RD SHARMA

Solutions

Class 9 Maths

Chapter 3

Ex 3.1

1. Simplify each of the following:

(i) $\sqrt[3]{4} \times \sqrt[3]{16}$

(ii) $\frac{\sqrt[4]{1250}}{\sqrt[4]{2}}$

Sol:

(i) $\frac{\sqrt[4]{1250}}{\sqrt[4]{2}}$

(Note: $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b}$)

$= \sqrt[3]{4 \times 16}$

$= \sqrt[3]{64}$

$= \sqrt[3]{4^3}$

$= (4^3)^{\frac{1}{3}}$

$= 4^{(3 \times \frac{1}{3})}$

$= 4^1$

= 4

(ii) $\frac{\sqrt[4]{1250}}{\sqrt[4]{2}}$

(Note: $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$)

$= \sqrt[4]{\frac{1250}{2}}$

$= \sqrt[4]{\frac{2 \times 625}{2}}$

$= \sqrt[4]{625}$

$= \sqrt[4]{15^4}$

$= 15^{(4 \times \frac{1}{4})}$

= 15

2. Simplify the following expressions:

(i) $(4 + \sqrt{7})(3 + \sqrt{2})$

(ii) $(3 + \sqrt{3})(5 - \sqrt{2})$

(iii) $(\sqrt{5} - 2)(\sqrt{3} - \sqrt{5})$

Solution:

(i) $(4+\sqrt{7})(3+\sqrt{2})$

= $12 + 4\sqrt{2} + 3\sqrt{7} + \sqrt{7\times 2}$

= $12 + 4\sqrt{2} + 3\sqrt{7} + \sqrt{14}$

(ii) $(3+\sqrt{3})(5-\sqrt{2})$

= $15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{3\times 2}$

= $15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{6}$

(iii) $(\sqrt{5}-2)(\sqrt{3}-\sqrt{5})$

= $\sqrt{15}-\sqrt{25}-2\sqrt{3}+2\sqrt{5}$

= $\sqrt{15}-\sqrt{5\times 5}-2\sqrt{3}+2\sqrt{5}$

= $\sqrt{15}-5-2\sqrt{3}+2\sqrt{5}$

3. Simplify the following expressions:

(i) $(11+\sqrt{11})(11-\sqrt{11})$

(ii) $(5+\sqrt{7})(5-\sqrt{7})$

(iii) $(\sqrt{8}-\sqrt{2})(\sqrt{8}+\sqrt{2})$

(iv) $(3+\sqrt{3})(3-\sqrt{3})$

(v) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Solution:

(i) $(11+\sqrt{11})(11-\sqrt{11})$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

So, $11^2 - 11$

$121 - 11 = 110$

(ii) $(5+\sqrt{7})(5-\sqrt{7})$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

So, $5^2 - 7$

$25 - 7 = 18$

(iii) $(\sqrt{8}-\sqrt{2})(\sqrt{8}+\sqrt{2})$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$\sqrt{8\times 8}-\sqrt{2\times 2} = 8 - 2$

= 6

(iv) $(3+\sqrt{3})(3-\sqrt{3})$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

= 9 − $\sqrt{3 \times 3}$

= 6

(v) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

= $\sqrt{5 \times 5} - \sqrt{2 \times 2}$

= 5 − 2

= 3

4. Simplify the following expressions:

(i) $(\sqrt{3} + \sqrt{7})^2$

(ii) $(\sqrt{5} - \sqrt{3})^2$

(iii) $(2\sqrt{5} + 3\sqrt{2})^2$

Solution:

(i) $(\sqrt{3} + \sqrt{7})^2$

As we know , (a + b) ^{2} = $(a^2 + 2 \times a \times b + b^2)$

= $\sqrt{3^2} + 2 \times \sqrt{3} \times \sqrt{7} + \sqrt{7^2}$

= 3 + $2 \times \sqrt{3 \times 7}$ + 7

= 10 + $2 \times \sqrt{21}$

(ii) $(\sqrt{5} - \sqrt{3})^2$

As we know , (a − b) ^{2} = $(a^2 - 2 \times a \times b + b^2)$

(iii) $(2\sqrt{5} + 3\sqrt{2})^2$

As we know , (a + b) ^{2} = $(a^2 + 2 \times a \times b + b^2)$

= $4\sqrt{5 \times 5} + 2 \times 2\sqrt{5} \times 3\sqrt{2} + 9\sqrt{2 \times 2}$

= 20 + $12\sqrt{10}$ + 18

= 28 + $12\sqrt{10}$

RD SHARMA
Solutions
Class 9 Maths
Chapter 3
Ex 3.2

1. Rationalize the denominator of each of the following:

(i) $\frac{3}{\sqrt{5}}$

(ii) $\frac{3}{2\sqrt{5}}$

(iii) $\frac{1}{\sqrt{12}}$

(iv) $\frac{\sqrt{2}}{\sqrt{3}}$

(v) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

(vi) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

(vii) $\frac{3\sqrt{2}}{\sqrt{5}}$

Solution:

(i) $\frac{3}{\sqrt{5}}$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{5}$

$= \frac{3\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}}$

$= \frac{3\times\sqrt{5}}{5}$

(ii) $\frac{3}{2\sqrt{5}}$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{5}$

$= \frac{3\times\sqrt{5}}{2\sqrt{5}\times\sqrt{5}}$

$= \frac{3\sqrt{5}}{2\times\sqrt{5\times5}}$

$= \frac{3\sqrt{5}}{2\times5}$

$= \frac{3\sqrt{5}}{10}$

(iii) $\frac{1}{\sqrt{12}}$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{12}$

$= \frac{1\times\sqrt{12}}{\sqrt{12}\times\sqrt{12}}$

$= \frac{\sqrt{12}}{\sqrt{12\times 12}}$

$= \frac{\sqrt{12}}{12}$

(iv) $\frac{\sqrt{2}}{\sqrt{3}}$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{3}$

$= \frac{\sqrt{2}\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$= \frac{\sqrt{2\times 3}}{\sqrt{3\times 3}}$

$= \frac{\sqrt{6}}{3}$

(v) $\frac{\sqrt{3}+1}{\sqrt{2}}$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{2}$

$= \frac{(\sqrt{3}+1)\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$

$= \frac{(\sqrt{3}\times\sqrt{2})+\sqrt{2}}{\sqrt{2\times 2}}$

$= \frac{\sqrt{6}+\sqrt{2}}{2}$

(vi) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{3}$

$= \frac{(\sqrt{2}+\sqrt{5})\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$= \frac{(\sqrt{2}\times\sqrt{3})+(\sqrt{5}\times\sqrt{3})}{\sqrt{3}\times\sqrt{3}}$

$= \frac{\sqrt{6}+\sqrt{15}}{3}$

(vii) $\frac{3\sqrt{2}}{\sqrt{5}}$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{5}$

$= \frac{3\sqrt{2}\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}}$

$= \frac{3\sqrt{2\times5}}{\sqrt{5\times5}}$

$= \frac{3\sqrt{10}}{5}$

2. Find the value to three places of decimals of each of the following.

It is given that $\sqrt{2}$ = 1.414, $\sqrt{3}$ = 1.732, $\sqrt{5}$ = 2.236, $\sqrt{10}$ = 3.162.

(i) $\frac{2}{\sqrt{3}}$

(ii) $\frac{3}{\sqrt{10}}$

(iii) $\frac{\sqrt{5}+1}{\sqrt{2}}$

(iv) $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$

(v) $\frac{2+\sqrt{3}}{3}$

(vi) $\frac{\sqrt{2}-1}{\sqrt{5}}$

Solution:

Given, $\sqrt{2}$ = 1.414, $\sqrt{3}$ = 1.732, $\sqrt{5}$ = 2.236, $\sqrt{10}$ = 3.162.

(i) $\frac{2}{\sqrt{3}}$

Rationalizing the denominator by multiplying both numerator and denominator with $\sqrt{3}$

$= \frac{2\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$= \frac{2\sqrt{3}}{\sqrt{3\times3}}$

$= \frac{2\sqrt{3}}{3}$

$= \frac{2\times1.732}{3}$

$= \frac{3.464}{3}$

= 1.154666666

(ii) $\frac{3}{\sqrt{10}}$

Rationalizing the denominator by multiplying both numerator and denominator with $\sqrt{10}$

$= \frac{3\sqrt{10}}{\sqrt{10}\times\sqrt{10}}$

$= \frac{3\sqrt{10}}{\sqrt{10\times10}}$

$= \frac{3\sqrt{10}}{10}$

$= \frac{9.486}{10}$

= 0. 9486

(iii) $\frac{\sqrt{5}+1}{\sqrt{2}}$

Rationalizing the denominator by multiplying both numerator and denominator with $\sqrt{3}$

$= \frac{(\sqrt{5}\times\sqrt{2})+\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$

$= \frac{\sqrt{10}+\sqrt{2}}{2}$

$= \frac{4.576}{2}$

= 2.288

(iv) $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$

Rationalizing the denominator by multiplying both numerator and denominator with $\sqrt{2}$

$= \frac{(\sqrt{10}\times\sqrt{2})+(\sqrt{15}\times\sqrt{2})}{\sqrt{2}\times\sqrt{2}}$

$= \frac{\sqrt{20}+\sqrt{30}}{2}$

$= \frac{(\sqrt{10}\times\sqrt{2})+(\sqrt{10}\times\sqrt{3})}{2}$

$= \frac{(3.162\times1.414)+(3.162\times1.732)}{2}$

$= \frac{(4.471068)+(5.476584)}{2}$

$= \frac{9.947652}{2}$

= 4.973826

(v) $\frac{2+\sqrt{3}}{3}$

$= \frac{2+1.732}{3}$

$= \frac{3.732}{3}$

= 1.244

(vi) $\frac{\sqrt{2}-1}{\sqrt{5}}$

Rationalizing the denominator by multiplying both numerator and denominator with $\sqrt{5}$

$= \frac{(\sqrt{2}\times\sqrt{5})-\sqrt{5}}{\sqrt{5}\times\sqrt{5}}$

$= \frac{\sqrt{10}-\sqrt{5}}{5}$

$= \frac{3.162-2.236}{5}$

$= \frac{0.926}{5}$

= 0.1852

3. Express each one of the following with rational denominator:

(i) $\frac{1}{3+\sqrt{2}}$

(ii) $\frac{1}{\sqrt{6}-\sqrt{5}}$

(iii) $\frac{16}{\sqrt{41}-5}$

(iv) $\frac{30}{5\sqrt{3}-3\sqrt{5}}$

(v) $\frac{1}{2\sqrt{5}-\sqrt{3}}$

(vi) $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$

(vii) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

(viii) $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$

(ix) $\frac{b^2}{\sqrt{(a^2+b^2)}+a}$

Solution:

(i) $\frac{1}{3+\sqrt{2}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3-\sqrt{2}$

$= \frac{3-\sqrt{2}}{(3+\sqrt{2})(3-\sqrt{2})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{3-\sqrt{2}}{9-2}$

$= \frac{3-\sqrt{2}}{7}$

(ii) $\frac{1}{\sqrt{6}-\sqrt{5}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{6}+\sqrt{5}$

$= \frac{\sqrt{6}+\sqrt{2}}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{\sqrt{6}+\sqrt{2}}{6-2}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

(iii) $\frac{16}{\sqrt{41}-5}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{41}+5$

$= \frac{16\times(\sqrt{41}+5)}{(\sqrt{41}-5)(\sqrt{41}+5)}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{16\sqrt{41}+80}{41-5}$

$= \frac{16\sqrt{41}+80}{16}$

$= \frac{16(\sqrt{41}+5)}{16}$

$= \sqrt{41}+5$

(iv) $\frac{30}{5\sqrt{3}-3\sqrt{5}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $5\sqrt{3}+3\sqrt{5}$

$= \frac{30\times(5\sqrt{3}+3\sqrt{5})}{(5\sqrt{3}-3\sqrt{5})(5\sqrt{3}+3\sqrt{5})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{30\times(5\sqrt{3}+3\sqrt{5})}{75-45}$

$= \frac{30\times(5\sqrt{3}+3\sqrt{5})}{30}$

$= 5\sqrt{3} + 3\sqrt{5}$

(v) $\frac{1}{2\sqrt{5}-\sqrt{3}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2\sqrt{5} + \sqrt{3}$

$= \frac{2\sqrt{5}+\sqrt{3}}{(2\sqrt{5}-\sqrt{3})(2\sqrt{5}+\sqrt{3})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{2\sqrt{5}+\sqrt{3}}{20-3}$

$= \frac{2\sqrt{5}+\sqrt{3}}{17}$

(vi) $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2\sqrt{2} + \sqrt{3}$

$= \frac{(\sqrt{3}+1)(2\sqrt{2}+\sqrt{3})}{(2\sqrt{2}+\sqrt{3})(2\sqrt{2}-\sqrt{3})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{(2\sqrt{6}+3+2\sqrt{2}+\sqrt{3})}{8-3}$

$= \frac{(2\sqrt{6}+3+2\sqrt{2}+\sqrt{3})}{5}$

(vii) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $6 - 4\sqrt{2}$

$= \frac{(6-4\sqrt{2})(6-4\sqrt{2})}{(6+4\sqrt{2})(6-4\sqrt{2})}$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$= \frac{(6-4\sqrt{2})^2}{36-32}$

As we know , $(a - b)^2 = (a^2 - 2 \times a \times b + b^2)$

$= \frac{36-48\sqrt{2}+32}{4}$

$= \frac{68-48\sqrt{2}}{4}$

$= \frac{4(17-12\sqrt{2})}{4}$

$= 17 - 12\sqrt{2}$

(viii) $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2\sqrt{5} - 3$

$= \frac{(3\sqrt{2}+1)\times(2\sqrt{5}-3)}{(2\sqrt{5}-3)(2\sqrt{5}-3)}$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$= \frac{6\sqrt{10}-9\sqrt{2}+2\sqrt{5}-3}{(20-9)}$

$= \frac{6\sqrt{10}-9\sqrt{2}+2\sqrt{5}-3}{11}$

(ix) $\frac{b^2}{\sqrt{(a^2+b^2)}+a}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{(a^2 + b^2)} - a$

$= \frac{b^2(\sqrt{(a^2+b^2)}-a)}{(\sqrt{(a^2+b^2)}+a)(\sqrt{(a^2+b^2)}-a)}$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$= \frac{b^2(\sqrt{(a^2+b^2)}-a)}{(a^2+b^2)-a^2)}$

$= \frac{b^2(\sqrt{(a^2+b^2)}-a)}{b^2}$

4. Rationalize the denominator and simplify:

(i) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(ii) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$

(iii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$

(iv) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

(v) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

(vi) $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

Solution:

(i) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{3}-\sqrt{2}$

= $\frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

= $\frac{(\sqrt{3}-\sqrt{2})^2}{3-2}$

As we know , $(a - b)^2$ = $(a^2 - 2 \times a \times b + b^2)$

= $\frac{3-2\sqrt{3}\sqrt{2}+2}{1}$

= $5 - 2\sqrt{6}$

(ii) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $7 - 4\sqrt{3}$

= $\frac{(5+2\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

= $\frac{(5+2\sqrt{3})(7-4\sqrt{3})}{49-48}$

= 35 − $20\sqrt{3}$ + $14\sqrt{3}$ − 24

= 11 − $6\sqrt{3}$

(iii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3+2\sqrt{2}$

= $\frac{(1+\sqrt{2})(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

= $\frac{(1+\sqrt{2})(3+2\sqrt{2})}{9-8}$

= $3+2\sqrt{2}+3\sqrt{2}+4$

= 7 + $5\sqrt{2}$

(iv) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3\sqrt{5}+2\sqrt{6}$

= $\frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{(3\sqrt{5}-2\sqrt{6})(3\sqrt{5}+2\sqrt{6})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

= $\frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{45-24}$

= $\frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{21}$

= $\frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{21}$

= $\frac{6\sqrt{30}+24-15-2\sqrt{30}}{21}$

= $\frac{4\sqrt{30}+9}{21}$

(v) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{48}-\sqrt{18}$

= $\frac{(4\sqrt{3}+5\sqrt{2})(\sqrt{48}-\sqrt{18})}{(\sqrt{48}+\sqrt{18})(\sqrt{48}-\sqrt{18})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{(4\sqrt{3}+5\sqrt{2})(\sqrt{48}-\sqrt{18})}{48-18}$

$= \frac{48-12\sqrt{6}+20\sqrt{6}-30}{30}$

$= \frac{18+8\sqrt{6}}{30}$

$= \frac{9+4\sqrt{6}}{15}$

(vi) $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2\sqrt{2}-3\sqrt{3}$

$= \frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{3})}$

$= \frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{8-27}$

$= \frac{(4\sqrt{6}-2\sqrt{10})-18+3\sqrt{15})}{-19}$

$= \frac{(18-4\sqrt{6}+2\sqrt{10}-3\sqrt{15})}{19}$

5. Simplify:

(i) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$

(ii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

(iii) $\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$

(iv) $\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}}$

(v) $\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$

Solution:

(i) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3\sqrt{2}-2\sqrt{3}$ for $\frac{1}{3\sqrt{2}+2\sqrt{3}}$ and the rationalizing factor $\sqrt{3}+\sqrt{2}$ for $\frac{1}{\sqrt{3}-\sqrt{2}}$

$= \frac{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}-2\sqrt{3})}{(3\sqrt{2}+2\sqrt{3})(3\sqrt{2}-2\sqrt{3})} + \frac{\sqrt{12}(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$

Now, (a + b) (a − b) = $(a^2 - b^2)$

= $\frac{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}-2\sqrt{3})}{18-12} + \frac{\sqrt{12}(\sqrt{3}+\sqrt{2})}{3-2}$

As we know , (a − b) ^{2} = $(a^2 - 2 \times a \times b + b^2)$

= $\frac{(3\sqrt{2})^2-(2\times3\sqrt{2}\times2\sqrt{3})+(2\sqrt{3})^2}{6} + 2\sqrt{3}(\sqrt{3}+\sqrt{2})$

= $\frac{(18-12\sqrt{6}+12)}{6} + (6+2\sqrt{6})$

= $3 - 2\sqrt{6} + 2 + (6+2\sqrt{6})$

= $5 - 2\sqrt{6} + (6+2\sqrt{6})$

= 11

(ii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{5}+\sqrt{3}$ for $\frac{1}{\sqrt{5}-\sqrt{3}}$ and the rationalizing factor $\sqrt{5}-\sqrt{3}$ for $\frac{1}{\sqrt{5}+\sqrt{3}}$

= $\frac{(\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} + \frac{(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

Now as we know, (a + b) (a − b) = $(a^2 - b^2)$, (a − b) ^{2} = $(a^2 - 2 \times a \times b + b^2)$ and $(a+b)^2 = (a^2 + 2 \times a \times b + b^2)$

= $\frac{5+2\times\sqrt{5}\times\sqrt{3}+3}{5-3} + \frac{5-2\times\sqrt{3}\times\sqrt{5}+3}{5-3}$

= $\frac{8+2\sqrt{15}+8-2\sqrt{15}}{2}$

= $\frac{16}{2}$

= 8

(iii) $\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3-\sqrt{5}$ for $\frac{1}{3+\sqrt{5}}$ and the rationalizing factor $3+\sqrt{5}$ for $\frac{1}{3-\sqrt{5}}$

$\frac{(7+3\sqrt{5})(3-\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} - \frac{(7-3\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3-\sqrt{5})}$

Now as we know, (a + b) (a − b) = $(a^2 - b^2)$

= $\frac{(7+3\sqrt{5})(3-\sqrt{5})}{9-5} - \frac{(7-3\sqrt{5})(3+\sqrt{5})}{9-5}$

$= \frac{(21-7\sqrt{5}+9\sqrt{5}-15)}{4} - \frac{(21+7\sqrt{5}-9\sqrt{5}-15)}{4}$

$= \frac{(6+2\sqrt{5})}{4} - \frac{(6-2\sqrt{5})}{4}$

$= \frac{4\sqrt{5}}{4}$

$= \sqrt{5}$

(iv) $\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2-\sqrt{3}$ for $\frac{1}{2+\sqrt{3}}$, the rationalizing factor $\sqrt{5}+\sqrt{3}$ for $\frac{1}{\sqrt{5}-\sqrt{3}}$, and the rationalizing factor $2+\sqrt{5}$ for $\frac{1}{2-\sqrt{5}}$

$= \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} + \frac{2\times(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} + \frac{2+\sqrt{5}}{(2-\sqrt{5})(2+\sqrt{5})}$

Since, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{2-\sqrt{3}}{4-3} + \frac{2\times(\sqrt{5}+\sqrt{3})}{5-3} + \frac{2+\sqrt{5}}{4-5}$

$= \frac{2-\sqrt{3}}{1} + \frac{2\sqrt{5}+2\sqrt{3}}{2} + \frac{2+\sqrt{5}}{-1}$

$= \frac{4-2\sqrt{3}+2\sqrt{5}+2\sqrt{3}-4-2\sqrt{5}}{2}$

$= \frac{0}{2}$

$= 0$

(v) $\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{5}-\sqrt{3}$ for $\frac{1}{\sqrt{5}+\sqrt{3}}$, the rationalizing factor $\sqrt{3}-\sqrt{2}$ for $\frac{1}{\sqrt{3}+\sqrt{2}}$, and the rationalizing factor $\sqrt{5}-\sqrt{2}$ for $\frac{1}{\sqrt{5}+\sqrt{2}}$

$= \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} + \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} - \frac{3\times(\sqrt{5}-\sqrt{2})}{\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$

Since, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{2(\sqrt{5}-\sqrt{3})}{5-3} + \frac{\sqrt{3}-\sqrt{2}}{3-2} - \frac{3\times(\sqrt{5}-\sqrt{2})}{5-2}$

$= \frac{2\sqrt{5}-2\sqrt{3}}{2} + \frac{\sqrt{3}-\sqrt{2}}{1} - \frac{3\times\sqrt{5}-3\sqrt{2}}{3}$

$= \frac{6\sqrt{5}-6\sqrt{3}+6\sqrt{3}-6\sqrt{2}-6\sqrt{5}+6\sqrt{2}}{3}$

$= \frac{0}{3}$

= 0

6. **In each of the following determine rational numbers a and b:**

(i) $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$

(ii) $\frac{4+\sqrt{2}}{2+\sqrt{2}} = a - \sqrt{b}$

(iii) $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$

(iv) $\frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$

(v) $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$

(vi) $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$

Solution:

(i) Given,

$\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{3} - 1$

$= \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{3-2\sqrt{3}+1}{3-1}$

$= \frac{4-2\sqrt{3}}{2}$

$= 2 - \sqrt{3}$

$2 - \sqrt{3} = a - b\sqrt{3}$

On comparing the rational and irrational parts of the above equation, we get,

a = 2 and b = 1

(ii) Given;

$\frac{4+\sqrt{2}}{2+\sqrt{2}} = a - \sqrt{b}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2 - \sqrt{2}$

$= \frac{(4+\sqrt{2})(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{(8-4\sqrt{2}+2\sqrt{2}-2)}{4-2}$

$= \frac{(6-2\sqrt{2})}{2}$

$= 3 - \sqrt{2}$

$3 - \sqrt{2} = a - \sqrt{b}$

On comparing the rational and irrational parts of the above equation, we get,

a = 3 and b = 2

(iii) Given,

$\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3 + \sqrt{2}$

$= \frac{(3+\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{(9+6\sqrt{2}+2)}{9-2}$

$= \frac{(11+6\sqrt{2})}{7}$

$= \frac{11}{7} + \frac{6\sqrt{2}}{7}$

$\frac{11}{7} + \frac{6\sqrt{2}}{7} = a + b\sqrt{2}$

On comparing the rational and irrational parts of the above equation, we get,

a = $\frac{11}{7} + \frac{6\sqrt{2}}{7}$ and

b = $\frac{6}{7}$

(iv) Given,

$\frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $7 - 4\sqrt{3}$

$= \frac{(5+3\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{(35-20\sqrt{3}+21\sqrt{3}-36)}{49-48}$

= -1 + $\sqrt{3}$

-1 + $\sqrt{3}$ = $a + b\sqrt{3}$

On comparing the rational and irrational parts of the above equation, we get,

a = -1 and
b = 1

(v) $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}$ = $a - b\sqrt{77}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{11} - \sqrt{7}$

$= \frac{(\sqrt{11}-\sqrt{7})(\sqrt{11}-\sqrt{7})}{(\sqrt{11}+\sqrt{7})(\sqrt{11}-\sqrt{7})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{(11-\sqrt{77}-\sqrt{77}+7)}{11-7}$

$= \frac{(18-2\sqrt{77})}{4}$

$= \frac{9}{2} - \frac{\sqrt{77}}{2}$

$\frac{9}{2} - \frac{\sqrt{77}}{2}$ = $a - b\sqrt{77}$

On comparing the rational and irrational parts of the above equation, we get,

a = $\frac{9}{2}$ and
b = $\frac{1}{2}$

(vi) Given,

$= \frac{4+3\sqrt{5}}{4-3\sqrt{5}}$ = $a + b\sqrt{5}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $4 + 3\sqrt{5}$

$= \frac{(4+3\sqrt{5})(4+3\sqrt{5})}{(4-3\sqrt{5})(4+3\sqrt{5})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{(16+24\sqrt{5}+45)}{-29}$

$= \frac{(61+24\sqrt{5})}{-29}$

$= \frac{-61}{29} - \frac{(24\sqrt{5})}{29}$

$\frac{-61}{29} - \frac{(24\sqrt{5})}{29} = a + b\sqrt{5}$

On comparing the rational and irrational parts of the above equation, we get,

$a = \frac{-61}{29}$, and
$b = \frac{-24}{29}$

7. If x = 2 + $\sqrt{3}$, find the value of $x^3 + \frac{1}{x^3}$

Solution:

Given,

x = 2 + $\sqrt{3}$,

To find the value of $x^3 + \frac{1}{x^3}$

We have, x = 2 + $\sqrt{3}$,

$\frac{1}{x} = \frac{1}{2+\sqrt{3}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2 - \sqrt{3}$ for $\frac{1}{2+\sqrt{3}}$

$= \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$

Since, (a + b) (a − b) = $(a^2 - b^2)$

$\frac{1}{x} = \frac{2-\sqrt{3}}{4-3}$

$x + \frac{1}{x} = 2 + \sqrt{3} + 2 - \sqrt{3}$

$x + \frac{1}{x} = 4$

We know that, $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

$(x^3 + \frac{1}{x^3}) = (x + \frac{1}{x})(x^2 - x.\frac{1}{x} + \frac{1}{x^2})$

$(x^3 + \frac{1}{x^3}) = (x + \frac{1}{x})(x^2 + \frac{1}{x}^2 - 1)$

$(x^3 + \frac{1}{x^3}) = (x + \frac{1}{x})(x^2 + \frac{1}{x^2} + 2 - 2 - 1)$

$(x^3 + \frac{1}{x^3}) = (x + \frac{1}{x})(x^2 + \frac{1}{x^2} + 2(x.\frac{1}{x}) - 2 - 1)$

$(x^3 + \frac{1}{x^3}) = (x + \frac{1}{x})((x + \frac{1}{x})^2 - 3)$

Putting the value of $x + \frac{1}{x}$ in the above equation, we get,

$$(x^3 + \frac{1}{x^3}) = (4)(4^2 - 3)$$
$$(x^3 + \frac{1}{x^3}) = 52$$

8. If x = 3 + $\sqrt{8}$, find the value of $(x^2 + \frac{1}{x^2})$

Solution:

Given,

x = 3 + $\sqrt{8}$,

To find the value of $(x^2 + \frac{1}{x^2})$

We have, x = 3 + $\sqrt{8}$,

$$\frac{1}{x} = \frac{1}{3+\sqrt{8}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3 - \sqrt{8}$ for $\frac{1}{3+\sqrt{8}}$

$$\frac{1}{x} = \frac{3-\sqrt{8}}{(3+\sqrt{8})(3-\sqrt{8})}$$

Since, (a + b) (a − b) = $(a^2 - b^2)$

$$\frac{1}{x} = \frac{3-\sqrt{8}}{9-8}$$
$$\frac{1}{x} = 3 - \sqrt{8}$$
$$(x^2 + \frac{1}{x^2}) = ((3 + \sqrt{8})^2(3 - \sqrt{8})^2)$$
$$(x^2 + \frac{1}{x^2}) = ((9 + 8 + 6\sqrt{8}) + (9 + 8 - 6\sqrt{8}))$$

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9. Find the value of $\frac{6}{\sqrt{5}-\sqrt{3}}$, it being given that $\sqrt{3}$ = 1.732 and $\sqrt{5}$ = 2.236.

Given,

$$\frac{6}{\sqrt{5}-\sqrt{3}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{5} + \sqrt{3}$ for $\frac{1}{\sqrt{5}-\sqrt{3}}$

$$= \frac{6(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$$

Since, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{6\sqrt{5}+6\sqrt{3}}{5-3}$

$= \frac{6\sqrt{5}+6\sqrt{3}}{2}$

$= 3(\sqrt{5} + \sqrt{3})$

= 3(2.236+1.732)

= 3(3.968)

= 11.904

10. Find the values of each of the following correct to three places of decimals, it being given that

$\sqrt{2}$ **= 1.414,** $\sqrt{3} = 1.732,$ $\sqrt{5}$ **= 2.236,** $\sqrt{6}$ **= 2.4495,** $\sqrt{10}$ **= 3.162**

(i) $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3 - 2\sqrt{5}$

$= \frac{(3-\sqrt{5})(3-2\sqrt{5})}{(3+2\sqrt{5})(3+2\sqrt{5})}$

Since, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{(3-\sqrt{5})(3-2\sqrt{5})}{9-20}$

$= \frac{(9-6\sqrt{5}-3\sqrt{5}+10)}{-11}$

$= \frac{(19-9\sqrt{5})}{-11}$

$= \frac{(9\sqrt{5}-19)}{11}$

$= \frac{(9(2.236))-19)}{11}$

$= \frac{(20.124-19)}{11}$

$= \frac{1.124}{11}$

= 0.102

(ii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3 + 2\sqrt{2}$

$= \frac{(1+\sqrt{2})(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})}$

As we know, (a + b) (a − b) = $(a^2 - b^2)$

$= \frac{(1+\sqrt{2})(3+2\sqrt{2})}{9-8}$

$= 3 + 2\sqrt{2} + 3\sqrt{2} + 4$

$= 7 + 5\sqrt{2}$

= 7 + 7.07

= 14.07

11. If x = $\frac{\sqrt{3}+1}{2}$, find the value of $4x^3 + 2x^2 - 8x + 7$.

Solution:

Given,

x = $\frac{\sqrt{3}+1}{2}$ and given to find the value of $4x^3 + 2x^2 - 8x + 7$

$2x = \sqrt{3} + 1$
$2x - 1 = \sqrt{3}$

Now, squaring on both the sides, we get,

$(2x - 1)^2 = 3$

$4x^2 - 4x + 1 = 3$
$4x^2 - 4x + 1 - 3 = 0$
$4x^2 - 4x - 2 = 0$
$2x^2 - 2x - 1 = 0$

Now taking $4x^3 + 2x^2 - 8x + 7$

$2x(2x^2 - 2x - 1) + 4x^2 + 2x + 2x^2 - 8x + 7$
$2x(2x^2 - 2x - 1) + 6x^2 - 6x + 7$

As, $2x^2 - 2x - 1 = 0$

$2x(0)+3(2x^2 - 2x - 1))+7+3$
0+3(0)+10
10