RD SHARMA

Solutions

Class 9 Maths

Chapter 2

Ex 2.1

1. Simplify the following:

(i) $3(a^4 b^3)^{10} \times 5 (a^2 b^2)^3$

Solution:

$= 3(a^{40} b^{30}) \times 5 (a^6 b^6)$

$= 15 (a^{46} b^{36})$

(ii) $(2x^{-2} y^3)^3$

Solution:

$= (2^3 x^{-2\times3} y^{3\times3})$

$= 8x^{-6} y^9$

(iii) $\frac{(4\times10^7)(6\times10^{-5})}{8\times10^4}$

Solution:

$\frac{(4\times10^7)(6\times10^{-5})}{8\times10^4}$

$= \frac{(24\times10^7\times10^{-5})}{8\times10^4}$

$= \frac{(24\times10^{7-5})}{8\times10^4}$

$= \frac{(24\times10^2)}{8\times10^4}$

$= \frac{(3\times10^2)}{10^4}$

$= \frac{3}{100}$

(iv) $\frac{4ab^2(-5ab^3)}{10a^2b^2}$

Solution:

$= \frac{-20a^2b^5}{10a^2b^2}$

$= -2b^3$

(v) $(\frac{x^2y^2}{a^2b^3})^n$

Solution:

$= \frac{x^{2n}y^{2n}}{a^{2n}b^{3n}}$

(vi) $\frac{(a^{3n-9})^6}{a^{2n-4}}$

Solution:

$= \frac{a^{18n-54}}{a^{2n-4}}$

$= a^{18n-2n-54+4}$

$= a^{16n-50}$

2. If a = 3 and b = -2, find the values of:

(i) $a^a + b^b$

(ii) $a^b + b^a$

(iii) $a^b + b^a$

Solution:

(i) We have,

$a^a + b^b$

$= 3^3 + (-2)^{-2}$

$= 3^3 + (-\frac{1}{2})^2$

$= 27 + \frac{1}{4}$

$= \frac{109}{4}$

(ii) $a^b + b^a$

$= 3^{-2} + (-2)^3$

$= (\frac{1}{3})^2 + (-2)^3$

$= \frac{1}{9} - 8$

$= -\frac{71}{9}$

(iii) We have,

$a^b + b^a$

$= (3 + (-2))^{3(-2)}$

$= (3-2))^{-6}$

$= 1^{-6}$

= 1

3.Prove that:

(i) $(\frac{x^a}{x^b})^{a^2+ab+b^2} \times (\frac{x^b}{x^c})^{b^2+bc+c^2} \times (\frac{x^c}{x^a})^{c^2+ca+a^2}$ **= 1**

(ii) $\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2}$ **=** $x^{2(a^3+b^3+c^3)}$

(iii) $\left(\frac{x^a}{x^b}\right)^c \times \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b$ **= 1**

Solution:

(i) To prove

$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$ = 1

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{x^{a^3+a^2b+ab^2}}{x^{a^2b+ab^2+b^3}} \times \frac{x^{b^3+b^2c+bc^2}}{x^{b^2c+bc^2+c^3}} \times \frac{x^{c^3+c^2a+ca^2}}{x^{c^2a+ca^2+a^3}}$$
$$x^{a^3+a^2b+ab^2-(b^3+a^2b+ab^2)} \times x^{b^3+b^2c+bc^2-(c^3+b^2c+bc^2)} \times x^{c^3+c^2a+ca^2-(a^3+c^2a+ca^2)}$$
$$x^{a^3-b^3} \times x^{b^3-c^3} \times x^{c^3-a^3}$$
$$x^{a^3-b^3+b^3-c^3+c^3-a^3}$$
$$x^0$$
$$1$$

Or,

Therefore, LHS = RHS

Hence proved

(ii) To prove,

$\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2} = x^{2(a^3+b^3+c^3)}$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$x^{(a+b)(a^2-ab+b^2)} \times x^{(b+c)(b^2-bc+c^2)} \times x^{(c+a)(c^2-ca+a^2)}$$
$$x^{a^3+b^3} \times x^{b^3+c^3} \times x^{c^3+a^3}$$
$$x^{a^3+b^3+b^3+c^3+c^3+a^3}$$
$$x^{2(a^3+b^3+c^3)}$$

Therefore, LHS = RHS

Hence proved

(iii) To prove,

$\left(\frac{x^a}{x^b}\right)^c \times \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b$ =1

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\left(\frac{x^{ac}}{x^{bc}}\right) \times \left(\frac{x^{ba}}{x^{ca}}\right) \times \left(\frac{x^{bc}}{x^{ab}}\right)$$

$x^{ac-bc} \times x^{ba-ca} \times x^{bc-ab}$

$x^{ac-bc+ba-ca+bc-ab}$

x^0

1

Therefore, LHS = RHS

Hence proved

4. Prove that:

(i) $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$ **= 1**

(ii) $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$

Solution:

(i) $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$ = 1

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{1+\frac{x^a}{x^b}} + \frac{1}{1+\frac{x^b}{x^a}}$$

$$\frac{x^b}{x^b+x^a} + \frac{x^a}{x^a+x^b}$$

$$\frac{x^b+x^a}{x^a+x^b}$$

1

Therefore, LHS = RHS

Hence proved

(ii) $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$$

$$\frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^a+x^c} + \frac{x^c}{x^c+x^b+x^a}$$

$$\frac{x^a+x^b+x^c}{x^a+x^b+x^c}$$

1

Therefore, LHS = RHS

Hence proved

5.Prove that:

(i) $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$ **= abc**

(ii) $(a^{-1} + b^{-1})^{-1}$

Solution:

(i) To prove,

$\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$ = abc

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{a+b+c}{\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}}$$

$$\frac{a+b+c}{\frac{a+b+c}{abc}}$$

abc

Therefore, LHS = RHS

Hence proved

(ii) To prove,

$(a^{-1} + b^{-1})^{-1}$ = $\frac{ab}{a+b}$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{(a^{-1}+b^{-1})}$$

$$\frac{1}{(\frac{1}{a}+\frac{1}{b})}$$

$$\frac{1}{(\frac{a+b}{ab})}$$

$$\frac{ab}{a+b}$$

Therefore, LHS = RHS

Hence proved

6. If abc = 1, show that $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$ **= 1**

Solution:

To prove,

$\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$ = 1

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{1+a+\frac{1}{b}} + \frac{1}{1+b+\frac{1}{c}} + \frac{1}{1+c+\frac{1}{a}}$$

$$\frac{b}{b+ab+1} + \frac{c}{c+bc+1} + \frac{a}{a+ac+1} \quad ....(1)$$

We know abc = 1

$c = \frac{1}{ab}$

By substituting the value c in equation (1), we get

$$\frac{b}{b+ab+1} + \frac{\frac{1}{ab}}{\frac{1}{ab}+b(\frac{1}{ab})+1} + \frac{a}{a+a(\frac{1}{ab})+1}$$

$$\frac{b}{b+ab+1} + \frac{\frac{1}{ab}\times ab}{1+b+ab} + \frac{ab}{1+ab+b}$$

$$\frac{b}{b+ab+1} + \frac{1}{1+b+ab} + \frac{ab}{1+ab+b}$$

$$\frac{1+ab+b}{b+ab+1}$$

1

Therefore, LHS = RHS

Hence proved

7. Simplify:

(i) $\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

Solution:

$= \frac{3^n \times 9^n \times 9}{\frac{3^n}{3} \times \frac{9^n}{9}}$

= 9 x 3 x 9

= 243

(ii) $\frac{(5\times 25^{n+1})(25\times 5^{2n})}{(5\times 5^{2n+3})-(25)^{n+1}}$

Solution:

$= \frac{(5\times 25^n\times 25)-(25\times 25^n)}{(5\times 25^n\times 125)(25^n\times 25)}$

$= \frac{25^n\times 25(5-1)}{25^n\times 25(25-1)}$

$= \frac{4}{24}$

$= \frac{1}{6}$

(iii) $\frac{(5^{n+3})-(6\times 5^{n+1})}{(9\times 5^n)-(2^2\times 5^n)}$

Solution:

$= \frac{(5^{n+3})-(6\times5^{n+1})}{(9\times5^{n})-(2^{2}\times5^{n})}$

$= \frac{(5^{n}\times5^{3})-(6\times5^{n}\times5)}{(9\times5^{n})-(2^{2}\times5^{n})}$

$= \frac{5^{n}(125-30)}{5^{n}(9-4)}$

$= \frac{95}{5}$

= 19

(iv) $\frac{(6\times8^{n+1})+(16\times2^{3n-2})}{(10\times2^{3n+1})-7\times(8)^{n}}$

Solution:

$= \frac{(6\times8^{n}\times8)+(16\times8^{n}\times\frac{1}{4})}{(10\times8^{n}\times2)-(7\times(8)^{n})}$

$= \frac{8^{n}(48+4)}{8^{n}(20-7)}$

$= \frac{52}{13}$

= 4

Level 2

8. Solve the following equations for x:

(i) 7^{2x+3} **= 1**

(ii) $2^{x+1} = 4^{x-3}$

(iii) $2^{5x+3} = 8^{x+3}$

(iv) $4^{2x} = \frac{1}{32}$

(v) $4^{x-1} \times (0.5)^{3-2x} = (\frac{1}{8})^{x}$

(vi) 2^{3x-7} **= 256**

Solution:

(i) We have,

=> 7^{2x+3} = 1

=> 7^{2x+3} = 7^{0}

=> 2x + 3 = 0

=> 2x = -3

=> x = $-\frac{3}{2}$

(ii) We have,

$2^{x+1} = 4^{x-3}$
$2^{x+1} = 2^{2x-6}$
x + 1 = 2x − 6
x = 7

(iii) We have,

$2^{5x+3} = 8^{x+3}$
$2^{5x+3} = 2^{3x+9}$
5x + 3 = 3x + 9
2x = 6
x = 3

(iv) We have,

$4^{2x} = \frac{1}{32}$
$2^{4x} = \frac{1}{2^5}$
$2^{4x} = 2^{-5}$
4x = -5
x = $\frac{-5}{4}$

(v) We have,

$4^{x-1} \times (0.5)^{3-2x} = (\frac{1}{8})^x$
$2^{2x-2} \times (\frac{1}{2})^{3-2x} = (\frac{1}{2})^{3x}$
$2^{2x-2} \times 2^{2x-3} = (\frac{1}{2})^{3x}$
$2^{2x-2+2x-3} = (\frac{1}{2})^{3x}$
$2^{4x-5} = 2^{-3x}$
4x-5 = -3x
7x = 5
x = $\frac{5}{7}$

(vi) 2^{3x-7} = 256

2^{3x-7} = 2^8
3x − 7 = 8
3x = 15
x = 5

9. Solve the following equations for x:

(i) $2^{2x} - 2^{x+3} + 2^4 = 0$

(ii) $3^{2x+4} + 1 = 2 \times 3^{x+2}$

Solution:

(i) We have,

=> $2^{2x} - 2^{x+3} + 2^4 = 0$

=> $2^{2x} + 2^4 = 2^x.2^3$

=> Let $2^x = y$

=> $y^2 + 2^4 = y \times 2^3$

=> $y^2 - 8y + 16 = 0$

=> $y^2 - 4y - 4y + 16 = 0$

=> y(y-4) -4(y-4) = 0

=> y = 4

=> $x^2 = 2^2$

=> x = 2

(ii) We have,

$3^{2x+4} + 1 = 2 \times 3^{x+2}$
$(3^{x+2})^2 + 1 = 2 \times 3^{x+2}$
Let 3^{x+2} = y
$y^2 + 1 = 2y$
$y^2 - 2y + 1 = 0$
$y^2 - y - y + 1 = 0$
$y(y-1) - 1(y-1) = 0$
$(y-1)(y-1) = 0$
y = 1

10. If 49392 = $a^4b^2c^3$, find the values of a, b and c, where a, b and c, where a, b, and c are different positive primes.

Solution:

Taking out the LCM , the factors are $2^4, 3^2 and 7^3$

$a^4b^2c^3$ = $2^4, 3^2 and 7^3$

a = 2, b = 3 and c = 7 [Since, a, b and c are primes]

11. If 1176 = $2^a \times 3^b \times 7^c$, Find a, b, and c.

Solution:

Given that 2, 3 and 7 are factors of 1176.

Taking out the LCM of 1176, we get

$2^3 \times 3^1 \times 7^2$ = $2^a \times 3^b \times 7^c$

By comparing, we get

a = 3, b = 1 and c = 2.

12. Given 4725 = $3^{a} \times 5^{b} \times 7^{c}$, find

(i) The integral values of a, b and c

Solution:

Taking out the LCM of 4725, we get

$3^{3} \times 5^{2} \times 7^{1} = 3^{a} \times 5^{b} \times 7^{c}$

By comparing, we get

a = 3, b = 2 and c = 1.

(ii) The value of $2^{-a} \times 3^{b} \times 7^{c}$

Solution:

\(2^{-a} \times 3^{b}\times 7^{c} = [latex]2^{-3}\times 3^{2}\times 7^{1}\)\(2^{-3}\times 3^{2}\times 7^{1}\]”>
[latex]2^{-3}\times 3^{2}\times 7^{1}\) = $\frac{1}{8} \times 9 \times 7$

$\frac{63}{8}$

13. If a = xy^{p-1}, b = xy^{q-1} and c = xy^{r-1}, prove that $a^{q-r}b^{r-p}c^{p-q}$ = 1

Solution:

Given,

a = xy^{p-1}, b = xy^{q-1} and c = xy^{r-1}

To prove, $a^{q-r}b^{r-p}c^{p-q}$ = 1

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

= $a^{q-r}b^{r-p}c^{p-q}$ ……(i)

By substituting the value of a, b and c in equation (i), we get

= $(xy^{p-1})^{q-r}(xy^{q-1})^{r-p}(xy^{r-1})^{p-q}$

= $xy^{pq-pr-q+r}xy^{qr-pq-r+p}xy^{rp-rq-p+q}$

= $xy^{pq-pr-q+r+qr-pq-r+p+rp-rq-p+q}$

= xy^{0}

= 1

RD SHARMA
Solutions
Class 9 Maths
Chapter 2
Ex 2.2

1. Assuming that x,y,z are positive real numbers, simplify each of the following

(i)

$$(\sqrt{(x^{-3})})^5$$

$$(\sqrt{(x^{-3})})^5 = (\sqrt{\frac{1}{x^3}})^5$$

$$(\frac{1}{x^{\frac{3}{2}}})^5 = \frac{1}{x^{\frac{15}{2}}}$$

$$(\sqrt{(x^{-3})})^5 = \frac{1}{x^{\frac{15}{2}}}$$

(ii)

$$\sqrt{x^3y^{-2}}$$

$$\sqrt{x^3y^{-2}} = \sqrt{\frac{x^3}{y^2}}$$

$$= (\frac{x^3}{y^2})^{\frac{1}{2}}$$

$$= \frac{x^{3\times\frac{1}{2}}}{y^{2\times\frac{1}{2}}}$$

$$\frac{x^{\frac{3}{2}}}{y}$$

$$\sqrt{x^3y^{-2}} = \frac{x^{\frac{3}{2}}}{y}$$

(iii)

$$(x^{-\frac{2}{3}}y^{-\frac{1}{2}})^2$$

$$= (x^{-\frac{2}{3}}y^{-\frac{1}{2}})^2 = (\frac{1}{x^{\frac{2}{3}}y^{\frac{1}{2}}})^2$$

$$= (\frac{1}{x^{\frac{2}{3}\times 2}y^{\frac{1}{2}\times 2}})$$

$$= \frac{1}{x^{\frac{4}{3}}y}$$

(iv)

$$(\sqrt{x})^{-\frac{2}{3}}\sqrt{y^4} \div \sqrt{xy^{\frac{1}{2}}}$$

$$= (x^{\frac{1}{2}})^{-\frac{2}{3}}(y^2) \div \sqrt{xy^{\frac{1}{2}}}$$

$$= \frac{x^{\frac{1}{2}\times\frac{2}{3}}y^2}{(xy^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$= \frac{x^{-\frac{1}{3}}y^2}{x^{\frac{1}{2}}y^{-\frac{1}{2}\times\frac{1}{2}}}$$

$$= (x^{-\frac{1}{3}} \times x^{-\frac{1}{2}}) \times (y^2 \times y^{\frac{1}{4}})$$

$$= (x^{-\frac{1}{3}-\frac{1}{2}})(y^{2+\frac{1}{4}})$$

$$= (x^{\frac{-2-3}{6}})(y^{\frac{8+1}{4}})$$

$$= (x^{-\frac{5}{6}})(y^{-\frac{9}{4}})$$

$$= \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}}$$

(v)

$$\sqrt[5]{243x^{10}y^5z^{10}}$$

$$= (243x^{10}y^5z^{10})^{\frac{1}{5}}$$

$$= (243)^{\frac{1}{5}}x^{\frac{10}{5}}y^{\frac{5}{5}}z^{\frac{10}{5}}$$

$$= (3^5)^{\frac{1}{5}}x^2yz^2$$

$$= 3x^2yz^2$$

(vi)

$$\left(\frac{x^{-4}}{y^{-10}}\right)^{\frac{5}{4}}$$

$$= \left(\frac{y^{10}}{x^4}\right)^{\frac{5}{4}}$$

$$= \left(\frac{y^{10\times\frac{5}{4}}}{x^{4\times\frac{5}{4}}}\right)$$

$$= \left(\frac{y^{\frac{25}{2}}}{x^5}\right)$$

(vii)

$$\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^{5}\left(\frac{6}{7}\right)^{2}$$

$$= \left(\sqrt{\frac{2}{3}}\right)^{5}\left(\frac{6}{7}\right)^{\frac{4}{2}}$$

$$= \left(\frac{2}{3}\right)^{\frac{5}{2}}\left(\frac{6}{7}\right)^{\frac{4}{2}}$$

$$= \left(\frac{2^5}{3^5}\right)^{\frac{1}{2}}\left(\frac{6^4}{7^4}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^5}{3^5} \times \frac{6^4}{7^4}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2\times2\times2\times2\times2}{3\times3\times3\times3\times3} \times \frac{6\times6\times6\times6}{7\times7\times7\times7}\right)$$

$$= \left(\frac{512}{7203}\right)$$

2. Simplify

(i)

$$\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}$$

$$= (16)^{-\frac{1}{5}\times\frac{5}{2}}$$

$$= (16)^{-\frac{1}{2}}$$

$$= (4^2)^{-\frac{1}{2}}$$

$$= (4^{2\times-\frac{1}{2}})$$

$$= (4^{-1})$$

$$= \frac{1}{4}$$

(ii) $\sqrt[5]{(32)^{-3}}$

$$\sqrt[5]{(32)^{-3}}$$

$$= [(2^5)^{-3}]^{\frac{1}{5}}$$

$$= (2^{-15})^{\frac{1}{5}}$$

$$= 2^{-3}$$

$$= \frac{1}{2^3}$$

$$= \frac{1}{8}$$

(iii)

$$\sqrt[3]{(343)^{-2}}$$

$$= [(343)^{-2}]^{\frac{1}{3}}$$

$$= (343)^{-2\times\frac{1}{3}}$$

$$= (7^3)^{-\frac{2}{3}}$$

$$= (7^{-2})$$

$$= (\frac{1}{7^2})$$

$$= (\frac{1}{49})$$

(iv)

$$(0.001)^{\frac{1}{3}}$$

$$= (\frac{1}{1000})^{\frac{1}{3}}$$

$$= (\frac{1}{10^3})^{\frac{1}{3}}$$

$$= (\frac{1^{\frac{1}{3}}}{(10^3)^{\frac{1}{3}}})$$

$$\frac{1}{10^{3\times\frac{1}{3}}}$$

$$= \frac{1}{10} = 0.1$$

(v)

$$\frac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$$

$$= \frac{((5^2))^{\frac{3}{2}}\times((3^5))^{\frac{3}{5}}}{((4^2))^{\frac{5}{4}}\times((4^2))^{\frac{4}{3}}}$$

$$= \frac{5^{2\times\frac{3}{2}}\times3^{5\times\frac{3}{5}}}{2^{4\times\frac{5}{4}}\times2^{3\times\frac{4}{3}}}$$

$$= \frac{5^3\times3^3}{2^5\times2^4}$$

$$= \frac{125\times27}{32\times16}$$

$$= \frac{3375}{512}$$

(vi) $\left(\frac{\sqrt{2}}{5}\right)^{8} \div \left(\frac{\sqrt{2}}{5}\right)^{13}$

$= \frac{\left(\frac{\sqrt{2}}{5}\right)^{8}}{\left(\frac{\sqrt{2}}{5}\right)^{13}}$

$= \left(\frac{\sqrt{2}}{5}\right)^{8-13}$

$= \left(\frac{\sqrt{2}}{5}\right)^{-5}$

$= \frac{\left(2^{\frac{1}{2}}\right)^{-5}}{(5)^{-5}}$

$= \frac{\left(2^{\frac{1}{2}\times -5}\right)}{(5)^{-5}}$

$= \frac{1}{2^{\frac{5}{2}}} \times \frac{5^5}{1}$

$= \frac{5^5}{2^{\frac{5}{2}}}$

$= \frac{3125}{4\sqrt{2}}$

(vii)

$$\left[\frac{5^{-1}\times 7^2}{5^2\times 7^{-4}}\right]^{\frac{7}{2}} \times \left[\frac{5^{-2}\times 7^3}{5^3\times 7^{-5}}\right]^{\frac{-5}{2}}$$

$$= \frac{(5^{-1}\times 7^2)^{\frac{7}{2}}}{(5^2\times 7^{-4})^{\frac{7}{2}}} \times \frac{(5^{-2}\times 7^3)^{\frac{-5}{2}}}{(5^3\times 7^{-5})^{\frac{-5}{2}}}$$

$$= \frac{(5^{-1})^{\frac{7}{2}}\times (7^2)^{\frac{7}{2}}}{(5^2)^{\frac{7}{2}}\times (7^{-4})^{\frac{7}{2}}} \times \frac{(5^{-2})^{\frac{-5}{2}}\times (7^3)^{\frac{-5}{2}}}{(5^3)^{\frac{-5}{2}}\times (7^{-5})^{\frac{-5}{2}}}$$

$$= \frac{5^{-\frac{7}{2}}\times 7^7}{5^7\times 7^{-14}} \times \frac{5^5\times 7^{-\frac{15}{2}}}{5^{-\frac{15}{2}}\times 7^{-\frac{25}{2}}}$$

$$= \frac{7^{7+\frac{7}{14}}}{5^{7+\frac{7}{2}}} \times \frac{5^{5+\frac{15}{2}}}{7^{\frac{15}{2}+\frac{25}{2}}}$$

$$= \frac{7^{21}}{5^{\frac{21}{2}}} \times \frac{5^{\frac{25}{2}}}{7^{\frac{40}{2}}}$$

$$= \frac{7^{21}}{7^{20}} \times \frac{5^{\frac{25}{2}}}{5^{\frac{21}{2}}}$$

$$= 7^{21-20} \times 5^{\frac{25}{2}-\frac{21}{2}}$$

$$= 7^{1} \times 5^{\frac{4}{2}}$$

$$= 7^{1} \times 5^{2}$$

$$= 7 \times 25$$

$$= 175$$

3. Prove that

(i) $(\sqrt{3 \times 5^{-3}} \div \sqrt[3]{3^{-1}}\sqrt{5}) \times \sqrt[6]{3 \times 5^{6}} = \frac{3}{5}$

$$(\sqrt{3 \times 5^{-3}} \div \sqrt[3]{3^{-1}}\sqrt{5}) \times \sqrt[6]{3 \times 5^{6}}$$

$$= ((3 \times 5^{-3})^{\frac{1}{2}} \div (3^{-1})^{\frac{1}{3}}(5)^{\frac{1}{2}}) \times (3 \times 5^{6})^{\frac{1}{6}}$$

$$= ((3)^{\frac{1}{2}}(5^{-3})^{\frac{1}{2}} \div (3^{-1})^{\frac{1}{3}}(5)^{\frac{1}{2}}) \times (3 \times 5^{6})^{\frac{1}{6}}$$

$$= ((3)^{\frac{1}{2}}(5)^{\frac{-3}{2}} \div (3)^{\frac{-1}{3}}(5)^{\frac{1}{2}}) \times ((3)^{\frac{1}{6}} \times (5)^{\frac{6}{6}})$$

$$= ((3)^{\frac{1}{2}-(-\frac{1}{3})} \times (5)^{-\frac{3}{2}-\frac{1}{2}}) \times ((3)^{\frac{1}{6}} \times (5))$$

$$= ((3)^{\frac{3+2}{6}} \times (5)^{-\frac{4}{2}}) \times ((3)^{\frac{1}{6}} \times (5))$$

$$= ((3)^{\frac{5}{6}} \times (5)^{-2}) \times ((3)^{\frac{1}{6}} \times (5))$$

$$= ((3)^{\frac{5}{6}+\frac{1}{6}} \times (5)^{-2+1})$$

$$= ((3)^{\frac{6}{6}} \times (5)^{-1})$$

$$= ((3)^{1} \times (5)^{-1})$$

$$= ((3) \times (5)^{-1})$$

$$= ((3) \times (\frac{1}{5}))$$

$$= (\frac{3}{5})$$

(ii)

$$9^{\frac{3}{2}} - 3 \times 5^{0} - (\frac{1}{81})^{-\frac{1}{2}}$$

$$= (3^{2})^{\frac{3}{2}} - 3 - (\frac{1}{9^{2}})^{-\frac{1}{2}}$$

$$= 3^{2\times\frac{3}{2}} - 3 - (9^{-2})^{-\frac{1}{2}}$$

$$= 3^{3} - 3 - (9)^{-2\times-\frac{1}{2}}$$

$$= 27 - 3 - 9$$

$$= 15$$

(iii)

$$\frac{1}{4}^{2} - 3 \times 8^{\frac{2}{3}} \times 4^{0} + (\frac{9}{16})^{-\frac{1}{2}}$$

$$= (\frac{1}{2^{2}})^{-2} - 3 \times 8^{\frac{2}{3}} \times 1 + (\frac{3^{2}}{4^{2}})^{-\frac{1}{2}}$$

$$= (2^{-2})^{-2} - 3 \times 8^{\frac{2}{3}} \times 1 + (\frac{3^{2\times-\frac{1}{2}}}{4^{2\times-\frac{1}{2}}})$$

$$= 2^{4} - 3 \times 2^{3\times\frac{2}{3}} + \frac{4}{3}$$

$$= 16 - 3 \times 2^{2} + \frac{4}{3}$$

$$= 16 - 3 \times 4 + \frac{4}{3}$$

$$= 16 - 12 + \frac{4}{3}$$

$$= \frac{12+4}{3}$$

$$= \frac{16}{3}$$

(iv)

$$\frac{2^{\frac{1}{2}}\times3^{\frac{1}{3}}\times4^{\frac{1}{4}}}{10^{-\frac{1}{5}}\times5^{\frac{3}{5}}} \div \frac{4^{\frac{4}{3}}\times5^{-\frac{7}{5}}}{4^{-\frac{3}{5}}\times6}$$

$$= \frac{2^{\frac{1}{2}}\times3^{\frac{1}{3}}\times(2^{2})^{\frac{1}{4}}(2^{2})^{-\frac{3}{5}}\times(2\times3)}{(2\times5)^{-\frac{1}{5}}\times5^{\frac{3}{5}}\times3^{\frac{4}{3}}\times5^{-\frac{7}{5}}}$$

$$= \frac{2^{\frac{1}{2}}\times2^{\frac{1}{2}}\times(2^{2})^{-\frac{6}{5}}\times2^{1}\times3^{\frac{1}{3}}\times3}{2^{-\frac{1}{5}}\times5^{-\frac{1}{5}}\times5^{\frac{3}{5}}\times3^{\frac{4}{3}}\times5^{-\frac{7}{5}}}$$

$$= \frac{2^{\frac{1}{5}}\times2^{\frac{1}{2}}\times2^{\frac{1}{2}}\times2^{-\frac{6}{5}}\times2\times3^{\frac{1}{3}}\times3\times3^{-\frac{4}{3}}}{5^{-\frac{1}{5}}\times5^{\frac{3}{5}}\times5^{-\frac{7}{5}}}$$

$$= \frac{(2)^{\frac{1}{2}+\frac{1}{2}-\frac{6}{5}+1+\frac{1}{5}}\times(3)^{\frac{1}{3}+1-\frac{4}{3}}}{5^{-\frac{1}{5}}\times5^{\frac{3}{5}}\times5^{-\frac{7}{5}}}$$

$$= \frac{(2)^{\frac{1}{5}+1-\frac{6}{5}+1} \times (3)^{1-\frac{3}{3}}}{5^{-\frac{5}{5}}}$$

$$= \frac{(2)^{\frac{1}{5}+2-\frac{6}{5}} \times (3)^{1-1}}{5^{-1}}$$

$$= \frac{(2)^{2-1} \times (3)^{1-1}}{5^{-1}}$$

$$= \frac{(2)^{1} \times (3)^{0}}{5^{-1}}$$

$= 2 \times 1 \times 5$

$=10$

(v)

$$\sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}}$$

$$= \frac{1}{2} + \frac{1}{(0.01)^{\frac{1}{2}}} - (3^3)^{\frac{2}{3}}$$

$$= \frac{1}{2} + \frac{1}{(0.1)^{2\times\frac{1}{2}}} - (3)^{3\times\frac{2}{3}}$$

$$= \frac{1}{2} + \frac{1}{(0.1)^{1}} - (3)^{2}$$

$$= \frac{1}{2} + \frac{1}{(0.1)} - 9$$

$$= \frac{1}{2} + 10 - 9$$

$$= \frac{1}{2} + 1$$

$$= \frac{3}{2}$$

(vi)

$$\frac{2^{n}+2^{n-1}}{2^{n+1}-2^{n}}$$

$$= \frac{2^{n}+2^{n}\times2^{-1}}{2^{n}\times2^{1}-2^{n}}$$

$$= \frac{2^{n}[1+2^{-1}]}{2^{n}[2-1]}$$

$$= \frac{1+\frac{1}{2}}{1}$$

$$= 1 + \frac{1}{2}$$

$$= \frac{3}{2}$$

(vii)

$$\left(\frac{64}{125}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{-\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)$$

$$= \left(\frac{125}{64}\right)^{\frac{2}{3}} + \frac{1}{\left(\frac{4^4}{5^4}\right)^{\frac{1}{4}}} + \left(\frac{5}{(64)^{\frac{1}{3}}}\right)$$

$$= \left(\frac{5^3}{4^3}\right)^{\frac{2}{3}} + \frac{1}{\left(\frac{4}{5}\right)} + \left(\frac{5}{(4^3)^{\frac{1}{3}}}\right)$$

$$= \left(\frac{5}{4}\right)^2 + \frac{5}{4} + \frac{5}{4}$$

$$= \frac{25}{16} + \frac{10}{4}$$

$$= \frac{25}{16} + \frac{40}{16}$$

$$= \frac{26+40}{16}$$

$$= \frac{65}{16}$$

(viii)

$$\frac{3^{-3} \times 6^2 \times \sqrt{98}}{5^2 \times \sqrt[3]{\frac{1}{25}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times \sqrt{7 \times 7 \times 2}}{5^2 \times \left(\frac{1}{25}\right)^{\frac{1}{3}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{5^2 \times \left(\frac{1}{5^{2 \times \frac{1}{3}}}\right) \times \frac{1}{(15)^{\frac{4}{3}}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{5^2 \times 5^{-\frac{2}{3}} \times \frac{1}{(5 \times 3)^{\frac{4}{3}}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{5^2 \times 5^{-\frac{2}{3}} \times 5^{\frac{4}{3}} \times 3^{\frac{4}{3}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3}\times36\times7\sqrt{2}}{(5^{2}\times5^{-\frac{2}{3}}\times5^{-\frac{4}{3}})\times3^{-\frac{4}{3}}\times3^{\frac{1}{3}}}$$

$$= \frac{3^{-3}\times36\times7\sqrt{2}\times3^{\frac{4}{3}}\times3^{\frac{1}{3}}}{(5)^{2-\frac{2}{3}-\frac{4}{3}}}$$

$$= \frac{3^{-3}\times36\times7\sqrt{2}\times3^{\frac{4}{3}}\times3^{\frac{1}{3}}}{(5)^{\frac{6-2-4}{3}}}$$

$$= \frac{3^{-3+\frac{4}{3}-\frac{1}{3}}\times36\times7\sqrt{2}}{(5)^{0}}$$

$$= 3^{-3+\frac{3}{3}} \times 36 \times 7\sqrt{2}$$

$$= 3^{-3+1} \times 36 \times 7\sqrt{2}$$

$$= 3^{-2} \times 36 \times 7\sqrt{2}$$

$$= \frac{1}{3^{2}} \times 36 \times 7\sqrt{2}$$

$$= \frac{1}{9} \times 36 \times 7\sqrt{2}$$

$$= 4 \times 7\sqrt{2}$$

$$= 28\sqrt{2}$$

(ix)

$$\frac{(0.6)^{0}-(0.1)^{-1}}{(\frac{3}{8})^{-1}(\frac{3}{2})^{3}+(-3)^{1}}$$

$$= \frac{1-\frac{1}{0.1}}{\frac{8}{3}\times(\frac{3}{2})^{3}-3}$$

$$= \frac{1-10}{\frac{8}{3}\times\frac{3^{3}}{2^{3}}-3}$$

$$= \frac{-9}{3^{2}-3}$$

$$= \frac{-9}{9-3}$$

$$= \frac{-9}{6}$$

$$= -\frac{3}{2}$$

4. Show that

(i) $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$ *= 1*

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{1+\frac{x^a}{x^b}} + \frac{1}{1+\frac{x^b}{x^a}}$$
$$\frac{x^b}{x^b+x^a} + \frac{x^a}{x^a+x^b}$$
$$\frac{x^b+x^a}{x^a+x^b}$$
1

Therefore, LHS = RHS

Hence proved

(ii) $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$$
$$\frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^a+x^c} + \frac{x^c}{x^c+x^b+x^a}$$
$$\frac{x^a+x^b+x^c}{x^a+x^b+x^c}$$
1

Therefore, LHS = RHS

Hence proved

(ii)

$$[(\frac{x^{a(a-b)}}{x^{a(a+b)}}) \div (\frac{x^{b(b-a)}}{x^{b(b+a)}})]^{a+b}$$

$$[(\frac{x^{a(a-b)}}{x^{a(a+b)}}) \div (\frac{x^{b(b-a)}}{x^{b(b+a)}})]^{a+b}$$

$$= [(\frac{x^{a^2-ab}}{x^{a^2+ab}}) \div (\frac{x^{b^2-ab}}{x^{b^2+ab}})]^{a+b}$$

$$= [x^{(a^2-ab)-(a^2-ab)} \div x^{(b^2-ab)-(b^2-ab)}]^{a+b}$$

$$= [x^{-2ab} \div x^{-2ab}]^{a+b}$$

$$= [x^{-2ab-(-2ab)}]^{a+b}$$

$$= [x^{-2ab+2ab}]^{a+b}$$

$$= [x^0]^{a+b}$$

$$= [1]^{a+b}$$

$$= 1$$

(iii)

$$\left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}}\left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}}\left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}}$$

$$= \left(x^{\frac{1}{(a-b)(a-c)}}\right)\left(x^{\frac{1}{(b-c)(b-a)}}\right)\left(x^{\frac{1}{(c-a)(c-b)}}\right)$$

$$= x^{\frac{1}{(a-b)(a-c)}+\frac{1}{(b-c)(b-a)}+\frac{1}{(c-a)(c-b)}}$$

$$= x^{\frac{1}{(a-b)(a-c)}+\frac{-1}{(b-c)(a-b)}+\frac{1}{(a-c)(b-c)}}$$

$$= x^{\frac{(b-c)}{(a-b)(a-c)(b-c)}+\frac{-(a-c)}{(b-c)(a-b)(a-c)}+\frac{(a-b)}{(a-c)(b-c)(a-b)}}$$

$$= x^{\frac{b-c-a+c+a-b}{(a-b)(a-c)(b-c)}}$$

$$= x^{\frac{0}{(a-b)(a-c)(b-c)}}$$

$$= x^0 = 1$$

(iv) $\left(\frac{x^{a^2+b^2}}{x^{ab}}\right)^{a+b}\left(\frac{x^{b^2+c^2}}{x^{bc}}\right)^{b+c}\left(\frac{x^{c^2+a^2}}{x^{ac}}\right)^{a+c} = 2\left(a^3+b^3+c^3\right)$

$$\left(\frac{x^{a^2+b^2}}{x^{ab}}\right)^{a+b}\left(\frac{x^{b^2+c^2}}{x^{bc}}\right)^{b+c}\left(\frac{x^{c^2+a^2}}{x^{ac}}\right)^{a+c}$$

$$= \left(x^{a^2+b^2-ab}\right)^{a+b}\left(x^{b^2+c^2-bc}\right)^{b+c}\left(x^{c^2+a^2-ac}\right)^{a+c}$$

$$= \left(x^{a+b(a^2+b^2-ab)}\right)\left(x^{b+c(b^2+c^2-bc)}\right)\left(x^{a+c(c^2+a^2-ac)}\right)$$

$$= \left(x^{a^3+ab^2-a^2b+ab^2+b^3-ab^2}\right)\left(x^{b^3+bc^2-b^2c+cb^2+c^3-bc^2}\right)\left(x^{ac^2+a^3-a^2c+c^3+a^2c-ac^2}\right)$$

$$= \left(x^{a^3+b^3}\right)\left(x^{b^3+c^3}\right)\left(x^{a^3+c^3}\right)$$

$$= \left(x^{a^3+b^3+b^3+c^3+a^3+c^3}\right)$$

$$= \left(x^{2a^3+2b^3+2c^3}\right)$$

$$= \left(x^{2(a^3+b^3+c^3)}\right)$$

(v) $\left(x^{a-b}\right)^{a+b}\left(x^{b-c}\right)^{b+c}\left(x^{c-a}\right)^{c+a} = 1$

$$\left(x^{a-b}\right)^{a+b}\left(x^{b-c}\right)^{b+c}\left(x^{c-a}\right)^{c+a}$$

$$= x^{a^2-b^2}x^{b^2-c^2}x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$= x^0$$

$$= 1$$

(vi) $\left[\left(x^{a-a^{-1}}\right)^{\frac{1}{a-1}}\right]^{\frac{a}{a+1}} = x$

$$\left[\left(x^{a-a^{-1}}\right)^{\frac{1}{a-1}}\right]^{\frac{a}{a+1}}$$

$$= \left[\left(x^{\frac{a-a^{-1}}{a-1}}\right)\right]^{\frac{a}{a+1}}$$

$$= \left[\left(x^{\frac{a-a^{-1}}{a-1}}\right)\right]^{\frac{a}{a+1}}$$

$$= \left(x^{\frac{a(a-a^{-1})}{a^2-1}}\right)$$

$$= \left(x^{\frac{a^2-a^{-1+1}}{a^2-1}}\right)$$

$$= \left(x^{\frac{a^2-a^0}{a^2-1}}\right)$$

$$= \left(x^{\frac{a^2-1}{a^2-1}}\right)$$

$$= x^1 = x$$

(vii) $\left[\frac{a^{x+1}}{a^{y+1}}\right]^{x+y}\left[\frac{a^{y+2}}{a^{z+2}}\right]^{y+z}\left[\frac{a^{z+3}}{a^{x+3}}\right]^{z+x} = 1$

$$\left[\frac{a^{x+1}}{a^{y+1}}\right]^{x+y}\left[\frac{a^{y+2}}{a^{z+2}}\right]^{y+z}\left[\frac{a^{z+3}}{a^{x+3}}\right]^{z+x}$$

$$= \left[a^{(x+1)-(y+1)}\right]^{x+y}\left[a^{(y+2)-(z+2)}\right]^{y+z}\left[a^{(z+3)-(x+3)}\right]^{z+x}$$

$$= \left[a^{x-y}\right]^{x+y}\left[a^{y-z}\right]^{y+z}\left[a^{z-x}\right]^{z+x}$$

$$= \left[a^{x^2-y^2}\right]\left[a^{y^2-z^2}\right]\left[a^{z^2-x^2}\right]$$

$$= a^{x^2-y^2+y^2-z^2+z^2-x^2} = a^0$$

$$= 1$$

(viii) $\left(\frac{3^a}{3^b}\right)^{a+b}\left(\frac{3^b}{3^c}\right)^{b+c}\left(\frac{3^c}{3^a}\right)^{c+a} = 1$

$$\left(\frac{3^a}{3^b}\right)^{a+b}\left(\frac{3^b}{3^c}\right)^{b+c}\left(\frac{3^c}{3^a}\right)^{c+a}$$

$$= (3^{a-b})^{a+b}(3^{b-c})^{b+c}(3^{c-a})^{c+a}$$

$$= 3^{a^2-b^2} \times 3^{b^2-c^2} \times 3^{c^2-a^2}$$

$$= 3^{a^2-b^2+b^2-c^2+c^2-a^2}$$

$$= 3^0 = 1$$

Level 2

5. If $2^x = 3^y = 12^z$ *,show that* $\frac{1}{z} = \frac{1}{y} + \frac{2}{x}$

$$2^x = 3^y = (2 \times 3 \times 2)^z$$

$$2^x = 3^y = (2^2 \times 3)^z$$

$$2^x = 3^y = (2^{2z} \times 3^z)$$

$$2^x = 3^y = 12^z = k$$

$$2 = k^{\frac{1}{x}}$$

$$3 = k^{\frac{1}{y}}$$

$$12 = k^{\frac{1}{z}}$$

$$12 = 2 \times 3 \times 2$$

$$12 = k^{\frac{1}{z}} = k^{\frac{1}{y}} \times k^{\frac{1}{x}} \times k^{\frac{1}{x}}$$

$$k^{\frac{1}{z}} = k^{\frac{2}{x}+\frac{1}{y}}$$

$$\frac{1}{z} = \frac{1}{y} + \frac{2}{x}$$

6. If $2^x = 3^y = 6^{-z}$ *,show that* $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

$2^x = 3^y = 6^{-z}$

$2^x = k$

$2 = k^{\frac{1}{x}}$

$3^y = k$

$3 = k^{\frac{1}{y}}$

$6^{-z} = k$

$k = \frac{1}{6^z}$

$6 = k^{-\frac{1}{z}}$

$2 \times 3 = 6$

$k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{-\frac{1}{z}}$

$\frac{1}{x} + \frac{1}{y} = -\frac{1}{z}$ [by equating exponents]

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

7. If $a^x = b^y = c^z$ *and* $b^2 = ac$*, then show that* $y = \frac{2zx}{z+x}$

Let $a^x = b^y = c^z = k$

$a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}, c = k^{\frac{1}{z}}$

Now,

$b^2 = ac$

$(k^{\frac{1}{y}})^2 = k^{\frac{1}{x}} \times k^{\frac{1}{z}}$

$k^{\frac{2}{y}} = k^{\frac{1}{x}+\frac{1}{z}}$

$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$

$\frac{2}{y} = \frac{x+z}{xz}$

$y = \frac{2xz}{x+z}$

8. If $3^x = 5^y = (75)^z$*, show that* $z = \frac{xy}{2x+y}$

$3^x = k$

$3 = k^{\frac{1}{x}}$

$5^y = k$

$5 = k^{\frac{1}{y}}$

$75^z = k$

$75 = k^{\frac{1}{z}}$

$3^1 \times 5^2 = 75^1$

$k^{\frac{1}{x}} \times k^{\frac{2}{y}} = k^{\frac{1}{z}}$

$\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$

$\frac{y+2x}{xy} = \frac{1}{z}$

$z = \frac{xy}{2x+y}$

9. If $(27)^x = \frac{9}{3^x}$*, find x*

We have,

$(27)^x = \frac{9}{3^x}$

$(3^3)^x = \frac{9}{3^x}$

$3^{3x} = \frac{9}{3^x}$

$3^{3x} = \frac{3^2}{3^x}$

$3^{3x} = 3^{2-x}$

$3x = 2 - x$ [On equating exponents]

$3x + x = 2$

$4x = 2$

$x = \frac{2}{4}$

$x = \frac{1}{2}$

Here the value of x is $\frac{1}{2}$

10. Find the values of x in each of the following

(i). $2^{5x} \div 2^x = \sqrt[5]{2^{20}}$

We have

$$2^{5x} \div 2^{x} = \sqrt[5]{2^{20}}$$

$$= \frac{2^{5x}}{2^{x}} = (2^{20})^{\frac{1}{5}}$$

$$= 2^{5x-x} = 2^{20\times\frac{1}{5}}$$

$$= 2^{4x} = 2^{4}$$

$= 4x = 4$ [On equating exponent]

$x = 1$

Hence the value of x is 1

(ii). $(2^3)^4 = (2^2)^x$

We have

$$(2^3)^4 = (2^2)^x$$

$$= 2^{3\times4} = 2^{2\times x}$$

$12 = 2x$

$2x = 12$ [On equating exponents]

$x = 6$

Hence the value of x is 6

(iii). $\left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{27}$

We have

$$\left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{27}$$

$$\Rightarrow \frac{(3)^x}{(5)^x}\frac{(5)^{2x}}{(3)^{2x}} = \frac{5^3}{3^3}$$

$$\Rightarrow \frac{5^{2x-x}}{3^{2x-x}} = \frac{5^3}{3^3}$$

$$\Rightarrow \frac{5^x}{3^x} = \frac{5^3}{3^3}$$

$$\Rightarrow \left(\frac{5}{3}\right)^x = \left(\frac{5}{3}\right)^3$$

$x = 3$ [on equating exponents]

Hence the value of x is 3

(iv)

$$5^{x-2} \times 3^{2x-3} = 135$$

We have,

$$5^{x-2} \times 3^{2x-3} = 135$$
$$\Rightarrow 5^{x-2} \times 3^{2x-3} = 5 \times 27$$
$$\Rightarrow 5^{x-2} \times 3^{2x-3} = 5^1 \times 3^3$$
$$\Rightarrow x - 2 = 1, 2x - 3 = 3 \text{ [On equating exponents]}$$
$$\Rightarrow x = 2 + 1, 2x = 3 + 3$$
$$\Rightarrow x = 3, 2x = 6 \Rightarrow x = 3$$

Hence the value of x is 3

(v). $2^{x-7} \times 5^{x-4} = 1250$

We have

$$2^{x-7} \times 5^{x-4} = 1250$$
$$\Rightarrow 2^{x-7} \times 5^{x-4} = 2 \times 625$$
$$\Rightarrow 2^{x-7} \times 5^{x-4} = 2 \times 5^4$$
$$\Rightarrow x - 7 = 1 \Rightarrow x = 8, x - 4 = 4 \Rightarrow x = 8$$

Hence the value of x is 8

(vi).

$$(\sqrt[3]{4})^{2x+\frac{1}{2}} = \frac{1}{32}$$

$$(4^{\frac{1}{3}})^{2x+\frac{1}{2}} = \frac{1}{32}$$

$$(4)^{\frac{1}{3}(2x+\frac{1}{2})} = \frac{1}{32}$$

$$(4)^{\frac{1}{3}(2x+\frac{1}{2})} = \frac{1}{2^5}$$

$$(4)^{\frac{2}{3}x+\frac{1}{6}} = \frac{1}{2^5}$$

$$(2^2)^{\frac{2}{3}x+\frac{1}{6}} = \frac{1}{2^5}$$

$$(2)^{2(\frac{2}{3}x+\frac{1}{6})} = \frac{1}{2^5}$$

$$(2)^{\frac{4}{3}x+\frac{2}{6}} = \frac{1}{2^5}$$

$$(2)^{\frac{4}{3}x+\frac{1}{3}} = 2^{-5}$$

$$\frac{4}{3}x + \frac{1}{3} = -5$$

$4x + 1 = -15$

$4x = -15 - 1$

$4x = -16$

$x = \frac{-16}{4}$

$x = -4$

Hence the value of x is 4

(vii).

$5^{2x+3} = 1$

$5^{2x+3} = 1 \times 5^0$

$2x + 3 = 0$ [By equating exponents]

$2x = -3$

$x = \frac{-3}{2}$

Hence the value of x is $\frac{-3}{2}$

(viii).

$(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$

$(13)^{\sqrt{x}} = 256 - 81 - 6$

$(13)^{\sqrt{x}} = 256 - 87$

$(13)^{\sqrt{x}} = 169$

$(13)^{\sqrt{x}} = 13^2$

$\sqrt{x} = 2$ [By equating exponents]

$(\sqrt{x})^2 = (2)^2$

$x = 4$

Hence the value of x is 4

(ix).

$$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$$

$$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{5^3}{3^3}$$

$$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \left(\frac{5}{3}\right)^3$$

$$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \left(\frac{3}{5}\right)^{-3}$$

$$\left(\frac{3}{5}\right)^{\frac{1}{2}(x+1)} = \left(\frac{3}{5}\right)^{-3}$$

$$\frac{1}{2}(x+1) = -3$$
$$x + 1 = -6$$
$$x = -6 - 1$$
$$x = -7$$

Hence the value of x is 7

11. If x = $2^{\frac{1}{3}} + 2^{\frac{2}{3}}$, *show that* $x^3 - 6x = 6$

$x^3 - 6x = 6$

x = $2^{\frac{1}{3}} + 2^{\frac{2}{3}}$

Putting cube on both the sides, we get

$x^3 = (2^{\frac{1}{3}} + 2^{\frac{2}{3}})^3$

As we know, $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$x^3 = (2^{\frac{1}{3}})^3 + (2^{\frac{2}{3}})^3 + 3(2^{\frac{1}{3}})(2^{\frac{2}{3}})(2^{\frac{1}{3}} + 2^{\frac{2}{3}})$
$x^3 = (2^{\frac{1}{3}})^3 + (2^{\frac{2}{3}})^3 + 3(2^{\frac{1}{3}+\frac{2}{3}})(x)$
$x^3 = (2^{\frac{1}{3}})^3 + (2^{\frac{2}{3}})^3 + 3(2)(x)$
$x^3 = 6 + 6x$
$x^3 - 6x = 6$

Hence proved

12. Determine $(8x)^x$, if $9^{x+2} = 240 + 9^x$.

$9^{x+2} = 240 + 9^x$
$9^x . 9^2 = 240 + 9^x$

Let 9^x be y

81y = 240 + y
81y − y = 240
80y = 240

$y = 3$

Since, $y = 3$

Then,

$9^x = 3$

$3^{2x} = 3$

Therefore, $x = \frac{1}{2}$

$(8x)^x = (8 \times \frac{1}{2})^{\frac{1}{2}}$

$= (4)^{\frac{1}{2}}$

$= 2$

Therefore $(8x)^x = 2$

13. If $3^{x+1} = 9^{x-2}$, find the value of 2^{1+x}

$3^{x+1} = 9^{x-2}$

$3^{x+1} = 3^{2x-4}$
$x + 1 = 2x - 4$
$x = 5$

Therefore the value of $2^{1+x} = 2^{1+5} = 2^6 = 64$

14. If $3^{4x} = (81)^{-1}$ and $(10)^{\frac{1}{y}} = 0.0001$, find the value of 2^{-x+4y}.

$3^{4x} = (81)^{-1}$ and $(10)^{\frac{1}{y}} = 0.0001$

$3^{4x} = (3)^{-4}$
$x = -1$

And, $(10)^{\frac{1}{y}} = 0.0001$

$(10)^{\frac{1}{y}} = (10)^{-4}$

$\frac{1}{y} = -4$
$y = \frac{1}{-4}$

To find the value of 2^{-x+4y}, we need to substitute the value of x and y

$2^{-x+4y} = 2^{1+4(\frac{1}{-4})} = 2^{1-1} = 2^0 = 1$

15. If $5^{3x} = 125$ and $10^y = 0.001$. Find x and y.

$5^{3x} = 125$ and $10^y = 0.001$

$5^{3x} = 5^3$
$x = 1$

Now,

$10^y = 0.001$
$10^y = 10^{-3}$

$y = -3$

Therefore, the value of $x = 1$ and the value of $y = -3$

16. Solve the following equations

(i)

$$3^{x+1} = 27 \times 3^4$$
$$3^{x+1} = 3^3 \times 3^4$$
$$3^{x+1} = 3^{3+4}$$
$$x + 1 = 3 + 4 \qquad \text{[By equating exponents]}$$
$$x + 1 = 7$$
$$x = 7 - 1$$
$$x = 6$$

(ii)

$$4^{2x} = (\sqrt[3]{16})^{-\frac{6}{y}} = (\sqrt{8})^2$$
$$(2^2)^{2x} = (16^{\frac{1}{3}})^{-\frac{6}{y}} = (\sqrt{8})^2$$
$$2^{4x} = [(2^4)^{\frac{1}{3}}]^{-\frac{6}{y}} = (2^{\frac{3}{2}})^2$$
$$2^{4x} = (2^{\frac{4}{3}})^{-\frac{6}{y}} = (2^{\frac{3}{2}})^2$$
$$2^{4x} = (2^{\frac{4}{3}})^{-\frac{6}{y}} = 2^3$$
$$2^{4x} = 2^3$$
$$4x = 3 \qquad \text{(By equating exponents)}$$
$$x = \frac{3}{4}$$
$$2^{-\frac{8}{y}} = 2^3$$
$$-\frac{8}{y} = 3 \qquad \text{(By equating exponents)}$$
$$y = \frac{-8}{3}$$

(iii).

$$3^{x-1} \times 5^{2y-3} = 225$$
$$3^{x-1} \times 5^{2y-3} = 3^2 \times 5^2$$
$x - 1 = 2$ [By equating exponents]

$x = 3$

$$3^{x-1} \times 5^{2y-3} = 3^2 \times 5^2$$
$2y - 3 = 2$ [By equating exponents]

$2y = 5$

$y = \frac{5}{2}$

(iv).

$$8^{x+1} = 16^{y+2} \text{ and } (\tfrac{1}{2})^{3+x} = (\tfrac{1}{4})^{3y}$$
$$(2^3)^{x+1} \text{ and } (2^{-1})^{3+x} = (2^{-2})^{3y}$$
$$3x + 3 = 4y + 8 \text{ and } -3 - x = -6y$$
$$3x + 3 = 4y + 8 \text{ and } 3 + x = 6y$$
$$3x + 3 = 4y + 8 \text{ and } y = \frac{3+x}{6}$$

3x+3=4y+8—eq1

$y = \frac{3+x}{6}$—-eq2

Substitute eq2 in eq1

$$3x + 3 = 4\left(\frac{3+x}{6}\right) + 8$$
$$3x + 3 = 2\left(\frac{3+x}{3}\right) + 8$$
$$3x + 3 = \left(\frac{6+2x}{3}\right) + \frac{24}{3}$$
$$3(3x + 3) = 6 + 2x + 24$$
$$9x + 9 = 30 + 2x$$
$$7x = 21$$
$$x = \frac{21}{7}$$
$$x = 3$$

Putting value of x in eq2

$$\frac{3+3}{6} = yy = 1$$

(v).

$$4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^{x}$$

$$2^{2x-2} \times \left(\frac{5}{10}\right)^{3-2x} = \left(\frac{1}{2^3}\right)^{x}$$

$$2^{2x-2} \times \left(\frac{1}{2}\right)^{3-2x} = 2^{-3x}$$

$$2^{2x-2} \times 2^{-3+2x} = 2^{-3x}$$

$$2x - 2 - 3 + 2x = -3x \qquad \text{[By equating exponents]}$$

$$4x + 3x = 5$$

$$7x = 5$$

$$x = \frac{5}{7}$$

(vi).

$$\sqrt{\frac{a}{b}} = \left(\frac{b}{a}\right)^{1-2x}$$

$$\left(\frac{a}{b}\right)^{\frac{1}{2}} = \left(\frac{a}{b}\right)^{-(1-2x)} \quad \frac{1}{2} = -1 + 2x \qquad \text{[By equating exponents]}$$

$$\frac{1}{2} + 1 = 2x$$

$$2x = \frac{3}{2}$$

$$x = \frac{3}{4}$$

17. If a and b are distinct positive primes such that $\sqrt[3]{a^6b^{-4}} = a^xb^{2y}$*, find x and y*

$$\sqrt[3]{a^6b^{-4}} = a^xb^{2y}$$

$$(a^6b^{-4})^{\frac{1}{3}} = a^xb^{2y}$$

$$a^{\frac{6}{3}}b^{\frac{-4}{3}} = a^xb^{2y}$$

$$a^2b^{\frac{-4}{3}} = a^xb^{2y}$$

$$x = 2, 2y = \frac{-4}{3}$$

$$y = \frac{\frac{-4}{3}}{2}$$

$$y = -\frac{2}{3}$$

18.If a and b are different positive primes such that

(i).

$$\left(\frac{a^{-1}b^2}{a^2b^{-4}}\right)^7 \div \left(\frac{a^3b^{-5}}{a^{-2}b^3}\right) = a^xb^y$$

find x and y

$$\left(\frac{a^{-1}b^{2}}{a^{2}b^{-4}}\right)^{7} \div \left(\frac{a^{3}b^{-5}}{a^{-2}b^{3}}\right) = a^{x}b^{y}$$

$$(a^{-1-2}b^{2+4})^{7} \div (a^{3+2}b^{-5-3}) = a^{x}b^{y}$$

$$(a^{-3}b^{6})^{7} \div (a^{5}b^{-8}) = a^{x}b^{y}$$

$$(a^{-21}b^{42}) \div (a^{5}b^{-8}) = a^{x}b^{y}$$

$$(a^{-21-5}b^{42+8}) = a^{x}b^{y}$$

$$(a^{-26}b^{50}) = a^{x}b^{y}$$

$$x = -26, y = 50$$

(ii) $(a+b)^{-1}(a^{-1}+b^{-1}) = a^{x}b^{y}$, find x and y

$$(a+b)^{-1}(a^{-1}+b^{-1})$$

$$= \left(\frac{1}{a+b}\right)\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$= \left(\frac{1}{a+b}\right)\left(\frac{b+a}{ab}\right)$$

$$= \frac{1}{ab}$$

$$= (ab)^{-1} = a^{-1}b^{-1}$$

By equating exponents

$$x = -1, y = -1$$

Therefore $x + y + 2 = -1 - 1 + 2 = 0$

19. If $2^{x} \times 3^{y} \times 5^{z} = 2160$ *, find x,y and z. Hence compute the value of* $3^{x} \times 2^{-y} \times 5^{-z}$

$$2^{x} \times 3^{y} \times 5^{z} = 2160$$

$$2^{x} \times 3^{y} \times 5^{z} = 2^{4} \times 3^{3} \times 5^{1}$$

$$x = 4, y = 3, z = 1$$

$$3^{x} \times 2^{-y} \times 5^{-z} = 3^{4} \times 2^{-3} \times 5^{-1}$$

$$= \frac{3\times3\times3\times3}{2\times2\times2\times5}$$

$$= \frac{81}{40}$$

20. If $1176 = 2^{a} \times 3^{b} \times 7^{c}$*, find the values of a,b and c. Hence compute the value of* $2^{a} \times 3^{b} \times 7^{-c}$ *as a fraction*

$$1176 = 2^a \times 3^b \times 7^c$$
$$2^3 \times 3^1 \times 7^2 = 2^a \times 3^b \times 7^c$$
$$a = 3, b = 1, c = 2$$
We have to find the value of $2^a \times 3^b \times 7^{-c}$
$$2^a \times 3^b \times 7^{-c} = 2^3 \times 3^1 \times 7^{-2}$$
$$= \frac{2\times2\times2\times3}{7\times7}$$
$$= \frac{24}{49}$$

21. Simplify

(i)

$$\left(\frac{x^{a+b}}{x^c}\right)^{a-b}\left(\frac{x^{b+c}}{x^a}\right)^{b-c}\left(\frac{x^{c+a}}{x^b}\right)^{c-a}$$
$$(x^{a+b-c})^{a-b}(x^{b+c-a})^{b-c}(x^{c+a-b})^{c-a}$$
$$(x^{a^2-b^2-ca+cb})(x^{b^2-c^2-ab+ac})(x^{c^2-a^2-bc+ab})$$
$$x^{a^2-b^2-ca+cb+b^2-c^2-ab+ac+c^2-a^2-bc+ab}$$
$$x^0 = 1$$

(ii)

$$\sqrt[lm]{\frac{x^l}{x^m}} \times \sqrt[mn]{\frac{x^m}{x^n}} \times \sqrt[nl]{\frac{x^n}{x^l}}$$
$$\sqrt[lm]{x^{l-m}} \times \sqrt[mn]{x^{m-n}} \times \sqrt[nl]{x^{n-l}}$$
$$(x^{l-m})^{\frac{1}{lm}} \times (x^{m-n})^{\frac{1}{mn}} \times (x^{n-l})^{\frac{1}{nl}}$$
$$(x)^{\frac{l-m}{lm}} \times (x)^{\frac{m-n}{mn}} \times (x)^{\frac{n-l}{nl}}$$
$$(x)^{\frac{l-m}{lm}+\frac{m-n}{mn}+\frac{n-l}{nl}}$$
$$(x)^{n(\frac{l-m}{lm})+l(\frac{m-n}{mn})+m(\frac{n-l}{nl})}$$
$$(x)^{\frac{nl-mn+lm-nl+mn-ml}{mnl}}$$
$$(x)^{\frac{0}{mnl}}$$
$$x^0 = 1$$

22. Show that

$$\frac{(a+\frac{1}{b})^m \times (a-\frac{1}{b})^n}{(b+\frac{1}{a})^m \times (b-\frac{1}{a})^n} = (\frac{a}{b})^{m+n}$$

$$= \frac{(\frac{ab+1}{b})^m \times (\frac{ab-1}{b})^n}{(\frac{ab+1}{a})^m \times (\frac{ab+1}{a})^n}$$

$$= (\frac{a}{b})^m \times (\frac{a}{b})^n$$

$$= (\frac{a}{b})^{m+n}$$

Hence LHS = RHS

23.

(i). If $a = x^{m+n}y^l, b = x^{n+l}y^m$ and $c = x^{l+m}y^n$, prove that $a^{m-n}b^{n-l}c^{l-m} = 1$

$$(x^{m+n}y^l)^{m-n}(x^{n+l}y^m)^{n-l}(x^{l+m}y^n)^{l-m}$$

$$= (x^{(m+n)(m-n)}y^{l(m-n)})(x^{(n+l)(n-l)}y^{m(n-l)})(x^{(l+m)(l-m)}y^{n(l-m)})$$

$$= (x^{m^2-n^2}y^{lm-ln})(x^{n^2-l^2}y^{mn-ml})(x^{l^2-m^2}y^{nl-nm})$$

$$= x^{m^2-n^2+n^2-l^2+l^2-m^2}y^{lm-ln+mn-ml+nl-nm}$$

$$= x^0y^0$$

$$= 1$$

(ii). If $x = a^{m+n}, y = a^{n+l}$ and $z = a^{l+m}$, prove that $x^my^nz^l = x^ny^lz^m$

LHS $= x^my^nz^l$

$$(a^{m+n})^m(a^{n+l})^n(a^{l+m})^l$$

$$= a^{m^2+nm} \times a^{n^2+ln} \times a^{l^2+ml}$$

$$= a^{n^2+nm} \times a^{l^2+ln} \times a^{m^2+ml}$$

$$= a^{(m+n)n}a^{(n+l)l}a^{(l+m)m}$$

$$= x^ny^lz^m$$