow$ $a * b = b * a$

$\therefore$ $*$ is commutative on Q.

Associative:

Let $a, b, c \in Q$, then

$$(a * b) * c = (a^2 + b^2) * c = (a^2 + b^2)^2 + c^2 \quad ---(i)$$

and, $$a * (b * c) = a * (a^2 + b^2) = a^2 + (b^2 + c^2)^2 \quad ---(ii)$$

From (i) & (ii),

$$(a * b) * c \neq a * (b * c)$$

$\therefore$ $*$ is not associative on Q.

Binary Operations Ex 3.2 Q4(v)

Binary operation '$\circ$' defined on Q, given by $a \circ b = \frac{ab}{2}$ for all $a, b \in Q$

Commutative:

Let $a, b \in Q$, then

$$a \circ b = \frac{ab}{2} = \frac{ba}{2} = b \circ a$$

$\Rightarrow$ $a \circ b = b \circ a$

$\therefore$ $\circ$ is commutative on Q.

Associativity:

Let $a, b, c \in Q$, then

$$(a \circ b) \circ c = \left(\frac{ab}{2}\right) \circ c = \frac{abc}{4} \quad ---(i)$$

$$a \circ (b \circ c) = a \circ \left(\frac{bc}{2}\right) = \frac{abc}{4} \quad ---(ii)$$

From (i) & (ii) we get

$$(a \circ b) \circ c = a \circ (b \circ c)$$

$\therefore$ '$\circ$' is associative on Q.

Binary Operations Ex 3.2 Q4(vi)

Commutative:

Let $a, b \in Q$, then

$$a * b = ab^2 \neq ba^2 = b * a$$

$\Rightarrow$ $a * b \neq b * a$

$\therefore$ $*$ is not commutative on Q

Associativity:

Let $a, b, c \in Q$, then

$$(a * b) * c = ab^2 * c = ab^2c^2 \quad ---(i)$$

& $$a * (b * c) = a * bc^2 = a(bc^2)^2 \quad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\therefore$ $*$ is not associative on Q

Binary Operations Ex 3.2 Q4(vii)

Commutativity:

Let $a, b \in Q$, then

$$a * b = a + ab \quad ---(i)$$

$$b * a = b + ab \quad ---(ii)$$

From (i) & (ii)

$$a * b \neq b * a$$

$\Rightarrow$ $*$ is not commutative on Q

Associativity:

Let $a, b, c \in Q$, then

$$(a * b) * c = (a + ab) * c = a + ab + ac + abc \quad ---(i)$$

$$a * (b * c) = a * (b + bc)$$

$$= a + ab + abc \quad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow$ $*$ is not associative on Q

Binary Operations Ex 3.2 Q4(viii)

Commutativity: Let $a, b \in R$, then

$$a * b = a + b - 7$$
$$= b + a - 7$$
$$= b * a$$

$\Rightarrow$ $a * b = b * a$

$\Rightarrow$ $*$ is commutative on R

Associativity: Let $a, b, c \in Q$, then

$$(a * b) * c = (a + b - 7) * c$$
$$= a + b - 7 + c - 7$$
$$= a + b + c - 17 \quad ---(i)$$

and, $$a * (b * c) = a * (b + c - 7)$$
$$= a + b + c - 7 - 7$$
$$= a + b + c - 17 \quad ---(ii)$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow$ $*$ is associative on R

Binary Operations Ex 3.2 Q4(ix)

Commutativity:

Let $a, b \in R - \{-1\}$, then

$$a * b = \frac{a}{b+1} \neq \frac{b}{a+1} = b * a$$

$\Rightarrow \quad a * b \neq b * a$

$\Rightarrow \quad *$ is not commutative on $R - \{-1\}$

Associativity:

Let $a, b, c \in R - \{-1\}$, then

$$(a * b) * c = \left(\frac{a}{b+1}\right) * c$$

$$= \frac{\frac{a}{b+1}}{c+1} = \frac{a}{(b+1)(c+1)} \qquad ---(i)$$

& $\quad a * (b * c) = a * \left(\frac{b}{c+1}\right)$

$$= \frac{a}{\frac{b}{c+1}+1} = \frac{a(c+1)}{b+c+1} \qquad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow \quad *$ is not associative on $R - \{-1\}$

Binary Operations Ex 3.2 Q4(x)

Commutativity:

Let $a, b \in Q$, then

$$a * b = ab + 1 = ba + 1 = b * a$$

$\Rightarrow \quad a * b = b * a$

$\Rightarrow \quad *$ is commutative on Q

Associativity:

Let $a, b, c \in Q$, then

$$(a * b) * c = (ab + 1) * c$$

$$= abc + c + 1 \qquad ---(i)$$

$$a * (b * c) = a * (bc + 1)$$

$$= abc + a + 1 \qquad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow \quad *$ is not associative on Q.

Binary Operations Ex 3.2 Q4(xi)

Commutativity:

Let $a, b \in N$, then

$$a * b = a^b \neq b^a = b * a$$

$\Rightarrow \quad a * b \neq b * a$

$\Rightarrow \quad$ '*' is not commutative on N

Associativity:

Let $a, b, c \in N$, then

$$(a * b) * c = a^b * c = \left(a^b\right)^c = a^{bc} \qquad ---(i)$$

$$a * (b * c) = a * b^c = (a)^{b^c} \qquad ---(ii)$$

From (i) and (ii)

$$a^{bc} \neq (a)^{b^c}$$

$\Rightarrow (a * b) * c \neq a * (b * c)$

$\Rightarrow \quad$ '*' is not associative on N.

Binary Operations Ex 3.2 Q4(xii)

Commutativity:

Let $a, b \in N$, then

$$a * b = a^b \neq b^a = b * a$$

$\Rightarrow \quad a * b \neq b * a$

$\Rightarrow \quad$ '*' is not commutative on N

Associativity:

Let $a, b, c \in N$, then

$$(a * b) * c = a^b * c = \left(a^b\right)^c = a^{bc} \qquad ---(i)$$

$$a * (b * c) = a * b^c = (a)^{b^c} \qquad ---(ii)$$

From (i) and (ii)

$$a^{bc} \neq (a)^{b^c}$$

$\Rightarrow (a * b) * c \neq a * (b * c)$

$\Rightarrow \quad$ '*' is not associative on N.

Binary Operations Ex 3.2 Q4(xiii)

Commutativity:

Let $a, b \in Z$ then,

$$a * b = a - b \neq b - a = b * a$$

$\Rightarrow \quad a * b \neq b * a$

$\Rightarrow \quad *$ is not commutative on Z

Associativity:

Let $a, b, c \in Z$, then

$$(a * b) * c = (a - b) * c = (a - b - c) \qquad ---(i)$$

& $\quad a * (b * c) = a * (b - c) = (a - b + c) \qquad ---(ii)$

From (i) & (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow \quad$ '*' is not associative on Z.

Binary Operations Ex 3.2 Q4(xiv)

Commutativity:

Let $a, b \in Q$ then,

$$a * b = \frac{ab}{4} = \frac{ba}{4} = b * a$$

$\Rightarrow \quad a * b = b * a$

$\therefore \quad *$ is commutative on Q

Associativity:

Let $a, b, c \in Q$ then,

$$(a * b) * c = \frac{ab}{4} * c = \frac{abc}{16} \qquad ---(i)$$

and, $\quad a * (b * c) = a * \frac{bc}{4} = \frac{abc}{16} \qquad ---(ii)$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$\therefore \quad$ '*' is associative on Q.

Binary Operations Ex 3.2 Q4(xv)

Commutativity:

Let $a, b \in Q$ then,

$$a * b = (a-b)^2 = (b-a)^2 = b * a$$

$\Rightarrow \quad a * b = b * a$

$\therefore \quad$ '$*$' is commutative on Q.

Associativity:

Let $a, b, c \in Q$ then,

$$(a * b) * c = (a-b)^2 * c = \left[(a-b)^2 - c\right]^2 \qquad ---(i)$$

and, $$a * (b * c) = a * (b-c)^2 = \left[a - (b-c)^2\right]^2 \qquad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\therefore \quad *$ is not associative on Q.

Binary Operations Ex 3.2 Q5

The binary operator o defined on $Q - \{-1\}$ is given by

$$a \circ b = a + b - ab \text{ for all } a, b \in Q - \{-1\}$$

Commutativity:

Let $a, b \in Q - \{-1\}$, then

$$a \circ b = a + b - ab = b + a - ba = b \circ a$$

$\Rightarrow \quad a \circ b = b \circ a$

$\Rightarrow \quad$ 'o' is commutative on $Q - \{-1\}$.

Binary Operations Ex 3.2 Q6

The binary operator $*$ defined on Z and is given by

$$a * b = 3a + 7b$$

Commutativity: Let $a, b \in Z$, then

$$a * b = 1a + 7b \text{ and}$$
$$b * a = 3b + 7a$$

$\therefore \quad a * b \neq b * a$

Hence, '$*$' is not commutative on Z.

Binary Operations Ex 3.2 Q7

We have, $*$ is a binary operator defined on Z is given by

$$a*b = ab+1 \text{ for all } a,b \in Z$$

Associativity: Let $a,b,c \in Z$, then

$$(a*b)*c = (ab+1)*c$$
$$= abc+c+1 \quad ---(i)$$

and, $a*(b*c) = a*(bc+1)$
$$= abc+a+1 \quad ---(ii)$$

From (i) & (ii)

$\therefore \quad (a*b)*c \neq a*(b*c)$

Hence, '$*$' is not associative on Z.

Binary Operations Ex 3.2 Q8

We have, set of real numbers except -1 and $*$ is an operator given by

$$a*b = a+b+ab \text{ for all } a,b \in S = R-\{-1\}$$

Now, $\forall\ a,b \in S$

$$a*b = a+b+ab \in S$$

$\because$ if $\quad a+b+ab = -1$

$\Rightarrow \quad a+b(1+a)+1 = 0$

$\Rightarrow \quad (a+1)(b+1) = 0$

$\Rightarrow \quad a = -1$ or $b = -1$

but $a \neq -1$ and $b \neq -1$ (given)

$\therefore \quad a+b+ab \neq -1$

$\Rightarrow \quad a*b \in S$ for $ab \in S$

$\Rightarrow \quad$ '$*$' is a binary operator on S

Commutativity: Let $a,b \in S$

$\Rightarrow \quad a*b = a+b+ab = b+a+ba = b*a$

$\Rightarrow \quad a*b = b*a$

and, $a*(b*c) = a*(b+c+bc)$

$= a+b+c+bc+ab+ac+abc \quad ---(ii)$

From (i) and (ii)

$(a*b)*c = a*(b*c)$

$\therefore$ '$*$' is associative on S.

Now, $(2*x)*3 = 7$

$\Rightarrow (2+x+2x)*3 = 7$

$\Rightarrow 2+x+2x+3+6+3x+6x = 7$

$\Rightarrow 11+12x = 7$

$\Rightarrow 12x = -4$

$\Rightarrow x = \frac{-4}{12} \quad \Rightarrow x = \frac{-1}{3}$

Binary Operations Ex 3.2 Q9

The binary operator $*$ defined as

$a*b = \frac{a-b}{2}$ for all $a,b \in Q$.

Now,

Associativity: Let $a,b,c \in Q$, then

$(a*b)*c = \frac{a-b}{2}*c = \frac{\frac{a-b}{2}-c}{2}$

$= \frac{a-b-2c}{4} \quad ---(i)$

and, $a*(b*c) = a*\frac{b-c}{2} = \frac{a-\frac{b-c}{2}}{2}$

$= \frac{2a-b+c}{4} = \quad ---(ii)$

From (i) & (ii)

$(a*b)*c \neq a*(b*c)$

Hence, '$*$' is not associative on Q.

Binary Operations Ex 3.2 Q10

The binary operator $*$ defined as

$$a*b = a+3b-4 \text{ for all } a,b \in Z$$

Now,

Commutativity: Let $a,b \in Z$, then

$$a*b = a+3b-4 \neq b+3a-4 = b*a$$

$\Rightarrow$ $a*b \neq b*a$

$\Rightarrow$ '$*$' is not commutative on Z.

Associativity: Let $a,b,c \in Z$, then

$$(a*b)*c = (a+3b-4)*c = a+3b-4+3c-4$$
$$= a+3b+3c-8 \quad ---(i)$$

and, $$a*(b*c) = a*(b+3c-4) = a+3(b+3c-4)-4$$
$$= a+3b+9c-16 \quad ---(ii)$$

From (i) & (ii)

$$(a*b)*c \neq a*(b*c)$$

Hence, '$*$' is not associative on Z.

Binary Operations Ex 3.2 Q11

Q be the set of rational numbers and $*$ be a binary operation defined as

$$a*b = \frac{ab}{5} \text{ for all } a,b \in Q$$

Now,

Associativity: Let $a,b,c \in Q$, then

$$(a*b)*c = \frac{ab}{5}*c = \frac{abc}{25} \quad ---(i)$$

and, $$a*(b*c) = a*\frac{bc}{5} = \frac{abc}{25} \quad ---(ii)$$

From (i) & (ii)

$\therefore$ $(a*b)*c = a*(b*c)$

$\Rightarrow$ $*$ is associative on Q.

Binary Operations Ex 3.2 Q12

The binary operator $*$ is defined as

$$a*b=\frac{ab}{7} \text{ for all } a,b\in Q$$

Now,

Associativity: Let $a,b,c\in Q$, then

$$(a*b)*c=\frac{ab}{7}*c=\frac{abc}{49} \qquad ---(i)$$

and, $$a*(b*c)=a*\frac{bc}{7}=\frac{abc}{49} \qquad ---(ii)$$

From (i) & (ii)

$$(a*b)*c=a*(b*c)$$

$\Rightarrow$ '$*$' is associative on Q.

Binary Operations Ex 3.2 Q13

The binary operator $*$ defined as

$$a*b=\frac{a+b}{2} \text{ for all } a,b\in Q.$$

Now,

Associativity: Let $a,b,c\in Q$, then

$$(a*b)*c=\frac{a+b}{2}*c=\frac{\frac{a+b}{2}+c}{2}$$

$$=\frac{a+b+2c}{4} \qquad ---(i)$$

and, $$a*(b*c)=a*\frac{b+c}{2}$$

$$=\frac{a+\frac{b+c}{2}}{2}$$

$$=\frac{2a+b+c}{4}= \qquad ---(ii)$$

From (i) & (ii)

$$(a*b)*c\neq a*(b*c)$$

Hence, '$*$' is not associative on Q.

RD Sharma Solutions Class 12 Maths Chapter 3 Ex 3.3

Binary Operations Ex 3.3 Q1

The binary operator $*$ is defined on I^+
and is given by,

$$a * b = a + b \text{ for all } a, b \in I^+$$

Let $a \in I^+$ and $e \in I^+$ be the identity element with respect to $*$.
by identity property, we have,

$$a * e = e * a = a$$

$\Rightarrow \quad a + e = a$

$\Rightarrow \quad e = 0$

Thus the required identity element is 0.

Binary Operations Ex 3.3 Q2

Let $R - \{-1\}$ be the set and $*$ be a binary operator, given by

$$a * b = a + b + ab \text{ for all } a, b \in R - \{-1\}$$

Now,
Let $a \in R - \{-1\}$ and $e \in R - \{-1\}$ be the identity element with respect to $*$.
by identity property, we have,

$$a * e = e * a = a$$

$\Rightarrow \quad a + e + ae = a$

$\Rightarrow \quad e(1 + a) = 0$

$\Rightarrow \quad e = 0 \qquad [\because 1 + a \neq 0 \text{ as } a \neq -1]$

$\therefore$ The required identity element is 0.

Binary Operations Ex 3.3 Q3

We are given the binary operator $*$ defined on Z as

$$a * b = a + b - 5 \text{ for all } a, b \in Q.$$

Let e be the identity element with respect to $*$

Then, $a * e = e * a = a$ [By identity property]

$\Rightarrow$ $a + e - 5 = a$

$\Rightarrow$ $e = 5$

Hence, the required identity element with respect to $*$ is 5.

Binary Operations Ex 3.3 Q4

The binary operator $*$ is defined on Z, and is given by

$$a * b = a + b + 2 \text{ for all } a, b \in Z.$$

Let $a \in Z$ and $e \in Z$ be the identity element with respect to $*$, then

$a * e = e * a = a$ [By identity property]

$\Rightarrow$ $a + e + 2 = a$

$\Rightarrow$ $e = -2 \in Z$

Hence, the identity element with respect to $*$ is -2.

RD Sharma Solutions Class 12 Maths Chapter 3 Ex 3.4

Binary Operations Ex 3.4 Q1

Given,

$$a * b = a + b - 4 \text{ for all } a, b \in Z$$

(i)

Commutative: Let $a, b \in Z$, then

$\Rightarrow \quad a * b = a + b - 4 = b + a - 4 = b * a$

$\Rightarrow \quad a * b = b * a$

So, '$*$' is commutative on Z.

Associativity: Let $a, b, c \in Z$, then

$$(a * b) * c = (a + b - 4) * c = a + b - 4 + c - 4$$
$$= a + b + c - 8 \qquad ---(i)$$

and, $a * (b * c) = a * (b + c - 4) = a + b + c - 8 \qquad ---(ii)$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '$*$' is associative on Z.

(ii)

Let $e \in Z$ be the identity element with respect to $*$.

By identity property, we have

$$a * e = e * a = a \text{ for all } a \in Z$$

$\Rightarrow \quad a + e - 4 = a$

$\Rightarrow \quad e = 4$

So, $e = 4$ will be the identity element with respect to $*$

(iii)

Let $b \in Z$ be the inverse element of $a \in Z$

Then, $a * b = b * a = e$

$\Rightarrow \quad a + b - 4 = e$

$\Rightarrow \quad a + b - 4 = 4 \qquad [\because e = 4]$

$\Rightarrow \quad b = 8 - a$

Thus, $b = 8 - a$ will be the inverse element of $a \in Z$.

Binary Operations Ex 3.4 Q2

We have,

$$a * b = \frac{3ab}{5} \text{ for all } a, b \in Q_0$$

(i)

Commutative: Let $a, b \in Q_0$, then

$$a * b = \frac{3ab}{5} = \frac{3ba}{5} = b * a$$

$\Rightarrow \quad a * b = b * a$

So, '$*$' is commutative on Q_0

Associativity: Let $a, b, c \in Q_0$, then

$$(a * b) * c = \frac{3ab}{5} * c$$
$$= \frac{9abc}{25} \qquad ---(i)$$

and, $$a * (b * c) = a * \frac{3bc}{5}$$
$$= \frac{9abc}{25} \qquad ---(ii)$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '$*$' is associative on Q_0

(ii)

Let $e \in Q_0$ be the identity element with respect to $*$, then

$a * e = e * a = a$ for all $a \in Q_0$

$\Rightarrow \quad \frac{3ae}{5} = a$

$\Rightarrow \quad e = \frac{5}{3}$

will be the identity element with respect to $*$.

(iii)

Let $b \in Q_0$ be the inverse element of $a \in Q_0$, then

$a * b = b * a = e$

$\Rightarrow \quad \frac{3}{5}ab = e$

$\Rightarrow \quad \frac{3}{5}ab = \frac{5}{3} \qquad \left[\because e = \frac{5}{3}\right]$

$\Rightarrow \quad b = \frac{25}{9a}$

$\therefore \quad b = \frac{25}{9a}$ is the inverse of $a \in Q_0$.

Binary Operations Ex 3.4 Q3

We have,

$a * b = a + b + ab$ for all $a, b \in Q - \{-1\}$

(i)

Commutativity: Let $a, b \in Q - \{-1\}$

$\Rightarrow \quad a * b = a + b + ab = b + a + ba = b * a$

$\Rightarrow \quad a * b = b * a$

$\Rightarrow \quad$ '$*$' is commutative on $Q - \{-1\}$

Associativity: Let $a, b, c \in Q - \{-1\}$, then

$\Rightarrow \quad (a * b) * c = (a + b + ab) * c$

$= a + b + ab + c + ac + bc + abc \qquad ---(i)$

and, $\quad a * (b * c) = a * (b + c + bc)$

$= a + b + c + bc + ab + ac + abc \qquad ---(ii)$

From (i) & (ii)

$(a * b) * c = a * (b * c)$

$\Rightarrow \quad *$ is associative on $Q - \{-1\}$

(ii)

Let e be identity element with respect to $*$.

By identity property,

$a * e = a = e * a$ for all $a \in Q - \{-1\}$

$\Rightarrow \quad a + e + ae = a$

$\Rightarrow \quad e(1 + a) = 0 \quad \Rightarrow e = 0 \qquad [\because 1 + a \neq 0 \text{ as } a \neq -1]$

$\therefore \quad e = 0$ is the identity element with respect to $*$

(iii)

Let b be the inverse of $a \in Q - \{-1\}$

Then, $\quad a * b = b * a = e \qquad$ [e is the identity element]

$\Rightarrow \quad a + b + ab = e$

$\Rightarrow \quad a + b + ab = 0$

$\Rightarrow \quad b(1 + a) = -a$

$\Rightarrow \quad b = \frac{-a}{1+a} \qquad \left[\because \frac{-a}{1+a} \neq -1, \text{ because if } \frac{-a}{1+a} = -1 \Rightarrow a = 1 + a \Rightarrow 1 = 0 \text{ Not possible}\right]$

$\therefore \quad b = \frac{-a}{1+a}$ is the inverse of a with respect to $*$

Binary Operations Ex 3.4 Q4

We have,

$$(a,b)\odot(c,d) = (ac, bc+d) \text{ for all } (a,b),(c,d) \in R_0 \times R$$

(i)

Commutativity: Let $(a,b),(c,d) \in R_0 \times R$, then

$$\Rightarrow \quad (a,b)\odot(c,d) = (ac, bc+d) \qquad ---(i)$$

and, $$(c,d)\odot(a,b) = (ca, da+b) \qquad ---(ii)$$

From (i) & (ii)

$$(a,b)\odot(c,d) \neq (c,d)\odot(a,b)$$

$\Rightarrow$ '$\odot$' is not commutative on $R_0 \times R$.

Associativity: Let $(a,b),(c,d),(e,f) \in R_0 \times R$, then

$$\Rightarrow \quad ((a,b)\odot(c,d))\odot(e,f) = (ac, bc+d)\odot(e,f)$$
$$= (ace, bce, de+f) \qquad ---(i)$$

and, $$(a,b)\odot(c,d\odot(e,f)) = (a,b)\odot(ce, de+f)$$
$$= (ace, bce+de+f) \qquad ---(ii)$$

$\Rightarrow \quad ((a,b)\odot(c,d))\odot(e,f) = (a,b)\odot((c,d)\odot(e,f))$

$\Rightarrow \quad$ '$\odot$' is associative on $R_0 \times R$.

(ii)

Let $(x,y) \in R_0 \times R$ be the identity element with respect to $\odot$, then

$$(a,b)\odot(x,y) = (x,y)\odot(a,b) = (a,b) \text{ for all } (a,b) \in R_0 \times R$$

$\Rightarrow \quad (ax, bx + y) = (a,b)$

$\Rightarrow \quad ax = a$ and $bx + y = b$

$\Rightarrow \quad x = 1,$ and $y = 0$

$\therefore \quad (1,0)$ will be the identity element with respect to $\odot$.

(iii)

Let $(c,d) \in R_0 \times R$ be the inverse of $(a,b) \in R_0 \times R$, then

$$(a,b)\odot(c,d) = (c,d)\odot(a,b) = e$$

$\Rightarrow \quad (ac, bc + d) = (1,0) \qquad [\because e = (1,0)]$

$\Rightarrow \quad ac = 1$ and $bc + d = 0$

$\Rightarrow \quad c = \frac{1}{a}$ and $d = -\frac{b}{a}$

$\therefore \quad \left(\frac{1}{a}, -\frac{b}{a}\right)$ will be the inverse of (a,b).

Binary Operations Ex 3.4 Q5

We have,

$$a * b = \frac{ab}{2} \text{ for all } a, b \in Q_0$$

(i)

Commutativity: Let $a, b \in Q_0$, then

$\Rightarrow \quad a * b = \frac{ab}{2} = \frac{ba}{2} = b * a$

$\Rightarrow \quad a * b = b * a$

Hence, '*' is commutative on Q_0.

Associativity: Let $a, b, c \in Q_0$, then

$\Rightarrow \quad (a * b) * c = \frac{ab}{2} * c = \frac{abc}{4} \quad$ ---(i)

and, $\quad a * (b * c) = a * \frac{bc}{2} = \frac{abc}{4} \quad$ ---(ii)

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow \quad *$ is associative on Q_0.

(ii)

Let $e \in Q_0$ be the identity element with respect to $*$.

By identity property, we have,

$$a * e = e * a = a \text{ for all } a \in Q_0$$

$\Rightarrow \quad \frac{ae}{2} = a \quad \Rightarrow e = 2$

Thus, the required identity element is 2.

(iii)

Let $b \in Q_0$ be the inverse of $a \in Q_0$ with respect to $*$, then,

$$a * b = b * a = e \text{ for all } a \in Q_0$$

$\Rightarrow \quad \frac{ab}{2} = e \quad \Rightarrow \frac{ab}{2} = 2$

$\Rightarrow b = \frac{4}{a}$

Thus, $\quad b = \frac{4}{a}$ is the inverse of a with respect to $*$.

Binary Operations Ex 3.4 Q6

We have,

$$a * b = a + b - ab \text{ for all } a, b \in R - \{+1\}$$

(i)

Commutative: Let $a, b \in R - \{+1\}$, then,

$\Rightarrow \quad a * b = a + b - ab = b + a - ba = b * a$

$\Rightarrow \quad a * b = b * a$

So, '$*$' is commutative on $R - \{+1\}$.

Associativity: Let $a, b, c \in R - \{+1\}$, then

$$\begin{aligned}(a * b) * c &= (a + b - ab) * c \\ &= a + b - ab + c - ac - bc + abc \\ &= a + b + c - ab - ac - bc + abc \qquad ---(i)\end{aligned}$$

and,

$$\begin{aligned}a * (b * c) &= a * (b + c - bc) \\ &= a + b + c - bc - ab - ac + abc \qquad ---(ii)\end{aligned}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '$*$' is associative on $R - \{+1\}$.

(ii)

Let $e \in R - \{+1\}$ be the identity element with respect to $*$, then

$$a * e = e * a = a \text{ for all } a \in R - \{+1\}$$

$\Rightarrow \quad a + e - ae = a$

$\Rightarrow \quad e(1 - a) = 0$

$\Rightarrow \quad e = 0 \qquad [\because a \neq 1 \Rightarrow 1 - a \neq 0]$

$\therefore \quad e = 0$ will be the identity element with respect to $*$.

(iii)

Let $b \in R - \{1\}$ be the inverse element of $a \in R - \{1\}$, then

$$a * b = b * a = e$$

$\Rightarrow \quad a + b - ab = 0 \qquad [\because e = 0]$

$\Rightarrow \quad b(1 - a) = -a$

$\Rightarrow \quad b = \frac{-a}{1 - a} \neq 1 \qquad \left[\because \text{if } \frac{-a}{1-a} = 1 \Rightarrow -a = 1 - a \Rightarrow 1 = 0 \text{ Not possible}\right]$

$\therefore \quad b = \frac{-a}{1 - a}$ is the inverse of $a \in R - \{1\}$ with respect to $*$.

Binary Operations Ex 3.4 Q7

We have,

$$(a,b)*(c,d)=(ac,bd) \text{ for all } (a,b),(c,d)\in A$$

(i)

Let $(a,b),(c,d)\in A$, then

$$(a,b)*(c,d)=(ac,bd)$$
$$=(ca,db) \qquad [\because ac=ca \text{ and } bd=db]$$
$$=(c,d)*(a,b)$$

$\Rightarrow \quad (a,b)*(c,d)=(c,d)*(a,b)$

So, '$*$' is commutative on A

Associativity: Let $(a,b),(c,d),(e,f)\in A$, then

$\Rightarrow \quad ((a,b)*(c,d))*(e,f)=(ac,bd)*(e,f)$
$$=(ace,bdf) \qquad ---(i)$$

and, $\quad (a,b)*((c,d)*(e,f))=(a,b)*(ce,df)$
$$=(ace,bdf) \qquad ---(ii)$$

From (i) & (ii)

$\Rightarrow \quad ((a,b)*(c,d))*(e,f)=(a,b)*((c,d)*(e,f))$

So, '$*$' is associative on A.

(ii)

Let $(x,y)\in A$ be the identity element with respect to $*$.

$$(a,b)*(x,y)=(x,y)*(a,b)=(a,b) \text{ for all } (a,b)\in A$$

$\Rightarrow \quad (ax,by)=(a,b)$

$\Rightarrow \quad ax=a$ and $by=b$

$\Rightarrow \quad x=1$, and $y=1$

$\therefore \quad (1,1)$ will be the identity element

(iii)

Let $(c,d)\in A$ be the inverse of $(a,b)\in A$, then

$$(a,b)*(c,d)=(c,d)*(a,b)=e$$

$\Rightarrow \quad (ac,bd)=(1,1) \qquad [\because e=(1,1)]$

$\Rightarrow \quad ac=1$ and $bd=1$

$\Rightarrow \quad c=\frac{1}{a}$ and $d=\frac{1}{b}$

$\therefore \quad \left(\frac{1}{a}, \frac{1}{b}\right)$ will be the inverse of (a, b) with respect to $*$.

Binary Operations Ex 3.4 Q8

The binary operation * on **N** is defined as:

$a * b$ = H.C.F. of a and b

It is known that:

H.C.F. of a and b = H.C.F. of b and a, $a, b \in$ **N**.

Therefore, $a * b = b * a$

Thus, the operation * is commutative.

For $a, b, c \in$ **N**, we have:

$(a * b) * c$ = (H.C.F. of a and b) $* c$ = H.C.F. of a, b, and c

$a * (b * c) = a *$(H.C.F. of b and c) = H.C.F. of a, b, and c

Therefore, $(a * b) * c = a * (b * c)$

Thus, the operation * is associative.

Now, an element $e \in$ **N** will be the identity for the operation * if $a * e = a = e * a$, $\forall a \in$ **N**.

But this relation is not true for any $a \in$ **N**.

Thus, the operation * does not have any identity in **N**.

RD Sharma Solutions Class 12 Maths Chapter 6 Ex 6.5

Chapter 6 Determinants Ex 6.5 Q1

Here $D = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

$= 1(3) - 1(-3) - 2(3)$

$= 3 + 3 - 6$

$= 0$

Since $D = 0$, so the system has infinite solutions:

Now let $z = k$,

$x + y = 2k$

$2x + y = 3k$

Solving there equations by cramer's Rule

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} 2k & 1 \\ 3k & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-k}{-1} = k$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 1 & 2k \\ 2 & 3k \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-k}{-1} = k$$

thus, we have $x = k, y = k, z = k$

and there values satisfy eq.(3)

Hence $x = k, y = k, z = k$

Chapter 6 Determinants Ex 6.5 Q2

Here $D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix}$

$= 2(4) - 3(1) + 4(-3)$

$= 8 - 3 - 7$

$= -2$

$\neq 0$

So, the given system of equations has only the trivial solutions i.e $x = 0 = y = z$:

Hence $x = 0$

$y = 0$

$z = 0$

Chapter 6 Determinants Ex 6.5 Q3

Here $D = \begin{vmatrix} 3 & 1 & 1 \\ 1 & -4 & 3 \\ 2 & 5 & -2 \end{vmatrix}$

$= 3(8-15) - 1(-2-6) + 1(13)$

$= -21 + 8 + 13$

$= 0$

So, the system has infinite solutions:

Let $z = k$,

so, $3x + y = -k$

$x - 4y = -3k$

Now,

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} -k & 1 \\ -3k & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \frac{7k}{-13}$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 3 & -k \\ 1 & -3k \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \frac{-8k}{-13}$$

$$x = \frac{-7k}{13}, y = \frac{8k}{13}, z = k$$

and there values satisfy eq.(3)

Hence $x = -7k, y = 8k, z = 13k$

Chapter 6 Determinants Ex 6.5 Q4

$$D = \begin{vmatrix} 2\lambda & -2 & 3 \\ 1 & \lambda & 2 \\ 2 & 0 & \lambda \end{vmatrix}$$

$$= 3\lambda^3 + 2\lambda - 8 - 6\lambda$$
$$= 2\lambda^3 - 4\lambda - 8$$

which is satisfied by $\lambda = 2$ $\left[\because \quad \text{for non-trivial solutions } \lambda = 2\right]$

Now Let $z = k$,

$$4x - 2y = -3k$$
$$x + 2y = -3k$$

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} -3k & -2 \\ -2k & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix}} = \frac{-10k}{10} = -k$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 4 & -3k \\ 1 & -2k \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix}} = \frac{-5k}{10} = \frac{-k}{2}$$

Hence solution is $x = -k, y = \frac{-k}{2}, z = k$

Chapter 6 Determinants Ex 6.5 Q5

$$D = \begin{vmatrix} (a-1) & -1 & -1 \\ -1 & (b-1) & -1 \\ -1 & -1 & (c-1) \end{vmatrix}$$

Now for non-trivial solution, $D = 0$

$$0 = (a-1)\left[(b-1)(c-1) - 1\right] + 1\left[-c + \cancel{1} - \cancel{1}\right] - \left[\cancel{1} + b - \cancel{1}\right]$$
$$0 = (a-1)\left[bc - b - c + \cancel{1} - \cancel{1}\right] - c - b$$
$$0 = abc - ab - ac + \cancel{b} + \cancel{c} - \cancel{c} - \cancel{b}$$
$$ab + bc + ac = abc$$

Hence proved