RD SHARMA

Solutions

Class 10 Maths

Chapter 8

Ex 8.7

Question 1: Find the consecutive numbers whose squares have the same sum of 85.

Solution:

Let the two consecutive two natural numbers be (x) and (x+1) respectively.

Given,

That the sum of their squares is 85.

Then, by hypothesis, we get,

$= x^2 + (x+1)^2 = 85$

$= x^2 + x^2 + 2x + 1 = 85$

$= 2x^2 + 2x + 1 - 85 = 0$

$= 2x^2 + 2x + -84 = 0$

$= 2(x^2 + x + -42) = 0$

Now applying factorization method, we get,

$= x^2 + 7x - 6x - 42 = 0$

$= x(x+7) - 6(x+7) = 0$

$= (x-6)(x+7) = 0$

Either,

$x-6 = 0$ therefore , $x=6$

$x+7 = 0$ therefore $x= -7$

Hence the consecutive numbers whose sum of squares is 85 are 6 and -7 respectively.

Question 2: Divide 29 into two parts so that the sum of the squares of the parts is 425.

Solution:

Let the two parts be (x) and (29-x) respectively.

According to the question, the sum of the two parts is 425.

Then by hypothesis,

$= x^2 + (29-x)^2 = 425$

= $x^2+x^2+841+-58x = 425$

= $2x^2-58x+841 -425 =0$

= $2x^2-58x+416 =0$

= $x^2-29x +208 =0$

Now, applying the factorization method

= $x^2 -13x-16x+208 =0$

= $x(x-13) -16(x-13) =0$

= $(x-13)(x-16) =0$

Either $x-13 =0$ therefore $x=13$

Or, $x-16 =0$ therefore $x=16$

The two parts whose sum of the squares is 425 are 13 and 16 respectively.

Question 3: Two squares have sides x cm and (x+4) cm. The sum of their areas is 656 cm^2.find the sides of the squares.

Solution:

Given,

The sum of the sides of the squares are = x cm and (x+4) cm respectively.

The sum of the areas = 656 cm^2

We know that,

Area of the square = side * side

Area of the square = $x(x+4)$ cm^2

Given that the sum of the areas is 656 cm^2

Hence by hypothesis,

= $x(x+4) +x(x+4) = 656$

= $2x(x+4) = 656$

= $x^2 +4x =328$

Now by applying factorization method,

$= x^2 + 20x - 16x - 328 = 0$

$= x(x+20) - 16(x+20) = 0$

$= (x+20)(x16) = 0$

Either $x+20 = 0$ therefore $x = -20$

Or, $x-16 = 0$ therefore $x = 16$

No negative value is considered as the value of the side of the square can never be negative.

Therefore, the side of the square is 16.

Therefore, $x+4 = 16+4 = 20$ cm

Hence, the side of the square is 20cm.

Question 4: The sum of two numbers is 48 and their product is 432. Find the numbers.

Solution:

Given the sum of two numbers is 48.

Let the two numbers be x and 48-x also the sum of their product is 432.

According to the question

$= x(48-x) = 432$

$= 48x - x^2 = 432$

$= x^2 - 48x + 432 = 0$

$= x^2 - 36x - 12x + 432 = 0$

$= x(x-36) - 12(x-36) = 0$

$= (x-36)(x-12) = 0$

Either $x-36 = 0$ therefore $x = 36$

Or, $x-12 = 0$ therefore $x = 12$

The two numbers are 12 and 36 respectively.

Question 5: If an integer is added to its square, the sum is 90. Find the integer with the help of quadratic equation.

Solution:

Let the integer be x

Given that if an integer is added to its square , the sum is 90

= x+ x^2 = 90

= x^2 +x- 90=0

= x^2 +10x-9x- 90=0

= x(x+10)-9(x+10) =0

= (x+10)(x-9) =0

Either x+10 = 0

Therefore x= -10

Or, x-9 =0

Therefore x =9

The values of the integer are 9 and -10 respectively.

Question 6: Find the whole numbers which when decreased by 20 is equal to69 times the reciprocal of the numbers.

Solution:

Let the whole number be x cm

As it is decreased by 20 = (x-20) = $69x \frac{69}{x}$

x-20 = $69x \frac{69}{x}$

x(x-20) =69

x^2 -20x -69 =0

Now by applying factorization method ,

x^2 -23x+3x-69 =0

x(x-23) +3(x-23)=0

(x-23)(x+3) =0

Either, x= 23

Or, x= -3

As the whole numbers are always positive x= -3 is not considered.

The whole number is 23.

Question 7: Find the consecutive natural numbers whose product is 20

Solution:

Let the two consecutive natural number be x and x+1 respectively.

Given that the product of natural numbers is 20

= x(x+1) =20

= x^2+x-20 =0

= x^2+5x-4x-20 =0

= x(x+5)-4(x+5) =0

= (x+5)(x-4) =0

Either x+5 =0

Therefore x =-5

Considering the positive value of x.

Or, x-4 =0

Therefore x =4

The two consecutive natural numbers are 4 and 5 respectively.

Question 8: The sum of the squares of two consecutive odd positive integers is 394. Find the two numbers?

Solution:

Let the consecutive odd positive integer are 2x-1 and 2x+1 respectively.

Given, that the sum of the squares is 394.

According to the question,

$(2x-1)^2 + (2x+1)^2 = 394$

$4x^2$ +1-4x+$4x^2$+1+4x = 394

Now cancelling out the equal and opposite terms ,

$8x^2$+ 2 = 394

$8x^2$ = 392

X^2 = 49

X= 7 and -7

Since the value of the edge of the square cannot be negative so considering only the positive value.

That is 7

Now, 2x-1 = 14-1 = 13

2x+1 = 14 +1 = 15

The consecutive odd positive numbers are 13 and 15 respectively.

Question 9: The sum of two numbers is 8 and 15 times the sum of the reciprocal is also 8 . Find the numbers.

Solution:

Let the numbers be x and 8-x respectively.

Given that the sum of the numbers is 8 and 15 times the sum of their reciprocals.

According to the question,

= 15(1x + 18−x)=8$15(\frac{1}{x}+\frac{1}{8-x}) = 8$

= 15 8−x+xx(8−x) =8$15\frac{8-x+x}{x(8-x)} = 8$

= 15× 88x−x2 =8$15 \times \frac{8}{8x-x^2} = 8$

= 120=8(8x-x^2)

= 120 = 64x-$8x^2$

=$8x^2$-64x+120=0

= 8(x^2-8x+15)=0

= x^2-8x+15=0

$= x^2-5x-3x+15=0$

$=x(x-5)-3(x-5) =0$

$= (x-5)(x-3) =0$

Either $x-5 = 0$ therefore $x =5$

Or, $x-3 =0$ therefore $x =3$

The two numbers are 5 and 3 respectively.

Question 10: The sum of a number and its positive square root is $625\frac{6}{25}$. Find the numbers.

Solution:

Let the number be x

By the hypothesis, we have

$x+\sqrt{x}=625\, x+\sqrt{x}=\frac{6}{25}$

Let us assume that $x=y^2$, we get

$y+y^2=625\, y+y^2=\frac{6}{25}$

$= 25y^2+25y-6 =0$

The value of y can be determined by:

$y=-b\pm\sqrt{b^2-4ac}2a\, y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Where a = 25, b = 25 , c =-6

$y=-25\pm\sqrt{625+600}50\, y=\frac{-25\pm\sqrt{625+600}}{50}$ $y=-25\pm3550\, y=\frac{-25\pm35}{50}$ $y=15\text{and}y=-1110$
$y=\frac{1}{5}\text{ and } y=\frac{-11}{10}$

$= x=y^2= 15^2=125\, \frac{1}{5}^2=\frac{1}{25}$

The number x is $125\frac{1}{25}$

Question 11: There are three consecutive integers such that the square of the first increased by the product of the other two integers gives 154. What are the integers?

Solution:

Let the three consecutive numbers be x, x+1, x+2 respectively.

X^2+(x+1)(x+2) =154

= x^2+x^2+3x+2 =154

= $3x^2+3x-152=0$

The value of x can be obtained by the formula

x=−b±√b2−4ac2a $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Here a =3 , b =3 , c =152

x= x=−3±√9−12166 $x = \frac{-3 \pm \sqrt{9-1216}}{6}$

x=8andx=−192 $x = 8 \text{ and } x = \frac{-19}{2}$

Considering the value of x

If x=8

x+1 =9

x+2 = 10

The three consecutive numbers are 8 , 9 , 10 respectively.

Question 12: The product of two successive integral multiples of 5 is 300. Determine the multiples.

Solution:

Given that the product of two successive integral multiples of 5 is 300

Let the integers be 5x and 5(x+1)

According to the question,

5x[5(x+1)] = 300

= 25x(x+1) =300

= x^2+x =12

= $x^2+x-12=0$

$= x^2+4x-3x-12=0$

$= x(x+4)-3(x+4) = 0$

$=(x+4)(x-3) = 0$

Either $x+4 = 0$

Therefore $x=-4$

Or, $x-3 = 0$

Therefore $x = 3$

$x = -4$

$5x = -20$

$5(x+1) = -15$

$x=3$

$5x = 15$

$5(x+1) = 20$

The two successive integral multiples are 15,20 and -15 and -20 respectively.

Question 13: The sum of the squares of two numbers is 233 and one of the numbers is 3 less than the other number. Find the numbers.

Solution:

Let the number is x

Then the other number is 2x-3

According to the question:

$x^2+(2x-3)^2 = 233$

$= x^2+4x^2+9-12x = 233$

$= 5x^2-12x-224 = 0$

The value of x can be obtained by $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Here a= 5 , b= -12 , c =-224

$x = \frac{12\pm\sqrt{144+20(224)}}{2(5)}$

x=8andx=−285 $x = 8 \text{ and } x = \frac{-28}{5}$

Considering the value of x =8

2x-3 = 15

The two numbers are 8 and 15 respectively.

Question 14: The difference of two number is 4 . If the difference of the reciprocal is 421 $\frac{4}{21}$. find the numbers.

Solution:

Lethe two numbers be x and x-4 respectively.

Given, that the difference of two numbers is 4 .

By the given hypothesis we have,

= 1x−4 − 1x = 421 $\frac{1}{x-4} - \frac{1}{x} = \frac{4}{21}$

= x−x+4x(x−4) = 421 $\frac{x-x+4}{x(x-4)} = \frac{4}{21}$

= 84 = 4x(x-4)

= x^2-4x-21 =0

Applying factorization theorem,

= x^2 -7x+3x-21 =0

=(x-7)(x+3) =0

Either x-7 =0 therefore x= 7

Or, x+3 = 0 therefore x = -3

Hence the required numbers are -3 and 7 respectively.

Question 15: Let us find two natural numbers which differ by 3 and whose squares have the sum 117.

Solution:

Let the numbers be x and x-3

According to the question

$x^2+(x-3)^2=117$

$= x^2+x^2+9-6x-117 =0$

$= 2x^2-6x-108 =0$

$= x^2-3x-54 =0$

$= x^2-9x+6x-54 =0$

$= x(x-9)+6(x-9) =0$

$=(x-9)(x+6) =0$

Either $x-9 =0$ therefore $x=9$

Or ,$x+6 =0$ therefore $x=-6$

Considering the positive value of x that is 9

$x=9$

$x-3 = 6$

The two numbers are 6 and 9 respectively.

Question 16: The sum of the squares of these consecutive natural numbers is 149. Find the numbers.

Solution:

Let the numbers be x , x+1, and x+2 respectively.

According to given hypothesis

$X^2+ (x+1)^2+(x+2)^2 =149$

$X^2+ X^2 + X^2 +1+2x+4+4x = 149$

$3x^2 +6x-144 =0$

$X^2+2x-48=0$

Now applying factorization method,

$X^2 +8x-6x-48=0$

$X(x+8)-6(x+8) =0$

(x+8)(x-6) =0

Either x+8 =0 therefore x= -8

Or, x-6 =0 therefore x= 6

Considering only the positive value of x that is 6 and discarding the negative value.

x=6

x+1 = 7

x+2 = 8

The three consecutive numbers are 6 , 7 , and 8 respectively.

Question 17: Sum of two numbers is 16. The sum of their reciprocal is 13 $\frac{1}{3}$.find the numbers.

Solution:

Given that the sum of the two natural numbers is 16

Let the two natural numbers be x and 16-x respectively

According to the question

= 1x+116−x=13 $\frac{1}{x}+\frac{1}{16-x}=\frac{1}{3}$

= 16−x+xx(16−x)=13 $\frac{16-x+x}{x(16-x)}=\frac{1}{3}$

= 16x(16−x)=13 $\frac{16}{x(16-x)}=\frac{1}{3}$

= $16x-x^2$ =48

= $-16x+x^2+48$ =0

= $+x^2-16x+48=0$

= $+x^2-12x-4x+48=0$

= x(x-12)-4(x-12) =0

= (x-12)(x-4) =0

Either x-12 =0 therefore x= 12

Or , x-4 =0 therefore x= 4

The two numbers are 4 and 12 respectively.

Question 18: Determine the two consecutive multiples of 3 whose product is 270

Solution:

Let the consecutive multiples of 3 are 3xand 3x+3

According to the question

3x(3x+3) = 270

= x(3x+3) =90

= $3x^2+3x$ =90

= $3x^2+3x$ -90=0

= x^2+x -30=0

= $x^2+6x-5x$ -30=0

=x(x+6)-5(x+6) =0

= (x+6)(x-5) =0

Either x+6 = 0 therefore x=-6

Or , x-5 = 0 therefore x=5

Considering the positive value of x

x=5

3x = 15

3x+3 = 18

The two consecutive multiples of 3 are 15 and 18 respectively.

Question 19: The sum of a number and its reciprocal is 174 $\frac{17}{4}$. find the numbers.

Solution:

Lethe number be x

According to the question

$x^2+1x = 174 \frac{x^2+1}{x} = \frac{17}{4}$

$= 4(x^2+1)=17x$

$= 4x^2+4-17x=0$

$= 4x^2+4-16x-x=0$

= 4x(x-4)-1(x-4) =0

=(4x-1)(x-4) =0

Either x-4 =0 therefore x=4

Or, 4x-1 =0 therefore x= 14 $x = \frac{1}{4}$

The value of x is 4

Question 20: A two digit is such that the products of its digits is 8when 18 is subtracted from the number, the digits interchange their places. Find the number?

Solution:

Let the digits be x and x-2 respectively.

The product of the digits is 8

According to the question

x(x-2) = 8

$= x^2-2x-8 =0$

$= x^2-4x+2x-8 =0$

= x(x-4)+2(x-4) =0

Either x-4 =0 therefore x=4

Or , x+2 =0 therefore x= -2

Considering the positive value of x = 4

x-2 = 2

The two digit number is 42.

Question 21: A two digit number is such that the product of the digits is 12, when 36 is added to the number, the digits interchange their places .find the number.

Solution:

Let the tens digit be x

Then, the unit digit = 12x $\frac{12}{x}$

Therefore the number = 10x+ 12x $10x + \frac{12}{x}$

And, the number obtained by interchanging the digits = x+ 120x $x + \frac{120}{x}$

= 10x+ 12x +36=x+ 120x $10x + \frac{12}{x} + 36 = x + \frac{120}{x}$

= 9x+ 12−120x +36=0 $9x + \frac{12-120}{x} + 36 = 0$

= 9x2+12−120x+36xx =0 $\frac{9x^2+12-120x+36x}{x} = 0$

= 9x2+−108x+36xx =0 $\frac{9x^2+-108x+36x}{x} = 0$

= $9(x^2+4x-12)=0$

= $(x^2+4x-12)=0$

= $x^2+6x-2x-12=0$

= x(x+6)-2(x+6) =0

=(x-2)(x+6) =0

Either x-2 = 0 therefore x=2

Or, x+6 =0 therefore x= -6

Since a digit can never be negative. So x=2

The number is 26.

Question 22: A two digit number is such that the product of the digits is 16 when 54 is subtracted from the number, the digits are interchanged. Find the number.

Solution:

Let the two digits be:

Tens digit be x

Units digit be 16x $\frac{16}{x}$

Numbers = 10x+ 16x $10x + \frac{16}{x}$ ……………………….(i)

Number obtained by interchanging = $10(10x+ 16x)10(10x+\frac{16}{x})$

$10x+ 16x\,10x+\frac{16}{x}$ – $10(10x+ 16x)10(10x+\frac{16}{x})$ =54

= $10x^2+16-160+x^2 = 54$

= $9x^2-54x-144= 0$

= $x^2-6x-16 =0$

= $x^2-8x+2x-16 =0$

= x(x-8)+2(x-8) =0

=(x-8)(x+2)=0

Either x-8 =0 therefore x=8

Or, x+2 =0 therefore x =-2

A digit can never be negative so x = 8

Hence by putting the value of x in the above equation (i) the number is 82.

Question 23: Two numbers differ by 3 and their product is 504. Find the numbers.

Solution:

Let the numbers be x and x-3 respectively.

According to the question

= x(x-3) =504

=$x^2-3x-504 =0$

= $x^2-24x+21x-504 =0$

= x(x-24)+21 (x-24) =0

=(x-24)(x+21) =0

Either x-24 = 0 therefore x =24

Or , x+21 =0 , therefore x =-21

x = 24 and x= -21

x-3 = 21 and x-3 = -24

The two numbers are 21 a nd 24 and -21 and -24 respectively.

Question 24: Two numbers differ by 4 and their product is 192. Find the numbers.

Solution:

Let the two numbers be x and x-4 respectively

Given that the product of the numbers is 192

According to the question

= $x(x-4) = 192$

= $x^2-4x-192=0$

= $x^2-16x+12x-192=0$

= $x(x-16)+12(x-16)=0$

= $(x-16)(x+12)=0$

Either $x-16=0$ therefore $x=16$

Or, $x+12=0$ therefore $x=-12$

Considering only the positive value of x

x=16S

$x-4 = 12$

The two numbers are 12 and 16 respectively.

Question 25: A two digit number is 4 times the sum of its digits and twice the product of its digits. Find the numbers.

Solution:

Let the digit in the tens and the units place be x and y respectively.

Then it is represented by $10x+y$

According to the question,

$10x+y = 4$(sum of the digits) and $2xy$

$10x+y = 4(x+y)$ and $10x+y = 2xy$

$10x+y = 4x+4y$ and $10x+y = 2xy$

6x-3y =0 and 10x+y-2xy =0

y= 2x and 10x + 2x -2x(2x) = 0

12x= $4x^2$

4x(x-3) =0

Either 4x=0 therefore x= 0

Or, x-3 =0 therefore x= 3

We have y = 2x

When x= 3 , y= 6

Question 26: The sum of the squares of two positive integers is 208. If the square of the large number is 18 times the smaller. Find the numbers.

Solution:

Let the smaller number be x

Then, square of the large number be = 18x

Also, square of the smaller number be = x^2

It is given that the sum of the square of the integer is 208.

Therefore,

= x^2 + 18x =208

= x^2+18x -208 =0

Applying factorization theorem,

= x^2 +26x-8x-208 =0

= x(x+26)-8(x+26) =0

= (x+26)(x-8) =0

Either x+26=0 therefore x=-26

Or, x-8 =0 therefore x= 8

Considering the positive number, therefore x= 8.

Square of the largest number =18x = 18*8 = 144

Largest number = $\sqrt{144}$=12$\sqrt{144} = 12$

Hence the numbers are 8 and 12 respectively.

Question 27: The sum of two numbers is 18. The sum of their reciprocal is 14 $\frac{1}{4}$ .find the numbers.

Solution:

Let the numbers be x and (18-x) respectively.

According to the given hypothesis,

1x−118−x=14 $\frac{1}{x} - \frac{1}{18-x} = \frac{1}{4}$ 18−x+xx(18−x)=14 $\frac{18-x+x}{x(18-x)} = \frac{1}{4}$ 18−x2+18x=14 $\frac{18}{-x^2+18x} = \frac{1}{4}$

= 72 = $18x-x^2$

= $x^2-18x+72 =0$

Applying factorization theorem, we get,

= $x^2 -6x-12x +72=0$

= x(x-6)-12(x-6) =0

= (x-6)(x-12) =0

Either, x= 6

Or, x=12

The two numbers are 6 and 12 respectively.

Question 28: The sum of two numbers a and b is 15 and the sum of their reciprocals 1a $\frac{1}{a}$ and 1b $\frac{1}{b}$ is 310 $\frac{3}{10}$. Find the numbers a and b.

Solution:

Let us assume a number x such that

1x−115−x=310 $\frac{1}{x} - \frac{1}{15-x} = \frac{3}{10}$ 15−x+xx(15−x)=310 $\frac{15-x+x}{x(15-x)} = \frac{3}{10}$ 1515x−x2=310 $\frac{15}{15x-x^2} = \frac{3}{10}$

= $3x^2-45x+150=0$

= $x^2 -15 x+50 = 0$

Applying factorization theorem,

$= x^2 - 10x - 5x + 50 = 0$

$= x(x-10) - 5(x-10) = 0$

$= (x-10)(x-5) = 0$

Either, $x-10 = 0$ therefore $x=10$

Or, $x-5=0$ therefore $x=5$

Case (i)

If $x = a$, $a=5$ and $b= 15-x$, $b= 10$

Case (ii)

If $x = 15-a = 15-10 = 5$,

$x=a=10$, $b= 15-10 =5$

Hence when $a=5$, $b=10$

$a=10$, $b= 5$

Question 29: The sum of two numbers is 9. The sum of their reciprocal is $12\frac{1}{2}$.find the numbers.

Solution:

Given that the sum of the two numbers is 9

Let the two number be x and 9-x respectively

According to the question

$1x + 19-x = 12\ \frac{1}{x} + \frac{1}{9-x} = \frac{1}{2}$

$= 9-x+xx(9-x) = 12\ \frac{9-x+x}{x(9-x)} = \frac{1}{2}$

$= 99x-x^2 = 12\ \frac{9}{9x-x^2} = \frac{1}{2}$

$= 9x - x^2 = 18$

$= x^2 - 9x + 18 = 0$

$= x^2 - 6x - 3x + 18 = 0$

$= x(x-6) - 3(x-6) = 0$

$= (x-6)(x-3) = 0$

Either x-6 =0 therefore x= 6

Or x-3 =0 therefore x=3

The two numbers are 3 and 6 respectively

Question 30: Three consecutive positive integers are such that the sum of the squares of the first and the product of the other two is 46. Find the integers.

Solution:

Let the consecutive positive integers be x , x+1, x+2 respectively

According to the question

$X^2+(x+1)(x+2) = 46$

$= x^2+x^2+3x+2 = 46$

$=2 x^2+3x+2 = 46$

$= 2 x^2+3x+2 -46=0$

$= 2 x^2-8x+11x+ -44=0$

$= 2x(x-4)+11(x-4) =0$

$= (x-4)(2x+11) =0$

Either x-4 =0 therefore x=4

Or, 2x+11 =0 therefore x=−112 $x = \frac{-11}{2}$

Considering the positive value of x that is x= 4

The three consecutive numbers are 4 , 5 and 6 respectively

Question 31: The difference of squares of two numbers is 88. If the large number is 5 less than the twice of the smaller, then find the two numbers

Solution:

Let the smaller number be x and larger number is 2x-5

It is given that the difference of the squares of the number is 88

According to the question

$(2x-5)^2-x^2= 88$

$= 4x^2+25-20x- x^2 =88$

$= 3x^2-20x-63 =0$

$= 3x^2-27x+7x-63 =0$

= 3x(x-9)+7(x-9) =0

= (x-9)(3x+7)=0

Either x-9 =0 therefore x=9

Or, 3x+7 =0 therefore **x=**−73 $x = \frac{-7}{3}$

Since a digit can never be negative so x= 9

Hence the number is 2x-5 = 13

The required numbers are 9 and 13 respectively

Question 32: The difference of squares of two numbers is180. The square of the smaller number is 8 times the larger number. Find the two numbers

Solution:

Let the number be x

According to the question

$X^2-8x = 180$

$X^2-8x-180 =0$

$= X^2+10x-18x-180 =0$

= x(x+10)-18(x-10) =0

= (x-18)(x+10) =0

Either x-18 = 0 therefore x= 18

Or, x+10 =0 therefore x=-10

Case (i)

X=18

8x= 144

Larger number = $\sqrt{144} = 12\sqrt{144} = 12$

Case (ii)

X= -10

Square of the larger number 8x= -80

Here in this case no perfect square exist

Hence the numbers are 18 and 12 respectively .