ANSWER 1. We can write (x^4+1) as $(x^4+0x^3+0x^2+0x+1)$

$x-1)\overline{x^4+0x^3+0x^2+0x+1}(x^3+x^2+x+1$

x^4-x^3

$\underline{-\quad+}$

x^3+0x^2+0x+1

x^3-x^2

$\underline{-\quad+}$

x^2+0x+1

x^2-x

$\underline{-\quad+}$

$x+1$

$x-1$

$\underline{-\quad+}$

2

quotient $=(x^3+x^2+x+1)$ and remainder $=2$.

By verification:

$f(x)=x^4+1$

By substituting 1 in the place of x

$f(1)=1^4+1$

$f(1)=1+1$

so we get

$f(1)=2$, which is remainder.

ANSWER 2. $x+2)\overline{2x^4-6x^3+2x^2-x+2}(2x^3-10x^2+22x-45$

$2x^4+4x^3$

$\underline{-\quad-}$

$-10x^3+2x^2$

$-10x^3-20x^2$

$\underline{+\quad+}$

$22x^2-x$

$22x^2-44x$

$\underline{-\quad+}$

$-45x+2$

$-45x-90$

$\underline{+\quad+}$

$\underline{92}$

We know that ,

$(x+2)(2x^3-10x^2+22x-45)+92$

So we get ,

$=2x^4-10x^3+22x^2-45x+4x^3-20x^2+44x-90+92$

$=2x^4-6x^3+2x^2-x+2$

$=p(x)$

Therefore ,the division algorithm is verified.

ANSWER 3. Given,

$p(x) = x^3 - 6x^2 + 9x + 3$

To find the value of x,

Consider,

$g(x) = 0$

$x - 1 = 0$

So we get

$x = 1$

According to the remainder theorem,

p(x) divided by (x-1) obtains the remainder as g(1).

Calculating g(1)

$= 1^3 - 6(1)^2 + 9(1) + 3$

On further simplification

$= 1 - 6 + 9 + 3$

So we get

$= (-5) + 12$

$= 7$

Therefore , the remainder of the given expression is 7.

ANSWER 4. Given,

$p(x) = 2x^3 - 7x^2 + 9x - 13$

To find the value of x,

Consider,

$g(x) = 0$

$x - 3 = 0$

so we get

$x = 3$

According to the remainder theorem,

p(x) divided by (x-3) obtains the remainder as g(3).

Calculating g(3)

$= 2(3)^3 - 7(3)^2 + 9(3) - 13$

On further simplification
$= 2(27) - 7(9) + 27 - 13$

So we get
$= 54 - 63 + 27 - 13$
$= 5$

Therefore , the remainder of the given expression is 5.

ANSWER 5. Given,
$p(x) = 3x^4 - 6x^2 - 8x - 2$

To find the value of x,
Consider,
$g(x) = 0$

$x - 2 = 0$

so we get
$x = 2$
According to the remainder theorem,
p(x) divided by (x-2) obtains the remainder as g(2).

Calculating g(2)
$= 3(2)^4 - 6(2)^2 - 8(2) - 2$

On further simplification
$= 3(16) - 6(4) - 16 - 2$

So we get
$= 48 - 24 - 16 - 2$
$= 6$

Therefore , the remainder of the given expression is 6.

ANSWER 6. Given,
$p(x) = 2x^3 - 9x^2 + x + 15$

To find the value of x,
Consider,
$g(x) = 0$

$2x - 3 = 0$

so we get

2x= 3

By dividing

x= $\frac{3}{2}$

According to the remainder theorem,

p(x) divided by (2x-3) obtains the remainder as g($\frac{3}{2}$).

Calculating g($\frac{3}{2}$)

= $2(\frac{3}{2})^3 - 9(\frac{3}{2})^2 + (\frac{3}{2}) + 15$

On further simplification

=$2(\frac{27}{8}) - 9(\frac{9}{4}) + (\frac{3}{2}) + 15$

$= \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + 15$

$= \frac{27-81+6+60}{4}$

So we get

$= \frac{12}{4}$

= 3

Therefore , the remainder of the given expression is 3.

ANSWER 7. Given,

p(x)= $x^3 - 2x^2 - 8x - 1$

To find the value of x,

Consider,

g(x) = 0

x +1 = 0

so we get

x = -1

According to the remainder theorem,

p(x) divided by (x+ 1) obtains the remainder as g(- 1).

Calculating g(-1)

= $(-1)^3 - 2(-1)^2 - 8(-1) - 1$

On further simplification
= - 1 – 2 + 8 – 1

So we get
= - 3 +7
= 4

Therefore , the remainder of the given expression is 4.

ANSWER 8. Given,
$p(x)= 2x^3 + x^2 - 15x -12$

To find the value of x,
Consider,
g(x) = 0

x + 2= 0

So we get

x= (-2)
According to the remainder theorem,
p(x) divided by (x+ 2) obtains the remainder as g(- 2).

Calculating g(-2)
$= 2 (-2)^3 + (-2)^2 - 15(-2) – 12$

On further simplification
= - 16 + 4 +30 – 12

So we get
= -12 +18
= 6

Therefore , the remainder of the given expression is 6.

ANSWER 9. Given,
$p(x)= 6x^3 + 13x^2 + 3$

To find the value of x,
Consider,
g(x) = 0

3x+2 = 0

So we get

x = $(\frac{-2}{3})$

According to the remainder theorem,
p(x) divided by (3x+ 2) obtains the remainder as g$(\frac{-2}{3})$.

Calculating g$(\frac{-2}{3})$
$= 6\,(\frac{-2}{3})^3 + 13\,(\frac{-2}{3})^2 + 3$

On further simplification
$= 6(\frac{-8}{27}) + 13\,(\frac{4}{9}) + 3$

$= \frac{-48}{27} + \frac{52}{9} + 3$

$= \frac{-16}{9} + \frac{52}{9} + 3$

So we get
$= \frac{63}{9}$

= 7

Therefore , the remainder of the given expression is 7.

ANSWER 10. Given,
p(x)= $x^3 - 6x^2 + 2x - 4$

To find the value of x,
Consider,
g(x) = 0

$1 - \frac{3}{2}x = 0$

So we get

x= $\frac{2}{3}$
According to the remainder theorem,
p(x) divided by$(1 - \frac{3}{2}x)$ obtains the remainder as g$(\frac{2}{3})$.

Calculating g($\frac{2}{3}$)

= ($\frac{2}{3}$)³ - 6 ($\frac{2}{3}$)² + 2 ($\frac{2}{3}$) – 4

On further simplification

= $\frac{8}{27}$ - 6($\frac{4}{9}$) +$\frac{4}{3}$ - 4

= $\frac{8}{27}$ - $\frac{8}{3}$ + $\frac{4}{3}$ – 4

= $\frac{8-72+36-108}{27}$

So we get
= $\frac{-136}{27}$

Therefore , the remainder of the given expression is ($\frac{-136}{27}$).

ANSWER 11. Given,
$p(x)= 2x^3 + 3x^2 - 11x - 3$

To find the value of x,
Consider,
g(x) = 0

x+ $\frac{1}{2}$ = 0

So we get

x = $\frac{-1}{2}$
According to the remainder theorem,
p(x) divided by(x+ $\frac{1}{2}$) obtains the remainder as g($\frac{-1}{2}$).

Calculating g($\frac{-1}{2}$)

= 2($\frac{-1}{2}$)³ + 3($\frac{-1}{2}$)² - 11($\frac{-1}{2}$) – 3

On further simplification

$= 2\,(\frac{-1}{8}) + 3(\frac{1}{4}) + \frac{11}{2} - 3$

$= -\frac{1}{4} + \frac{3}{4} + \frac{11}{2} - 3$

$= \frac{-1+3+22-12}{2}$

So we get

$= \frac{12}{4}$

$= 3$

Therefore , the remainder of the given expression is 3 .

ANSWER 12. Given,

$p(x) = x^3 - ax^2 + 6x - a$

To find the value of x,

Consider,

$g(x) = 0$

$x - a = 0$

So we get

$x = a$

According to the remainder theorem,

p(x) divided by(x+ a) obtains the remainder as g(a).

Calculating g(a),

$= a^3 - a(a)^2 + 6a - a$

On further simplification

$= a^3 - a^3 + 5a$

So we get

$= 5a$

Therefore , the remainder of the given expression is 5a .

ANSWER 13. Consider $p(x) = (2x^3 + x^2 - ax + 2)$ and $q(x) = (2x^3 - 3x^2 - 3x + a)$

When p(x) and q(x) are divided by (x-2) the remainder obtained is p(2) and q(2).

To find a ,

Let us take

p(2) = q(2)

$2x^3 + x^2 - ax + 2 = 2x^3 - 3x^2 - 3x + a$

By substituting 2 in the place of x

$\Rightarrow 2(2)^3 + (2)^2 - a(2) + 2 = 2(2)^3 - 3(2)^2 - 3(2) + a$

On further calculation:

$\Rightarrow 2(8) + 4 - 2a + 2 = 2(8) - 3(4) - 6 + a$

$\Rightarrow 16 + 4 - 2a + 2 = 16 - 12 - 6 + a$

So we get

$\Rightarrow 22 - 2a = -2 + a$

$\Rightarrow 22 + 2 = 2a + a$

$\Rightarrow 24 = 3a$

By dividing

$a = 8$

Thus, the value of a is 8.

ANSWER 14. Given,

$p(x) = x^4 - 2x^3 + 3x^2 - ax + b$

Consider (x-1) = 0 where x = 1 and the remainder is 5

p(1) = 5

by substituting 1 in the place of x

$1^4 - 2(1)^3 + 3(1)^2 - a + b = 5$

So we get

$2 - a + b = 5$ ………….(1)

Consider (x +1) = 0 where x = -1 and the remainder is 19 .

p(-1) = 19

by substituting (-1) in the place of x

$(-1)^4 - 2(-1)^3 + 3(-1)^2 - a + b = 19$

So we get

$1 + 2 + 3 - a + b = 19$

$6 - a + b = 19$ ……………(2)

By adding equation (1) and (2)

8 + 2b = 24
2b = 24 – 8

By dividing 16 by 2 we get
b = 8 ………………..(3)

Now applying (3) in (1)

2 – a + 8 = 5

So we get

10 – a = 5
a = 5

Substituting the value of a and b in p(x) when divided by (x-2)
$p(x) = x^4 - 2x^3 + 3x^2 - ax + b$

by substituting 2 in the place of x

$p(2) = 2^4 - 2(2)^3 + 3(2)^2 - (5)(2) + 8$

On further calculation:
p(2) = 16 – 16 + 12 – 10 + 8

p(2) = 10

Therefore , the remainder when p(x) is divided by (x-2) is 10 .

ANSWER 15. Consider,
g(x) = 0

which means
x-2 = 0

x = 2
Now applying x= 2 in p (x) , we obtain
$p(x) = x^3 - 5x^2 + 4x – 3$

By substituting the value 2 in the place of x

$p(2)= 2(2)^3 - 5 (2)^2 +4(2)- 3$

On further calculation:

p(2)= 8-20 + 8 – 3

So we get

p(2)= -4 -3

p(2)= - 7 ≠ 0

Therefore, it is proved that p(x) is not a multiple of g(x).

ANSWER 16. Consider,

g(x) = 0

2x + 1 = 0

So we get

2x = - 1

x = $\frac{-1}{2}$

Now apply x = $\frac{-1}{2}$ in p(x)

p (x) = $2x^3 - 11x^2 - 4x + 5$

By substituting $\frac{-1}{2}$ in the place of x

p ($\frac{-1}{2}$) = $2(\frac{-1}{2})^3 - 11(\frac{-1}{2})^2 - 4(\frac{-1}{2}) + 5$

So we get

p($\frac{-1}{2}$) = $\frac{-1}{4} - \frac{11}{2} + 7$

p($\frac{-1}{2}$) = $\frac{-1-11+28}{4}$

by dividing 16 by 4

p($\frac{-1}{2}$) = ($\frac{16}{4}$)

So we get

$p(\frac{-1}{2}) = 4 \neq 0$

Hence , it is shown that g(x) is not a factor of p (x).