

**RD Sharma**  
**Solutions**  
**Class 11 Maths**  
**Chapter 3**  
**Ex 3.1**

### Functions Ex 3.1 Q1

Function = Let  $A$  and  $B$  be two non-empty sets. A relation  $f$  from  $A$  to  $B$ , i.e., a sub-set of  $A \times B$ , is called a function (or a mapping or a map) from  $A$  to  $B$ , if

- (i) for each  $a \in A$  there exists  $b \in B$  such that  $(a, b) \in f$
- (ii)  $(a, b) \in f$  and  $(a, c) \in f \Rightarrow b = c$

If  $(a, b) \in f$ , then ' $b$ ' is called the image of ' $a$ ' under  $f$

If a function  $f$  is expressed as the set of ordered pairs, the domain  $f$  is the set of all first components of members of  $f$  and the range of  $f$  is the set of second components of members of  $f$ .

### Functions Ex 3.1 Q2

Function = Let  $A$  and  $B$  be two non-empty sets. Then a function ' $f$ ' from set  $A$  to set  $B$  is a rule or method or correspondence which associates elements of set  $A$  to elements of set  $B$  such that:

- (i) all elements of set  $A$  are associated to element in set  $B$ .
- (ii) an element of set  $A$  is associated to a unique element in set  $B$ .

In other words, a function ' $f$ ' from a set  $A$  to set  $B$  associates each element of set  $A$  to a unique element of set  $B$ .

### Functions Ex 3.1 Q3

Function is a type of relation. But in a function no two ordered pairs have the same first element. For eg:  $R_1$  and  $R_2$  are two relations.

Clearly,  $R_1$  is a function, but  $R_2$  is not a function because two ordered pairs  $(1, 2)$  and  $(1, 4)$  have the same first element.

This means every function is a relation but every relation is not a function.

### Functions Ex 3.1 Q4

We have,

$$f(x) = x^2 - 2x - 3$$

Now,

$$\begin{aligned} f(-2) &= (-2)^2 - 2(-2) - 3 \\ &= 4 + 4 - 3 \\ &= 5 \end{aligned}$$

$$\begin{aligned} f(-1) &= (-1)^2 - 2(-1) - 3 \\ &= 1 + 2 - 3 \\ &= 0 \end{aligned}$$

$$\begin{aligned} f(0) &= (0)^2 - 2 \times 0 - 3 \\ &= -3 \end{aligned}$$

$$\begin{aligned} f(1) &= (1)^2 - 2 \times 1 - 3 \\ &= 1 - 2 - 3 \\ &= -4 \end{aligned}$$

$$\begin{aligned} f(2) &= (2)^2 - 2 \times 2 - 3 \\ &= 4 - 4 - 3 \\ &= -3 \end{aligned}$$

(a)  $\text{Rang}(f) = \{-4, -3, 0, 5\}$

(b) Clearly, pre-images of 6, -3 and 5 is  $\emptyset$ ,  $\{0, 2\}$ ,  $-2$  respectively.

### Functions Ex 3.1 Q5

We have,

$$f(x) = \begin{cases} 3x - 2, & x < 0 \\ 1, & x = 0 \\ 4x + 1, & x > 0 \end{cases}$$

Now,

$$f(1) = 4 \times 1 + 1 = 5,$$

$$f(-1) = 3 \times (-1) - 2 = -3 - 2 = -5,$$

$$f(0) = 1,$$

and,  $f(2) = 4 \times 2 + 1 = 9$

$$\therefore \begin{aligned} f(1) &= 5, & f(-1) &= -5, \\ f(0) &= 1, & f(2) &= 9, \end{aligned}$$

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### Functions Ex 3.1 Q6

We have,

$$f(x) = x^2 \quad \text{--- (i)}$$

(a) clearly range of  $f = R^+$  (set of all real numbers greater than or equal to zero)

(b) we have,

$$\begin{aligned} & \{x : f(x) = 4\} \\ \Rightarrow & f(x) = 4 \quad \text{--- (ii)} \end{aligned}$$

Using equation (i) and equation (ii), we get

$$\begin{aligned} & x^2 = 4 \\ \Rightarrow & x = \pm 2 \\ \therefore & \{x : f(x) = 4\} = \{-2, 2\} \end{aligned}$$

$$\begin{aligned} & \{y : f(y) = -1\} \\ \Rightarrow & f(y) = -1 \quad \text{--- (iii)} \end{aligned}$$

Clearly,  $x^2 \neq -1$  or  $x^2 \geq 0$

$$\Rightarrow f(y) \neq -1$$

$$\therefore \{y : f(y) = -1\} = \emptyset$$

### Functions Ex 3.1 Q7

We have,

$$f : R^+ \rightarrow R$$

$$\text{and } f(x) = \log_e x \quad \text{--- (i)}$$

(a) Now,

$$f : R^+ \rightarrow R$$

$\therefore$  the image set of the domain of  $f = R$

(b) Now,

$$\begin{aligned} & \{x : f(x) = -2\} \\ \Rightarrow & f(x) = -2 \quad \text{--- (ii)} \end{aligned}$$

Using equation (i) and equation (ii), we get

$$\begin{aligned} & \log_e x = -2 \\ \Rightarrow & x = e^{-2} \quad \left[ \because \log_e b = c \Rightarrow b = e^c \right] \end{aligned}$$

$$\therefore \{x : f(x) = -2\} = \{e^{-2}\}$$

(c) Now,

$$\begin{aligned} f(xy) &= \log_e(xy) & [f(x) &= \log_e x] \\ &= \log_e x + \log_e y & [\because \log mn &= \log m + \log n] \end{aligned}$$

$$\begin{aligned} & f(x) + f(y) \\ \therefore & f(xy) = f(x) + f(y) \end{aligned}$$

Yes,  $f(xy) = f(x) + f(y)$ .

### Functions Ex 3.1 Q8

(a) we have,

$$\{(x, y) = y = 3x, x \in \{1, 2, 3\}, y \in \{3, 6, 9, 12\}\}$$

Putting  $x = 1, 2, 3$  in  $y = 3x$ , we get

$$y = 3, 6, 9 \text{ respectively}$$

$$\therefore R = \{(1, 3), (2, 6), (3, 9)\}$$

Yes, it is a function.

(b) we have,

$$\{(x, y) : y > x + 1, x = 1, 2 \text{ and } y = 2, 4, 6\}$$

Putting  $x = 1, 2$  in  $y > x + 1$ , we get

$$y > 2, y > 3 \text{ respectively.}$$

$$\therefore R = \{(1, 4), (1, 6), (2, 4), (2, 6)\}$$

It is not a function from  $A$  to  $B$  because two ordered pairs in  $R$  have the same first element.

(c) we have,

$$\{(x, y) = x + y = 3, x, y \in \{0, 1, 2, 3\}\}$$

Now,

$$y = 3 - x$$

Putting  $x = 0, 1, 2, 3$ , we get

$$y = 3, 2, 1, 0 \text{ respectively}$$

$$\therefore R = \{(0, 3), (1, 2), (2, 1), (3, 0)\}$$

Yes, this relation is a function.

### Functions Ex 3.1 Q9

We have,

$$f : R \rightarrow R \text{ and } g : C \rightarrow C$$

$$\therefore \text{Domain } (f) = R \text{ and Domain } (g) = C$$

$$\therefore \text{Domain } (f) \neq \text{Domain } (g) = C$$

$$\therefore f(x) \text{ and } g(x) \text{ are not equal functions.}$$

### Functions Ex 3.1 Q10

(i) We have,

$$f(x) = x^2$$

Range of  $f(x) = R^+$  (set of all real numbers greater than or equal to zero)

$$= \{x \in R \mid x \geq 0\}$$

(ii) We have,

$$g(x) = \sin x$$

Range of  $g(x) = \{x \in R : -1 \leq x \leq 1\}$

(iii) We have,

$$h(x) = x^2 + 1$$

Range of  $h(x) = \{x \in R : x \geq 1\}$

### Functions Ex 3.1 Q11

(a) We have,

$$f_1 = \{(1, 1), (2, 11), (3, 1), (4, 15)\}$$

$f_1$  is a function from  $X$  to  $Y$ .

(b) We have,

$$f_2 = \{(1, 1), (2, 7), (3, 5)\}$$

$f_2$  is not a function from  $X$  to  $Y$  because there is an element  $4 \in X$  which is not associated to any element of  $Y$ .

(c) We have,

$$f_3 = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$$

$f_3$  is not a function from  $X$  to  $Y$  because an element  $2 \in X$  is associated to two elements 9 and 11 in  $Y$ .

### Functions Ex 3.1 Q12

We have,

$$f(x) = \text{highest prime factor of } x.$$

$$\therefore 12 = 3 \times 4,$$

$$13 = 13 \times 1,$$

$$14 = 7 \times 2,$$

$$15 = 5 \times 3,$$

$$16 = 2 \times 8,$$

$$17 = 17 \times 1$$

$$\therefore f = \{(12, 3), (13, 13), (14, 7), (15, 5), (16, 2), (17, 17)\}$$

$$\therefore \text{Range}(f) = \{3, 13, 7, 5, 2, 17\}$$

### Functions Ex 3.1 Q13

We know that,

$$\text{if } f : A \rightarrow B$$

such that  $y \in B$ . Then,

$$f^{-1}(y) = \{x \in A : f(x) = y\}. \text{ In other words, } f^{-1}(y) \text{ is the set of pre-images of } y.$$

Let  $f^{-1}\{17\} = x$ . Then,  $f(x) = 17$

$$\Rightarrow x^2 + 1 = 17$$

$$\Rightarrow x^2 = 17 - 1 = 16$$

$$\Rightarrow x = \pm 4$$

Let  $f^{-1}\{-3\} = x$ . Then,  $f(x) = -3$

$$\Rightarrow x^2 + 1 = -3$$

$$\Rightarrow x^2 = -3 - 1 = -4$$

$$\Rightarrow x = \sqrt{-4}$$

$$\therefore f^{-1}\{-3\} = \emptyset$$

### Functions Ex 3.1 Q14

We have,

$$A = \{p, q, r, s\} \text{ and } B = \{1, 2, 3\}$$

(a) Now,

$$R_1 = \{(p, 1), (q, 2), (r, 1), (s, 2)\}$$

$R_1$  is a function

(b) Now,

$$R_2 = \{(p, 1), (q, 1), (r, 1), (s, 1)\}$$

$R_2$  is a function

(c) Now,

$$R_3 = \{(p, 1), (q, 2), (p, 2), (s, 3)\}$$

$R_3$  is not a function because an element  $p \in A$  is associated to two elements 1 and 2 in  $B$ .

(d) Now,

$$R_4 = \{(p, 2), (q, 3), (r, 2), (s, 2)\}$$

$R_4$  is a function.

### Functions Ex 3.1 Q15

We have,

$$f(n) = \text{the highest prime factor of } n.$$

Now,

$$9 = 3 \times 3,$$

$$10 = 5 \times 2,$$

$$11 = 11 \times 1,$$

$$12 = 3 \times 4,$$

$$13 = 13 \times 1$$

$$\therefore f = \{(9, 3), (10, 5), (11, 11), (12, 3), (13, 13)\}$$

Clearly,  $\text{range}(f) = \{3, 5, 11, 13\}$

### Functions Ex 3.1 Q16

We have,

$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$$

$$\text{and, } g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$$

$$\text{Now, } f(3) = (3)^2 = 9 \text{ and } f(3) = 3 \times 3 = 9$$

$$\text{and, } g(2) = (2)^2 = 4 \text{ and } g(2) = 3 \times 2 = 6$$

We observe that  $f(x)$  takes unique value at each point in its domain  $[0, 10]$ . However  $g(x)$  does not take unique value at each point in its domain  $[0, 10]$ .

Hence,  $g(x)$  is not a function.

### Functions Ex 3.1 Q17

Given  $f(x) = x^2$

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$$f(1.1) = 1.21$$

$$f(1) = 1$$

$$\frac{f(1.1) - f(1)}{(1.1) - 1} = \frac{1.21 - 1}{1.1 - 1}$$

$$= \frac{0.21}{0.1}$$

$$= 2.1$$

### Functions Ex 3.1 Q18

$f: X \rightarrow \mathbb{R}$  given by  $f(x) = x^3 + 1$

$$f(-1) = (-1)^3 + 1 = -1 + 1 = 0$$

$$f(0) = (0)^3 + 1 = 0 + 1 = 1$$

$$f(3) = (3)^3 + 1 = 27 + 1 = 28$$

$$f(9) = (9)^3 + 1 = 81 + 1 = 82$$

$$f(7) = (7)^3 + 1 = 343 + 1 = 344$$

Set of ordered pairs are  $\{(-1, 0), (0, 1), (3, 28), (9, 82), (7, 344)\}$

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**Functions Ex 3.1 Q18**

$f : X \rightarrow \mathbb{R}$  given by  $f(x) = x^3 + 1$

$$f(-1) = (-1)^3 + 1 = -1 + 1 = 0$$

$$f(0) = (0)^3 + 1 = 0 + 1 = 1$$

$$f(3) = (3)^3 + 1 = 27 + 1 = 28$$

$$f(9) = (9)^3 + 1 = 81 + 1 = 82$$

$$f(7) = (7)^3 + 1 = 343 + 1 = 344$$

Set of ordered pairs are  $\{(-1,0), (0,1), (3,28), (9,82), (7,344)\}$

**Functions Ex 3.2 Q1**

We have,

$$f(x) = x^2 - 3x + 4$$

Now,

$$\begin{aligned} f(2x+1) &= (2x+1)^2 - 3(2x+1) + 4 \\ &= 4x^2 + 1 + 4x - 6x - 3 + 4 \\ &= 4x^2 - 2x + 2 \end{aligned}$$

It is given that

$$f(x) = f(2x+1)$$

$$\Rightarrow x^2 - 3x + 4 = 4x^2 - 2x + 2$$

$$\Rightarrow 0 = 4x^2 - x^2 - 2x + 3x + 2 - 4$$

$$\Rightarrow 3x^2 + x - 2 = 0$$

$$\Rightarrow 3x^2 + 3x - 2x - 2 = 0$$

$$\Rightarrow 3x(x+1) - 2(x+1) = 0$$

$$\Rightarrow (x+1)(3x-2) = 0$$

$$\Rightarrow x+1 = 0 \quad \text{or} \quad 3x-2 = 0$$

$$\Rightarrow x = -1 \quad \text{or} \quad x = \frac{2}{3}$$

**Functions Ex 3.2 Q2**

We have,

$$f(x) = (x-a)^2(x-b)^2$$

Now,

$$\begin{aligned} f(a+b) &= (a+b-a)^2(a+b-b)^2 \\ &= b^2a^2 \end{aligned}$$

$$\Rightarrow f(a+b) = a^2b^2$$

**Functions Ex 3.2 Q3**

We have,

$$y = f(x) = \frac{ax-b}{bx-a}$$

$$\Rightarrow y = \frac{ax-b}{bx-a}$$

$$\Rightarrow y(bx-a) = ax-b$$

$$\Rightarrow xyb - ay = ax - b$$

$$\Rightarrow xyb - ax = ay - b$$

$$\Rightarrow x(by-a) = ay-b$$

$$\Rightarrow x = \frac{ay-b}{by-a}$$

$$\Rightarrow x = f(y)$$

Hence, proved

### Functions Ex 3.2 Q4

We have,

$$f(x) = \frac{1}{1-x}$$

Now,

$$\begin{aligned} f\{f(x)\} &= f\left\{\frac{1}{1-x}\right\} \\ &= \frac{1}{1 - \frac{1}{1-x}} \\ &= \frac{1}{\frac{1-x-1}{1-x}} \\ &= \frac{1-x}{-x} \\ &= \frac{x-1}{x} \end{aligned}$$

$$\begin{aligned} \therefore f[f(x)] &= f\left\{\frac{x-1}{x}\right\} \\ &= \frac{1}{1 - \left(\frac{x-1}{x}\right)} \\ &= \frac{1}{\frac{x-x+1}{x}} \\ &= \frac{x}{1} \\ &= x \end{aligned}$$

$\therefore f[f(x)] = x$  Hence, proved.

### Functions Ex 3.2 Q5

We have,

$$f(x) = \frac{x+1}{x-1}$$

Now,

$$\begin{aligned} f[f(x)] &= f\left(\frac{x+1}{x-1}\right) \\ &= \frac{\left(\frac{x+1}{x-1}\right) + 1}{\left(\frac{x+1}{x-1}\right) - 1} \\ &= \frac{\frac{x+1+x-1}{x-1}}{\frac{x+1-1(x-1)}{x-1}} \\ &= \frac{2x}{\frac{x+1-x+1}{x-1}} \\ &= \frac{2x}{2} \\ &= x \end{aligned}$$

$\therefore f[f(x)] = x$  Hence, proved.

### Functions Ex 3.2 Q6

We have,

$$f(x) = \begin{cases} x^2, & \text{when } x < 0 \\ x, & \text{when } 0 \leq x < 1 \\ \frac{1}{x}, & \text{when } x \geq 1 \end{cases}$$

(a)  $f(1/2) = \frac{1}{2}$

(b)  $f(-2) = (-2)^2 = 4$

(c)  $f(1) = \frac{1}{1} = 1$

(d)  $f(5) = \frac{1}{5}$

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$$(d) f(\sqrt{3}) = \frac{1}{\sqrt{3}}$$

$$(e) f(\sqrt{-3}) = \text{does not exist because } \sqrt{-3} \notin \text{domain}(f).$$

### Functions Ex 3.2 Q7

We have,

$$f(x) = x^3 - \frac{1}{x^3} \quad \text{---(i)}$$

Now,

$$f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 - \frac{1}{\left(\frac{1}{x}\right)^3}$$

$$= \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3 \quad \text{---(ii)}$$

Adding equation (i) and equation (ii), we get

$$\begin{aligned} f(x) + f\left(\frac{1}{x}\right) &= \left(x^3 - \frac{1}{x^3}\right) + \left(\frac{1}{x^3} - x^3\right) \\ &= x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 \\ &= 0 \end{aligned}$$

$$\therefore f(x) + f\left(\frac{1}{x}\right) = 0 \quad \text{Hence, proved.}$$

### Functions Ex 3.2 Q8

We have,

$$f(x) = \frac{2x}{1+x^2}$$

Now,

$$\begin{aligned} f(\tan \theta) &= \frac{2(\tan \theta)}{1 + \tan^2 \theta} \\ &= \sin 2\theta \end{aligned}$$

$$\left[ \because \sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta} \right]$$

$$\therefore f(\tan \theta) = \sin 2\theta \quad \text{Hence, proved.}$$

### Functions Ex 3.2 Q9

$$\begin{aligned} f\left(\frac{1}{x}\right) &= \frac{\frac{x-1}{x+1}}{\frac{x}{x+1}} = \frac{1-x}{1+x} = -f(x) \end{aligned}$$

$$\text{ii. } f(x) = \frac{x-1}{x+1}$$

$$f\left(-\frac{1}{x}\right) = \frac{\frac{-1}{x} - 1}{\frac{-1}{x} + 1} = \frac{-1-x}{-1+x} = -\frac{1+x}{x-1} = -\frac{1}{f(x)}$$

### Functions Ex 3.2 Q10

We have,

$$f(x) = (a - x^n)^{1/n}, \quad a > 0$$

Now,

$$\begin{aligned} f(f(x)) &= f(a - x^n)^{1/n} \\ &= \left[ a - \left\{ (a - x^n)^{1/n} \right\}^n \right]^{1/n} \\ &= \left[ a - (a - x^n) \right]^{1/n} \\ &= \left[ a - a + x^n \right]^{1/n} \end{aligned}$$


---

$$\begin{aligned}
 &= (x^n)^{1/n} \\
 &= (x)^{n \times \frac{1}{n}} \\
 &= x
 \end{aligned}$$

$\therefore f(f(x)) = x$  Hence, proved.

### Functions Ex 3.2 Q11

We have,

$$af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5 \quad \text{--- (i)}$$

$$\Rightarrow af\left(\frac{1}{x}\right) + bf(x) = \frac{1}{\frac{1}{x}} - 5$$

$$= x - 5$$

$$\Rightarrow af\left(\frac{1}{x}\right) + bf(x) = x - 5 \quad \text{--- (ii)}$$

Adding equations (i) and (ii), we get

$$af(x) + bf(x) + bf\left(\frac{1}{x}\right) + af\left(\frac{1}{x}\right) = \frac{1}{x} - 5 + x - 5$$

$$\Rightarrow (a+b)f(x) + f\left(\frac{1}{x}\right)(a+b) = \frac{1}{x} + x - 10$$

$$\Rightarrow f(x) + f\left(\frac{1}{x}\right) = \frac{1}{a+b} \left[ \frac{1}{x} + x - 10 \right] \quad \text{--- (iii)}$$

Subtracting equation (ii) from equation (i), we get

$$af(x) - bf(x) + bf\left(\frac{1}{x}\right) - af\left(\frac{1}{x}\right) = \frac{1}{x} - 5 - x + 5$$

$$\Rightarrow (a-b)f(x) - f\left(\frac{1}{x}\right)(a-b) = \frac{1}{x} - x$$

$$\Rightarrow f(x) - f\left(\frac{1}{x}\right) = \frac{1}{a-b} \left[ \frac{1}{x} - x \right]$$

Adding equations (iii) and (iv), we get

$$2f(x) = \frac{1}{a+b} \left[ \frac{1}{x} + x - 10 \right] + \frac{1}{a-b} \left[ \frac{1}{x} - x \right]$$

$$\Rightarrow 2f(x) = \frac{(a-b) \left[ \frac{1}{x} + x - 10 \right] + (a+b) \left[ \frac{1}{x} - x \right]}{(a+b)(a-b)}$$

$$\Rightarrow 2f(x) = \frac{\frac{a}{x} + ax - 10a - \frac{b}{x} - bx + 10b + \frac{a}{x} - ax + \frac{b}{x} - bx}{a^2 - b^2}$$

$$\Rightarrow 2f(x) = \frac{2a - 10a + 10b - 2bx}{a^2 - b^2}$$

$$\Rightarrow f(x) = \frac{1}{a^2 - b^2} \times \frac{1}{2} \left[ \frac{2a}{x} - 10a + 10b - 2bx \right]$$

$$= \frac{1}{a^2 - b^2} \left[ \frac{a}{x} - 5a + 5b - bx \right]$$

$$\therefore f(x) = \frac{1}{a^2 - b^2} \left[ \frac{a}{x} - bx - 5a + 5b \right]$$

$$= \frac{1}{a^2 - b^2} \left[ \frac{a}{x} - bx \right] - \frac{5(a-b)}{a^2 - b^2}$$

$$= \frac{1}{a^2 - b^2} \left[ \frac{a}{x} - bx \right] - \frac{5(a-b)}{(a-b)(a+b)}$$

$$= \frac{1}{a^2 - b^2} \left[ \frac{a}{x} - bx \right] - \frac{5}{a+b}$$

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**Ex 3.3**

## Functions Ex 3.3 Q1

We have,

$$f(x) = \frac{1}{x}$$

Clearly,  $f(x)$  assumes real values for all real values for all  $x$  except for the values of  $x = 0$

Hence, Domain  $(f) = \mathbb{R} - \{0\}$

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We have,

$$f(x) = \frac{1}{x-7}$$

Clearly,  $f(x)$  assumes real values for all real values of  $x$  except for the values of  $x$  satisfying  $x-7=0$  i.e.,  $x=7$

Hence,  $\text{Domain}(f) = R - \{7\}$

We have,

$$f(x) = \frac{3x-2}{x+1}$$

We observe that  $f(x)$  is a rational function of  $x$  as  $\frac{3x-2}{x+1}$  is a rational expression.

Clearly,  $f(x)$  assumes real values for all  $x$  except for the values of  $x$  for which  $x+1=0$  i.e.,  $x=-1$

Hence,  $\text{Domain} = R - \{-1\}$

We have,

$$\begin{aligned} f(x) &= \frac{2x+1}{x^2-9} \\ &= \frac{2x+1}{(x^2-3^2)} \\ &= \frac{2x+1}{(x-3)(x+3)} \end{aligned} \quad [\because a^2 - b^2 = (a-b)(a+b)]$$

We observe that  $f(x)$  is a rational function of  $x$  as  $\frac{2x+1}{x^2-9}$  is a rational expression.

Clearly,  $f(x)$  assumes real values for all  $x$  except for all those values of  $x$  for which  $x^2-9=0$  i.e.,  $x=-3, 3$

Hence,  $\text{Domain}(f) = R - \{-3, 3\}$ .

We have,

$$\begin{aligned} f(x) &= \frac{x^2+2x+1}{x^2-8x+12} \\ &= \frac{x^2+2x+1}{x^2-6x-2x+12} \\ &= \frac{x^2+2x+1}{x(x-6)-2(x-6)} \\ &= \frac{x^2+2x+1}{(x-6)(x-2)} \end{aligned}$$

Clearly,  $f(x)$  is a rational function of  $x$  as  $\frac{x^2+2x+1}{x^2-8x+12}$  is a rational expression in  $x$ .

We observe that  $f(x)$  assumes real values for all  $x$  except for all those values of  $x$  for which  $x^2-8x+12=0$  i.e.,  $x=2, 6$

$\therefore \text{Domain}(f) = R - \{2, 6\}$

### Functions Ex 3.3 Q2

(i) We have,

$$f(x) = \sqrt{x-2}$$

Clearly,  $f(x)$  assumes real values, if

$$\begin{aligned} x-2 &\geq 0 \\ \Rightarrow x &\geq 2 \\ \Rightarrow x &\in [2, \infty) \end{aligned}$$

Hence,  $\text{Domain}(f) = [2, \infty)$

(ii) We have,

$$f(x) = \frac{1}{\sqrt{x^2-1}}$$

Clearly,  $f(x)$  assumes real values, if

$$\begin{aligned} x^2-1 &> 0 \\ \Rightarrow (x-1)(x+1) &> 0 & [\because a^2 - b^2 = (a-b)(a+b)] \\ \Rightarrow x &< -1 \text{ or } x > 1 \\ \Rightarrow x &\in (-\infty, -1) \cup (1, \infty) \end{aligned}$$

Hence,  $\text{domain}(f) = (-\infty, -1) \cup (1, \infty)$

(iii) We have,

$$f(x) = \sqrt{9 - x^2}$$

Clearly,  $f(x)$  assumes real values, if

$$9 - x^2 \geq 0$$

$$\Rightarrow 9 \geq x^2$$

$$\Rightarrow x^2 \leq 9$$

$$\Rightarrow -3 \leq x \leq 3$$

$$\Rightarrow x \in [-3, 3]$$

Hence,  $\text{domain}(f) = [-3, 3]$

(iv) We have,

$$f(x) = \sqrt{\frac{x-2}{3-x}}$$

Clearly,  $f(x)$  assumes real values, if

$$x - 2 \geq 0 \quad \text{and} \quad 3 - x > 0$$

$$\Rightarrow x \geq 2 \quad \text{and} \quad 3 > x$$

$$\Rightarrow x \in [2, 3]$$

Hence,  $\text{domain}(f) = [2, 3]$ .

### Functions Ex 3.3 Q3

We have,

$$f(x) = \frac{ax + b}{bx - a}$$

We observe that  $f(x)$  is a rational function of  $x$  as  $\frac{ax + b}{bx - a}$  is a rational expression.

Clearly,  $f(x)$  assumes real values for all  $x$  except for the values of  $x$  for which

$$bx - a = 0 \text{ i.e., } bx = a$$

$$\Rightarrow x = \frac{a}{b}$$

$$\therefore \text{Domain}(f) = R - \left\{ \frac{a}{b} \right\}$$

Range of  $f$ : Let  $f(x) = y$

$$\Rightarrow \frac{ax + b}{bx - a} = y$$

$$\Rightarrow ax + b = y(bx - a)$$

$$\Rightarrow ax + b = bxy - ay$$

$$\Rightarrow b + ay = bxy - ax$$

$$\Rightarrow b + ay = x(by - a)$$

$$\Rightarrow \frac{b + ay}{b - ay} = x$$

$$\Rightarrow x = \frac{b + ay}{by - a}$$

Clearly,  $x$  will take real value for all  $x \in R$  except for

$$by - a = 0$$

$$\Rightarrow by = a$$

$$\Rightarrow y = \frac{a}{b}$$

$$\therefore \text{Range}(f) = R - \left\{ \frac{a}{b} \right\}.$$

We have,

$$f(x) = \frac{ax - b}{cx - d}$$

We observe that  $f(x)$  is a rational function of  $x$  as  $\frac{ax - b}{cx - d}$  is a rational expression.

Clearly,  $f(x)$  assumes real values for all  $x$  except for all those values of  $x$  for which

$$cx - d = 0 \text{ i.e., } cx = d$$

$$\Rightarrow x = \frac{d}{c}$$

$$\therefore \text{Domain}(f) = R - \left\{ \frac{d}{c} \right\}$$

Range: Let  $f(x) = y$

---

$$\begin{aligned}
\Rightarrow \frac{ax - b}{cx - d} &= y \\
\Rightarrow ax - b &= y(cx - d) \\
\Rightarrow ax - b &= cy - dy \\
\Rightarrow dy - b &= cxy - 9x \\
\Rightarrow dy - b &= x(cy - a) \\
\Rightarrow \frac{dy - b}{cy - a} &= x
\end{aligned}$$

Clearly,  $x$  assumes real values for all  $y$  except

$$cy - a = 0 \text{ i.e., } y = \frac{a}{c}$$

$$\text{Hence, range}(f) = \mathbb{R} - \left\{ \frac{a}{c} \right\}$$

We have,

$$f(x) = \sqrt{x-1}$$

Clearly,  $f(x)$  assumes real values, if

$$\begin{aligned}
x - 1 &\geq 0 \\
\Rightarrow x &\geq 1 \\
\Rightarrow x &\in [1, \infty)
\end{aligned}$$

$$\text{Hence, domain}(f) = [1, \infty)$$

Range: For  $x \geq 1$ , we have,

$$\begin{aligned}
x - 1 &\geq 0 \\
\Rightarrow \sqrt{x-1} &\geq 0 \\
\Rightarrow f(x) &\geq 0
\end{aligned}$$

Thus,  $f(x)$  takes all real values greater than zero.

$$\text{Hence, range}(f) = [0, \infty)$$

We have,

$$f(x) = \sqrt{x-3}$$

Clearly,  $f(x)$  assumes real values, if

$$\begin{aligned}
x - 3 &\geq 0 \\
\Rightarrow x &\geq 3 \\
\Rightarrow x &\in [3, \infty)
\end{aligned}$$

$$\text{Hence, domain}(f) = [3, \infty)$$

Range: For  $x \geq 3$ , we have,

$$\begin{aligned}
x - 3 &\geq 0 \\
\Rightarrow \sqrt{x-3} &\geq 0 \\
\Rightarrow f(x) &\geq 0
\end{aligned}$$

Thus,  $f(x)$  takes all real values greater than zero.

$$\text{Hence, range}(f) = [0, \infty)$$

We have,

$$f(x) = \frac{x-2}{2-x}$$

Domain of  $f$ : Clearly,  $f(x)$  is defined for all  $x \in \mathbb{R}$  except for which

$$2-x \neq 0 \text{ i.e., } x \neq 2$$

$$\text{Hence, domain}(f) = \mathbb{R} - \{2\}$$

Range of  $f$ : Let  $f(x) = y$

$$\begin{aligned}
\Rightarrow \frac{x-2}{2-x} &= y \\
\Rightarrow \frac{-1(2-x)}{2-x} &= y \\
\Rightarrow -1 &= y \\
\Rightarrow y &= -1
\end{aligned}$$

$$\therefore \text{Range}(f) = \{-1\}$$

We have,

$$f(x) = |x-1|$$

Clearly,  $f(x)$  is defined for all  $x \in \mathbb{R}$

Clearly,  $f(x)$  is defined for all  $x \in \mathbb{R}$ .

$$\Rightarrow \text{Domain}(f) = \mathbb{R}$$

Range: Let  $f(x) = y$

$$\Rightarrow |x - 1| = y$$

$$\Rightarrow f(x) \geq 0 \quad \forall x \in \mathbb{R}$$

It follows from the above relation that  $y$  takes all real values greater or equal to zero.

$$\therefore \text{Range}(f) = [0, \infty)$$

As  $|x|$  is defined for all real numbers, its domain is  $\mathbb{R}$  and range is only negative numbers because,  $|x|$  is always positive real number for all real numbers and  $-|x|$  is always negative real numbers.

In order to have  $F(x)$  has defined value, term inside square root should always be greater than or equal to zero which gives domain as  $-3 \leq x \leq 3$

Where as Range of above function is limited to  $[0, 3]$

**RD Sharma**  
**Solutions**  
**Class 11 Maths**  
**Chapter 3**  
**Ex 3.4**

### Functions Ex 3.4 Q1

We have,

$$f(x) = x^3 + 1 \text{ and } g(x) = x + 1$$

Now,

$$f + g : R \rightarrow R \text{ given by } (f + g)(x) = x^3 + x + 2$$

$$f - g : R \rightarrow R \text{ given by } (f - g)(x) = x^3 + 1 - (x + 1) \\ = x^3 - x$$

$$cf : R \rightarrow R \text{ given by } (cf)(x) = c(x^3 + 1)$$

$$fg : R \rightarrow R \text{ given by } (fg)(x) = (x^3 + 1)(x + 1) \\ = x^4 + x^3 + x + 1$$

$$\frac{1}{f} : R - \{-1\} \rightarrow R \text{ given by } \left(\frac{1}{f}\right)(x) = \frac{1}{x^3 + 1}$$

$$\frac{f}{g} : R - \{-1\} \rightarrow R \text{ given by } \left(\frac{f}{g}\right)(x) = \frac{(x + 1)(x^2 - x + 1)}{x + 1} \\ = x^2 - x + 1$$

We have,

$$f(x) = \sqrt{x - 1} \text{ and } g(x) = \sqrt{x + 1}$$

Now,

$$f + g : (1, \infty) \rightarrow R \text{ defined by } (f + g)(x) = \sqrt{x - 1} + \sqrt{x + 1},$$

$$f - g : (1, \infty) \rightarrow R \text{ defined by } (f - g)(x) = \sqrt{x - 1} - \sqrt{x + 1},$$

$$cf : (1, \infty) \rightarrow R \text{ defined by } (cf)(x) = c\sqrt{x - 1},$$

$$fg : (1, \infty) \rightarrow R \text{ defined by } (fg)(x) = (\sqrt{x - 1})(\sqrt{x + 1}) \\ = \sqrt{x^2 - 1}$$

$$\frac{1}{f} : (1, \infty) \rightarrow R \text{ defined by } \left(\frac{1}{f}\right)(x) = \frac{1}{\sqrt{x - 1}}$$

$$\frac{f}{g} : (1, \infty) \rightarrow R \text{ defined by } \left(\frac{f}{g}\right)(x) = \sqrt{\frac{x - 1}{x + 1}}$$

### Functions Ex 3.4 Q2

We have,

$$f(x) = 2x + 5 \text{ and } g(x) = x^2 + x$$

We observe that  $f(x) = 2x + 5$  is defined for all  $x \in R$ .

So,  $\text{domain}(f) = R$

Clearly  $g(x) = x^2 + x$  is defined for all  $x \in R$

So,  $\text{domain}(g) = R$

$\therefore \text{Domain}(f) \cap \text{Domain}(g) = R$

(i) Clearly,  $(f + g) : R \rightarrow R$  is given by

$$(f + g)(x) = f(x) + g(x) \\ = 2x + 5 + x^2 + x \\ = x^2 + 3x + 5$$

$$\text{Domain}(f+g) = R$$

(ii) We find that  $f-g: R \rightarrow R$  is defined as

$$\begin{aligned}(f-g)(x) &= f(x) - g(x) \\ &= 2x + 5 - (x^2 + x) \\ &= 2x + 5 - x^2 - x \\ &= -x^2 + x + 5\end{aligned}$$

$$\text{Domain}(f-g) = R$$

(iii) We find that  $fg: R \rightarrow R$  is given by

$$\begin{aligned}(fg)(x) &= f(x) \times g(x) \\ &= (2x + 5) \times (x^2 + x) \\ &= 2x^3 + 2x^2 + 5x^2 + 5x \\ &= 2x^3 + 7x^2 + 5x\end{aligned}$$

$$\text{Domain}(fg) = R$$

(iv) We have,

$$g(x) = x^2 + x$$

$$\therefore f(x) = 0 \Rightarrow x^2 + x = 0$$

$$\Rightarrow x(x+1) = 0$$

$$\Rightarrow x = 0 \text{ or, } x = -1$$

$$\begin{aligned}\text{So, } \text{domain}\left(\frac{f}{g}\right) &= \text{domain}(f) \cap \text{domain}(g) - \{x : g(x) = 0\} \\ &= R - \{-1, 0\}\end{aligned}$$

$$\text{We find that, } \frac{f}{g}: R - \{-1, 0\} \rightarrow R \text{ is given by } \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x+5}{x^2+x}$$

$$\text{Domain}\left(\frac{f}{g}\right) = R - \{-1, 0\}$$

### Functions Ex 3.4 Q3

We have,

$$f(x) = \begin{cases} -1, & -2 \leq x \leq 0 \\ x-1, & 0 < x \leq 2 \end{cases}$$

Now,

$$f(|x|) = |x| - 1, \text{ where } -2 \leq x \leq 2$$

$$\text{and } |f(x)| = \begin{cases} 1, & -2 \leq x \leq 0 \\ -(x-1), & 0 \leq x < 1 \\ (x-1), & 1 \leq x \leq 2 \end{cases}$$

$$\begin{aligned}\therefore g(x) &= f(|x|) + |f(x)| \\ &= \begin{cases} -x & -2 \leq x \leq 0 \\ 0, & 0 < x < 1 \\ 2(x-1), & 1 \leq x \leq 2 \end{cases}\end{aligned}$$

### Functions Ex 3.4 Q4

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that  $f(x) = \sqrt{x+1}$  is defined for all  $x \geq -1$

$$\text{So, } \text{domain}(f) = [-1, \infty]$$

Clearly,  $g(x) = \sqrt{9-x^2}$  is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now

$$\begin{aligned}\text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\ &= [-1, 3]\end{aligned}$$

$$f + g : [-1, 3] \rightarrow \mathbb{R} \text{ is given by } (f + g)(x) = f(x) + g(x) = \sqrt{x+1} + \sqrt{9-x^2}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that  $f(x) = \sqrt{x+1}$  is defined for all  $x \geq -1$

$$\text{So, domain}(f) = [-1, \infty]$$

Clearly,  $g(x) = \sqrt{9-x^2}$  is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned}\text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\ &= [-1, 3]\end{aligned}$$

$$g - f : [-3, 3] \rightarrow \mathbb{R} \text{ is given by } (g - f)(x) = g(x) - f(x) = \sqrt{9-x^2} - \sqrt{x+1}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that  $f(x) = \sqrt{x+1}$  is defined for all  $x \geq -1$

$$\text{So, domain}(f) = [-1, \infty)$$

Clearly,  $g(x) = \sqrt{9-x^2}$  is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned}\text{domain}(f) \cap \text{domain}(g) &= [-1, \infty) \cap [-3, 3] \\ &= [-1, 3]\end{aligned}$$

$$\begin{aligned}fg : [-3, 3] \rightarrow \mathbb{R} \text{ is given by } (fg)(x) &= f(x) \times g(x) = \sqrt{x+1} \times \sqrt{9-x^2} \\ &= \sqrt{9+9x-x^2-x^3}\end{aligned}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that  $f(x) = \sqrt{x+1}$  is defined for all  $x \geq -1$

$$\text{So, domain}(f) = [-1, \infty]$$

Clearly,  $g(x) = \sqrt{9-x^2}$  is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned}\text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\ &= [-1, 3]\end{aligned}$$

$$\text{We have, } g(x) = \sqrt{9-x^2}$$

$$\therefore 9 - x^2 = 0 \Rightarrow x^2 - 9 = 0$$

$$\Rightarrow (x-3)(x+3) = 0$$

$$\Rightarrow x = \pm 3$$

$$\text{So, domain}\left(\frac{f}{g}\right) = [-1, 3] - [-3, 3] = [-1, 3]$$

$$\therefore \frac{f}{g} : [-1, 3] \rightarrow \mathcal{R} \text{ is given by } \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+1}}{\sqrt{9-x^2}}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that  $f(x) = \sqrt{x+1}$  is defined for all  $x \geq -1$

$$\text{So, } \text{domain}(f) = [-1, \infty]$$

Clearly,  $g(x) = \sqrt{9-x^2}$  is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned} \text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\ &= [-1, 3] \end{aligned}$$

We have,

$$f(x) = \sqrt{x+1}$$

$$\therefore \sqrt{x+1} = 0$$

$$\Rightarrow x+1 = 0$$

$$\Rightarrow x = -1$$

$$\begin{aligned} \text{So, } \text{domain}\left(\frac{g}{f}\right) &= [-1, 3] - \{-1\} \\ &= [-1, 3] \end{aligned}$$

$$\therefore \frac{g}{f} : [-1, 3] \rightarrow \mathcal{R} \text{ is given by } \frac{g}{f}(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{9-x^2}}{\sqrt{x+1}}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that  $f(x) = \sqrt{x+1}$  is defined for all  $x \geq -1$

$$\text{So, } \text{domain}(f) = [-1, \infty]$$

Clearly,  $g(x) = \sqrt{9-x^2}$  is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned} \text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\ &= [-1, 3] \end{aligned}$$

$$\begin{aligned} 2f - \sqrt{5}g : [-1, 3] \rightarrow \mathcal{R} \text{ defined by } (2f - \sqrt{5}g)(x) &= 2\sqrt{x+1} - \sqrt{5}\sqrt{9-x^2} \\ &= 2\sqrt{x+1} - \sqrt{45-5x^2}. \end{aligned}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that  $f(x) = \sqrt{x+1}$  is defined for all  $x \geq -1$

$$\text{So, } \text{domain}(f) = [-1, \infty]$$

Clearly,  $g(x) = \sqrt{9-x^2}$  is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$$\Rightarrow x^2 - 3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned} \text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\ &= [-1, 3] \end{aligned}$$

$$f^2 + 7f : [-1, \infty] \rightarrow \mathcal{R} \text{ defined by } (f^2 + 7f)(x) = f^2(x) + 7f(x)$$

$$[\therefore D(f) = [-1, \infty]]$$

$$\begin{aligned}
 &= (\sqrt{x+1})^2 + 7\sqrt{x+1} \\
 &= x+1+7\sqrt{x+1}
 \end{aligned}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that  $f(x) = \sqrt{x+1}$  is defined for all  $x \geq -1$

$$\text{So, domain}(f) = [-1, \infty]$$

Clearly,  $g(x) = \sqrt{9-x^2}$  is defined for

$$9-x^2 \geq 0 \Rightarrow x^2-9 \leq 0$$

$$\Rightarrow x^2-3^2 \leq 0$$

$$\Rightarrow (x-3)(x+3) \leq 0$$

$$\Rightarrow x \in [-3, 3]$$

$$\therefore \text{domain}(g) = [-3, 3]$$

Now,

$$\begin{aligned}
 \text{domain}(f) \cap \text{domain}(g) &= [-1, \infty] \cap [-3, 3] \\
 &= [-1, 3]
 \end{aligned}$$

We have,

$$g(x) = \sqrt{9-x^2}$$

$$\therefore 9-x^2 = 0 \Rightarrow x^2-9 = 0$$

$$\Rightarrow (x-3)(x+3) = 0$$

$$\Rightarrow x = \pm 3$$

$$\begin{aligned}
 \text{So, domain}\left(\frac{1}{g}\right) &= [-3, 3] - \{-3, 3\} \\
 &= (-3, 3)
 \end{aligned}$$

$$\therefore \frac{5}{g} = (-3, 3) \rightarrow \mathbb{R} \text{ defined by } \left(\frac{5}{g}\right)(x) = \frac{5}{\sqrt{9-x^2}}$$

### Functions Ex 3.4 Q5

We have,

$$f(x) = \log_e(1-x)$$

$$\text{and } g(x) = [x]$$

$f(x) = \log_e(1-x)$  is defined, if  $1-x > 0$

$$\Rightarrow 1 > x$$

$$\Rightarrow x < 1$$

$$\Rightarrow x \in (-\infty, 1)$$

$$\therefore \text{Domain}(f) = (-\infty, 1)$$

$$g(x) = [x] \text{ is defined for all } x \in \mathbb{R}$$

$$\therefore \text{Domain}(g) = \mathbb{R}$$

$$\begin{aligned}
 \therefore \text{Domain}(f) \cap \text{Domain}(g) &= (-\infty, 1) \cap \mathbb{R} \\
 &= (-\infty, 1)
 \end{aligned}$$

$$\begin{aligned}
 \text{(i) } f+g : (-\infty, 1) \rightarrow \mathbb{R} \text{ defined by } (f+g)(x) &= f(x) + g(x) \\
 &= \log_e(1-x) + [x]
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) } fg : (-\infty, 1) \rightarrow \mathbb{R} \text{ defined by } (fg)(x) &= f(x) \times g(x) \\
 &= \log_e(1-x) \times [x] \\
 &= [x] \log_e(1-x)
 \end{aligned}$$

$$\text{(iii) } g(x) = [x]$$

$$\therefore [x] = 0$$

$$\Rightarrow x \in (0, 1)$$

$$\begin{aligned}
 \text{So, domain}\left(\frac{f}{g}\right) &= \text{domain}(f) \cap \text{domain}(g) - \{x : g(x) = 0\} \\
 &= (-\infty, 0)
 \end{aligned}$$

$$\therefore \frac{f}{g} : (-\infty, 0) \rightarrow \mathbb{R} \text{ defined by } \left(\frac{f}{g}\right)(x) = \frac{\log_e(1-x)}{[x]}$$

(iv) We have

$$f(x) = \log_e(1-x)$$

$$\Rightarrow \frac{1}{f(x)} = \frac{1}{\log_e(1-x)}$$

$$\therefore \frac{1}{f(x)} \text{ is defined if } \log_e(1-x) \text{ is defined and } \log_e(1-x) \neq 0$$

$$\Rightarrow 1-x > 0 \quad \text{and} \quad 1-x \neq 0$$

$$\Rightarrow x < 1 \quad \text{and} \quad x \neq 0$$

$$\Rightarrow x \in (-\infty, 0) \cup (0, 1)$$

$$\therefore \text{domain}\left(\frac{g}{f}\right) = (-\infty, 0) \cup (0, 1)$$

$$\frac{g}{f} : (-\infty, 0) \cup (0, 1) \rightarrow \mathbb{R} \text{ defined by } \left(\frac{g}{f}\right)(x) = \frac{[x]}{\log_e(1-x)}$$

Now,

$$\begin{aligned} (f+g)(-1) &= f(-1) + g(-1) \\ &= \log_e(1-(-1)) + [-1] \\ &= \log_e 2 - 1 \end{aligned}$$

$$\Rightarrow (f+g)(-1) = \log_e 2 - 1$$

$$\text{(v)} \quad fg(0) = \log_e(1-0) \times [0] = 0$$

$$\text{(vi)} \quad \left(\frac{f}{g}\right)\left(\frac{1}{2}\right) = \text{does not exist}$$

$$\text{(vii)} \quad \left(\frac{g}{f}\right)\left(\frac{1}{2}\right) = \frac{\left[\frac{1}{2}\right]}{\log_e\left(1-\frac{1}{2}\right)} = 0$$

### Functions Ex 3.4 Q6

We have,

$$f(x) = \sqrt{x+1}, \quad g(x) = \frac{1}{x}$$

$$\text{and } h(x) = 2x^2 - 3$$

Clearly,  $f(x)$  is defined for  $x+1 \geq 0$

$$\Rightarrow x \geq -1$$

$$\Rightarrow x \in [-1, \infty]$$

$$\therefore \text{Domain}(f) = [-1, \infty]$$

$g(x)$  is defined for  $x \neq 0$

$$\Rightarrow x \in \mathbb{R} - \{0\}$$

and,  $h(x)$  is defined for all  $x \in \mathbb{R}$

$$\therefore \text{Domain}(f) \cap \text{Domain}(g) \cap \text{Domain}(h) = [-1, \infty] - \{0\}$$

Clearly,

$$2f+g-h : [-1, \infty] - \{0\} \rightarrow \mathbb{R} \text{ is given by}$$

$$\begin{aligned} (2f+g-h)(x) &= 2f(x) + g(x) - h(x) \\ &= 2\sqrt{x+1} + \frac{1}{x} - 2x^2 + 3 \end{aligned}$$

$$\begin{aligned} \therefore (2f+g-h)(1) &= 2\sqrt{1+1} + \frac{1}{1} - 2 \times (1)^2 + 3 \\ &= 2\sqrt{2} + 1 - 2 + 3 \\ &= 2\sqrt{2} + 4 - 2 \\ &= 2\sqrt{2} + 2 \end{aligned}$$

and,  $(2f+g-h)(0)$  does not exist, it is not lies in the domain  $x \in [-1, \infty] - \{0\}$ .

### Functions Ex 3.4 Q7

Let,

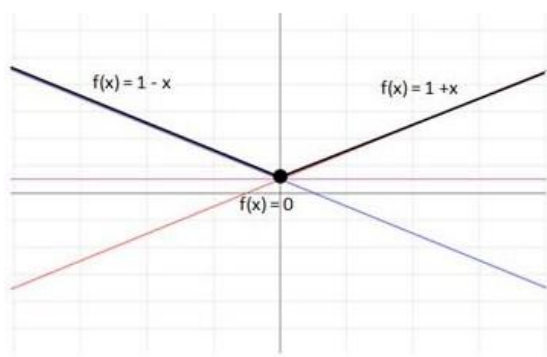
$$y = f(x) = \begin{cases} 1-x, & x < 0 \\ 1, & x = 0 \\ x+1, & x > 0 \end{cases}$$

The graph of  $f(x)$  for  $x < 0$  is the part of the line  $y = 1-x$  that lies to the left of origin.

The graph of  $f(x)$  for  $x > 0$  is the part of the line  $y = 1+x$  that lies to the right of origin.

For  $x = 0$ , the graph of  $f(x)$  represents the point  $(0,1)$

The graph of  $f(x)$  is shown below.



### Functions Ex 3.4 Q8

$f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $(f + g)(x) = 3x - 2$

$f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $(f - g)(x) = -x + 4$

$f: \mathbb{R} - \left\{ \frac{3}{2} \right\} \rightarrow \mathbb{R}$  defined by  $\frac{f}{g}(x) = \frac{x + 1}{2x - 3}$

### Functions Ex 3.4 Q9

$f + g: [0, \infty) \rightarrow \mathbb{R}$  defined by  $(f + g)(x) = \sqrt{x} + x$ ;

$f - g: [0, \infty) \rightarrow \mathbb{R}$  defined by  $(f - g)(x) = \sqrt{x} - x$ ;

$fg: [0, \infty) \rightarrow \mathbb{R}$  defined by  $(fg)(x) = x^{3/2}$ ;

$\frac{f}{g}: [0, \infty) \rightarrow \mathbb{R}$  defined by  $\left( \frac{f}{g} \right)(x) = \frac{1}{\sqrt{x}}$ ;

### Functions Ex 3.4 Q10

$(f + g): \mathbb{R} \rightarrow [0, \infty)$  defined by  $(f + g)(x) = x^2 + 2x + 1 = (x + 1)^2$

$(f - g): \mathbb{R} \rightarrow \mathbb{R}$  defined by  $(f - g)(x) = x^2 - 2x - 1$

$(fg): \mathbb{R} \rightarrow \mathbb{R}$  defined by  $(fg)(x) = 2x^3 + x^2$

$\left( \frac{f}{g} \right): \mathbb{R} \rightarrow \mathbb{R}$  defined by  $\left( \frac{f}{g} \right)(x) = \frac{x^2}{2x + 1}$