RD SHARMA

Solutions

Class 10 Maths

Chapter 6

Ex 6.1

Prove the following trigonometric identities

Q1: $(1-Cos^2 A) Cosec^2 A = 1$

Ans: $(1-Cos^2 A) Cosec^2 A = Sin^2 A Cosec^2 A$

$= (Sin A Cosec A)^2$

$= (Sin A \times (1/Sin A))^2$

$= (1)^2 = 1$

Q2: $(1 + Cot^2 A) Sin^2 A = 1$

Ans: We know, $Cosec^2 A - Cot^2 A = 1$

So,

$(1 + Cot^2 A) Sin^2 A = Cosec^2 A Sin^2 A$

$= (Cosec A Sin A)^2$

$= ((1/Sin A) \times Sin A)^2$

$= (1)^2 = 1$

Q3: $\mathbf{tan^2θcos^2θ}$$\tan^2θ \cos^2θ$ **=** $\mathbf{1-cos^2θ}$$1-\cos^2θ$

A3: We know ,

$sin^2θ+cos^2θ=1$$\sin^2θ+\cos^2θ=1$

So,

$tan^2θcos^2θ$$\tan^2θ \cos^2θ$ = $(tanθ×cosθ)^2$$(\tanθ \times \cosθ)^2$

=($sinθcosθ$ ×$cosθ)^2$= $(\frac{\sinθ}{\cosθ} \times \cosθ)^2$ =$(sinθ)^2$= $(\sinθ)^2$ =$sin^2θ$= $\sin^2θ$ $1-cos^2θ$$1-\cos^2θ$

Q4: $\mathbf{cosecθ\sqrt{1-cos^2θ}=1}$$\text{cosec}θ\sqrt{1-\cos^2θ}=1$

A4: We know ,

$sin^2θ+cos^2θ=1$$\sin^2θ+\cos^2θ=1$

So,

$cosecθ\sqrt{1-cos^2θ}=cosecθ\sqrt{sin^2θ}$$\text{cosec}θ\sqrt{1-\cos^2θ}=\text{cosec}θ\sqrt{\sin^2θ}$

= =$cosecθsinθ$= $\text{cosec}θ \sinθ$

= = $1sinθ$ $sinθ$= $\frac{1}{\sinθ} \sinθ$

= 1

Q5 : $\mathbf{(sec^2θ-1)(cosec^2θ-1)=1}$$(\sec^2θ-1)(\text{cosec}^2θ-1)=1$

A5: We know that,

$(\sec^2\theta-\tan^2\theta)=1$ $(\sec^2\theta - \tan^2\theta) = 1$ $(\text{cosec}^2\theta-\cot^2\theta)=1$ $(\text{cosec}^2\theta - \cot^2\theta) = 1$

So,

$(\sec^2\theta-1)(\text{cosec}^2\theta-1)=\tan^2\theta\times\cot^2\theta$ $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1) = \tan^2\theta \times \cot^2\theta$ $=(\tan\theta\times\cot\theta)^2$ $= (\tan\theta \times \cot\theta)^2$ $=$ $(\tan\theta\times 1\tan\theta)^2$ $= (\tan\theta \times \frac{1}{\tan\theta})^2$

$= 1^2 = 1$

Q6: $\tan\theta+1\tan\theta=\sec\theta\,\text{cosec}\theta$ $\tan\theta + \frac{1}{\tan\theta} = \sec\theta\ \text{cosec}\theta$

A6: We know that,

$(\sec^2\theta-\tan^2\theta)=1$ $(\sec^2\theta - \tan^2\theta) = 1$

So,

$\tan\theta+1\tan\theta=\tan^2\theta+1\tan\theta$ $\tan\theta + \frac{1}{\tan\theta} = \frac{\tan^2\theta+1}{\tan\theta}$ $=\sec^2\theta\tan\theta$ $= \frac{\sec^2\theta}{\tan\theta}$ $=\sec\theta\sec\theta\tan\theta$ $= \sec\theta\frac{\sec\theta}{\tan\theta}$ $=\sec\theta\,1\cos\theta\,\sin\theta\cos\theta$ $= \sec\theta\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$ $=\sec\theta\,1\sin\theta$ $= \sec\theta\frac{1}{\sin\theta}$ Undefined control sequence \thetacosec

Q7: $\cos\theta1-\sin\theta=1+\sin\theta\cos\theta$ $\frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$

A7: We know ,

$\sin^2\theta+\cos^2\theta=1$ $\sin^2\theta + \cos^2\theta = 1$

So, Multiplying both numerator and denominator by $(1+\sin\theta)$ $(1 + \sin\theta)$, we have

$\cos\theta1-\sin\theta=\cos\theta(1+\sin\theta)(1-\sin\theta)(1+\sin\theta)$ $\frac{\cos\theta}{1-\sin\theta} = \frac{\cos\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$ $=\cos\theta(1+\sin\theta)(1-\sin^2\theta)$ $= \frac{\cos\theta(1+\sin\theta)}{(1-\sin^2\theta)}$ $=\cos\theta(1+\sin\theta)\cos^2\theta$ $= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$ $=$ $(1+\sin\theta)\cos\theta$ $= \frac{(1+\sin\theta)}{\cos\theta}$

Q8: $\cos\theta1+\sin\theta=1-\sin\theta\cos\theta$ $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$

A8: We know ,

$\sin^2\theta+\cos^2\theta=1$ $\sin^2\theta + \cos^2\theta = 1$

Multiplying both numerator and denominator by $(1-\sin\theta)$ $(1 - \sin\theta)$, we have

$\cos\theta1+\sin\theta=\cos\theta(1-\sin\theta)(1+\sin\theta)(1-\sin\theta)$ $\frac{\cos\theta}{1+\sin\theta} = \frac{\cos\theta(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$ $=\cos\theta(1-\sin\theta)(1-\sin^2\theta)$ $= \frac{\cos\theta(1-\sin\theta)}{(1-\sin^2\theta)}$ $=\cos\theta(1-\sin\theta)(\cos^2\theta)$ $= \frac{\cos\theta(1-\sin\theta)}{(\cos^2\theta)}$ $=$ $(1-\sin\theta)\cos\theta$ $= \frac{(1-\sin\theta)}{\cos\theta}$ $=(1-\sin\theta)\cos\theta$ $= \frac{(1-\sin\theta)}{\cos\theta}$

Q 9: $\cos^2 A + 11+\cot^2 A$ $\frac{1}{1+\cot^2 A}$ **= 1**

A9: We know that,

$\text{Sin}^2 A + \cos^2 A = 1$

$\text{cosec}^2 A - \cot^2 A = 1$

So, $\cos^2A + 11+\cot^2A = \cos^2A + 1\mathrm{cosec}^2A$ $\cos^2A + \frac{1}{1+\cot^2A} = \cos^2A + \frac{1}{\mathrm{cosec}^2A}$

$= \cos^2A + (1\mathrm{cosec}A)^2 = \cos^2A + (\frac{1}{\mathrm{cosec}A})^2$ $= \cos^2A + \sin A^2 = \cos^2A + \sin A^2$

= 1

Q10: $\sin A^2 + 11+\tan^2A = 1$ $\sin A^2 + \frac{1}{1+\tan^2A} = 1$

A10: We know,

$\mathrm{Sin}^2A + \cos^2A = 1$

$\sec^2A - \tan^2A = 1$

So,

$\sin A^2 + 11+\tan^2A = \sin A^2 + 1\sec^2A$ $\sin A^2 + \frac{1}{1+\tan^2A} = \sin A^2 + \frac{1}{\sec^2A}$ $= \sin A^2 + (1\sec A)^2 = \sin A^2 + (\frac{1}{\sec A})^2$ $= \sin A^2 + \cos^2A$
$= \sin A^2 + \cos^2A$

= 1

Q11: $\sqrt{1-\cos\theta 1+\cos\theta} = \mathrm{cosec}\theta - \cot\theta$ $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \mathrm{cosec}\theta - \cot\theta$

A11: We know ,

$\sin^2\theta + \cos^2\theta = 1$ $\sin^2\theta + \cos^2\theta = 1$

Multiplying both numerator and denominator by $(1-\cos\theta)$ $(1-\cos\theta)$, we have

$\sqrt{1-\cos\theta 1+\cos\theta} = \sqrt{(1-\cos\theta)(1-\cos\theta)(1+\cos\theta)(1-\cos\theta)}$ $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$ $= \sqrt{(1-\cos\theta)^2 1-\cos^2\theta} = \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$ $= \sqrt{(1-\cos\theta)^2 \sin^2\theta}$
$= \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$ $= (1-\cos\theta)\sin\theta = \frac{(1-\cos\theta)}{\sin\theta}$ $= 1\sin\theta - \cos\theta\sin\theta = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

\(=\frac{1}{sin\theta}-\frac{cos\theta}{sin\theta}[/latex\]

Q12: [latex]\frac{1-cos\theta }{sin\theta}=\frac{sin\theta}{1+cos\theta}\)

A12: We know ,

$\sin^2\theta + \cos^2\theta = 1$ $\sin^2\theta + \cos^2\theta = 1$

Multiplying both numerator and denominator by $(1+\cos\theta)$ $(1+\cos\theta)$, we have

$= (1-\cos^2\theta)(1+\cos\theta)(\sin\theta) = \frac{(1-\cos^2\theta)}{(1+\cos\theta)(\sin\theta)}$ $= (\sin^2\theta)(1+\cos\theta)(\sin\theta) = \frac{(\sin^2\theta)}{(1+\cos\theta)(\sin\theta)}$ $= (\sin\theta)(1+\cos\theta) = \frac{(\sin\theta)}{(1+\cos\theta)}$

Q13. $\sin\theta 1-\cos\theta$ $\frac{\sin\theta}{1-\cos\theta}$ **= cosec$\theta\theta$ + cot$\theta\theta$**

Ans:

Given, L.H.S = $\sin\theta 1-\cos\theta$ $\frac{\sin\theta}{1-\cos\theta}$

Rationalize both nr and dr with 1+cos $\theta\theta$

$= \frac{\sin\theta}{1-\cos\theta} * \frac{1+\cos\theta}{1+\cos\theta}$

We know that, (a-b)(a+b) = $a^2 - b^2$

=> $\frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$

Here, $(1-\cos^2\theta) = \sin^2\theta$

=> $\frac{\sin\theta + (\sin\theta*\cos\theta)}{\sin^2\theta}$

=> $\frac{\sin\theta}{\sin^2\theta} + \frac{\sin\theta*\cos\theta}{\sin^2\theta}$

=> $\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$

=> $\text{cosec}\,\theta + \cot\theta$

Hence, L.H.S = R.H.S

Q14. $\frac{1 - \sin\theta}{1 + \sin\theta}$ **=** $(\sec\theta - \tan\theta)^2$

Ans:

Given, L.H.S = $\frac{1 - \sin\theta}{1 + \sin\theta}$

Rationalize with nr and dr with $1 - \sin\theta$

=> $\frac{1 - \sin\theta}{1 + \sin\theta} * \frac{1 - \sin\theta}{1 - \sin\theta}$

Here, $(1-\sin\theta)(1+\sin\theta) = \cos^2\theta$

=> $\frac{(1 - \sin\theta)^2}{\cos^2\theta}$

=> $\left(\frac{1 - \sin\theta}{\cos\theta}\right)^2$

=> $\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$

=> $(\sec\theta - \tan\theta)^2$

Hence, L.H.S = R.H.S

Q15. $\frac{(1 + \cot^2\theta)\tan\theta}{\sec^2\theta}$ **= cot\theta**

Ans:

Given, L.H.S = $\frac{(1 + \cot^2\theta)\tan\theta}{\sec^2\theta}$

Here, $1 + \cot^2\theta = \text{cosec}^2\theta$

=> $\frac{\text{cosec}^2\theta*\tan\theta}{\sec^2\theta}$

=> $\frac{1}{\sin^2\theta} * \frac{\cos^2\theta}{1} * \frac{\sin\theta}{\cos\theta}$

=> $\frac{\cos\theta}{\sin\theta}$

=> $\cot\theta$

Hence, L.H.S = R.H.S

Q16. $\tan^2\theta - \sin^2\theta = \tan^2\theta * \sin^2\theta$

Ans:

Given, L.H.S = $\tan^2\theta - \sin^2\theta$

Here, $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

=> $\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$

=> $\sin^2\theta [\frac{1}{\cos^2\theta} - 1]$

=> $\sin^2\theta [\frac{1 - \cos^2\theta}{\cos^2\theta}]$

=> $\frac{\sin^2\theta}{\cos^2\theta} * \sin^2\theta$

=> $\tan^2\theta * \sin^2\theta$

Hence, L.H.S = R.H.S

Q17. $(\text{cosec}\ \theta + \sin\theta)(\text{cosec}\ \theta - \sin\theta) = \cot^2\theta + \cos^2\theta$

Ans:

Given, L.H.S = $(\text{cosec}\ \theta + \sin\theta)(\text{cosec}\ \theta - \sin\theta)$

Here, $(a + b)(a - b) = a^2 - b^2$

$\text{cosec}^2\theta$ can be written as $1 + \cot^2\theta$ and $\sin^2\theta$ can be written as $1 - \cos^2\theta$

=> $1 + \cot^2\theta - (1 - \cos^2\theta)$

=> $1 + \cot^2\theta - 1 + \cos^2\theta$

=> $\cot^2\theta + \cos^2\theta$

Hence, L.H.S = R.H.S

Q18. $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$

Ans:

Given, L.H.S = $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta)$

Here, $(a + b)(a - b) = a^2 - b^2$

$\sec^2\theta$ can be written as $1 + \tan^2\theta$ and $\cos^2\theta$ can be written as $1 - \sin^2\theta$

=> $1 + \tan^2\theta - (1 - \sin^2\theta)$

=> $1 + \tan^2\theta - 1 + \sin^2\theta$

=> $\tan^2\theta + \sin^2\theta$

Hence, L.H.S = R.H.S

Q19. secA(1 – sinA)(secA + tanA) = 1

Ans:

Given, L.H.S = secA(1 – sinA)(secA + tanA)

Here, secA = 1cosA $\frac{1}{\cos A}$ and tanA = sinAcosA $\frac{\sin A}{\cos A}$

=> 1cosA $\frac{1}{\cos A}$ * (1 – sinA) * 1+sinAcosA $\frac{1+\sin A}{\cos A}$

=> \(\frac{cos^{2}A}{cos^{2}A}\)

=> 1

Hence, L.H.S = R.H.S

Q20. (cosecA – sinA)(secA – cosA)(tanA + cotA) = 1

Ans:

Given, L.H.S = (cosecA – sinA)(secA – cosA)(tanA + cotA)

Here, cosecA = 1sinA $\frac{1}{\sin A}$, secA = 1cosA $\frac{1}{\cos A}$, tanA = sinAcosA $\frac{\sin A}{\cos A}$, cotA = cosAsinA $\frac{\cos A}{\sin A}$

Substitute the above values in L.H.S

=> (1sinA $\frac{1}{\sin A}$ – sinA)(1cosA $\frac{1}{\cos A}$ – cosA)(sinAcosA $\frac{\sin A}{\cos A}$ + cosAsinA $\frac{\cos A}{\sin A}$)

=> (1−sin^{2}AsinA $\frac{1-\sin^2 A}{\sin A}$) * (1−cos^{2}AcosA $\frac{1-\cos^2 A}{\cos A}$) * (

sin^{2}A+cos^{2}AsinA∗cosA $\frac{\sin^2 A + \cos^2 A}{\sin A * \cos A}$

Here, (\(\frac{1\;-sin^{2}A}) = cos^{2}A, ([latex]\frac{1\;-cos^{2}A}) = sin^{2}A, sin^{2}A + cos^{2}A = 1

=> [latex]\frac{sin^{2}A\;*\;cos^{2}A\;*\;1}{sin^{2}A\;*\;cos^{2}A}\)

=> 1

Hence, L.H.S = R.H.S

Q21. (1 + tan^{2}θ$\tan^2\theta$)(1 – sin θθ)(1 + sin θθ) = 1

Ans:

Given, L.H.S = (1 + tan^{2}θ$\tan^2\theta$)(1 – sin\theta)(1 + sin\theta)

We know that,

Sin^2 θθ + cos^2 θθ = 1

And sec^2 θθ – tan^2 θθ = 1

So,

(1 + tan^{2}θ$\tan^2\theta$)(1 – sin θθ)(1 + sin θθ) = (1 + tan^{2}θ$\tan^2\theta$){(1 – sin θθ)(1 + sin θθ)}

= (1 + tan^{2}θ$\tan^2\theta$)((1 – sin^{2}θ$\sin^2\theta$)

= sec^{2}θ∗tan^{2}θ$\sec^2\theta * \tan^2\theta$

= (1cos^{2}θ)∗cos^{2}θ$(\frac{1}{\cos^2\theta}) * \cos^2\theta$

= 1

hence, L.H.S = R.H.S

Q22. $(sin^2A*cot^2A)+(cos^2A;*tan^2A)$ $(\sin^2 A \; * \; \cot^2 A) \; + \; (\cos^2 A; * \tan^2 A)$ **= 1**

Ans:

Given, L.H.S = Undefined control sequence \A

Here, \((sin^{2}A\;+\;cos^{2}A) = 1

So,

[latex](sin^{2}\A\;*\;cot^{2}A)\;+\;(cos^{2}A\;*\;tan^{2}A)\) = $\sin^2A($ cos^2Asin^2A $\frac{\cos^2 A}{\sin^2 A}$ $)$ + $\cos^2A($ sin^2Acos^2A $\frac{\sin^2 A}{\cos^2 A}$

= $\cos^2A + \sin^2A$

= 1

Hence , L.H.S = R.H.S

Q23:

1. **cot θθ – tan θθ = $2cos^2θ−1sinθ*cosθ$** $\frac{2\cos^2\theta - 1}{\sin\theta * \cos\theta}$

Ans:

Give, L.H.S = cot θθ – tan θθ

Here, $\sin^2$ θθ + $\cos^2$ θθ = 1

So,

=> cot θθ – tan θθ = $cosθsinθ$ $\frac{\cos\theta}{\sin\theta}$ – $sinθcosθ$ $\frac{\sin\theta}{\cos\theta}$

= $cos^2θ−sin^2θsinθ*cosθ$ $\frac{\cos^2\theta - \sin^2\theta}{\sin\theta * \cos\theta}$

= $cos^2θ−(1−cos^2θ)sinθ*cosθ$ $\frac{\cos^2\theta - (1 - \cos^2\theta)}{\sin\theta * \cos\theta}$

= $cos^2θ−1−cos^2θsinθ*cosθ$ $\frac{\cos^2\theta - 1 - \cos^2\theta}{\sin\theta * \cos\theta}$

= $($ $2cos^2θ−1sinθ*cosθ$ $)(\frac{2\cos^2\theta - 1}{\sin\theta * \cos\theta})$

Hence, L.H.S = R.H.S

1. **tanθ−cotθ=** $\tan\theta \; - \; \cot\theta \; =$ **$($ $2sin^2θ−1sinθ*cosθ$ $)$** $(\frac{2\sin^2\theta - 1}{\sin\theta * \cos\theta})$

Sol:

Given, L.H.S = \(tan\theta\;-\;cot\theta

We know that,

$\sin^2$ [latex]\theta\) + $\cos^2$ θθ = 1

tanθ−cotθ=[latex] $sinθcosθ$ $\tan\theta \; - \; \cot\theta = [latex]\frac{\sin\theta}{\cos\theta}$ – $cosθsinθ$ $\frac{\cos\theta}{\sin\theta}$

= $sin^2θ−cos^2θsinθcosθ$ $\frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$

= sin²θ−(1−sin²θ)sinθcosθ $\frac{\sin^2\theta - (1 - \sin^2\theta)}{\sin\theta\cos\theta}$

= sin²θ−1+sin²θsinθcosθ $\frac{\sin^2\theta - 1 + \sin^2\theta}{\sin\theta\cos\theta}$

= (2sin²θ−1sinθ∗cosθ)$\left(\frac{2\sin^2\theta - 1}{\sin\theta * \cos\theta}\right)$

Hence, L.H.S = R.H.S

Q24. **cos²θsinθ−cosecθ+sinθ** $\frac{\cos^2\theta}{\sin\theta} - \text{cosec}\theta + \sin\theta$ **= 0**

Ans:

Given, L.H.S cos²θsinθ−cosecθ+sinθ $\frac{\cos^2\theta}{\sin\theta} - \text{cosec}\theta + \sin\theta$

We know that,

$\text{Sin}^2\,\theta\theta + \cos^2\,\theta\theta = 1$

So,

cos²θsinθ−cosecθ+sinθ $\frac{\cos^2\theta}{\sin\theta} - \text{cosec}\theta + \sin\theta$ = (cos²θsinθ−cosecθ)+sinθ $\frac{\cos^2\theta}{\sin\theta} - \text{cosec}\theta) + \sin\theta$

= (cos²θsinθ−1sinθ)+sinθ$\left(\frac{\cos^2\theta}{\sin\theta} - \frac{1}{\sin\theta}\right) + \sin\theta$

= (cos²θ−1sinθ)+sinθ$\left(\frac{\cos^2\theta - 1}{\sin\theta}\right) + \sin\theta$

= (−sin²θsinθ)+sinθ$\left(\frac{-\sin^2\theta}{\sin\theta}\right) + \sin\theta$

= −sinθ+sinθ$-\sin\theta + \sin\theta$

= 0

Hence, L.H.S = R.H.S

Q 25 . **11+sinA** $\frac{1}{1+\sin A}$ **+ 11−sinA** $\frac{1}{1-\sin A}$ **= 2 sec² A**

Ans:

LHS = 11+sinA $\frac{1}{1+\sin A}$ + 11−sinA $\frac{1}{1-\sin A}$

(1−sinA)+(1+sinA)(1+sinA)(1−sinA) $\frac{(1-\sin A)+(1+\sin A)}{(1+\sin A)(1-\sin A)}$ 1−sinA+1+sinA1−sin²A $\frac{1-\sin A+1+\sin A}{1-\sin^2 A}$

⇒ 21−sin²A ⇒ $\frac{2}{1-\sin^2 A}$ [Since , (1 + sin A)(1 – sin A) = 1−sin²A$1 - \sin^2 A$]

⇒ 2cos²A ⇒ $\frac{2}{\cos^2 A}$ [Since , 1-sin² A = cos2 A]

⇒2sec²A⇒ $2\sec^2 A$

LHS = RHS Hence proved

Q 26 . **1+sinθcosθ+cosθ1+sinθ=2secθ** $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$

Ans:

LHS = 1+sinθcosθ+cosθ1+sinθ $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$

$= \frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$

$= \frac{1+\sin^2\theta+2\sin\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$

$\Rightarrow \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = 2\sec\theta$

LHS = RHS   Hence proved

Q 27 . $\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta} = \frac{1+\sin^2\theta}{1-\sin^2\theta}$

Ans:

We know that $\sin^2\theta + \cos^2\theta = 1$

So,

LHS =

$$\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}$$
$$= \frac{(1+2\sin\theta+\sin^2\theta)+(1-2\sin\theta+\sin^2\theta)}{2\cos^2\theta}$$
$$= \frac{1+2\sin\theta+\sin^2\theta+1-2\sin\theta+\sin^2\theta}{2\cos^2\theta}$$
$$= \frac{2+2\sin^2\theta}{2\cos^2\theta}$$
$$= \frac{2(1+\sin^2\theta)}{2(1-\sin^2\theta)}$$
$$= \frac{(1+\sin^2\theta)}{(1-\sin^2\theta)}$$

LHS = RHS   Hence proved

Q 28 . $\frac{1+\tan^2\theta}{1+\cot^2\theta} = \left[\frac{1-\tan\theta}{\cot\theta}\right]^2 - \tan^2\theta$

Ans :

LHS = $\frac{1+\tan^2\theta}{1+\cot^2\theta}$

$= \frac{\sec^2\theta}{\operatorname{cosec}^2\theta}$ [Since , $\tan^2\theta$ + 1 = $\sec^2\theta$, 1 + $\cot^2\theta$ = $\operatorname{cosec}^2\theta$]

$= \frac{1}{\cos^2\theta\cdot 1}\sin^2\theta$

$= \tan^2\theta$

LHS = RHS   Hence proved

Q 29 . $\frac{1+\sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1-\cos\theta}$

Ans :

LHS = $\frac{1+\sec\theta}{\sec\theta}$

= $\frac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$

= $\frac{\cos\theta+1}{\cos\theta} \cdot \cos\theta$

= $1+\cos\theta$

RHS = $\frac{\sin^2\theta}{1-\cos\theta}$

= $\frac{1-\cos^2\theta}{1-\cos\theta}$

= $\frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$

= $1+\cos\theta$

LHS = RHS Hence proved

Q 30 . $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$ **=** $1+\tan\theta+\cot\theta$

Ans:

LHS = $\frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\cot\theta}{1-\tan\theta}$

= $\frac{\tan^2\theta}{\tan\theta-1} + \frac{\cot\theta}{1-\tan\theta}$

= $\frac{1}{1-\tan\theta}\left[\frac{1}{\tan\theta} - \tan^2\theta\right]$

= $\frac{1}{1-\tan\theta}\left[\frac{1-\tan^3\theta}{\tan\theta}\right]$

= $\frac{1}{1-\tan\theta} \frac{(1-\tan\theta)(1+\tan\theta+\tan^2\theta)}{\tan\theta}$ [Since , $a^3-b^3=(a-b)(a^2+ab+b^2)$]

= $\frac{1+\tan\theta+\tan^2\theta}{\tan\theta}$

= $\frac{1}{\tan\theta} + \frac{\tan\theta}{\tan\theta} + \frac{\tan^2\theta}{\tan\theta}$

= $1+\tan\theta+\cot\theta$

LHS = RHS Hence proved

Q 31 . $\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$

Ans :

We know that $\sec^2\theta - \tan^2\theta = 1$

Cubing both sides

$(\sec^2\theta - \tan^2\theta)^3 = 1$ $(\sec^2\theta - \tan^2\theta)^3 = 1$

$\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta(\sec^2\theta - \tan^2\theta) = 1$ $\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta(\sec^2\theta - \tan^2\theta) = 1$ [Since , $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ $a^3 - b^3 = (a-b)(a^2+ab+b^2)$]

$\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta = 1$ $\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta = 1$ $\Rightarrow \sec^6\theta = \tan^6\theta + 3\sec^2\theta\tan^2\theta + 1$
$\Rightarrow \sec^6\theta = \tan^6\theta + 3\sec^2\theta\tan^2\theta + 1$

Hence proved.

Q 32 . $\mathrm{cosec}^6\theta = \cot^6\theta + 3\cot^2\theta\,\mathrm{cosec}^2\theta + 1$ $\mathrm{cosec}^6\theta = \cot^6\theta + 3\cot^2\theta\,\mathrm{cosec}^2\theta + 1$

Ans :

We know that $\mathrm{cosec}^2\theta - \cot^2\theta = 1$ $\mathrm{cosec}^2\theta - \cot^2\theta = 1$

Cubing both sides

$(\mathrm{cosec}^2\theta - \cot^2\theta)^3 = 1$ $(\mathrm{cosec}^2\theta - \cot^2\theta)^3 = 1$

$\mathrm{cosec}^6\theta - \cot^6\theta - 3\mathrm{cosec}^2\theta\cot^2\theta(\mathrm{cosec}^2\theta - \cot^2\theta) = 1$
$\mathrm{cosec}^6\theta - \cot^6\theta - 3\mathrm{cosec}^2\theta\cot^2\theta(\mathrm{cosec}^2\theta - \cot^2\theta) = 1$ [Since , $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
$a^3 - b^3 = (a-b)(a^2+ab+b^2)$]

$\mathrm{cosec}^6\theta - \cot^6\theta - 3\mathrm{cosec}^2\theta\cot^2\theta = 1$ $\mathrm{cosec}^6\theta - \cot^6\theta - 3\mathrm{cosec}^2\theta\cot^2\theta = 1$ $\Rightarrow \mathrm{cosec}^6\theta = \cot^6\theta + 3\mathrm{cosec}^2\theta\cot^2\theta + 1$
$\Rightarrow \mathrm{cosec}^6\theta = \cot^6\theta + 3\mathrm{cosec}^2\theta\cot^2\theta + 1$

Hence proved.

Q 33 . $(1+\tan^2\theta)\cot\theta\,\mathrm{cosec}^2\theta = \tan\theta$ $\frac{(1+\tan^2\theta)\cot\theta}{\mathrm{cosec}^2\theta} = \tan\theta$

Ans :

We know that $\sec^2\theta - \tan^2\theta = 1$ $\sec^2\theta - \tan^2\theta = 1$

Therefore , $\sec^2\theta = 1 + \tan^2\theta$ $\sec^2\theta = 1 + \tan^2\theta$

LHS = $\sec^2\theta \cdot \cot\theta\,\mathrm{cosec}^2\theta$ $\frac{\sec^2\theta \cdot \cot\theta}{\mathrm{cosec}^2\theta}$

= $1 \cdot \sin^2\theta\cos^2\theta \cdot \cos\theta\sin\theta$ $\frac{1 \cdot \sin^2\theta}{\cos^2\theta} \cdot \frac{\cos\theta}{\sin\theta}$ [$\because \sec\theta = 1\cos\theta$,$\mathrm{cosec}\theta = 1\sin\theta$,$\cot\theta = \cos\theta\sin\theta$]

$\Rightarrow \sin\theta\cos\theta = \tan\theta \Rightarrow$ $\frac{\sin\theta}{\cos\theta} = \tan\theta$

LHS = RHS Hence proved

Q 34 . \(\frac{ 1+cosA}{sin ^{2} A}\) = 11−cosA $\frac{1}{1-\cos A}$

Ans:

We know that $\sin^2 A + \cos^2 A$ $\sin^2 A + \cos^2 A$ = 1

$\sin^2 A = 1 - \cos^2 A$ $\sin^2 A = 1 - \cos^2 A$ $\Rightarrow \sin^2 A = (1-\cos A)(1+\cos A) \Rightarrow$ $\sin^2 A = (1-\cos A)(1+\cos A)$ $\Rightarrow$ LHS= $(1+\cos A)(1-\cos A)(1+\cos A) \Rightarrow$ $\text{LHS} = \frac{(1+\cos A)}{(1-\cos A)(1+\cos A)}$

$= \Rightarrow \text{LHS} = \dfrac{1}{(1-\cos A)}$

∴ LHS = RHS   Hence proved

Q 35 . $\dfrac{\sec A-\tan A}{\sec A+\tan A} = \dfrac{\cos^2 A}{(1+\sin A)^2}$

Ans:

LHS = $\dfrac{\sec A-\tan A}{\sec A+\tan A}$

Rationalizing the denominator by multiplying and dividing with sec A + tan A , we get

$\dfrac{\sec A-\tan A}{\sec A+\tan A} \times \dfrac{\sec A+\tan A}{\sec A+\tan A}$

$= \dfrac{\sec^2 A-\tan^2 A}{(\sec A+\tan A)^2}$

$= \dfrac{1}{(\sec A+\tan A)^2}$

$= \dfrac{1}{(\sec^2 A+\tan^2 A+2\sec A\tan A)}$

$= \dfrac{1}{\left(\frac{1}{\cos^2 A}+\frac{\sin^2 A}{\cos^2 A}+\frac{2\sin A}{\cos A}\right)}$

$\Rightarrow \dfrac{\cos^2 A}{1+\sin^2 A+2\sin A}$

$= \dfrac{\cos^2 A}{(1+\sin A)^2}$

∴ LHS = RHS   Hence proved

Q 36 . $\dfrac{1+\cos A}{\sin A} = \dfrac{\sin A}{1-\cos A}$

Ans:

LHS = $\dfrac{1+\cos A}{\sin A}$

Multiply both numerator and denominator with (1 – cos A) we get ,

$\dfrac{(1+\cos A)(1-\cos A)}{\sin A(1-\cos A)}$

$= \dfrac{1-\cos^2 A}{\sin A(1-\cos A)}$

$= \dfrac{\sin^2 A}{\sin A(1-\cos A)}$

$= \dfrac{\sin A}{1-\cos A}$

∴ LHS = RHS   Hence proved

37.

(i) $\sqrt{\dfrac{1+\sin A}{1-\sin A}}$ **= sec A + tan A**

Ans:

To prove,

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

Considering left hand side (LHS),

Rationalize the numerator and denominator with $\sqrt{1+\sin A}$

$$\sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \frac{(1+\sin A)}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

= sec A + tan A

Therefore, LHS = RHS

Hence proved

(ii) $\sqrt{\frac{(1-\cos A)}{(1+\cos A)}} + \sqrt{\frac{(1+\cos A)}{(1-\cos A)}}$ **= 2cosec A**

Ans:

To prove,

$$\sqrt{\frac{(1-\cos A)}{(1+\cos A)}} + \sqrt{\frac{(1+\cos A)}{(1-\cos A)}} = 2\operatorname{cosec} A$$

Considering left hand side (LHS),

Rationalize the numerator and denominator.

$$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}} + \sqrt{\frac{(1+\cos A)(1+\cos A)}{(1-\cos A)(1+\cos A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}} + \sqrt{\frac{(1+\cos A)^2}{(1-\cos^2 A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(\sin^2 A)}} + \sqrt{\frac{(1+\cos A)^2}{(\sin^2 A)}}$$

$$= \frac{(1-\cos A)}{(\sin A)} + \frac{(1+\cos A)}{(\sin A)}$$

$$= \frac{(1-\cos A+1+\cos A)}{(\sin A)}$$

$$= \frac{(2)}{(\sin A)}$$

= 2cosec A

Therefore, LHS = RHS

Hence proved

38. **Prove that:**

(i) $\sqrt{(\sec\Theta-1)(\sec\Theta+1)} + \sqrt{(\sec\Theta+1)(\sec\Theta-1)}\ \sqrt{\frac{(\sec\Theta-1)}{(\sec\Theta+1)}} + \sqrt{\frac{(\sec\Theta+1)}{(\sec\Theta-1)}}$ **= 2cosec ΘΘ**

Ans:

To prove,

$= \sqrt{(\sec\Theta-1)(\sec\Theta+1)} + \sqrt{(\sec\Theta+1)(\sec\Theta-1)}\ \sqrt{\frac{(\sec\Theta-1)}{(\sec\Theta+1)}} + \sqrt{\frac{(\sec\Theta+1)}{(\sec\Theta-1)}} = 2\text{cosec}\ \Theta\Theta$

Considering left hand side (LHS),

Rationalize the numerator and denominator.

$= \sqrt{(\sec\Theta-1)(\sec\Theta-1)(\sec\Theta+1)(\sec\Theta-1)} + \sqrt{(\sec\Theta+1)(\sec\Theta+1)(\sec\Theta-1)(\sec\Theta+1)}\ \sqrt{\frac{(\sec\Theta-1)(\sec\Theta-1)}{(\sec\Theta+1)(\sec\Theta-1)}} + \sqrt{\frac{(\sec\Theta+1)(\sec\Theta+1)}{(\sec\Theta-1)(\sec\Theta+1)}}$

$= \sqrt{(\sec\Theta-1)^2(\sec^2\Theta-1)} + \sqrt{(\sec\Theta+1)^2(\sec^2\Theta-1)}\ \sqrt{\frac{(\sec\Theta-1)^2}{(\sec^2\Theta-1)}} + \sqrt{\frac{(\sec\Theta+1)^2}{(\sec^2\Theta-1)}}$

$= \sqrt{(\sec\Theta-1)^2\tan^2\Theta} + \sqrt{(\sec\Theta+1)^2\tan^2\Theta}\ \sqrt{\frac{(\sec\Theta-1)^2}{\tan^2\Theta}} + \sqrt{\frac{(\sec\Theta+1)^2}{\tan^2\Theta}}$

$= (\sec\Theta-1)\tan\Theta + (\sec\Theta+1)\tan\Theta\ \frac{(\sec\Theta-1)}{\tan\Theta} + \frac{(\sec\Theta+1)}{\tan\Theta}$

$= (\sec\Theta-1+\sec\Theta+1)\tan\Theta\ \frac{(\sec\Theta-1+\sec\Theta+1)}{\tan\Theta}$

$= (2\cos\Theta)\cos\Theta\sin\Theta\ \frac{(2\cos\Theta)}{\cos\Theta\sin\Theta}$

$= 2\sin\Theta\ \frac{2}{\sin\Theta}$

= 2cosec ΘΘ

Therefore, LHS = RHS

Hence proved

(ii) $\sqrt{(1+\sin\Theta)(1-\sin\Theta)} + \sqrt{(1-\sin\Theta)(1+\sin\Theta)}\ \sqrt{\frac{(1+\sin\Theta)}{(1-\sin\Theta)}} + \sqrt{\frac{(1-\sin\Theta)}{(1+\sin\Theta)}}$ **= 2sec ΘΘ**

Ans:

To prove,

$= \sqrt{(1+\sin\Theta)(1-\sin\Theta)} + \sqrt{(1-\sin\Theta)(1+\sin\Theta)}\ \sqrt{\frac{(1+\sin\Theta)}{(1-\sin\Theta)}} + \sqrt{\frac{(1-\sin\Theta)}{(1+\sin\Theta)}} = 2\sec\ \Theta\Theta$

Considering left hand side (LHS),

Rationalize the numerator and denominator.

$= \sqrt{(1+\sin\Theta)(1+\sin\Theta)(1-\sin\Theta)(1+\sin\Theta)} + \sqrt{(1-\sin\Theta)(1-\sin\Theta)(1+\sin\Theta)(1-\sin\Theta)}\ \sqrt{\frac{(1+\sin\Theta)(1+\sin\Theta)}{(1-\sin\Theta)(1+\sin\Theta)}} + \sqrt{\frac{(1-\sin\Theta)(1-\sin\Theta)}{(1+\sin\Theta)(1-\sin\Theta)}}$

$= \sqrt{(1+\sin\Theta)^2(1-\sin^2\Theta)} + \sqrt{(1-\sin\Theta)^2(1-\sin^2\Theta)}\ \sqrt{\frac{(1+\sin\Theta)^2}{(1-\sin^2\Theta)}} + \sqrt{\frac{(1-\sin\Theta)^2}{(1-\sin^2\Theta)}}$

$= \sqrt{(1+\sin\Theta)^2(\cos^2\Theta)} + \sqrt{(1-\sin\Theta)^2(\cos^2\Theta)}\ \sqrt{\frac{(1+\sin\Theta)^2}{(\cos^2\Theta)}} + \sqrt{\frac{(1-\sin\Theta)^2}{(\cos^2\Theta)}}$

$= (1+\sin\Theta)(\cos\Theta) + (1-\sin\Theta)(\cos\Theta)\ \frac{(1+\sin\Theta)}{(\cos\Theta)} + \frac{(1-\sin\Theta)}{(\cos\Theta)}$

= (1+sinΘ+1−sinΘ)(cosΘ) $\frac{(1+\sin\Theta+1-\sin\Theta)}{(\cos\Theta)}$

= (2)(cosΘ) $\frac{(2)}{(\cos\Theta)}$

= 2secΘ$2\sec\Theta$

Therefore, LHS = RHS

Hence proved

(iii) $\sqrt{(1+\cos\Theta)(1-\cos\Theta)}$ $\sqrt{\frac{(1+\cos\Theta)}{(1-\cos\Theta)}}$ **\sqrt{\frac{(1-cos \Theta)}{(1+cos \Theta)}} = 2cosecΘ**$2\mathrm{cosec}\Theta$

Ans:

To prove,

$\sqrt{(1-\cos\Theta)(1+\cos\Theta)}+\sqrt{(1+\cos\Theta)(1-\cos\Theta)}$ $\sqrt{\frac{(1-\cos\Theta)}{(1+\cos\Theta)}}+\sqrt{\frac{(1+\cos\Theta)}{(1-\cos\Theta)}}$ = 2cosec \Theta

Considering left hand side (LHS),

Rationalize the numerator and denominator.

= $\sqrt{(1-\cos\Theta)(1-\cos\Theta)(1+\cos\Theta)(1-\cos\Theta)}+\sqrt{(1+\cos\Theta)(1+\cos\Theta)(1-\cos\Theta)(1+\cos\Theta)}$ $\sqrt{\frac{(1-\cos\Theta)(1-\cos\Theta)}{(1+\cos\Theta)(1-\cos\Theta)}}+\sqrt{\frac{(1+\cos\Theta)(1+\cos\Theta)}{(1-\cos\Theta)(1+\cos\Theta)}}$

= $\sqrt{(1-\cos\Theta)^2(1-\cos^2\Theta)}+\sqrt{(1+\cos\Theta)^2(1-\cos^2\Theta)}$ $\sqrt{\frac{(1-\cos\Theta)^2}{(1-\cos^2\Theta)}}+\sqrt{\frac{(1+\cos\Theta)^2}{(1-\cos^2\Theta)}}$

= $\sqrt{(1-\cos\Theta)^2(\sin^2\Theta)}+\sqrt{(1+\cos\Theta)^2(\sin^2\Theta)}$ $\sqrt{\frac{(1-\cos\Theta)^2}{(\sin^2\Theta)}}+\sqrt{\frac{(1+\cos\Theta)^2}{(\sin^2\Theta)}}$

= (1−cosΘ)(sinΘ) + (1+cosΘ)(sinΘ) $\frac{(1-\cos\Theta)}{(\sin\Theta)}+\frac{(1+\cos\Theta)}{(\sin\Theta)}$

= (1−cosΘ+1+cosΘ)(sinΘ) $\frac{(1-\cos\Theta+1+\cos\Theta)}{(\sin\Theta)}$

= (2)(sinΘ) $\frac{(2)}{(\sin\Theta)}$

= 2cosec \Theta

Therefore, LHS = RHS

Hence proved

(iv) secΘ−1secΘ+1 $\frac{\sec\Theta-1}{\sec\Theta+1}$ **= (sinΘ1+cosΘ)²**$(\frac{\sin\Theta}{1+\cos\Theta})^2$

Ans:

To prove,

secΘ−1secΘ+1 $\frac{\sec\Theta-1}{\sec\Theta+1}$ = (sinΘ1+cosΘ)²$(\frac{\sin\Theta}{1+\cos\Theta})^2$

Considering left hand side (LHS),

= secΘ−1secΘ+1 $\frac{\sec\Theta-1}{\sec\Theta+1}$

= 1−cosΘ1+cosΘ $\frac{1-\cos\Theta}{1+\cos\Theta}$

Multiply and divide with (1+cosΘΘ)

= (1−cosΘ)(1+cosΘ)(1+cosΘ)(1+cosΘ) $\frac{(1-\cos\Theta)(1+\cos\Theta)}{(1+\cos\Theta)(1+\cos\Theta)}$

= (1−cos²Θ)(1+cosΘ)² $\frac{(1-\cos^2\Theta)}{(1+\cos\Theta)^2}$

= (sin²Θ)(1+cosΘ)² $\frac{(\sin^2\Theta)}{(1+\cos\Theta)^2}$

= (sinΘ1+cosΘ)²$(\frac{\sin\Theta}{1+\cos\Theta})^2$

Therefore, LHS = RHS

Hence proved

39. **(sec A – tan A)² = 1−sinA1+sinA** $\frac{1-\sin A}{1+\sin A}$

Ans:

To prove,

(sec A – tan A)² = 1−sinA1+sinA $\frac{1-\sin A}{1+\sin A}$

Considering left hand side (LHS),

= (sec A – tan A)²

= [1cosA − sinAcosA]²$[\frac{1}{\cos A} - \frac{\sin A}{\cos A}]^2$

= (1−sinA)²cos²A $\frac{(1-\sin A)^2}{\cos^2 A}$

= (1−sinA)²1−sin²A $\frac{(1-\sin A)^2}{1-\sin^2 A}$

= (1−sinA)²(1+sinA)(1−sinA) $\frac{(1-\sin A)^2}{(1+\sin A)(1-\sin A)}$

= (1−sinA)(1+sinA) $\frac{(1-\sin A)}{(1+\sin A)}$

Therefore, LHS = RHS

Hence proved

40. **1−cosA1+cosA** $\frac{1-\cos A}{1+\cos A}$ **= (cot A – cosec A)²**

Ans:

To prove,

1−cosA1+cosA $\frac{1-\cos A}{1+\cos A}$ = (cot A – cosecA)

Considering left hand side (LHS),

Rationalize the numerator and denominator with (1 – cos A)

= (1−cosA)(1−cosA)(1+cosA)(1−cosA) $\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}$

= (1−cosA)²(1−cos²A) $\frac{(1-\cos A)^2}{(1-\cos^2 A)}$

= (1−cosA)²(sin²A) $\frac{(1-\cos A)^2}{(\sin^2 A)}$

= (1sinA − cosAsinA)²$(\frac{1}{\sin A} - \frac{\cos A}{\sin A})^2$

$= (\text{cosec } A - \cot A)^2$

$= (\cot A - \text{cosec})^2$

Therefore, LHS = RHS

Hence proved

41. $\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2\text{cosec}A\cot A$

Ans:

To prove,

$\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2\text{cosec}A\cot A$

Considering left hand side (LHS),

$= \frac{\sec A + 1 + \sec A - 1}{(\sec A + 1)(\sec A - 1)}$

$= \frac{2\sec A}{(\sec^2 A - 1)}$

$= \frac{2\sec A}{(\tan^2 A)}$

$= \frac{2\cos^2 A}{(\cos A \sin^2 A)}$

$= \frac{2\cos A}{(\sin^2 A)}$

$= \frac{2\cos A}{(\sin A)(\sin A))}$

$= 2\text{cosec } A \cot A$

Therefore, LHS = RHS

Hence proved

42. $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A}$ **= sin A + cos A**

Ans:

To prove,

$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$

Considering left hand side (LHS),

$= \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A}$

$= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}}$

$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$

$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$

$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A - \sin A}$

$= \cos A + \sin A$

Therefore, LHS = RHS

Hence proved

43. **(cosecA)(cosecA−1) + (cosecA)(cosecA+1)** $\frac{(\text{cosecA})}{(\text{cosecA}-1)} + \frac{(\text{cosecA})}{(\text{cosecA}+1)}$ **= 2sec^2 A**

Ans:

To prove,

(cosecA)(cosecA−1) + (cosecA)(cosecA+1) $\frac{(\text{cosecA})}{(\text{cosecA}-1)} + \frac{(\text{cosecA})}{(\text{cosecA}+1)}$ = 2sec^2 A

Considering left hand side (LHS),

= (cosecA)(cosecA+1+cosecA−1)(cosec^{2}A−1)) $\frac{(\text{cosecA})(\text{cosecA}+1+\text{cosecA}-1)}{(\text{cosec}^2\text{A}-1)}$)

= (2cosec^{2}A)cot^{2}A $\frac{(2\text{cosec}^2\text{A})}{\cot^2\text{A}}$

= (2sin^{2}A)sin^{2}A.cos^{2}A $\frac{(2\sin^2\text{A})}{\sin^2\text{A}.\cos^2\text{A}}$

= 2cos^{2}A $\frac{2}{\cos^2\text{A}}$

= 2sec^{2}A$2\sec^2\text{A}$

Therefore, LHS = RHS

Hence proved

44. tan^{2}A1+tan^{2}A + cot^{2}A1+cot^{2}A $\frac{\tan^2\text{A}}{1+\tan^2\text{A}} + \frac{\cot^2\text{A}}{1+\cot^2\text{A}}$ **= 1**

Ans:

To prove,

tan^{2}A1+tan^{2}A + cot^{2}A1+cot^{2}A $\frac{\tan^2\text{A}}{1+\tan^2\text{A}} + \frac{\cot^2\text{A}}{1+\cot^2\text{A}}$ = 1

Considering left hand side (LHS),

= sin^{2}Acos^{2}A cos^{2}A+sin^{2}Acos^{2}A + cos^{2}Asin^{2}A cos^{2}A+sin^{2}Asin^{2}A $\frac{\frac{\sin^2\text{A}}{\cos^2\text{A}}}{\frac{\cos^2\text{A}+\sin^2\text{A}}{\cos^2\text{A}}} + \frac{\frac{\cos^2\text{A}}{\sin^2\text{A}}}{\frac{\cos^2\text{A}+\sin^2\text{A}}{\sin^2\text{A}}}$

= sin^{2}Acos^{2}A+sin^{2}A + cos^{2}Acos^{2}A+sin^{2}A $\frac{\sin^2\text{A}}{\cos^2\text{A}+\sin^2\text{A}} + \frac{\cos^2\text{A}}{\cos^2\text{A}+\sin^2\text{A}}$

= sin^{2}A+cos^{2}Acos^{2}A+sin^{2}A $\frac{\sin^2\text{A}+\cos^2\text{A}}{\cos^2\text{A}+\sin^2\text{A}}$

= 1

Therefore, LHS = RHS

Hence proved

45. **cotA−cosAcotA+cosA** $\frac{\text{cotA}-\text{cosA}}{\text{cotA}+\text{cosA}}$ **= cosecA−1cosecA+1** $\frac{\text{cosecA}-1}{\text{cosecA}+1}$

Ans:

To prove,

cotA−cosAcotA+cosA $\frac{\cot A - \cos A}{\cot A + \cos A}$ = cosecA−1cosecA+1 $\frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

Considering left hand side (LHS),

= cosAsinA−cosAcosAsinA+cosA $\frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$

= cosAcosecA−cosAcosAcosecA+cosA $\frac{\cos A \operatorname{cosec} A - \cos A}{\cos A \operatorname{cosec} A + \cos A}$

= cosA(cosecA−1)cosA(cosecA+1) $\frac{\cos A(\operatorname{cosec} A - 1)}{\cos A(\operatorname{cosec} A + 1)}$

= (cosecA−1)(cosecA+1) $\frac{(\operatorname{cosec} A - 1)}{(\operatorname{cosec} A + 1)}$

Therefore, LHS = RHS

Hence proved

46. **1+cosΘ−sin²ΘsinΘ(1+cosΘ)** $\frac{1+\cos\Theta-\sin^2\Theta}{\sin\Theta(1+\cos\Theta)}$ **= cot Θ**Θ

Ans:

To prove,

1+cosΘ−sin²ΘsinΘ(1+cosΘ) $\frac{1+\cos\Theta-\sin^2\Theta}{\sin\Theta(1+\cos\Theta)}$ = cot ΘΘ

Considering left hand side (LHS),

= 1+cosΘ−(1−cos²Θ)sinΘ(1+cosΘ) $\frac{1+\cos\Theta-(1-\cos^2\Theta)}{\sin\Theta(1+\cos\Theta)}$

= 1+cosΘ−1+cos²ΘsinΘ(1+cosΘ) $\frac{1+\cos\Theta-1+\cos^2\Theta}{\sin\Theta(1+\cos\Theta)}$

= cosΘ+cos²ΘsinΘ(1+cosΘ) $\frac{\cos\Theta+\cos^2\Theta}{\sin\Theta(1+\cos\Theta)}$

= cosΘ(1+cosΘ)sinΘ(1+cosΘ) $\frac{\cos\Theta(1+\cos\Theta)}{\sin\Theta(1+\cos\Theta)}$

= (cosΘ)(sinΘ) $\frac{(\cos\Theta)}{(\sin\Theta)}$

= cotΘ$\cot\Theta$

Therefore, LHS = RHS

Hence, proved.

(i) 1+cosΘ+sinΘ1+cosΘ−sinΘ $\frac{1+\cos\Theta+\sin\Theta}{1+\cos\Theta-\sin\Theta}$ **= 1+sinΘcosΘ** $\frac{1+\sin\Theta}{\cos\Theta}$

Ans:

To prove,

1+cosΘ+sinΘ1+cosΘ−sinΘ $\frac{1+\cos\Theta+\sin\Theta}{1+\cos\Theta-\sin\Theta}$ = 1+sinΘcosΘ $\frac{1+\sin\Theta}{\cos\Theta}$

Dividing the numerator and denominator with cosΘ$\cos\Theta$

Considering LHS, we get,

= 1+cosΘ+sinΘcosΘ 1+cosΘ−sinΘcosΘ $\frac{\frac{1+\cos\Theta+\sin\Theta}{\cos\Theta}}{\frac{1+\cos\Theta-\sin\Theta}{\cos\Theta}}$

$= \frac{\sec\Theta+1+\tan\Theta}{\sec\Theta+1-\tan\Theta}$

$= \frac{1+\sec\Theta+\tan\Theta}{1+\sec\Theta-\tan\Theta}$

[As we know,

$$(\sec^2\Theta)-(\tan^2\Theta)=1$$
$$(\sec\Theta+\tan\Theta)(\sec\Theta-\tan\Theta)=1$$
$$(\sec\Theta+\tan\Theta)=\frac{1}{(\sec\Theta-\tan\Theta)}$$]

$= \frac{\frac{1}{(\sec\Theta-\tan\Theta)}+1}{1+\sec\Theta-\tan\Theta}$

$= \frac{1+\sec\Theta-\tan\Theta}{1+\sec\Theta-\tan\Theta} \times \frac{1}{\sec\Theta-\tan\Theta}$

$= \sec\Theta+\tan\Theta$

$= \frac{1+\sin\Theta}{\cos\Theta}$

Therefore, LHS = RHS

Hence proved

(ii) $\frac{\sin\Theta-\cos\Theta+1}{\sin\Theta+\cos\Theta-1} = \frac{1}{\sec\Theta-\tan\Theta}$

Ans:

To prove,

$\frac{\sin\Theta-\cos\Theta+1}{\sin\Theta+\cos\Theta-1} = \frac{1}{\sec\Theta-\tan\Theta}$

Considering LHS, we get,

$\frac{\sin\Theta-\cos\Theta+1}{\sin\Theta+\cos\Theta-1}$

Dividing the numerator and denominator with $\cos\Theta$, we get,

$= \frac{\tan\Theta+\sec\Theta-1}{\tan\Theta-\sec\Theta+1}$

[As we know, $(\sec\Theta+\tan\Theta)=\frac{1}{(\sec\Theta-\tan\Theta)}$]

$= \frac{\frac{1}{(\sec\Theta-\tan\Theta)}-1}{\tan\Theta-\sec\Theta+1}$

$= \frac{\tan\Theta-\sec\Theta+1}{\tan\Theta-\sec\Theta+1} \times \frac{1}{(\sec\Theta-\tan\Theta)}$

$= \frac{1}{(\sec\Theta-\tan\Theta)}$

Therefore, LHS = RHS

Hence proved

(iii) $\frac{\cos\Theta-\sin\Theta+1}{\cos\Theta+\sin\Theta-1} = \mathrm{cosec}\Theta+\cot\Theta$

Ans:

To prove,

$\frac{\cos\Theta-\sin\Theta+1}{\cos\Theta+\sin\Theta-1} = \mathrm{cosec}\Theta+\cot\Theta$

Considering LHS, we get,

Dividing the numerator and denominator with $\sin\Theta$, we get,

= $\frac{\frac{\cos\Theta-\sin\Theta+1}{\sin\Theta}}{\frac{\cos\Theta+\sin\Theta-1}{\sin\Theta}}$

= $\frac{\cot\Theta+\operatorname{cosec}\Theta-1}{\cot\Theta-\operatorname{cosec}\Theta+1}$

[As we know,

$(\operatorname{cosec}^2\Theta)-(\cot^2\Theta)=1$

$(\operatorname{cosec}\Theta+\cot\Theta)(\operatorname{cosec}\Theta-\cot\Theta)=1$

$(\operatorname{cosec}\Theta+\cot\Theta)=\frac{1}{(\operatorname{cosec}\Theta-\cot\Theta)}$]

= $\frac{\frac{1}{(\operatorname{cosec}\Theta-\cot\Theta)}-1}{\cot\Theta-\operatorname{cosec}\Theta+1}$

= $\frac{\cot\Theta-\operatorname{cosec}\Theta+1}{\cot\Theta-\operatorname{cosec}\Theta+1}\times\frac{1}{(\operatorname{cosec}\Theta-\cot\Theta)}$

= $\frac{1}{(\operatorname{cosec}\Theta-\cot\Theta)}$

= $\operatorname{cosec}\Theta+\cot\Theta$

Therefore, LHS = RHS

Hence proved

(iv) $(\sin\Theta+\cos\Theta)(\tan\Theta+\cot\Theta)$ = $\sec\Theta+\operatorname{cosec}\Theta$

Ans:

To prove,

$(\sin\Theta+\cos\Theta)(\tan\Theta+\cot\Theta)$ = $\operatorname{cosec}\Theta+\operatorname{cosec}\Theta$

Considering LHS, we get,

= $(\sin\Theta+\cos\Theta)(\frac{\sin\Theta}{\cos\Theta}+\frac{\cos\Theta}{\sin\Theta})$

= $(\frac{\sin^2\Theta}{\cos\Theta}+\cos\Theta+\sin\Theta+\frac{\cos^2\Theta}{\sin\Theta}$

= $\sin\Theta(\tan\Theta+1)+\cos\Theta(\frac{1}{\tan\Theta}+1)$

= $\sin\Theta(\tan\Theta+1)+\frac{\cos\Theta}{\tan\Theta}(\tan\Theta+1)$

= $(\sin\Theta+\frac{\cos\Theta}{\tan\Theta})(\tan\Theta+1)$

= $(\frac{\sin^2\Theta+\cos^2\Theta}{\sin\Theta})(\tan\Theta+1)$

= $(\frac{1}{\sin\Theta})(\tan\Theta+1)$

= Undefined control sequence \Thetasin

= $\sec\Theta+\operatorname{cosec}\Theta$

Therefore, LHS = RHS

Hence proved

50. **\frac{tanA}{1+secA}-\frac{tanA}{1-secA}= 2 cosec A**

Ans:

To prove,

\frac{tanA}{1+secA}-\frac{tanA}{1-secA}= 2 cosec A

Considering LHS, we get,

$= \frac{\frac{\sin A}{\cos A}}{\frac{\cos A+1}{\cos A}} - \frac{\frac{\sin A}{\cos A}}{\frac{\cos A-1}{\cos A}}$

$= \frac{\sin A}{\cos A+1} - \frac{\sin A}{\cos A-1}$

$= \sin A\left(\frac{1}{\cos A+1} - \frac{1}{\cos A-1}\right)$

$= \sin A\left(\frac{\cos A-1-\cos A-1}{\cos^2 A-1}\right)$

$= \sin A\left(\frac{\cos A-1-\cos A-1}{\cos^2 A-1}\right)$

$= \sin A\left(\frac{-2}{-\sin^2 A}\right)$

$= \frac{2}{\sin A})$

= 2 cosec A

Therefore, LHS = RHS

Hence proved

Q51: $1 + \frac{\cot^2\Theta}{1+\mathrm{cosec}\Theta} = \mathrm{cosec}\Theta$

Ans:

$1 + \frac{\mathrm{cosec}^2\Theta-1}{1+\mathrm{cosec}\Theta}$ $[\because \cot^2\Theta=\mathrm{cosec}^2\Theta-1]$

$1 + \frac{(\mathrm{cosec}\Theta-1)(\mathrm{cosec}\Theta+1)}{1+\mathrm{cosec}\Theta}$

$=1+\mathrm{cosec}\Theta-1$ $[\because (a+b)(a-b)=a^2-b^2)]$

$=\mathrm{cosec}\Theta$

Therefore, LHS = RHS

Hence, proved.

Q52: $\frac{\cos\Theta}{\mathrm{cosec}\Theta+1} + \frac{\cos\Theta}{\mathrm{cosec}\Theta-1} = 2\tan\Theta$

Ans:

$\frac{\cos\Theta}{\frac{1}{\sin\Theta}+1} + \frac{\cos\Theta}{\frac{1}{\sin\Theta}-1}$

$\frac{\cos\Theta}{\frac{1+\sin\Theta}{\sin\Theta}} + \frac{\cos\Theta}{\frac{1-\sin\Theta}{\sin\Theta}}$

$\frac{(\cos\Theta)(\sin\Theta)}{1+\sin\Theta} + \frac{(\cos\Theta)(\sin\Theta)}{1-\sin\Theta}$

$\frac{(1-\sin\Theta)(\sin\Theta\cos\Theta)+(\sin\Theta\cos\Theta)}{(1+\sin\Theta)(1-\sin\Theta)}$

$\frac{\sin\Theta\cos\Theta-\sin\Theta\cos\Theta+\sin\Theta\cos\Theta+\sin^2\Theta\cos^2\Theta}{1-\sin^2\Theta}$

$= \frac{\sin\Theta\cos\Theta}{\cos^2\Theta}$

$= \frac{2\sin\Theta}{\cos\Theta}$

$= 2\tan\Theta$

Therefore, LHS = RHS

Hence, proved

Q53) $(1+\tan^2 A)+(1+\frac{1}{\tan^2 A}) = \frac{1}{\sin^2 A-\sin^4 A}$

Ans:

LHS = $(1+\sin^2A\cos^2A)+(1+\cos^2A\sin^2A)(1+\frac{\sin^2A}{\cos^2A})+(1+\frac{\cos^2A}{\sin^2A})$

=> $\cos^2A+\sin^2A\cos^2A+\sin^2A+\cos^2A\sin^2A\ \frac{\cos^2A+\sin^2A}{\cos^2A}+\frac{\sin^2A+\cos^2A}{\sin^2A}$

=> $1\cos^2A+1\sin^2A$ [$\because \sin^2A+\cos^2A=1$

=> $\sin^2A+\cos^2A\sin^2A\cos^2A=1\sin^2A(1-\sin^2A)$ [$\because \cos^2A=1-\sin^2A$]

=> $1\sin^2A-\sin^4A\ \frac{1}{\sin^2A-\sin^4A}$

Therefore, LHS = RHS.

Hence Proved.

Q54) $\sin^2A\cos^2B - \cos^2A\sin^2B = \sin^2A - \sin^2B$

Ans:

LHS = $\sin^2A\cos^2B - \cos^2A\sin^2B$

= $\sin^2A(1-\sin^2B)-(1-\sin^2A)(\sin^2A)$[$\because \cos^2A=1-\sin^2A$]

= $\sin^2A-\sin^2A\sin^2B-\sin^2B+\sin^2A\sin^2B\sin^2A-\sin^2A\sin^2B-\sin^2B+\sin^2A\sin^2B$

= $\sin^2A-\sin^2B\sin^2A-\sin^2B$

= RHS

Hence Proved.

Q55: (i) $\cot A+\tan B\cot B+\tan A=\cot A\tan B\ \frac{\cot A+\tan B}{\cot B+\tan A}=\cot A\tan B$

Ans:

LHS = $\cot A+\tan B\cot B+\tan A\ \frac{\cot A+\tan B}{\cot B+\tan A}$

= $\cos A\sin A+\sin B\cos B\ \cos B\sin B+\sin A\cos A\ \frac{\frac{\cos A}{\sin A}+\frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B}+\frac{\sin A}{\cos A}}$

= $\cos A\cos B+\sin A\sin B\sin A\cos B\ \cos A\cos B+\sin A\sin B\cos A\sin B\ \frac{\frac{\cos A\cos B+\sin A\sin B}{\sin A\cos B}}{\frac{\cos A\cos B+\sin A\sin B}{\cos A\sin B}}$

= $\cos A\cos B+\sin A\sin B\sin A\cos B \times \cos A\sin B\cos A\cos B+\sin A\sin B\ \frac{\cos A\cos B+\sin A\sin B}{\sin A\cos B}\times\frac{\cos A\sin B}{\cos A\cos B+\sin A\sin B}$

= $\cos A\sin B\sin A\cos B\ \frac{\cos A\sin B}{\sin A\cos B}$

= $\cot A\tan B$

= RHS

Hence Proved.

(ii) $\tan A+\tan B\cot A+\cot B=\tan A\tan B\ \frac{\tan A+\tan B}{\cot A+\cot B}=\tan A\tan B$

Ans:

LHS = $\tan A+\tan B\cot A+\cot B\ \frac{\tan A+\tan B}{\cot A+\cot B}$

$= \dfrac{\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}}{\frac{\cos A}{\sin A}+\frac{\cos B}{\sin B}}$

$= \dfrac{\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\sin B+\cos B\sin A}{\sin A\sin B}}$

$= \dfrac{\sin A\cos B+\cos A\sin B}{\cos A\cos B} \times \dfrac{\sin A\sin B}{\cos A\sin B+\cos B\sin A}$

$= \dfrac{\sin A\sin B}{\cos A\cos B}$

$= \tan A\tan B$

= RHS

Hence Proved.

Q56) $\cot^2 A\,\mathrm{cosec}^2 B - \cot^2 B\,\mathrm{cosec}^2 A = \cot^2 A - \cot^2 B$

Ans:

LHS = $\cot^2 A\,\mathrm{cosec}^2 B - \cot^2 B\,\mathrm{cosec}^2 A$

$= \cot^2 A(1+\cot^2 B) - \cot^2 B(1+\cot^2 A)$ $[\because \mathrm{cosec}^2\theta = 1+\cot^2\theta]$

$= \cot^2 A + \cot^2 A\cot^2 B - \cot^2 B - \cot^2 B\cot^2 A$

$= \cot^2 A - \cot^2 B$

= RHS

Hence Proved.

Q57) $\tan^2 A\sec^2 B - \sec^2 A\tan^2 B = \tan^2 A - \tan^2 B$

Ans:

LHS = $\tan^2 A\sec^2 B - \sec^2 A\tan^2 B$

$= \tan^2 A(1+\tan^2 B) - \sec^2 A(\tan^2 A)$

$= \tan^2 A + \tan^2 A\tan^2 B - \tan^2 B(1+\tan^2 A)$ $[\because \sec^2 A = 1+\tan^2 A]$

$= \tan^2 A + \tan^2 A\tan^2 B - \tan^2 B - \tan^2 A\tan^2 B$

$= \tan^2 A - \tan^2 B$

= RHS

Hence Proved.

Q58) If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, prove that $x^2 - y^2 = a^2 - b^2$.

Ans:

LHS = $x^2 - y^2$

$= (a\sec\theta + b\tan\theta)^2 - (a\tan\theta + b\sec\theta)^2$

$= a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta - a^2\tan^2\theta - b^2\sec^2\theta - 2ab\sec\theta\tan\theta$

$= a^2\sec^2\theta + b^2\tan^2\theta - a^2\tan^2\theta - b^2\sec^2\theta$

$= a^2\sec^2\theta - b^2\sec^2\theta + b^2\tan^2\theta - a^2\tan^2\theta$

$= \sec^2\theta(a^2 - b^2) + \tan^2\theta(b^2 - a^2)$

$= \sec^2\theta(a^2 - b^2) - \tan^2\theta(a^2 - b^2)$

$= (\sec^2\theta - \tan^2\theta)(a^2 - b^2)$

$= a^2 - b^2$

= RHS

Hence Proved.

Q59) If $3\sin\theta + 5\cos\theta = 5$, prove that $5\sin\theta - 3\cos\theta = \pm 3$.

Ans:

Given $3\sin\theta + 5\cos\theta = 5$

$3\sin\theta = 5 - 5\cos\theta$ $3\sin\theta = 5(1 - \cos\theta)$ $3\sin\theta = \frac{5(1-\cos\theta)(1-\cos\theta)}{1+\cos\theta}$ $3\sin\theta = \frac{5(1-\cos^2\theta)}{1+\cos\theta}$ $3\sin\theta = \frac{5\sin^2\theta}{1+\cos\theta}$ $3 + 3\cos\theta = 5\sin\theta$ $3 = 5\sin\theta - 3\cos\theta$

= RHS

Hence Proved.

Q60) If $\text{cosec}\theta + \cot\theta$ =m and $\text{cosec}\theta - \cot\theta$ =n, prove that mn = 1.

Ans:

LHS = mn

$= (\text{cosec}\theta + \cot\theta)(\text{cosec}\theta - \cot\theta)$

$= \text{cosec}^2\theta - \cot^2\theta$

= 1

= RHS

Hence Proved.

Q 62 . If $T_n = \sin^n\theta + \cos_n\theta$, prove that $\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$.

Ans:

LHS = $\frac{(\sin^3\theta + \cos^3\theta) - (\sin^5\theta + \cos^5\theta)}{\sin\theta + \cos\theta}$

$= \frac{\sin^3\theta(1 - \sin^2\theta) + \cos^3\theta(1 - \cos^2\theta)}{\sin\theta + \cos\theta}$

$= \frac{\sin^3\theta \times \cos^2\theta + \cos^3\theta \times \sin^2\theta}{\sin\theta + \cos\theta}$

= $\frac{\sin^2\theta\cos^2\theta(\sin\theta+\cos\theta)}{\sin\theta+\cos\theta}$

= $\sin^2\theta\cos^2\theta$

RHS = Missing close brace

= Missing close brace

= $\frac{\sin^5\theta\times\cos^2\theta+\cos^5\theta\times\sin^2\theta}{\sin^3\theta+\cos^3\theta}$

= $\frac{\sin^2\theta\cos^2\theta(\sin^3\theta+\cos^3\theta)}{\sin\theta+\cos\theta}$

= $\sin^2\theta\cos^2\theta$

LHS = RHS Hence proved .

Q 63 . $(\tan\theta+\frac{1}{\cos\theta})^2+(\tan\theta-\frac{1}{\cos\theta})^2$ **=** $2(\frac{1+\sin^2\theta}{1-\sin^2\theta})$

Ans:

$(\tan\theta+\sec\theta)^2+(\tan\theta-\sec\theta)^2$

= $\tan^2\theta+\sec^2\theta+2\tan\theta\sec\theta+\tan^2\theta+\sec^2\theta-2\tan\theta\sec\theta$

= $2\tan^2\theta+2\sec^2\theta$

= $2[\tan^2\theta+\sec^2\theta]$

= $2[\frac{\sin^2\theta}{\cos^2\theta}+\frac{1}{\cos^2\theta}]$

= $2(\frac{1+\sin^2\theta}{\cos^2\theta})$

= $2(\frac{1+\sin^2\theta}{1-\sin^2\theta})$

= RHS

LHS = RHS Hence proved .

Q 64 . $(\frac{1}{\sec^2\theta-\cos^2\theta}+\frac{1}{\mathrm{cosec}^2\theta-\sin^2})\sin^2\theta\cos^2\theta$ **=** $\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$

Ans:

$[\frac{1}{\frac{1}{\cos^2\theta}-\cos^2\theta}+\frac{1}{\frac{1}{\sin^2\theta}-\sin^2\theta}]\sin^2\theta\cos^2$

= $[\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}}+\frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}]\sin^2\theta\cos^2\theta$

= $[\frac{\cos^2\theta}{1-\cos^4\theta}+\frac{\sin^2\theta}{1-\sin^4\theta}]\sin^2\theta\cos^2\theta$

= $[\frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta-\cos^4\theta}+\frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta-\sin^4\theta}]\sin^2\theta\cos^2\theta$

= $[\frac{\cos^2\theta}{\cos^2\theta(1-\cos^2\theta)+\sin^2\theta}+\frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta(1-\sin^2\theta)}]\sin^2\theta\cos^2\theta$

$= [\cos^2\theta\cos^2\theta\sin^2\theta+\sin^2\theta + \sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta]\sin^2\theta\cos^2\theta\left[\frac{\cos^2\theta}{\cos^2\theta\sin^2\theta+\sin^2\theta} + \frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta\cos^2\theta}\right]\sin^2\theta\cos^2\theta$

$= [\cos^2\theta\sin^2\theta(\cos^2\theta+1) + \sin^2\theta\cos^2\theta(\sin^2\theta+1)]\sin^2\theta\cos^2\theta\left[\frac{\cos^2\theta}{\sin^2\theta(\cos^2\theta+1)} + \frac{\sin^2\theta}{\cos^2\theta(\sin^2\theta+1)}\right]\sin^2\theta\cos^2\theta$

$= \cos^4\theta(\sin^2\theta+1)+\sin^4\theta(\cos^2\theta+1)\sin^2\theta\cos^2\theta(\cos^2\theta+1)(\sin^2\theta+1)\sin^2\theta\cos^2\theta\frac{\cos^4\theta(\sin^2\theta+1)+\sin^4\theta(\cos^2\theta+1)}{\sin^2\theta\cos^2\theta(\cos^2\theta+1)(\sin^2\theta+1)}\sin^2\theta\cos^2\theta$

$= \cos^4\theta(\sin^2\theta+1)+\sin^4\theta(\cos^2\theta+1)(\cos^2\theta+1)(\sin^2\theta+1)\frac{\cos^4\theta(\sin^2\theta+1)+\sin^4\theta(\cos^2\theta+1)}{(\cos^2\theta+1)(\sin^2\theta+1)}$

$= \cos^4\theta+\cos^4\theta\sin^2\theta+\sin^4\theta+\sin^4\theta\cos^2\theta1+\sin^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta\frac{\cos^4\theta+\cos^4\theta\sin^2\theta+\sin^4\theta+\sin^4\theta\cos^2\theta}{1+\sin^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}$

$= 1-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta(\cos^2\theta+\sin^2\theta)1+1+\cos^2\theta\sin^2\theta\frac{1-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta(\cos^2\theta+\sin^2\theta)}{1+1+\cos^2\theta\sin^2\theta}$

$= 1-\sin^2\theta\cos^2\theta2+\sin^2\theta\cos^2\theta\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$

LHS = RHS Hence proved .

Q 65 . (i) . $\left[1+\sin\theta-\cos\theta1+\sin\theta+\cos\theta\right]^2\left[\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right]^2$ **=** $1-\cos\theta1+\cos\theta\frac{1-\cos\theta}{1+\cos\theta}$

Ans:

$= \left(1+\sin\theta-\cos\theta1+\sin\theta+\cos\theta \times 1+\sin\theta-\cos\theta1+\sin\theta-\cos\theta\right)^2\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} \times \frac{1+\sin\theta-\cos\theta}{1+\sin\theta-\cos\theta}\right)^2$

$= \left[(1+\sin\theta-\cos\theta)^2(1+\sin\theta)^2-\cos^2\theta\right]^2\left[\frac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta)^2-\cos^2\theta}\right]$

$= \left[(1)^2+\sin^2\theta+\cos^2\theta+2\times1\times\sin\theta+2\times\sin\theta(-\cos\theta)-2\cos\theta1-\cos^2\theta+\sin^2\theta+2\sin\theta\right]^2\left[\frac{(1)^2+\sin^2\theta+\cos^2\theta+2\times1\times\sin\theta+2\times\sin\theta(-\cos\theta)-2\cos\theta}{1-\cos^2\theta+\sin^2\theta+2\sin\theta}\right]^2$

$= \left[1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta\sin^2\theta+\sin^2\theta+2\sin\theta\right]^2\left[\frac{1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{\sin^2\theta+\sin^2\theta+2\sin\theta}\right]^2$

$= \left[2+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta2\sin^2\theta+2\sin\theta\right]^2\left[\frac{2+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{2\sin^2\theta+2\sin\theta}\right]^2$

$= \left[2(1+\sin\theta)-2\cos\theta(\sin\theta+1)2\sin\theta(\sin\theta+1)\right]^2\left[\frac{2(1+\sin\theta)-2\cos\theta(\sin\theta+1)}{2\sin\theta(\sin\theta+1)}\right]^2$

$= \left[(1+\sin\theta)(2-2\cos\theta)2\sin\theta(\sin\theta+1)\right]^2\left[\frac{(1+\sin\theta)(2-2\cos\theta)}{2\sin\theta(\sin\theta+1)}\right]^2$

$= \left[(2-2\cos\theta)2\sin\theta\right]^2\left[\frac{(2-2\cos\theta)}{2\sin\theta}\right]^2$

$= \left[(1-\cos\theta)\sin\theta\right]^2\left[\frac{(1-\cos\theta)}{\sin\theta}\right]^2$

$= (1-\cos\theta)^21-\cos^2\theta\frac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$= (1-\cos\theta)\times(1-\cos\theta)(1-\cos\theta)(1+\cos\theta)\frac{(1-\cos\theta)\times(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= 1-\cos\theta1+\cos\theta\frac{1-\cos\theta}{1+\cos\theta}$

LHS = RHS Hence proved .

Q 65 (ii) . $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \frac{1-\sin\theta}{\cos\theta}$

Ans:

= LHS = $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$

= $\frac{(\sec^2\theta-\tan^2\theta)+(\sec\theta-\tan\theta)}{1+\sec\theta+\tan\theta}$ [since , $\sec^2\theta-\tan^2\theta=1$]

= $\frac{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)+(\sec\theta-\tan\theta)}{1+\sec\theta+\tan\theta}$

= $\frac{(\sec\theta-\tan\theta)(1+\sec\theta+\tan\theta)}{1+\sec\theta+\tan\theta}$

= $(\sec\theta-\tan\theta)$

= $\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

= $\frac{1-\sin\theta}{\cos\theta}$

LHS = RHS Hence proved .

Q 66 . (sec A + tan A – 1)(sec A – tan A + 1) = 2 tan A

Ans:

= $(\sec A+\tan A-\{\sec^2 A-\tan^2 A\})[\sec A-\tan A+(\sec^2 A-\tan^2 A)]$

= $(\sec A+\tan A-(\sec A+\tan A)(\sec A-\tan A))[\sec A-\tan A+(\sec A-\tan A)(\sec A+\tan A)]$

= $(\sec A+\tan A)(1-(\sec A-\tan A))(\sec A-\tan A)(1+(\sec A+\tan A))$

= $(\sec^2 A-\tan^2 A)(1-\sec A+\tan A)(1+\sec A+\tan A)$

= $(1-\frac{1}{\cos A}+\frac{\sin A}{\cos A})(1+\frac{1}{\cos A}+\frac{\sin A}{\cos A})$

= $(\frac{\cos A-1+\sin A}{\cos A})(\frac{\cos A+1+\sin A}{\cos A})$

= $(\frac{(\cos A+\sin A)^2-1}{\cos^2 A})$

= $(\frac{\cos^2 A+\sin^2 A+2\sin A\cos B-1}{\cos^2 A})$

= $(\frac{1+2\sin A\cos B-1}{\cos^2 A})$

= $(\frac{2\sin A\cos B}{\cos^2 A})$

= 2 tan A

LHS = RHS Hence proved .

Q 67 . (1 + cot A – cosec A)(1 + tan A + sec A) = 2

Ans:

LHS = (1 + cot A – cosec A)(1 + tan A + sec A)

= (1+ cosAsinA − 1sinA)(1+ sinAcosA + 1cosA) $(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A})(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A})$

= (sinA+cosA−1sinA)(cosA+sinA+1cosA) $(\frac{\sin A+\cos A-1}{\sin A})(\frac{\cos A+\sin A+1}{\cos A})$

= ((sinA−cosA)^2−1sinAcosA) $(\frac{(\sin A-\cos A)^2-1}{\sin A\cos A})$

= sin^{2}A+2sinAcosA+cos^{2}A−1sinAcosA $\frac{\sin^2 A+2\sin A\cos A+\cos^2 A-1}{\sin A\cos A}$

= (1+2sinAcosA−1sinAcosA) $(\frac{1+2\sin A\cos A-1}{\sin A\cos A})$

= 2

LHS = RHS Hence proved .

Q 68 . (cosecθ−secθ)(cotθ−tanθ) $(\operatorname{cosec}\theta - \sec\theta)(\cot\theta - \tan\theta)$ **= (cosecθ+secθ)(secθcosecθ–2)** $(\operatorname{cosec}\theta + \sec\theta)(\sec\theta\operatorname{cosec}\theta - 2)$

Ans:

LHS = (cosecθ−secθ)(cotθ−tanθ) $(\operatorname{cosec}\theta - \sec\theta)(\cot\theta - \tan\theta)$

[1sinθ − 1cosθ][cosθsinθ − sinθcosθ] $[\frac{1}{\sin\theta} - \frac{1}{\cos\theta}][\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}]$ [cosθ−sinθsinθcosθ][cos^{2}θ−sin^{2}θsinθcosθ]

$[\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}][\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}]$ [(cosθ−sinθ)^2(cosθ+sinθ)sin^{2}θcos^{2}θ] $[\frac{(\cos\theta-\sin\theta)^2(\cos\theta+\sin\theta)}{\sin^2\theta\cos^2\theta}]$

RHS = (cosecθ+secθ)(secθcosecθ–2) $(\operatorname{cosec}\theta + \sec\theta)(\sec\theta\operatorname{cosec}\theta - 2)$

[1sinθ + 1cosθ][1cosθ − 1sinθ −2] $[\frac{1}{\sin\theta} + \frac{1}{\cos\theta}][\frac{1}{\cos\theta} - \frac{1}{\sin\theta} - 2]$

= [sinθ+cosθsinθcosθ][1−2sinθcosθsinθcosθ] $[\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}][\frac{1-2\sin\theta\cos\theta}{\sin\theta\cos\theta}]$

= [sinθ+cosθsinθcosθ][cos^{2}θ+sin^{2}θ–2sinθcosθsinθcosθ] $[\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}][\frac{\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta}{\sin\theta\cos\theta}]$

= [(cosθ−sinθ)^2(cosθ+sinθ)sin^{2}θcos^{2}θ] $[\frac{(\cos\theta-\sin\theta)^2(\cos\theta+\sin\theta)}{\sin^2\theta\cos^2\theta}]$ [∵cos^{2}θ+sin^{2}θ=1]

LHS = RHS Hence proved .

Q 70 . cosAcosecA−sinAsecAcosA+sinA $\frac{\cos A\operatorname{cosec}A-\sin A\sec A}{\cos A+\sin A}$ **= cosec A – sec A**

Ans:

LHS = cosAcosecA−sinAsecAcosA+sinA $\frac{\cos A\operatorname{cosec}A-\sin A\sec A}{\cos A+\sin A}$

= cosA× 1sinA −sinA× 1cosA cosA+sinA $\frac{\cos A\times\frac{1}{\sin A}-\sin A\times\frac{1}{\cos A}}{\cos A+\sin A}$

= cosAsinA − sinAcosA cosA+sinA $\frac{\frac{\cos A}{\sin A}-\frac{\sin A}{\cos A}}{\cos A+\sin A}$

= cos^{2}A−sin^{2}AcosAsinA cosA+sinA $\frac{\frac{\cos^2 A-\sin^2 A}{\cos A\sin A}}{\cos A+\sin A}$

$= \frac{\cos^2A-\sin^2A}{\cos A\sin A} \times \frac{1}{\cos A+\sin A}$

$= \frac{(\cos A-\sin A)(\cos A+\sin A)}{\cos A\sin A\times(\cos A+\sin A)}$

$= \frac{(\cos A-\sin A)}{\cos A\sin A}$

$= \frac{\cos A}{\cos A\sin A} - \frac{\sin A}{\cos A\sin A}$

$= \frac{1}{\sin A} - \frac{1}{\cos A}$

$= \text{cosec}A - \sec A$

= RHS

∴ LHS = RHS Hence proved .

Q 71 . $\frac{\sin A}{\sec A+\tan A-1} + \frac{\cos A}{\text{cosec}A+\cot A-1}$ **= 1**

Ans:

LHS : $\frac{\sin A}{\sec A+\tan A-1} + \frac{\cos A}{\text{cosec}A+\cot A-1}$

$= \frac{\sin A}{\frac{1}{\cos A}+\frac{\sin A}{\cos A}-1} + \frac{\cos A}{\frac{1}{\sin A}+\frac{\cos A}{\sin A}-1}$

$= \frac{\sin A}{\frac{1+\sin A-\cos A}{\cos A}} + \frac{\cos A}{\frac{1+\cos A-\sin A}{\sin A}}$

$= \frac{\sin A\cos A}{1+\sin A-\cos A} + \frac{\cos A\sin A}{1+\cos A-\sin A}$

$= (\sin A\cos A)\left[\frac{1}{1+\sin A-\cos A} + \frac{1}{1+\cos A-\sin A}\right]$

$= (\sin A\cos A)\left[\frac{2}{\cos A-\sin A+\sin A+\sin A\cos A-\sin^2A-\cos A-\cos^2A+\cos A\sin A}\right]$

$= (\sin A\cos A)\left[\frac{2}{1-\sin^2A-\cos^2A+2\sin A\cos A}\right]$

$= (\sin A\cos A)\left[\frac{2}{1-(\sin^2A-\cos^2A)+2\sin A\cos A}\right]$

$= (\sin A\cos A)\left[\frac{2}{1-1+2\sin A\cos A}\right]$

$= (\sin A\cos A) \times \frac{2}{2\sin A\cos A}$

= 1

= RHS

∴ LHS = RHS Hence proved .

Q 72 . $\frac{\tan A}{(1+\tan^2A)^2} + \frac{\cot A}{(1+\cot^2A)^2}$ **= sin A cos A**

Ans:

$\frac{\tan A}{(\sec^2A)^2} + \frac{\cot A}{(\text{cosec}^2A)^2}$ [1 + $\tan^2 A = \sec^2 A$, 1 + $\cot^2 A = \text{cosec}^2 A$]

$= \frac{\frac{\sin A}{\cos A}}{\sec^4A} + \frac{\frac{\cos A}{\sin A}}{\text{cosec}^4A}$

= sinAcosA 1cos^{4}A + cosAsinA 1sin^{4}A $\frac{\frac{\sin A}{\cos A}}{\frac{1}{\cos^4 A}} + \frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin^4 A}}$

= sinAcosA × cos^{4}A1 + cosAsinA × sin^{4}A1 $\frac{\sin A}{\cos A} \times \frac{\cos^4 A}{1} + \frac{\cos A}{\sin A} \times \frac{\sin^4 A}{1}$

= sinA×cos^{3}A+cosA×sin^{3}A$\sin A \times \cos^3 A + \cos A \times \sin^3 A$

= sinAcosA(cos^{2}A+sin^{2}A)$\sin A \cos A\,(\cos^2 A + \sin^2 A)$

= sinAcosA$\sin A \cos A$

LHS = RHS   Hence proved .

Q73. sec^{4}A(1−sin^{4}A)−2tan^{2}A=1$\sec^4 A(1\;-\;\sin^4 A)\;-\;2\tan^2 A = 1$

Ans:

Given, L.H.S = \(sec^{4}A(1\;-\;sin^{4}A)\;-\;2tan^{2}A

= [latex]sec^{4}A\;-\;sec^{4}A\;\times \;sin^{4}A\;-\;2tan^{4}A\)

= sec^{4}A−(1cos^{4}A ×sin^{4}A)−2tan^{4}A$\sec^4 A\;-\;(\frac{1}{\cos^4 A}\;\times\;\sin^4 A)\;-\;2\tan^4 A$

= sec^{4}A−tan^{4}A−2tan^{4}A$\sec^4 A\;-\;\tan^4 A\;-\;2\tan^4 A$

= (sec^{2}A)^2−tan^{4}A−2tan^{4}A$(\sec^2 A)^2\;-\;\tan^4 A\;-\;2\tan^4 A$

= (1+tan^{2}A)^2−tan^{4}A−2tan^{4}A$(1\;+\;\tan^2 A)^2\;-\;\tan^4 A\;-\;2\tan^4 A$

= 1+tan^{4}A+2tan^{2}A−tan^{4}A−2tan^{4}A$1 + \tan^4 A + 2\tan^2 A -\;\tan^4 A\;-\;2\tan^4 A$

= 1

Hence, L.H.S = R.H.S

Q74. **cot^{2}A(secA−1)1+sinA** $\frac{\cot^2 A(\sec A\;-\;1)}{1\;+\;\sin A}$ **= sec^{2}A[1−sinA1+sinA]**$\sec^2 A[\frac{1\;-\;\sin A}{1\;+\;\sin A}]$

Ans:

Given, L.H.S = cot^{2}A(secA−1)1+sinA $\frac{\cot^2 A(\sec A\;-\;1)}{1\;+\;\sin A}$

Here, sin^{2}A+cos^{2}A$\sin^2 A\;+\;\cos^2 A$ = 1

= cos^{2}Asin^{2}A(1cosA −1)1+sinA $\frac{\frac{\cos^2 A}{\sin^2 A}(\frac{1}{\cos A}-1)}{1+\sin A}$

= cos^{2}Asin^{2}A(1−cosAcosA)1+sinA $\frac{\frac{\cos^2 A}{\sin^2 A}(\frac{1-\cos A}{\cos A})}{1+\sin A}$

= cosA×cosA(1−cos^{2}A)(1−cosAcosA)1+sinA $\frac{\frac{\cos A\times\cos A}{(1-\cos^2 A)}(\frac{1-\cos A}{\cos A})}{1+\sin A}$

= (cosA)(1+cosA) 11+sinA $\frac{(\cos A)}{(1+\cos A)}\,\frac{1}{1+\sin A}$

Solving,

RHS => sec^{2}a[1−sinA1+secA]$\sec^2 a[\frac{1-\sin A}{1+\sec A}]$

= 1cos^{2}A[1−sinA1+secA]$\frac{1}{\cos^2 A}[\frac{1-\sin A}{1+\sec A}]$

$= \frac{1}{\cos^2 A}\left[\frac{1-\sin A}{1+\sec A}\right]$

$= \frac{1}{\cos^2 A}\left[\frac{1-\sin A}{\cos A+1}\right]\cos A$

$= \frac{(1-\sin A)}{(\cos A+1)(\cos A)}$

Multiplying Nr. And Dr. with (1+SinA)

$= \frac{(1-\sin A)}{(\cos A+1)(\cos A)} \times \frac{1+\sin A}{1+\sin A}$

$= \frac{(1^2-\sin^2 A)}{(\cos A+1)(\cos A)(1+\sin A)}$

$= \frac{\cos^2 A}{(\cos A+1)(\cos A)(1+\sin A)}$

$= \frac{\cos A}{(\cos A+1)(1+\sin A)}$

Hence, LHS= RHS

Q75. $(1 + \cot A + \tan A)(\sin A - \cos A)$ **=** $\frac{\sec A}{\mathrm{cosec}^2 A}$ **–** $\frac{\mathrm{cosec} A}{\sec^2 A}$ **= sinAtanA – cotAcosA**

Ans:

Given, L.H.S = $(1 + \cot A + \tan A)(\sin A - \cos A)$

=> sinA – cosA + cotAsinA – cotAcosA + sinAtanA – tanAcosA

=> sinA – cosA + $\frac{\cos A}{\sin A} \times \sin A$ – cotAcosA + sinAtanA – $\frac{\sin A}{\cos A} \times \cos A$

=> sinA – cosA + cosA – cotAcosA + sinAtanA – sinA

=> sinAtanA – cosAcotA

=> $\frac{\sec A}{\mathrm{cosec}^2 A}$ – $\frac{\mathrm{cosec} A}{\sec^2 A}$

Here, secA = $\frac{1}{\cos A}$ and cosecA = $\frac{1}{\sin A}$

=> $\frac{\sin^2 A}{\cos A}$ – $\frac{\cos^2 A}{\sin A}$

=> $\frac{\sin^2 A - \cos^2 A}{\cos A \sin A}$

=> $(\sin A \times \frac{\sin A}{\cos A})$ – $(\cos A \times \frac{\cos A}{\cot A})$

=> sinAtanA – cosAcotA

Hence, L.H.S = R.H.S

Q76. If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta$ **= 1 and** $\frac{x}{a}\cos\theta - \frac{y}{b}\sin\theta$ **= 1, prove that** $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ **= 2**

Ans:

Given,

=> $(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta)^2 + (\frac{x}{a}\cos\theta - \frac{y}{b}\sin\theta)^2 = 1^2 + 1^2$

=> $\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{2xy}{ab}\cos\theta\sin\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2} - \frac{2xy}{ab}\sin\theta\cos\theta = 1 + 1$

=> $x^2a^2\cos^2\theta + y^2b^2\sin^2\theta + x^2a^2\sin^2\theta + y^2b^2\sin^2\theta \frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\sin^2\theta$ = 2

=> $\cos^2\theta[x^2a^2 + y^2b^2]\cos^2\theta[\frac{x^2}{a^2} + \frac{y^2}{b^2}]$ + $\sin^2\theta[x^2a^2 + y^2b^2]\sin^2\theta[\frac{x^2}{a^2} + \frac{y^2}{b^2}]$ = 2

=> $(\cos^2\theta + \sin^2\theta)[x^2a^2 + y^2b^2](\cos^2\theta + \sin^2\theta)[\frac{x^2}{a^2} + \frac{y^2}{b^2}]$ = 2

Here $\cos^2A + \sin^2A = 1$

=> (1) [\frac{x^{2}}{a^{2}}\;+\;\frac{y^{2}}{b^{2}}][/latex] = 2

=> [\frac{x^{2}}{a^{2}}\;+\;\frac{y^{2}}{b^{2}}][/latex] = 2

Q77. If $\text{cosec}\theta - \sin\theta = a^3 \text{cosec}\theta - \sin\theta = a^3$, $\sec\theta - \cos\theta = b^3 \sec\theta - \cos\theta = b^3$, prove that $a^2b^2(a^2+b^2) a^2b^2(a^2 + b^2)$ = 1

Ans:

Given, $\text{cosec}\theta - \sin\theta = a^3 \text{cosec}\theta - \sin\theta = a^3$

Here, $\text{cosec}\theta = 1\sin\theta\, \text{cosec}\theta = \frac{1}{\sin\theta}$

=> $1\sin\theta \frac{1}{\sin\theta} - \sin\theta\sin\theta = a^3a^3$

=> $1-\sin^2\theta\sin\theta \frac{1-\sin^2\theta}{\sin\theta} = a^3a^3$

Here $\cos^2A + \sin^2A = 1$

=> $\cos^2\theta\sin\theta \frac{\cos^2\theta}{\sin\theta} = a^3a^3$

=> $\cos^{23}\theta\sin^{13}\theta \frac{\cos^{\frac{2}{3}}\theta}{\sin^{\frac{1}{3}}\theta} = a$

Squaring on both sides

=> $a^2 = \cos^{43}\theta\sin^{23}\theta \frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta}$

$\sec\theta - \cos\theta = b^3 \sec\theta - \cos\theta = b^3$

=> $1\cos\theta \frac{1}{\cos\theta} - \cos\theta\cos\theta = b^3b^3$

=> $1-\cos^2\theta\cos\theta \frac{1-\cos^2\theta}{\cos\theta} = b^3b^3$

=> $\sin^2\theta\cos\theta \frac{\sin^2\theta}{\cos\theta} = b^3b^3$

=> $\sin^{23}\theta\cos^{13}\theta \frac{\sin^{\frac{2}{3}}\theta}{\cos^{\frac{1}{3}}\theta} = b$

Squaring on both sides

=> $b^2 = \sin^{43}\theta\cos^{23}\theta \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}$

Now, $a^2b^2(a^2+b^2)a^2b^2(a^2 + b^2)$

=> $\cos^{43}\theta\sin^{23}\theta \frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta} \times\times \sin^{43}\theta\cos^{23}\theta \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}(\cos^{43}\theta\sin^{23}\theta \frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta} + \sin^{43}\theta\cos^{23}\theta \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta})$

=> $\cos^{23}\theta\cos^{\frac{2}{3}}\theta \sin^{23}\theta\sin^{\frac{2}{3}}\theta$ $\sin^{23}\theta\sin^{\frac{2}{3}}\theta$}[/latex]

= 1

Hence, L.H.S = R.H.S

Q78. If $acos^3\theta \theta$ + $3acos\theta \theta$ $sin^2\theta sin^2\theta$ = m, $asin^3\theta sin^3\theta$ + $3acos^2\theta cos^2\theta$ $sin\theta sin\theta$ = n, prove that $(m+n)^{23}(m+n)^{\frac{2}{3}}$ + $(m-n)^{23}(m-n)^{\frac{2}{3}}$ = $2(a)^{23}(a)^{\frac{2}{3}}$

Ans:

Given, $(m+n)^{23}(m+n)^{\frac{2}{3}}$ + $(m-n)^{23}(m-n)^{\frac{2}{3}}$

Substitute the values of m and n in the above equation

=> (\\(($acos^3$ [latex]\theta\\) + $3acos\theta\theta$ $sin^2\theta sin^2\theta$\\;+\\; $asin^3\theta sin^3\theta$ + $3acos^2\theta cos^2\theta$ $sin\theta sin\theta$)^{\frac{2}{3}}[/latex] + (\\(($acos^3$[latex]\theta\\) + $3acos\theta\theta$ $sin^2\theta sin^2\theta$\\;-\\; $asin^3\theta sin^3\theta$ – $3acos^2\theta cos^2\theta$ $sin\theta sin\theta$)^{\frac{2}{3}}[/latex]

=> $(a)^{23}(a)^{\frac{2}{3}}$ ((\\((cos^3 [latex]\theta\\) + $3cos\theta\theta$ $sin^2\theta sin^2\theta$\\;+\\; $sin^3\theta sin^3\theta$ + $3cos^2\theta cos^2\theta$ $sin\theta sin\theta$)^{\frac{2}{3}}[/latex] + $(a)^{23}(a)^{\frac{2}{3}}$ ((\\((cos^3 [latex]\theta\\) + $3cos\theta\theta$ $sin^2\theta sin^2\theta$\\;-\\; $sin^3\theta sin^3\theta$ – $3cos^2\theta cos^2\theta$ $sin\theta sin\theta$)^{\frac{2}{3}}[/latex]

=> $(a)^{23}(a)^{\frac{2}{3}}$ $((cos\theta+sin\theta)^3)^{23}((cos\theta + sin\theta)^3)^{\frac{2}{3}}$ + $(a)^{23}(a)^{\frac{2}{3}}$ $((cos\theta-sin\theta)^3)^{23}((cos\theta - sin\theta)^3)^{\frac{2}{3}}$

=> $(a)^{23}(a)^{\frac{2}{3}}$ [$(cos\theta+sin\theta)^2(cos\theta + sin\theta)^2$] + $(a)^{23}(a)^{\frac{2}{3}}$ [$(cos\theta-sin\theta)^2(cos\theta - sin\theta)^2$]

=> $(a)^{23}(a)^{\frac{2}{3}}$ ($(cos^2\theta+sin^2\theta+2sin\theta cos\theta)(cos^2\theta + sin^2\theta + 2sin\theta cos\theta)$) + $(a)^{23}(a)^{\frac{2}{3}}$ ($(cos^2\theta+sin^2\theta-2sin\theta cos\theta)(cos^2\theta + sin^2\theta - 2sin\theta cos\theta)$)

=> $(a)^{23}(a)^{\frac{2}{3}}$ [1 + $2sin\theta cos\theta 2sin\theta cos\theta$] + $(a)^{23}(a)^{\frac{2}{3}}$ [1 – $2sin\theta cos\theta 2sin\theta cos\theta$]

=> $(a)^{23}(a)^{\frac{2}{3}}$ [1 + $2sin\theta cos\theta 2sin\theta cos\theta$] + 1 – $2sin\theta cos\theta 2sin\theta cos\theta$]

=> $(a)^{23}(a)^{\frac{2}{3}}$ (1 + 1)

=> $2(a)^{23}(a)^{\frac{2}{3}}$

Hence, L.H.S = R.H.S

Q79) $Ifx=acos^3\Theta,y=bsin^3\Theta,provethat(xa)^{23}+(yb)^{23}=1$ If $x = acos^3\Theta,\ y = bsin^3\Theta,$ prove that $(\frac{x}{a})^{\frac{2}{3}} + (\frac{y}{b})^{\frac{2}{3}} = 1$

Ans:

$x=acos^3\Theta:y=bsin^3\Theta x = acos^3\Theta :\ y = bsin^3\Theta$ $xa=cos^3\Theta:yb=sin^3\Theta \frac{x}{a} = cos^3\Theta :\ \frac{y}{b} = sin^3\Theta$

L.H.S = $[xa]^{23}+[yb]^{23}[\frac{x}{a}]^{\frac{2}{3}} + [\frac{y}{b}]^{\frac{2}{3}}$

$=[cos^3\Theta]^{23}+[sin^3\Theta]^{23}= [cos^3\Theta]^{\frac{2}{3}} + [sin^3\Theta]^{\frac{2}{3}}$ $=cos^2\Theta+sin^2\Theta(\because cos^2\Theta+sin^2\Theta=1)$

=1

Hence proved.

Q80) $Ifacos\Theta+bsin\Theta=mandasin\Theta-bcos\Theta=n,provethata^2+b^2=m^2+n^2$
If $acos\Theta + bsin\Theta = m$ and $asin\Theta - bcos\Theta = n,$ prove that $a^2 + b^2 = m^2 + n^2$

Ans:

$R.H.S=m^2+n^2 R.H.S = m^2 + n^2$

$=(a\cos\Theta+b\sin\Theta)^2+$

$(a\sin\Theta-b\cos\Theta)^2=a^2\cos^2\Theta+b^2\sin^2\Theta+2ab\sin\Theta\cos\Theta+a^2\sin^2\Theta+b^2\cos^2\Theta-2ab\sin\Theta\cos\Theta=a^2\cos^2\Theta+b^2\cos^2\Theta+b^2\sin^2\Theta+a^2\sin^2\Theta$

$=a^2(\sin^2\Theta+\cos^2\Theta)+b^2(\sin^2\Theta+\cos^2\Theta)=a^2+b^2[\because\sin^2\Theta+\cos^2\Theta=1]$

$= m^2 + n^2$

Q81: If $\cos A+\cos^2 A=1$, prove that $\sin^2 A+\sin^4 A=1$ If $\cos A + \cos^2 A = 1,$ prove that $\sin^2 A + \sin^4 A = 1$

Ans:

Given- $\cos A + \cos^2 A = 1$

We have to prove $\sin^2 A + \sin^4 A = 1$

Now, $\cos A + \cos^2 A = 1$

$\cos A = 1- \cos^2 A$

$\cos A = \sin^2 A$

$\sin^2 A = \cos A$

Therefore, we have $\sin^2 A + \sin^4 A = \cos A + (\cos A)^2 = \cos A + \cos^2 A =1$

Hence proved.

Q82:

If $\cos\Theta+\cos^2\Theta=1$, prove that $\sin^{12}\Theta+3\sin^{10}\Theta+3\sin^8\Theta+\sin^6\Theta+2\sin^4\Theta+2\sin^2\Theta-2=1$
If $\cos\Theta + \cos^2\Theta = 1,$ prove that $\sin^{12}\Theta + 3\sin^{10}\Theta + 3\sin^8\Theta + \sin^6\Theta + 2\sin^4\Theta + 2\sin^2\Theta - 2 = 1$

Ans:

$\cos\Theta+\cos^2\Theta=1$ $\cos\Theta + \cos^2\Theta = 1$ $\cos\Theta=1-\cos^2\Theta$ $\cos\Theta = 1 - \cos^2\Theta$

$\cos\Theta=\sin^2\Theta$ $\cos\Theta = \sin^2\Theta$ ……(i)

Now, $\sin^{12}\Theta+3\sin^{10}\Theta+3\sin^8\Theta+\sin^6\Theta+2\sin^4\Theta+2\sin^2\Theta-2$
Now, $\sin^{12}\Theta + 3\sin^{10}\Theta + 3\sin^8\Theta + \sin^6\Theta + 2\sin^4\Theta + 2\sin^2\Theta - 2$ $=(\sin^4\Theta)^3+3\sin^4\Theta.\sin^2\Theta(\sin^4\Theta+\sin^2\Theta)+(\sin^2\Theta)^3+2(\sin^2\Theta)^2+2\sin^2\Theta-2$
$= (\sin^4\Theta)^3 + 3\sin^4\Theta.\sin^2\Theta(\sin^4\Theta + \sin^2\Theta) + (\sin^2\Theta)^3 + 2(\sin^2\Theta)^2 + 2\sin^2\Theta - 2$ Using $(a+b)^3=a^3+b^3+3ab(a+b)$ and also from (i) $\cos\Theta=\sin^2\Theta$
Using $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ and also from (i) $\cos\Theta = \sin^2\Theta$ $(\sin^4\Theta+\sin^2\Theta)^3+2\cos^2\Theta+2\cos\Theta-2$
$(\sin^4\Theta + \sin^2\Theta)^3 + 2\cos^2\Theta + 2\cos\Theta - 2$ $((\sin^2\Theta)^2+\sin^2\Theta)^3+2\cos^2\Theta+2\cos\Theta-2$
$((\sin^2\Theta)^2 + \sin^2\Theta)^3 + 2\cos^2\Theta + 2\cos\Theta - 2$ $(\cos^2\Theta+\sin^2\Theta)^3+2\cos^2\Theta+2\cos\Theta-2$

$(\cos^2\Theta+\sin^2\Theta)^3+2\cos^2\Theta+2\cos\Theta-2$ $1+2\cos^2\Theta+2\sin^2\Theta-2$ [$\because \sin^2\Theta+\cos^2\Theta=1$]

$1+2(\cos^2\Theta+\sin^2\Theta)-2$ $1+2(\cos^2\Theta+\sin^2\Theta)-2$ $1+2(1)-2$ $1+2(1)-2$ $=1$ $=1$

L.H.S = R.H.S

Hence proved.

Q83: Given that: $(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)=(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$ $(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)=(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$. Show that one of the values of each member of this equality is $\sin\alpha\sin\beta\sin\gamma$ $\sin\alpha\ \sin\beta\ \sin\gamma$.

Ans:

We know that $1+\cos\Theta=1+\cos^2\frac{\Theta}{2}-\sin^2\frac{\Theta}{2}=2\cos^2\frac{\Theta}{2}$ $1+\cos\Theta=1+\cos^2\frac{\Theta}{2}-\sin^2\frac{\Theta}{2}=2\cos^2\frac{\Theta}{2}$

$\Rightarrow 2\cos^2\frac{\alpha}{2}.2\cos^2\frac{\beta}{2}.2\cos^2\frac{\gamma}{2}\ldots..(i)$ $\Rightarrow 2\cos^2\frac{\alpha}{2}.2\cos^2\frac{\beta}{2}.2\cos^2\frac{\gamma}{2}\ldots..(i)$ Multiply(i)withsinαsinβsinγanddivideitwithsameweget Multiply (i) with $\sin\alpha\ \sin\beta\ \sin\gamma$ and divide it with same we get $\frac{8\cos^2\frac{\alpha}{2}.\cos^2\frac{\beta}{2}.\cos^2\frac{\gamma}{2}}{\sin\alpha.\sin\beta.\sin\gamma}\times\sin\alpha.\sin\beta.\sin\gamma$

$\frac{8\cos^2\frac{\alpha}{2}.\cos^2\frac{\beta}{2}.\cos^2\frac{\gamma}{2}}{\sin\alpha.\sin\beta.\sin\gamma}\times\sin\alpha.\ \sin\beta.\ \sin\gamma$ $\Rightarrow\frac{2\cos^2\frac{\alpha}{2}.\cos^2\frac{\beta}{2}.\cos^2\frac{\gamma}{2}\times\sin\alpha.\sin\beta.\sin\gamma}{\sin\frac{\alpha}{2}.\sin\frac{\beta}{2}.\sin\frac{\gamma}{2}}$

$\Rightarrow\frac{2\cos^2\frac{\alpha}{2}.\ \cos^2\frac{\beta}{2}.\ \cos^2\frac{\gamma}{2}\times\sin\alpha.\ \sin\beta.\ \sin\gamma}{\sin\frac{\alpha}{2}.\ \sin\frac{\beta}{2}.\ \sin\frac{\gamma}{2}}$ $\sin\alpha.\sin\beta.\sin\gamma\times\cot\frac{\alpha}{2}.\cot\frac{\beta}{2}.\cot\frac{\gamma}{2}$ $\sin\alpha.\ \sin\beta.\ \sin\gamma\times\cot\frac{\alpha}{2}.\ \cot\frac{\beta}{2}.\ \cot\frac{\gamma}{2}$ RHS $(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$ RHS $(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$

We know that $1-\cos\Theta=1-\cos^2\frac{\Theta}{2}+\sin^2\frac{\Theta}{2}=2\sin^2\frac{\Theta}{2}$ $1-\cos\Theta=1-\cos^2\frac{\Theta}{2}+\sin^2\frac{\Theta}{2}=2\sin^2\frac{\Theta}{2}$

$\Rightarrow 2.\sin^2\frac{\alpha}{2}\ 2.\sin^2\frac{\beta}{2}\ 2.\sin^2\frac{\gamma}{2}$ $\Rightarrow 2.\sin^2\frac{\alpha}{2}\ 2.\sin^2\frac{\beta}{2}\ 2.\sin^2\frac{\gamma}{2}$ Multiply(i)withsinαsinβsinγanddivideitwithsameweget Multiply (i) with $\sin\alpha\ \sin\beta\ \sin\gamma$ and divide it with same we get $\frac{8\sin^2\frac{\alpha}{2}.\sin^2\frac{\beta}{2}.\sin^2\frac{\gamma}{2}}{\sin\alpha.\sin\beta.\sin\gamma}\times\sin\alpha.\sin\beta.\sin\gamma$

$\frac{8\sin^2\frac{\alpha}{2}.\sin^2\frac{\beta}{2}.\sin^2\frac{\gamma}{2}}{\sin\alpha.\sin\beta.\sin\gamma}\times\sin\alpha.\ \sin\beta.\ \sin\gamma$ $\Rightarrow\frac{8\sin^2\frac{\alpha}{2}.\sin^2\frac{\beta}{2}.\sin^2\frac{\gamma}{2}\times\sin\alpha.\sin\beta.\sin\gamma}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}.2\sin\frac{\beta}{2}\cos\frac{\beta}{2}.2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}}$

$\Rightarrow\frac{8\sin^2\frac{\alpha}{2}.\ \sin^2\frac{\beta}{2}.\ \sin^2\frac{\gamma}{2}\times\sin\alpha.\ \sin\beta.\ \sin\gamma}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}.\ 2\sin\frac{\beta}{2}\cos\frac{\beta}{2}.\ 2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}}$ $\Rightarrow\sin\alpha.\sin\beta.\sin\gamma\times\tan\frac{\alpha}{2}.\tan\frac{\beta}{2}.\tan\frac{\gamma}{2}$ $\Rightarrow\sin\alpha.\ \sin\beta.\ \sin\gamma\times\tan\frac{\alpha}{2}.\ \tan\frac{\beta}{2}.\ \tan\frac{\gamma}{2}$

Hence $\sin\alpha\sin\beta\sin\gamma$ $\sin\alpha\ \sin\beta\ \sin\gamma$ is the member of equality.

Q84: If $\sin\Theta+\cos\Theta=x$, prove that $\sin^6\Theta+\cos^6\Theta=\frac{4-3(x^2-1)^2}{4}$ $\sin\Theta+\cos\Theta=x$, prove that $\sin^6\Theta+\cos^6\Theta=\frac{4-3(x^2-1)^2}{4}$.

Ans:

$\sin\Theta+\cos\Theta=x$ $\sin\Theta+\cos\Theta=x$

Squaring on both sides

$(\sin\Theta+\cos\Theta)^2=x^2$ $(\sin\Theta+\cos\Theta)^2=x^2$ $\Rightarrow\sin\Theta^2+\cos\Theta^2+2\sin\Theta\cos\Theta=x^2$ $\Rightarrow\sin\Theta^2+\cos\Theta^2+2\sin\Theta\cos\Theta=x^2$ $\therefore\sin\Theta\cos\Theta=\frac{x^2-1}{2}\ldots\ldots(i)$ We know that $\sin^2\Theta+\cos^2\Theta=1$ We know that $\sin^2\Theta+\cos^2\Theta=1$

Cubing on both sides

$(\sin^2\Theta+\cos^2\Theta)^3=1^3$ $(\sin^2\Theta+\cos^2\Theta)^3=1^3$ $\sin^6\Theta+\cos^6\Theta+3\sin^2\Theta\cos^2\Theta(\sin^2\Theta+\cos^2\Theta)=1$ $\sin^6\Theta+\cos^6\Theta+3\sin^2\Theta\cos^2\Theta(\sin^2\Theta+\cos^2\Theta)=1$ $\Rightarrow\sin^6\Theta+\cos^6\Theta=1-3\sin^2\Theta\cos^2\Theta$ $\Rightarrow\sin^6\Theta+\cos^6\Theta=1-3\sin^2\Theta\cos^2\Theta$ $\Rightarrow\sin^6\Theta+\cos^6\Theta=1-\frac{3(x^2-1)^2}{4}$ $\Rightarrow\sin^6\Theta+\cos^6\Theta=1-\frac{3(x^2-1)^2}{4}$ $\therefore\sin^6\Theta+\cos^6\Theta=\frac{4-3(x^2-1)^2}{4}$

Q85. If x = $a\sec\theta\cos\phi$ $a\sec\theta\cos\phi$, y = $b\sec\theta\sin\phi$ $b\sec\theta\sin\phi$ and z= $c\tan\phi$ $c\ \tan\phi$, show that $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}$ $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}$ = 1

Ans:

Given, x = $a\sec\theta\cos\phi$

y = $b\sec\theta\sin\phi$

z = $c\tan\phi$

squaring x,y,z on the sides

$x^2 = a^2\sec^2\theta\cos^2\phi$

$\frac{x^2}{a^2} = \sec^2\theta\cos^2\phi$ — 1

$y^2 = b^2\sec^2\theta\sin^2\phi$

$\frac{y^2}{b^2} = \sec^2\theta\sin^2\phi$ — 2

$z^2 = c^2\tan^2\phi$

$\frac{z^2}{c^2} = \tan^2\phi$ —— 3

Substitute eq 1,2,3 in $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

=> $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

=> $\sec^2\theta\cos^2\phi + \sec^2\theta\sin^2\phi - \tan^2\phi$

=> $\sec^2\theta(\cos^2\phi + \sin^2\phi) - \tan^2\phi$

We know that, $\cos^2\phi + \sin^2\phi = 1$

=> $\sec^2\theta(1) - \tan^2\phi$

And, $\sec^2\theta - \tan^2\theta = 1$

=> 1

Hence, L.H.S= R.H.S

Q86. If $\sin\theta + 2\cos\theta$ prove that $2\sin\theta - \cos\theta = 2$

Ans:

Given, $\sin\theta + 2\cos\theta = 1$

Squaring on both sides

=> $(\sin\theta + 2\cos\theta)^2 = 1^2$

=> $\sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta = 1$

=> $4\cos^2\theta + 4\sin\theta\cos\theta = 1 - \sin^2\theta$

Here, $1 - \sin^2\theta = \cos^2\theta$

=> $4\cos^2\theta + 4\sin\theta\cos\theta - \cos^2\theta = 0$

=> $3\cos^2\theta + 4\sin\theta\cos\theta = 0$ — 1

We have, $2\sin\theta - \cos\theta = 2$

Squaring L.H.S

$(2\sin\theta - \cos\theta)^2$ = $4\sin^2\theta + \cos^2\theta - 4\sin\theta\cos\theta$

Here, $4\sin\theta\cos\theta$ = $3\cos^2\theta$

= $4\sin^2\theta + \cos^2\theta + 3\cos^2\theta$

= $4\sin^2\theta + 4\cos^2\theta$

= $4(\sin^2\theta + \cos^2\theta)$

= 4(1)

= 4

$(2\sin\theta - \cos\theta)^2$ = 4

=> $2\sin\theta - \cos\theta$ = 2

Hence proved