Exercise.1F

Answer.1. $\frac{1}{\sqrt{2}+\sqrt{3}} = \frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}^2-\sqrt{3}^2} = \frac{\sqrt{2}-\sqrt{3}}{2-3} = \frac{\sqrt{2}-\sqrt{3}}{-1} = \sqrt{3}-\sqrt{2}\{\sqrt{3}\times\sqrt{3}=\sqrt{9}=3\}$

$$\{\sqrt{2}\times\sqrt{2}=\sqrt{4}=2\}$$

Answer.2.

i) $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{\sqrt{7}^2} = \frac{\sqrt{7}}{7}$

ii) $\frac{\sqrt{5}}{2\sqrt{3}} = \frac{\sqrt{5}}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{2\sqrt{15}}{4\times\sqrt{3}^2} = \frac{2\sqrt{15}}{12} = \frac{\sqrt{15}}{6}$

iii) $\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-\sqrt{3}^2} = \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$

iv) $\frac{1}{\sqrt{5}-2} = \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{\sqrt{5}^2-2^2} = \frac{\sqrt{5}+2}{5-4} = \frac{\sqrt{5}+2}{1} = 5+\sqrt{2}$

v) $\frac{1}{5+3\sqrt{2}} = \frac{1}{5+3\sqrt{2}} \times \frac{5-3\sqrt{2}}{5-3\sqrt{2}} = \frac{5-3\sqrt{2}}{5^2-(3\sqrt{2})^2} = \frac{5-3\sqrt{2}}{25-9\times2} = \frac{5-3\sqrt{2}}{25-18} = \frac{5-3\sqrt{2}}{7}$

vi) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}^2-\sqrt{6}^2} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$

vii) $\frac{4}{\sqrt{11}-\sqrt{7}} = \frac{4}{\sqrt{11}-\sqrt{7}} \times \frac{\sqrt{11}+\sqrt{7}}{\sqrt{11}+\sqrt{7}} = \frac{4(\sqrt{11}+\sqrt{7})}{\sqrt{11}^2+\sqrt{7}^2} = \frac{4(\sqrt{11}+\sqrt{7})}{11-7} = \frac{4(\sqrt{11}+\sqrt{7})}{4} = \sqrt{11}+\sqrt{7}$

viii) $\frac{1+\sqrt{2}}{2-\sqrt{2}} = \frac{1+\sqrt{2}}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{1\times2+1\times\sqrt{2}+\sqrt{2}\times2+\sqrt{2}\times\sqrt{2}}{2^2-\sqrt{2}^2} = \frac{2+\sqrt{2}+2\sqrt{2}+2}{4-2}\{\sqrt{2}\times\sqrt{2}=\sqrt{4}=2\}$

$$= \frac{4+3\sqrt{2}}{4-2} = \frac{4+3\sqrt{2}}{2}$$

ix) $\frac{3-2\sqrt{2}}{3+2\sqrt{2}} = \frac{3-2\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{(3-2\sqrt{2})^2}{3^2-(2\sqrt{2})^2} = \frac{3^2+(2\sqrt{2})^2-2(3\times2\sqrt{2})}{9-4\times2} = \frac{9+4\times2-12\sqrt{2}}{9-8}$

$$= \frac{9+8-12\sqrt{2}}{1} = 17-12\sqrt{2}$$

Answer.3. Given, $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$ *and* $\sqrt{10} = 3.162$

i) $\frac{2}{\sqrt{5}} = \frac{2}{2.236} = \frac{2}{2.236} \times \frac{1000}{1000} = \frac{2000}{2236} = 0.894$

ii) $\frac{2-\sqrt{3}}{\sqrt{3}} = \frac{2-\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}-\sqrt{3}^2}{\sqrt{3}^2} = \frac{2\sqrt{3}-3}{3} = \frac{2\times1.732-3}{3} = \frac{3.464-3}{3} = \frac{0.464}{3} = 0.155(approx)$

iii) $\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}} = \frac{3.162-2.236}{1.414} = \frac{0.962}{1.414} = 0.655(approx.)$

Answer.4.

i) $\frac{\sqrt{2}-1}{\sqrt{2}+1} = a + b\sqrt{2}$

LHS,

$$\frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(\sqrt{2}-1)^2}{\sqrt{2}^2 - 1^2} = \frac{\sqrt{2}^2 + 1^2 - 2(\sqrt{2}\times 1)}{2-1}\{(a-b)^2 = a^2 + b^2 - 2ab\}$$

$$= \frac{2+1-2\sqrt{2}}{1} = 3 - 2\sqrt{2}$$

As LHS=RHS

$$\therefore 3 - 2\sqrt{2} = a + b\sqrt{2}$$

So, $a = 3$ and $-2\sqrt{2} = b\sqrt{2}; b = -2$

ii) $\frac{2-\sqrt{5}}{2+\sqrt{5}} = a\sqrt{5} + b$

LHS,

$$\frac{2-\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{(2-\sqrt{5})^2}{2^2 - \sqrt{5}^2} = \frac{2^2 - \sqrt{5}^2 - 2(2\times\sqrt{5})}{4-5}$$

$$= \frac{4+5-4\sqrt{5}}{-1}\{(a-b)^2 = a^2 + b^2 - 2ab\}$$

$$= \frac{9-4\sqrt{5}}{-1} = 4\sqrt{5} - 9$$

As LHS=RHS

$$\therefore 4\sqrt{5} - 9 = a\sqrt{5} + b$$

So, $4\sqrt{5} = a\sqrt{5}; a = 4$ *and* $b = -9$

iii) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}$

LHS,

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})^2}{\sqrt{3}^2 - \sqrt{2}^2} = \frac{\sqrt{3}^2 + \sqrt{2}^2 + 2(\sqrt{3}\times\sqrt{2})}{3-2} = \frac{3+2+2\sqrt{6}}{1}\{(a-b)^2$$
$$= a^2 + b^2 - 2ab\}$$

$$= 5 + 2\sqrt{6}$$

As LHS=RHS

$$\therefore 5 + 2\sqrt{6} = a + b\sqrt{6}$$

So, $a = 5$ *and* $2\sqrt{6} = b\sqrt{6}; b = 2$

iv) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$

LHS,

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{(5+2\sqrt{3})(7-4\sqrt{3})}{7^2-(4\sqrt{3})^2} = \frac{5\times 7 + 5\times -4\sqrt{3} + 2\sqrt{3}\times 7 + 2\sqrt{3}\times -4\sqrt{3}}{49 - 16\times 3}$$

$$= \frac{35 - 20\sqrt{3} + 14\sqrt{3} - 8\times 3}{49-48} = \frac{35 - 24 - 6\sqrt{3}}{1} = 11 - 6\sqrt{3}$$

As LHS=RHS

$$\therefore 11 - 6\sqrt{3} = a + b\sqrt{3}$$

So, $a = 11$ *and* $-6\sqrt{3} = b\sqrt{3}; b = -6$

Answer.5. Given, $\sqrt{3} = 1.732, \sqrt{5} = 2.236, \sqrt{6} = 2.449$ *and* $\sqrt{10} = 3.162$

i) $\frac{1}{\sqrt{6}+\sqrt{5}} = \frac{1}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}^2-\sqrt{5}^2} = \frac{\sqrt{6}-\sqrt{5}}{6-5} = \frac{2.449-2.236}{1} = 0.213$

ii) $\frac{6}{\sqrt{5}+\sqrt{3}} = \frac{6}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{6(\sqrt{5}-\sqrt{3})}{\sqrt{5}^2-\sqrt{3}^2} = \frac{6(\sqrt{5}-\sqrt{3})}{5-3} = \frac{6(\sqrt{5}-\sqrt{3})}{2} = 3(2.236 - 1.732) = 3 \times 0.504 = 1.512$

iii) $\frac{1}{4\sqrt{3}-3\sqrt{5}} = \frac{1}{4\sqrt{3}-3\sqrt{5}} \times \frac{4\sqrt{3}+3\sqrt{5}}{4\sqrt{3}+3\sqrt{5}} = \frac{4\sqrt{3}+3\sqrt{5}}{(4\sqrt{3})^2-(3\sqrt{5})^2} = \frac{4\times 1.732+3\times 2.236}{16\times 3-9\times 5} = \frac{6.928+6.708}{48-45} = \frac{13.636}{3} = 4.545$

iv) $\frac{3+\sqrt{5}}{3-\sqrt{5}} = \frac{3+\sqrt{5}}{3-\sqrt{5}} = \frac{(3+\sqrt{5})^2}{3^2-\sqrt{5}^2} = \frac{3^2+\sqrt{5}^2+2(3\times\sqrt{5})}{9-5} = \frac{9+5+6\sqrt{5}}{4} = \frac{14+6\sqrt{5}}{4} = \frac{14+6\times 2.236}{4} = \frac{14+13.416}{4} = \frac{27.416}{4} = 6.854$

v) $\frac{1+2\sqrt{3}}{2-\sqrt{3}} = \frac{1+2\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{(1+2\sqrt{3})(2+\sqrt{3})}{2^2-\sqrt{3}^2} = \frac{1\times 2+1\times\sqrt{3}+2\sqrt{3}\times 2+2\sqrt{3}\times\sqrt{3}}{4-3} = \frac{2+\sqrt{3}+4\sqrt{3}+2\times\sqrt{3}^2}{1} = 2 + 5\sqrt{3} + 6 = 8 + 5\times 1.732 = 8 + 8.66 = 16.66$

vi) $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}\times\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}=\frac{(\sqrt{5}+\sqrt{2})^2}{\sqrt{5}^2-\sqrt{2}^2}=\frac{\sqrt{5}^2+\sqrt{2}^2+2(\sqrt{5}\times\sqrt{2})}{5-2}=\frac{5+2+2\sqrt{10}}{3}=\frac{7+2\sqrt{10}}{3}=$
$\frac{7+2\times3.162}{3}=\frac{13.324}{3}=4.441$

Answer.6.

i) $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}=\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{16}\times\sqrt{3}+\sqrt{9}\times\sqrt{2}}=\frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}\times\frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}=$

$$\frac{7\sqrt{3}\times4\sqrt{3}+7\sqrt{3}\times-3\sqrt{2}-5\sqrt{2}\times4\sqrt{3}-5\sqrt{2}\times-3\sqrt{2}}{(4\sqrt{3})^2-(3\sqrt{2})^2}=\frac{28\times3-21\sqrt{6}-20\sqrt{6}-15\times2}{16\times3-9\times2}$$
$$=\frac{84-41\sqrt{6}+30}{48-18}=\frac{114-41\sqrt{6}}{30}$$

ii) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}=\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}\times\frac{3\sqrt{5}+2\sqrt{6}}{3\sqrt{5}+2\sqrt{6}}=\frac{2\sqrt{6}\times3\sqrt{5}+2\sqrt{6}\times2\sqrt{6}+(-\sqrt{5})\times3\sqrt{5}+(-\sqrt{5})\times2\sqrt{6}}{(3\sqrt{5})^2-(2\sqrt{6})^2}=$
$\frac{6\sqrt{30}+(2\sqrt{6})^2-3\times\sqrt{5}^2-2\sqrt{30}}{9\times5-4\times6}=\frac{4\sqrt{30}+4\times6-3\times5}{45-24}=\frac{4\sqrt{30}+24-15}{21}=\frac{4\sqrt{30}+9}{21}$

Answer.7.

i) $\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}=\frac{4+\sqrt{5}}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}\times\frac{4-\sqrt{5}}{4-\sqrt{5}}=\frac{(4+\sqrt{5})^2}{4^2-\sqrt{5}^2}+\frac{(4-\sqrt{5})^2}{4^2-\sqrt{5}^2}=\frac{4^2+\sqrt{5}^2+2(4\times\sqrt{5})}{16-5}+$
$\frac{4^2+\sqrt{5}^2-2(4\times\sqrt{5})}{16-5}=\frac{16+5+8\sqrt{11}}{11}+\frac{16+5-8\sqrt{11}}{11}=\frac{21+8\sqrt{11}+21-8\sqrt{11}}{11}=\frac{42}{11}$

ii) $\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{2}{\sqrt{5}-\sqrt{3}}-\frac{3}{\sqrt{2}-\sqrt{5}}$

$$\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}^2-\sqrt{2}^2}=\frac{\sqrt{3}-\sqrt{2}}{3-2}=\frac{\sqrt{3}-\sqrt{2}}{1}=\sqrt{3}-\sqrt{2}$$

$$\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{2(\sqrt{5}+\sqrt{3})}{\sqrt{5}^2-\sqrt{3}^2}=\frac{2(\sqrt{5}+\sqrt{3})}{5-3}=\frac{2(\sqrt{5}+\sqrt{3})}{2}=\sqrt{5}+\sqrt{3}$$

$$\frac{3}{\sqrt{2}-\sqrt{5}}\times\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}+\sqrt{5}}=\frac{3(\sqrt{2}+\sqrt{5})}{\sqrt{2}^2-\sqrt{5}^2}=\frac{3(\sqrt{2}+\sqrt{5})}{2-5}=\frac{3(\sqrt{2}+\sqrt{5})}{-3}=-\sqrt{2}-\sqrt{5}$$

Equate to question

$$=\sqrt{3}-\sqrt{2}-(\sqrt{5}+\sqrt{3})-\!\!-\sqrt{2}-\sqrt{5}=\sqrt{3}-\sqrt{2}-\sqrt{5}-\sqrt{3}+\sqrt{2}+\sqrt{5}=0$$

iii) $\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$$= \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{(2+\sqrt{3})^2}{2^2-\sqrt{3}^2} + \frac{(2-\sqrt{3})^2}{2^2-\sqrt{3}^2} + \frac{(\sqrt{3}-1)^2}{\sqrt{3}^2-1^2}$$

$$= \frac{2^2+\sqrt{3}^2+2(2\times\sqrt{3})}{4-3} + \frac{2^2+\sqrt{3}^2-2(2\times\sqrt{3})}{4-3} + \frac{\sqrt{3}^2+1^2-2(\sqrt{3}\times 1)}{3-1}$$

$$= \frac{4+3+4\sqrt{3}}{1} + \frac{4+3-4\sqrt{3}}{1} + \frac{3+1-2\sqrt{3}}{2}$$

$$= \frac{7+4\sqrt{3}}{1} + \frac{7-4\sqrt{3}}{1} + \frac{2(2-\sqrt{3})}{2} = 7 + 4\sqrt{3} + 7 - 4\sqrt{3} + 2 - \sqrt{3} = 16 - \sqrt{3}$$

iv) $\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

$$= \frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} + \frac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}} \times \frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}} - \frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$$

$$= \frac{2\sqrt{12}-2\sqrt{18}}{\sqrt{2}^2-\sqrt{3}^2} + \frac{6\sqrt{12}-6\sqrt{6}}{\sqrt{6}^2-\sqrt{3}^2} - \frac{8\sqrt{18}-8\sqrt{6}}{\sqrt{6}^2-\sqrt{2}^2}$$

$$= \frac{2\sqrt{12}-2\sqrt{18}}{2-3} + \frac{6\sqrt{12}-6\sqrt{6}}{6-3} - \frac{8\sqrt{18}-8\sqrt{6}}{6-2}$$

$$= \frac{2\sqrt{12}-2\sqrt{18}}{-1} + \frac{6\sqrt{12}-6\sqrt{6}}{3} - \frac{8\sqrt{18}-8\sqrt{6}}{4}$$

$$= \frac{-2\sqrt{12}+2\sqrt{18}}{1} + \frac{3(2\sqrt{12}-2\sqrt{6})}{3} - \frac{4(2\sqrt{18}-2\sqrt{6})}{4} = 2\sqrt{18} - 2\sqrt{12} + 2\sqrt{12} - 2\sqrt{6} - 2\sqrt{18} + 2\sqrt{6} = 0$$

Answer.8.

i) $\frac{1}{3+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+1} = 1$

$LHS,$

$$= \frac{1}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{3-\sqrt{7}}{3^2-\sqrt{7}^2} + \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}^2-\sqrt{5}^2} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}^2-\sqrt{3}^2} + \frac{\sqrt{3}-1}{\sqrt{3}^2-1^2}$$

$$= \frac{3-\sqrt{7}}{9-7} + \frac{\sqrt{7}-\sqrt{5}}{7-5} + \frac{\sqrt{5}-\sqrt{3}}{5-3} + \frac{\sqrt{3}-1}{3-1}$$

$$= \frac{3-\sqrt{7}}{2} + \frac{\sqrt{7}-\sqrt{5}}{2} + \frac{\sqrt{5}-\sqrt{3}}{2} + \frac{\sqrt{3}-1}{2} = \frac{3-\sqrt{7}+\sqrt{7}-\sqrt{5}+\sqrt{5}-\sqrt{3}+\sqrt{3}-1}{2}$$

$$= \frac{3-1}{2} = \frac{2}{2} = 1$$

As LHS=RHS, hence Proved

ii) $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}} = 2$

LHS

$$= \frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} \times \frac{\sqrt{3}-\sqrt{4}}{\sqrt{3}-\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} \times \frac{\sqrt{4}-\sqrt{5}}{\sqrt{4}-\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} \times \frac{\sqrt{5}-\sqrt{6}}{\sqrt{5}-\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} \times \frac{\sqrt{6}-\sqrt{7}}{\sqrt{6}-\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}} \times \frac{\sqrt{7}-\sqrt{8}}{\sqrt{7}-\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}} \times \frac{\sqrt{8}-\sqrt{9}}{\sqrt{8}-\sqrt{9}}$$

$$= \frac{1-\sqrt{2}}{1^2-\sqrt{2}^2} + \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}^2-\sqrt{3}^2} + \frac{\sqrt{3}-\sqrt{4}}{\sqrt{3}^2-\sqrt{4}^2} + \frac{\sqrt{4}-\sqrt{5}}{\sqrt{4}^2-\sqrt{5}^2} + \frac{\sqrt{5}-\sqrt{6}}{\sqrt{5}^2-\sqrt{6}^2} + \frac{\sqrt{6}-\sqrt{7}}{\sqrt{6}^2-\sqrt{7}^2} + \frac{\sqrt{7}-\sqrt{8}}{\sqrt{7}^2-\sqrt{8}^2} + \frac{\sqrt{8}-\sqrt{9}}{\sqrt{8}^2-\sqrt{9}^2}$$

$$= \frac{1-\sqrt{2}}{1-2} + \frac{\sqrt{2}-\sqrt{3}}{2-3} + \frac{\sqrt{3}-\sqrt{4}}{3-4} + \frac{\sqrt{4}-\sqrt{5}}{4-5} + \frac{\sqrt{5}-\sqrt{6}}{5-6} + \frac{\sqrt{6}-\sqrt{7}}{6-7} + \frac{\sqrt{7}-\sqrt{8}}{7-8} + \frac{\sqrt{8}-\sqrt{9}}{8-9}$$

$$= \frac{1-\sqrt{2}}{-1} + \frac{\sqrt{2}-\sqrt{3}}{-1} + \frac{\sqrt{3}-\sqrt{4}}{-1} + \frac{\sqrt{4}-\sqrt{5}}{-1} + \frac{\sqrt{5}-\sqrt{6}}{-1} + \frac{\sqrt{6}-\sqrt{7}}{-1} + \frac{\sqrt{7}-\sqrt{8}}{-1} + \frac{\sqrt{8}-\sqrt{9}}{-1}$$

$$= \frac{1-\sqrt{2}+\sqrt{2}-\sqrt{3}+\sqrt{3}-\sqrt{4}+\sqrt{4}-\sqrt{5}+\sqrt{5}-\sqrt{6}+\sqrt{6}-\sqrt{7}+\sqrt{7}-\sqrt{8}+\sqrt{8}-\sqrt{9}}{-1} = \frac{1-3}{-1} = \frac{-2}{-1} = 2$$

As LHS=RHS, hence Proved

Answer.9.

$$\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$$

Solve LHS

$$\frac{7+3\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$= \frac{7 \times 3 + 7 \times (-\sqrt{5}) + 3\sqrt{5} \times 3 + 3\sqrt{5} \times (-\sqrt{5})}{3^2 - \sqrt{5}^2} - \frac{7 \times 3 + 7 \times \sqrt{5} + (-3\sqrt{5}) \times 3 + (-3\sqrt{5}) \times \sqrt{5}}{3^2 - \sqrt{5}^2}$$

$$= \frac{21 - 7\sqrt{5} + 9\sqrt{5} - 3 \times \sqrt{5}^2}{9-5} - \frac{21 + 7\sqrt{5} - 9\sqrt{5} - 3 \times \sqrt{5}^2}{9-5}$$

$$= \frac{21 + 2\sqrt{5} - 3 \times 5}{4} - \frac{21 - 2\sqrt{5} - 3 \times 5}{4} = \frac{21 + 2\sqrt{5} - 15 - 21 + 2\sqrt{5} + 15}{4}$$

$$= \frac{4\sqrt{5}}{4}$$

Equate LHS to RHS

$$\frac{4\sqrt{5}}{4} = a + b\sqrt{5} \;\therefore a = 0; \quad b\sqrt{5} = \sqrt{5}, b = 1$$

Answer.10.

$$\frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}+\sqrt{11}} + \frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}} = \frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}+\sqrt{11}} \times \frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}-\sqrt{11}} + \frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}} \times \frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}+\sqrt{11}}$$

$$=\frac{(\sqrt{13}-\sqrt{11})^2}{\sqrt{13}^2-\sqrt{11}^2}+\frac{\left(\sqrt{13}+\sqrt{11}\right)^2}{\sqrt{13}^2-\sqrt{11}^2}$$

$$=\frac{\sqrt{13}^2+\sqrt{11}^2-2\left(\sqrt{13}\times\sqrt{11}\right)}{13-11}+\frac{\sqrt{13}^2+\sqrt{11}^2+2\left(\sqrt{13}\times\sqrt{11}\right)}{13-11}$$

$$=\frac{13+11-2\sqrt{143}}{2}+\frac{13+11+2\sqrt{143}}{2}=\frac{24-2\sqrt{143}+24+2\sqrt{143}}{2}=\frac{48}{2}=24$$

Answer.11. Given $x=3+2\sqrt{2}$

Put x value in $x+\frac{1}{x}$

$$x+\frac{1}{x}=3+2\sqrt{2}+\frac{1}{3+2\sqrt{2}}=3+2\sqrt{2}+\frac{1}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}=3+2\sqrt{2}+\frac{3-2\sqrt{3}}{3^2-2\sqrt{2}^2}$$

$$3+2\sqrt{2}+\frac{3-2\sqrt{3}}{9-4\times 2}=3+2\sqrt{2}+\frac{3-2\sqrt{2}}{9-8}=3+2\sqrt{2}+\frac{3-2\sqrt{2}}{1}=3+2\sqrt{2}+3-2\sqrt{2}=3+3$$
$$=6$$

It is Rational number.

Answer.12. Given$x=2-\sqrt{3}$

Put x value in $\left(x-\frac{1}{x}\right)^3$

$$(a-b)^3=a^3-b^3-3ab(a-b)$$

Here $a/x=2-\sqrt{3}$ *and* $b/\frac{1}{x}=\frac{1}{2-\sqrt{3}}$ *or* $\left(\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{2+\sqrt{3}}{2^2-\sqrt{3}^2}=\frac{2+\sqrt{3}}{4-3}=\frac{2+\sqrt{3}}{1}\right)=2+\sqrt{3}$

$$\left(x-\frac{1}{x}\right)^3=\left(2-\sqrt{3}-2-\sqrt{3}\right)^3=\left(-2\sqrt{3}\right)^3=-8\times\sqrt{27}=-8\times\sqrt{9}\times\sqrt{3}$$
$$=-8\times 3\times\sqrt{3}=-24\sqrt{3}$$

Answer.13. Given$x=9-4\sqrt{5}$

Put x value in $x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2\left(x\times\frac{1}{x}\right)=\left(x+\frac{1}{x}\right)^2-2$.....(i)

$$\frac{1}{x}=\frac{1}{9-4\sqrt{5}}\times\frac{9+4\sqrt{5}}{9+4\sqrt{5}}=\frac{9+4\sqrt{5}}{9^2-\left(4\sqrt{5}\right)^2}=\frac{9+4\sqrt{5}}{81-16\times\sqrt{5}^2}=\frac{9+4\sqrt{5}}{81-80}=\frac{9+4\sqrt{5}}{1}=9+4\sqrt{5}$$

Put in equation (i)

$$\left(9-4\sqrt{5}+9+4\sqrt{5}\right)^2-2=18^2-2=324-2=322$$

Answer.14. Given $x = \frac{5-\sqrt{21}}{2}$

$$\frac{1}{x} = \frac{1}{\frac{5-\sqrt{21}}{2}} = \frac{2}{5-\sqrt{21}} = \frac{2}{5-\sqrt{21}} \times \frac{5+\sqrt{21}}{5+\sqrt{21}} = \frac{2(5+\sqrt{21})}{5^2-\sqrt{21}^2} = \frac{2(5+\sqrt{21})}{25-21} = \frac{2(5+\sqrt{21})}{4}$$
$$= \frac{(5+\sqrt{21})}{2}$$

$$x + \frac{1}{x} = \frac{5-\sqrt{21}}{2} + \frac{5+\sqrt{21}}{2} = \frac{5-\sqrt{21}+\sqrt{21}+5}{2} = \frac{10}{2} = 5$$

Answer.15. Given $a = 3 - 2\sqrt{2}$

$$a^2 = (3-2\sqrt{2})^2 = 3^2 + (2\sqrt{2})^2 - 2(3 \times 2\sqrt{2}) = 9 + 4 \times 2 - 12\sqrt{2} = 17 - 12\sqrt{2}$$

$$\frac{1}{a^2} = \frac{1}{(3-2\sqrt{2})^2} = \frac{1}{3^2 + (2\sqrt{2})^2 - 2(3 \times 2\sqrt{2})} = \frac{1}{9 + 4 \times 2 - 12\sqrt{2}} = \frac{1}{9 + 8 - 12\sqrt{2}}$$
$$= \frac{1}{17 - 12\sqrt{2}}$$

Rationalise

$$\frac{1}{17-12\sqrt{2}} \times \frac{17+12\sqrt{2}}{17+12\sqrt{2}} = \frac{17+12\sqrt{2}}{17^2 - (12\sqrt{2})^2} = \frac{17+12\sqrt{2}}{289 - 144 \times 2} = \frac{17+12\sqrt{2}}{289-288} = \frac{17+12\sqrt{2}}{1}$$
$$= 17 + 12\sqrt{2}$$

$$a^2 - \frac{1}{a^2} = 17 - 12\sqrt{2} - (17 + 12\sqrt{2}) = 17 - 12\sqrt{2} - 17 - 12\sqrt{2} = -24\sqrt{2}$$

Answer.16. Given $x = \sqrt{13} + 2\sqrt{3}$

$$\frac{1}{x} = \frac{1}{\sqrt{13}+2\sqrt{3}} \times \frac{\sqrt{13}-2\sqrt{3}}{\sqrt{13}-2\sqrt{3}} = \frac{\sqrt{13}-2\sqrt{3}}{\sqrt{13}^2 - (2\sqrt{3})^2} = \frac{13-2\sqrt{3}}{13 - 4 \times 3} = \frac{13-2\sqrt{3}}{13-12} = 13 - 2\sqrt{3}$$

$$x - \frac{1}{x} = \sqrt{13} + 2\sqrt{3} - (\sqrt{13} - 2\sqrt{3}) = \sqrt{13} + 2\sqrt{3} - \sqrt{13} + 2\sqrt{3} = 4\sqrt{3}$$

Answer.17. Given $x = 2 + \sqrt{3}$

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2 - \sqrt{3}^2} = \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$$

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x\times\frac{1}{x}\right)\left(x+\frac{1}{x}\right)=\left(x+\frac{1}{x}\right)^3-3(1)\left(x+\frac{1}{x}\right)$$
$$=\left(2+\sqrt{3}+2-\sqrt{3}\right)^3-3\left(2+\sqrt{3}+2-\sqrt{3}\right)$$
$$=(4)^3-3(4)=64-12=52$$

Answer.18. Given $x=\frac{5-\sqrt{3}}{5+\sqrt{3}}$ and $y=\frac{5+\sqrt{3}}{5-\sqrt{3}}$

$$x=\frac{5-\sqrt{3}}{5+\sqrt{3}}\times\frac{5-\sqrt{3}}{5-\sqrt{3}}=\frac{\left(5-\sqrt{3}\right)^2}{5^2-\sqrt{3}^2}=\frac{5^2+\sqrt{3}^2-2\left(5\times\sqrt{3}\right)}{25-3}=\frac{25+3-10\sqrt{3}}{22}=\frac{28-10\sqrt{3}}{22}$$
$$=\frac{2\left(14-5\sqrt{3}\right)}{22}=\frac{14-5\sqrt{3}}{11}$$

$$y=\frac{5+\sqrt{3}}{5-\sqrt{3}}\times\frac{5+\sqrt{3}}{5+\sqrt{3}}=\frac{\left(5+\sqrt{3}\right)^2}{5^2-\sqrt{3}^2}=\frac{5^2+\sqrt{3}^2+2\left(5\times\sqrt{3}\right)}{25-3}=\frac{25+3+10\sqrt{3}}{22}=\frac{28+10\sqrt{3}}{22}$$
$$=\frac{2\left(14+5\sqrt{3}\right)}{22}=\frac{14+5\sqrt{3}}{11}$$

$$x-y=\frac{14-5\sqrt{3}}{11}-\frac{14+5\sqrt{3}}{11}=\frac{14-5\sqrt{3}-14-5\sqrt{3}}{11}=\frac{-10\sqrt{3}}{11}$$

Answer.19. Given $a=\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ and $b=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

Equation $3a^2+4ab-3b^2$

$$3\left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}\right)^2+4\left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}\right)-3\left(\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}\right)^2$$

$$3\left(\left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}\right)^2-\left(\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}\right)^2\right)+4=3\left(\frac{\left(7+2\sqrt{10}\right)^2-\left(7-2\sqrt{10}\right)^2}{7^2-\left(2\sqrt{10}\right)^2}\right)+4$$
$$=3\left(\frac{49+40+28\sqrt{10}-\left(49+40-28\sqrt{10}\right)}{49-40}\right)+4$$
$$=3\left(\frac{49+40+28\sqrt{10}-49-40+28\sqrt{10}}{9}\right)+4=3\left(\frac{56\sqrt{10}}{9}\right)+4$$
$$=\frac{56\sqrt{10}}{3}+4$$

Answer.20.

$a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}, b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Rationalise

$$a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}-\sqrt{2})^2}{\sqrt{3}^2-\sqrt{2}^2} = \frac{\sqrt{3}^2+\sqrt{2}^2-2(\sqrt{3}\times\sqrt{2})}{3-2} = \frac{3+2-2\sqrt{6}}{1} = 5-2\sqrt{6}$$

$$b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})^2}{\sqrt{3}^2-\sqrt{2}^2} = \frac{\sqrt{3}^2+\sqrt{2}^2+2(\sqrt{3}\times\sqrt{2})}{3-2} = \frac{3+2+2\sqrt{6}}{1} = 5+2\sqrt{6}$$

$$a^2+b^2-5ab = (5-2\sqrt{6})^2+(5+2\sqrt{6})^2-5[(5-2\sqrt{6})(5+2\sqrt{6})]$$

$$= 5^2+(2\sqrt{6})^2-2(5\times 2\sqrt{6})+5^2+(2\sqrt{6})^2+2(5\times 2\sqrt{6})-5\left(5^2-(2\sqrt{6})^2\right)$$

$$= 25+4\times 6-20\sqrt{6}+25+4\times 6+20\sqrt{6}-5(25-4\times 6)$$

$$= 25+24-20\sqrt{6}+25+24-20\sqrt{6}-5(25-24)$$

$$= 50+48-5 = 98-5 = 93$$

Answer.21 $p = \frac{3-\sqrt{5}}{3+\sqrt{5}}, q = \frac{3+\sqrt{5}}{3-\sqrt{5}}$

Rationalise

$$p = \frac{3-\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{(3-\sqrt{5})^2}{3^2-\sqrt{5}^2} = \frac{3^2+\sqrt{5}^2-2(3\times\sqrt{5})}{9-5} = \frac{9+5-6\sqrt{5}}{4} = \frac{14-6\sqrt{5}}{4}$$
$$= \frac{7-3\sqrt{5}}{2}$$

$$q = \frac{3+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{(3+\sqrt{5})^2}{3^2-\sqrt{5}^2} = \frac{3^2+\sqrt{5}^2+2(3\times\sqrt{5})}{9-5} = \frac{9+5+6\sqrt{5}}{4} = \frac{14+6\sqrt{5}}{4}$$
$$= \frac{7+3\sqrt{5}}{2}$$

$$p.q = \frac{7-3\sqrt{5}}{2} \times \frac{7+3\sqrt{5}}{2} = \frac{7^2-(3\sqrt{5})^2}{4} = \frac{49-45}{4} = \frac{4}{4} = 1$$

$$p^2+q^2 = (p+q)^2-2.p.q$$

$$= \left(\frac{7-3\sqrt{5}}{2}+\frac{7+3\sqrt{5}}{2}\right)^2-2\times 1 = \left(\frac{7-3\sqrt{5}+7+3\sqrt{5}}{2}\right)^2-2 = \left(\frac{14}{2}\right)^2-2 = 7^2-2 = 49-2$$
$$= 47$$

Answer.22.

i) $\frac{1}{\sqrt{7}+\sqrt{6}-\sqrt{13}} = \frac{1}{(\sqrt{7}+\sqrt{6})-\sqrt{13}} \times \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(\sqrt{7}+\sqrt{6})+\sqrt{13}} = \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(\sqrt{7}+\sqrt{6})^2-\sqrt{13}^2} = \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{\left(\sqrt{7}^2+\sqrt{6}^2+2(\sqrt{7}\times\sqrt{6})\right)-13}$

$$= \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(7+6+2\sqrt{42})-13} = \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{13+2\sqrt{42}-13} = \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{2\sqrt{42}} = \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{2\sqrt{42}} \times \frac{\sqrt{42}}{\sqrt{42}}$$

$$\frac{\sqrt{7}\times\sqrt{42}+\sqrt{6}\times\sqrt{42}+\sqrt{13}\times\sqrt{42}}{2\times\sqrt{42}^2} = \frac{\sqrt{294}+\sqrt{252}+\sqrt{546}}{2\times 42}$$

$$= \frac{\sqrt{49}\times\sqrt{6}+\sqrt{36}\times\sqrt{7}+\sqrt{546}}{84} = \frac{7\sqrt{6}+6\sqrt{7}+\sqrt{546}}{84}$$

ii) $\frac{3}{\sqrt{3}+\sqrt{5}-\sqrt{2}} = \frac{3}{(\sqrt{3}+\sqrt{5})-\sqrt{2}} \times \frac{(\sqrt{3}+\sqrt{5})+\sqrt{2}}{(\sqrt{3}+\sqrt{5})+\sqrt{2}} = \frac{3\left((\sqrt{3}+\sqrt{5})+\sqrt{2}\right)}{(\sqrt{3}+\sqrt{5})^2-\sqrt{2}^2} = \frac{3\left((\sqrt{3}+\sqrt{5})+\sqrt{2}\right)}{\left(\sqrt{3}^2+\sqrt{5}^2+2(\sqrt{3}+\sqrt{5})\right)-2}$

$$\Rightarrow \frac{3\left((\sqrt{3}+\sqrt{5})+\sqrt{2}\right)}{6+2\sqrt{3}\sqrt{5}} = \frac{3}{2}\times\frac{(\sqrt{3}+\sqrt{5})+\sqrt{2}}{3+\sqrt{3}\sqrt{5}} = \frac{3}{2}\times\frac{(\sqrt{3}+\sqrt{5})+\sqrt{2}}{3+\sqrt{15}}\times\frac{3-\sqrt{15}}{3-\sqrt{15}}$$

$$\Rightarrow \frac{3}{2}\times\frac{(\sqrt{3}+\sqrt{5}+\sqrt{2})(3-\sqrt{15})}{9-15} = \frac{3}{2}\times\frac{(\sqrt{3}+\sqrt{5}+\sqrt{2})(3-\sqrt{15})}{-6} = -\frac{1}{4}(3\sqrt{3}+3\sqrt{5}+3\sqrt{2}-3\sqrt{5}-5\sqrt{3}-\sqrt{30})$$

$$\Rightarrow -\frac{1}{4}(-2\sqrt{3}+3\sqrt{2}-\sqrt{30}) = \frac{2\sqrt{3}-3\sqrt{2}+\sqrt{30}}{4}$$

iii) $\frac{4}{2+\sqrt{3}+\sqrt{7}} = \frac{4}{(2+\sqrt{3})+\sqrt{7}} \times \frac{(2+\sqrt{3})-\sqrt{7}}{(2+\sqrt{3})-\sqrt{7}} = \frac{4\left((2+\sqrt{3})-\sqrt{7}\right)}{(2+\sqrt{3})^2-\sqrt{7}^2} = \frac{4\left((2+\sqrt{3})-\sqrt{7}\right)}{\left(2^2+\sqrt{3}^2+2(2\times\sqrt{3})\right)-7} = \frac{4\left((2+\sqrt{3})-\sqrt{7}\right)}{(4+3+4\sqrt{3})-7}$

$$= \frac{4\left((2+\sqrt{3})-\sqrt{7}\right)}{7+4\sqrt{3}-7} = \frac{4\left((2+\sqrt{3})-\sqrt{7}\right)}{4\sqrt{3}} = \frac{\left((2+\sqrt{3})-\sqrt{7}\right)}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2\times\sqrt{3}+\sqrt{3}^2-\sqrt{7}\times\sqrt{3}}{\sqrt{3}^2} = \frac{2\sqrt{3}+3-\sqrt{21}}{3}$$

Answer.23. Given $\sqrt{2} = 1.414$ *and* $\sqrt{6} = 2.449$,

$$\frac{1}{\sqrt{3}-\sqrt{2}-1} = \frac{1}{\sqrt{3}-(\sqrt{2}-1)}\times\frac{\sqrt{3}+(\sqrt{2}-1)}{\sqrt{3}+(\sqrt{2}-1)} = \frac{\sqrt{3}+(\sqrt{2}-1)}{\sqrt{3}^2-\left(\sqrt{2}^2+1^2-2.\sqrt{2}.1\right)}$$

$$= \frac{\sqrt{3}+(\sqrt{2}-1)}{3-(2+1-2\sqrt{2})} = \frac{\sqrt{3}+(\sqrt{2}-1)}{3-3+2\sqrt{2}}$$

$$\frac{\sqrt{3}+(\sqrt{2}-1)}{2\sqrt{2}}\times\frac{2\sqrt{2}}{2\sqrt{2}}=\frac{2\left(\sqrt{6}+\sqrt{2}^2-\sqrt{2}\right)}{4\times\sqrt{2}^2}=\frac{\sqrt{6}+2-\sqrt{2}}{2\times 2}=\frac{\sqrt{6}+2-\sqrt{2}}{4}$$

$$=\frac{2.449+2-1.414}{4}=\frac{3.035}{4}=0.758$$

Answer.24. Given $x=\frac{1}{2-\sqrt{3}}$

Rationalise

$$x=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{2+\sqrt{3}}{2^2-\sqrt{3}^2}=\frac{2+\sqrt{3}}{4-3}=2+\sqrt{3}$$

⇨ $x^3-2x^2-7x+5=(2+\sqrt{3})^3-2(2+\sqrt{3})^2-7(2+\sqrt{3})+5$

$$=2^3+\sqrt{3}^3+3[(2\times\sqrt{3})(2+\sqrt{3})]-2\left(2^2+\sqrt{3}^2+2(2\times\sqrt{3})\right)-14-7\sqrt{3}+5$$

$$=8+3\sqrt{3}+6\sqrt{3}(2+\sqrt{3})-2(4+3+4\sqrt{3})-9-7\sqrt{3}$$

$$=8+3\sqrt{3}+12\sqrt{3}+6.\sqrt{3}^2-2(7+4\sqrt{3})-9-7\sqrt{3}$$

$$=8+15\sqrt{3}+18-14-8\sqrt{3}-9-7\sqrt{3}=26-14-9+15\sqrt{3}-15\sqrt{3}=26-23=3$$

Answer.25. Given $\sqrt{5}=2.236$ *and* $\sqrt{10}=3.162$

$$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}=\frac{15}{\sqrt{10}+\sqrt{4}\times\sqrt{5}+\sqrt{4}+\sqrt{10}-\sqrt{5}-\sqrt{16}\times\sqrt{5}}$$

$$=\frac{15}{\sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-4\sqrt{5}}$$

$$=\frac{15}{3\sqrt{10}-3\sqrt{5}}=\frac{15}{3(\sqrt{10}-\sqrt{5})}=\frac{5}{\sqrt{10}-\sqrt{5}}=\frac{5}{3.162-2.236}=\frac{5}{0.926}=5.399(approx.)$$