RD SHARMA

Solutions

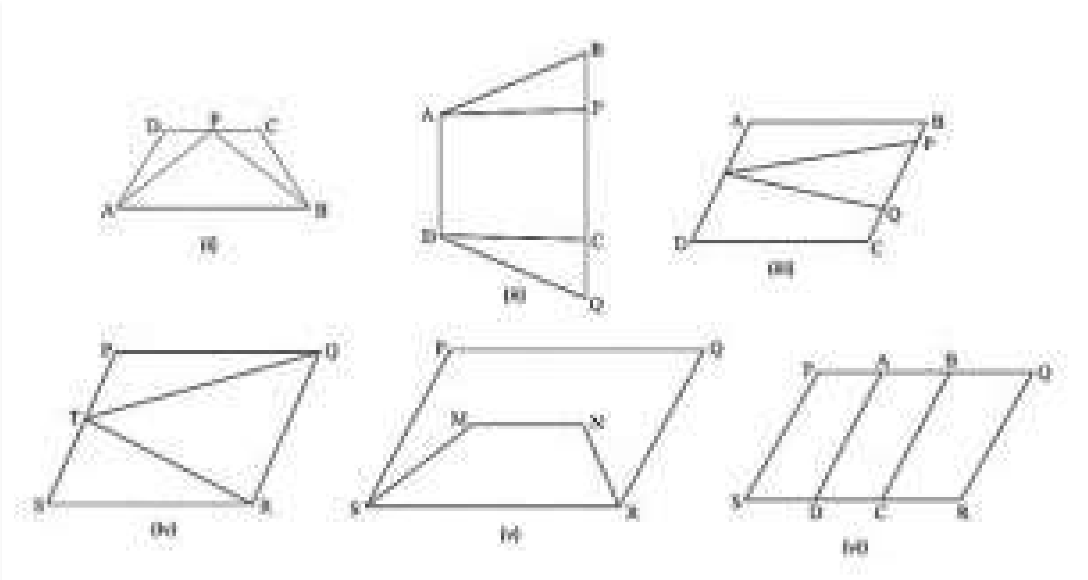
Class 9 Maths

Chapter 15

Ex *15.1*

Q 1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and two parallels:

SOLUTION :



(i) Δ PCD and trapezium ABCD are on the same base CD and between the same parallels AB and DC.

(ii) Parallelograms ABCD and APQD are on the same base AD and between the same parallels AD and BQ.

(iii) Parallelogram ABCD and ΔPQR are between the same parallels AD and BC but they are not on the same base.

(iv) ΔQQRT and parallelogram PQRS are on the same base QR and between the same parallels QR and PS.

(v) Parallelograms PQRS and trapezium SMNR are on the same base but not between the same parallels.

(vi) Parallelograms PQRS, AQRD, BCQR are between the same parallels. Also, parallelograms PQRS, BPSC, APSD are between the same parallels.

(vi) Parallelograms PQRS, AQRD, BCQR are between the same parallels. Also, parallelograms PQRS, BPSC, APSD are between the same parallels.

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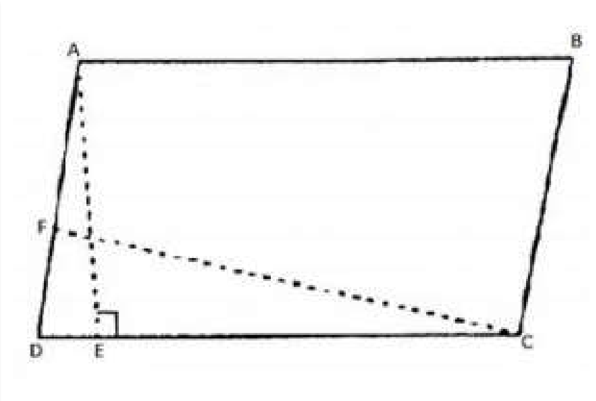
Ex *15.2*

Q 1. If figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm, and CF = 10 cm, Find AD.



Solution:

Given that,

In parallelogram ABCD, CD = AB = 16 cm
[∵ Opposite side of a parallelogram are equal]

We know that,

Area of parallelogram = Base × Corresponding altitude

Area of parallelogram ABCD = CD × AE = AD × CF

16 cm × cm = AD × 10 cm

AD = $\frac{16\times8}{10}$ cm = 12.8 cm

Thus, The length of AD is 12.8 cm.

Q 2. In Q 1, if AD = 6 cm, CF = 10 cm, and AE = 8 cm, Find AB.

Solution:

We know that,

Area of a parallelogram ABCD = AD × CF $\cdots\cdots\cdots(1)$

Again area of parallelogram ABCD = CD × AE$\cdots\cdots\cdots(2)$

Compare equation(1) and equation(2)

AD × CF = CD × AE

$\Rightarrow 6\times10 = D\times8$

$\Rightarrow D = \frac{60}{8}$ = 7.5 cm

$\therefore$ AB = DC = 7.5 cm [∵ Opposite side of a parallelogram are equal]

Q 3. Let ABCD be a parallelogram of area 124 cm^2. If E and F are the mid-points of sides AB and CD respectively, then find the area of parallelogram AEFD.

Solution:

Given,

Area of a parallelogram ABCD = 124 cm^2

Construction: Draw AP⊥DC

Proof:-

Area of a parallelogram AFED = DF × AP $\cdots\cdots\cdots(1)$

And area of parallelogram EBCF = FC × AP$\cdots\cdots\cdots(2)$

And DF = FC $\cdots\cdots\cdots(3)$ [F is the midpoint of DC]

Compare equation (1), (2) and (3)

Area of parallelogram AEFD = Area of parallelogram EBCF

$\therefore$ Area of parallelogram AEFD = $\frac{\text{Area of parallelogram ABCD}}{2}$ = $\frac{124}{2}$ = 62 cm^2

Q 4. If ABCD is a parallelogram, then prove that

Ar (ΔABD) = *Ar* (ΔBCD) = *Ar* (ΔABC) = *Ar* (ΔACD) = $\frac{1}{2}Ar\,(//^{gm}ABCD)$.

Solution:

Given:-

ABCD is a parallelogram,

To prove : − Ar (ΔABD) = Ar (ΔBCD) = Ar (ΔABC) = Ar (ΔACD) = $\frac{1}{2}Ar\,(//^{gm}ABCD)$.

Proof:- We know that diagonal of a parallelogram divides it into two equilaterals .

Since, AC is the diagonal.

Then, Ar (ΔABC) = Ar (ΔACD) = $\frac{1}{2}Ar\,(//^{gm}ABCD)\cdots\cdots\cdots(1)$

Since, BD is the diagonal.

Then , Ar (ΔABD) = Ar (ΔBCD) = $\frac{1}{2}Ar\,(//^{gm}ABCD)\cdots\cdots\cdots(2)$

Compare equation (1) and (2)

$\therefore$ Ar (ΔABC) = Ar (ΔACD) = Ar (ΔABD) = Ar (ΔBCD) = $\frac{1}{2}Ar\,(//^{gm}ABCD)$

RD SHARMA

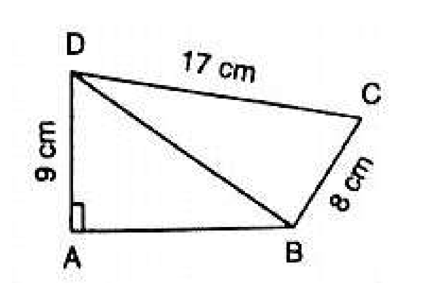
Solutions

Class 9 Maths

Chapter 15

Ex *15.3*

Q 1. In figure, compute the area of quadrilateral ABCD.



Solution:

Given:

DC = 17 cm, AD = 9 cm and BC = 8 cm

In ΔBCD we have

$CD^2 = BD^2 + BC^2$

$\Rightarrow 17^2 = BD^2 + 8^2$

$\Rightarrow BD^2 = 289 - 64$

$\Rightarrow BD = 15$

In ΔABD we have

$AB^2 + AD^2 = BD^2$

$\Rightarrow 15^2 = AB^2 + 9^2$

$\Rightarrow AB^2 = 225 - 81 = 144$

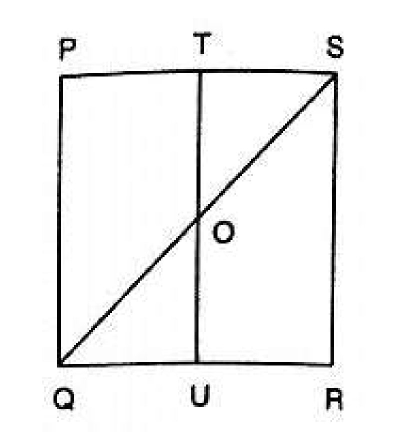
$\Rightarrow AB = 12$

$ar\,(quad\ ABCD) = ar\,(\Delta ABD) + ar\,(\Delta BCD)$

$ar\,(quad\ ABCD) = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 17) = 54 + 68 = 122\ cm^2$

$ar\,(quad\ ABCD) = \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 15) = 54 + 60 = 114\ cm^2$

Q2. In figure, PQRS is a square and T and U are, respectively, the midpoints of PS and QR . Find the area of ΔOTS if PQ = 8 cm.



Solution:

From the figure,

T and U are mid points of PS and QR respectively

$\therefore TU \| PQ$

$\Rightarrow TO \| PQ$

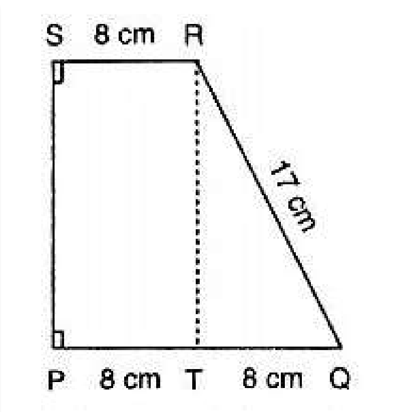
Thus , in ΔPQS , T is the mid point of PS and TO||PQ

$\therefore TO = \frac{1}{2} PQ = 4 \text{ cm}$

Also, TS = $\frac{1}{2} PS = 4 \text{ cm}$

$\therefore \text{ar}(\Delta OTS) = \frac{1}{2}(TO \times TS) = \frac{1}{2}(4 \times 4) \text{ cm}^2 = 8 \text{ cm}^2$

Q3. Compute the area of trapezium PQRS in figure



Solution:

We have,

$\text{ar}(\text{trap. } PQRS) = \text{ar}(\text{rect. } PSRT) + \text{ar}(\Delta QRT)$

$\Rightarrow \text{ar}(\text{trap. } PQRS) = PT \times RT + \frac{1}{2}(QT \times RT)$

= $= 8 \times RT + \frac{1}{2}(8 \times RT) = 12 \times RT$

In ΔQRT , we have

$QR^2 = QT^2 + RT^2$

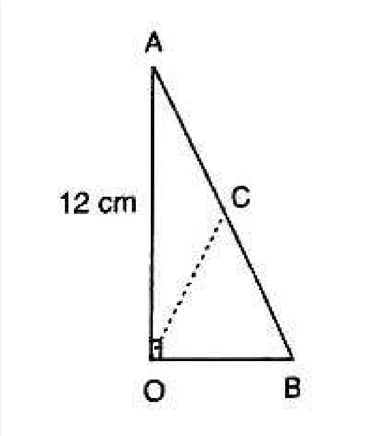
$\Rightarrow RT^2 = QR^2 - QT^2$

$\Rightarrow RT^2 = 17^2 - 8^2 = 225$

$\Rightarrow RT = 15$

Hence , area of trapezium = $12 \times 15 \text{ cm}^2$ = 180 cm^2

Q4. In figure, $\angle AOB = 90^\circ$, AC = BC , OA = 12 cm and OC = 6.5 cm .Find the area of ΔAOB.



Solution:

Since, the midpoint of the hypotenuse of a right triangle is equidistant from the vertices

$\therefore$ CA = CB = OC

$\Rightarrow$ CA = CB = 6.5 cm

$\Rightarrow$ AB = 13 cm

In right angled triangle OAB , we have

$AB^2 = OB^2 + OA^2$

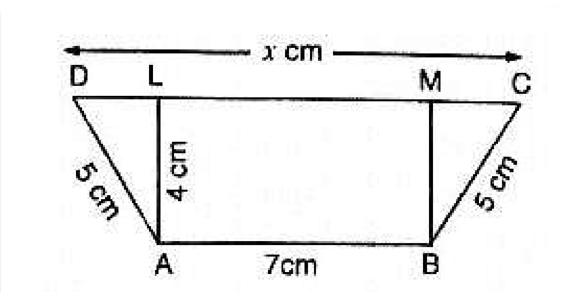
$\Rightarrow 13^2 = OB^2 + 12^2$

$\Rightarrow OB^2 = 13^2 - 12^2 = 169 - 144 = 25$

$\Rightarrow OB = 5$

$\therefore ar(\Delta AOB) = \frac{1}{2}(12 \times 5) = 30\ cm^2$

Q5. In figure, ABCD is a trapezium in which AB = 7 cm, AD = BC = 5 cm, DC = x cm , and distance between AB and DC is 4 cm . Find the value of x and area of trapezium ABCD.



Solution:

Draw AL ⊥ DC, BM ⊥ DC then ,

AL = BM = 4 cm and LM = 7 cm.

In Δ ADL , we have

$AD^2 = AL^2 + DL^2$

$\Rightarrow 25 = 16 + DL^2$

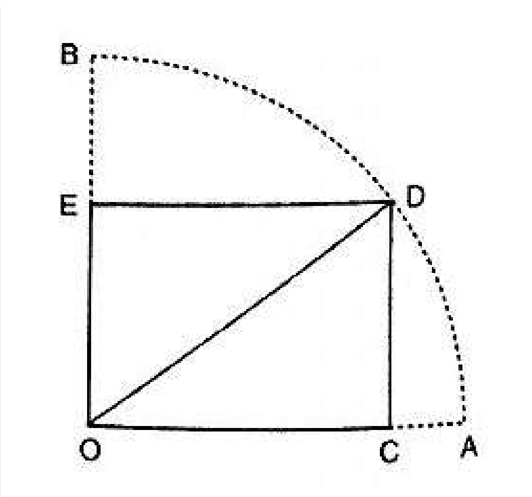
$\Rightarrow DL = 3$ cm

Similarly, MC = $\sqrt{BC^2 - BM^2}$ = $\sqrt{25 - 16} = 3$ cm

$\therefore x = CD = CM + ML + LD = (3 + 7 + 3)$ cm = 13 cm

$ar\,(\text{trap. } ABCD) = \frac{1}{2}(AB + CD) \times AL = \frac{1}{2}(7 + 13) \times 4 \text{ cm}^2 = 40 \text{ cm}^2$

Q 6. In figure, OCDE is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If OE = $2\sqrt{5}$ cm , find the area of the rectangle .



Solution:

Given OD = 10 cm and OE = $2\sqrt{5}$ cm

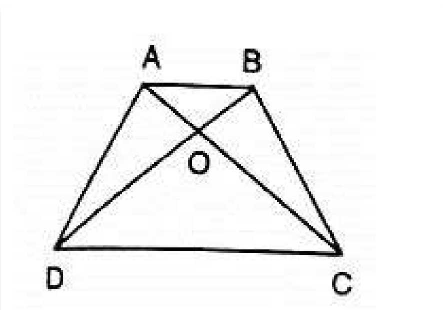
By using Pythagoras theorem

$\therefore OD^2 = OE^2 + DE^2$

$\Rightarrow DE = \sqrt{OD^2 - OE^2} = \sqrt{10^2 - (2\sqrt{5})^2} = 4\sqrt{5}$ cm

$\therefore$ Area of rectangle OCDE = $OE \times DE = 2\sqrt{5} \times 4\sqrt{5} \text{ cm}^2 = 40 \text{ cm}^2$

Q 7. In figure, ABCD is a trapezium in which AB || DC. Prove that $ar\,(\Delta AOD) = ar\,(\Delta BOC)$



Solution:

Given: ABCD is a trapezium in which AB || DC

To prove: $ar\,(\Delta AOD) = ar\,(\Delta BOC)$

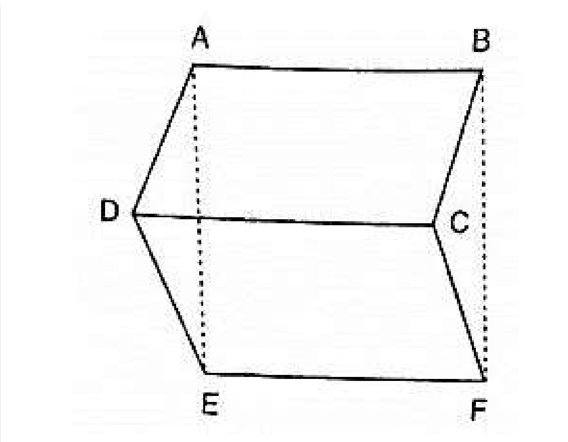
Proof:- Since , ΔADC and ΔBDC are on the same base DC and between same parallels AB and DC

Then, ar(ΔADC) = ar(ΔBDC)

$\Rightarrow \text{ar}(\Delta AOD) + \text{ar}(\Delta DOC) = \text{ar}(\Delta BOC) + \text{ar}(\Delta DOC)$

$\Rightarrow \text{ar}(\Delta AOD) = \text{ar}(\Delta BOC)$

Q 8. In figure, ABCD, ABFE and CDEF are parallelograms. Prove that $\text{ar}(\Delta ADE) = \text{ar}(\Delta BCF)$.



Solution:

Given that

ABCD is parallelogram $\Rightarrow AD = BC$

CDEF is parallelogram $\Rightarrow DE = CF$

ABFE is parallelogram $\Rightarrow AE = BF$

Thus, in Δs ADF and BCF , we have

AD = BC, DE = CF and AE = BF

So, by SSS criterion of congruence, we have

$\Delta ADE \cong \Delta BCF$

$\text{ar}(\Delta ADE) = \text{ar}(\Delta BCF)$

Q 9. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that :
$\text{ar}(\Delta APB) \times \text{ar}(\Delta CPD) = \text{ar}(\Delta APD) \times \text{ar}(\Delta BPC)$.

Solution:

Construction: – Draw BQ ⊥AC and DR ⊥ AC

Proof:-

L.H.S

= $\text{ar}(\Delta APB) \times \text{ar}(\Delta CDP)$

= $\frac{1}{2}[(AP \times BQ)] \times (\frac{1}{2} \times PC \times DR)$

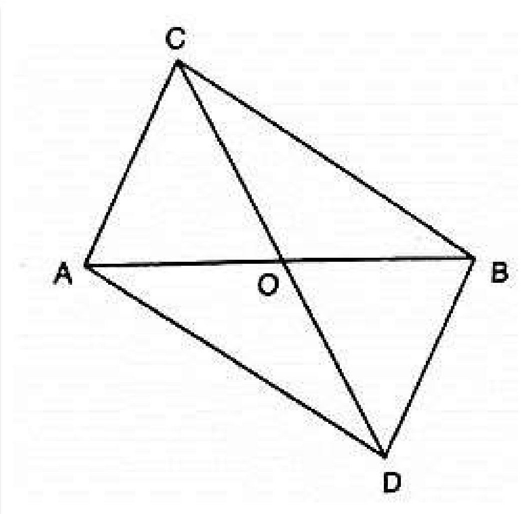
= $(\frac{1}{2} \times PC \times BQ) \times (\frac{1}{2} \times AP \times DR)$

= $\text{ar}(\Delta APD) \times \text{ar}(\Delta BPC)$.

= R.H.S

Hence proved.

Q 10. In figure, ABC and ABD are two triangles on the base AB. If line segment CD is bisected by AB at O, show that $ar(\Delta ABC) = ar(\Delta ABD)$.



Solution:

Given that CD is bisected by AB at O

To prove: $ar(\Delta ABC) = ar(\Delta ABD)$.

Construction: Draw CP ⊥ AB and DQ ⊥ AB .

Proof:

$ar(\Delta ABC) = \frac{1}{2} \times AB \times CP \cdots\cdots\cdot (1)$

$ar(\Delta ABD) = \frac{1}{2} \times AB \times DQ \cdots\cdots\cdot (2)$

In ΔCPO and ΔDQO

$\angle CPO = \angle DQO \qquad$ [each 90°]

Given that, CO = OD

$\angle COP = \angle DOQ \qquad$ [Vertically opposite angles are equal]

Then , $\Delta CP0 \cong \Delta DQO \qquad$ [By AAS condition]

$\therefore CP = DQ \qquad (3)$ [c. p. c. t]

Compare equation (1), (2) and (3)

$\therefore ar(\Delta ABC) = ar(\Delta ABD)$.

Q 11. If P is any point in the interior of a parallelogram ABCD , then prove that area of the triangle APB is less than half the area of parallelogram.

Solution:

Draw DN ⊥ AB and PM ⊥ AB

Now ,

$ar(||^{gm} ABCD) = AB \times DN, ar(\Delta APB) = \frac{1}{2}(AB \times PM)$

Now , PM < DN

$\Rightarrow AB \times PM < AB \times DN$

$\Rightarrow \frac{1}{2}(AB \times PM) < \frac{1}{2}(AB \times DN)$

$\Rightarrow ar(\Delta APB) < \frac{1}{2} ar(||^{gm} ABCD)$

Q 12. If AD is a median of a triangle ABC, then prove that triangles ADB and ADC are equal in area. If G is the mid-point of the median AD, prove that $ar(\Delta BGC) = 2ar(\Delta AGC)$.

Solution:

Draw AM ⊥ BC

Since, AD is the median of ΔABC

∴ BD = DC

$\Rightarrow BD = AM = DC \times AM$

$\Rightarrow \frac{1}{2}(BD \times AM) = \frac{1}{2}(DC \times AM)$

$\Rightarrow ar(\Delta ABD) = ar(\Delta ACD) \cdots\cdots\cdots (1)$

In ΔBGC , GD is the median

$\therefore ar(\Delta BGD) = ar(\Delta CGD) \cdots\cdots\cdots (2)$

In ΔACD , CG is the median

$\therefore ar(\Delta AGC) = ar(\Delta CGD) \cdots\cdots\cdots (3)$

From (2) and (3) we have,

$ar(\Delta BGD) = ar(\Delta AGC)$

But, $ar(\Delta BGC) = 2ar(\Delta BGD)$

$\therefore ar(\Delta BGC) = 2ar(\Delta AGC)$

Q 13. A point D is taken on the side BC of a ΔABC ***, such that BD = 2DC . Prove that*** $ar(\Delta ABD) = 2ar(\Delta ADC)$.

Solution:

Given that,

In ΔABC, BD = 2DC

To prove: $ar(\Delta ABD) = 2ar(\Delta ADC)$.

Construction:

Take a point E on BD such that BE = ED

Proof: Since, BE = ED and BD = 2 DC

Then, BE = ED = DC

We know that median of triangle divides it into two equal triangles.

$\therefore$ In ΔABD , AE is the median .

Then , ar $(\Delta ABD) = 2ar(\Delta AED) \cdots\cdots (1)$

In ΔAEC , AD is the median .

Then, ar $(\Delta AED) = 2ar(\Delta ADC) \cdots\cdots (2)$

Compare equation 1 and 2

$ar(\Delta ABD) = 2ar(\Delta ADC).$

Q 14. ABCD is a parallelogram whose diagonals intersect at O .If P is any point on BO, prove that :

(i) . $ar(\Delta ADO) = ar(\Delta CDO)$.

(ii) . $ar(\Delta ABP) = 2ar(\Delta CBP)$.

Solution:

Given that ABCD is the parallelogram

To Prove: (i) $ar(\Delta ADO) = ar(\Delta CDO)$.

(ii) $ar(\Delta ABP) = 2ar(\Delta CBP)$.

Proof: we know that diagonals of parallelogram bisect each other

$\therefore$ AO = OC and BO = OD

(i) . In Δ DAC , since DO is a median .

Then $ar(\Delta ADO) = ar(\Delta CDO)$.

(ii) . In Δ BAC , since BO is a median .

Then $ar(\Delta BAO) = ar(\Delta BCO) \cdots\cdots\cdots (1)$

In Δ PAC , since PO is a median .

Then $ar(\Delta PAO) = ar(\Delta PCO) \cdots\cdots\cdots (2)$

Subtract equation 2 from 1.

$\Rightarrow ar(\Delta BAO) - ar(\Delta PAO) = ar(\Delta BCO) - ar(\Delta PCO)$

$\Rightarrow ar(\Delta ABP) = 2ar(\Delta CBP)$.

Q 15. ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F.

(i) . Prove that $ar(\Delta ADF) = ar(\Delta ECF)$.

(ii) . If the area of ΔDFB = $3\ cm^2$ ***, find the area of*** $||^{gm}$ ***ABCD .***

Solution:

In triangles ADF and ECF, we have

$\angle ADF = \angle ECF$ [Alternate interior angles, Since $AD||BE$]

AD = EC [since $AD = BC = CE$]

And $\angle DFA = \angle CFA$ [Vertically opposite angles]

So, by AAS congruence criterion, we have

$\Delta ADF \cong \Delta ECF$

$\Rightarrow ar(\Delta ADF) = ar(\Delta ECF)$ and DF = CF .

Now, DF = CF

$\Rightarrow$ BF is a median in Δ BCD.

$\Rightarrow ar(\Delta BCD) = 2ar(\Delta BDF)$

$\Rightarrow ar(\Delta BCD) = 2 \times 3\ cm^2 = 6\ cm^2$

Hence, area of a parallelogram = $2ar(\Delta BCD) = 2 \times 6\ cm^2 = 12\ cm^2$

Q 16. ABCD is a parallelogram whose diagonals AC and BD intersect at O . A line through O intersects AB at P and DC at Q. Prove that $ar(\Delta POA) = ar(\Delta QOC)$**.**

Solution:

In triangles POA and QOC, we have

$\angle AOP = \angle COQ$

AO = OC

$\angle PAC = \angle QCA$

So, by ASA congruence criterion , we have

$\Delta POA \cong \Delta QOC$

$\Rightarrow ar(\Delta POA) = ar(\Delta QOC)$.

Q 17. ABCD is a parallelogram. E is a point on BA such that BE = 2EA and F is point on DC such that DF = 2FC. Prove that AECF is a parallelogram whose area is one third of the area of parallelogram ABCD.

Solution:

Draw FG $\perp$ AB

We have,

BE = 2 EA and DF = 2FC

$\Rightarrow$ AB − AE = 2 AE and DC − FC = 2 FC

$\Rightarrow$ AB = 3 AE and DC = 3 FC

$\Rightarrow AE = \frac{1}{3}AB$ and $FC = \frac{1}{3}DC \cdots\cdots\cdots(1)$

But AB = DC

Then, AE = FC [opposite sides of $||^{gm}$]

Thus, AE = FC and AE || FC

Then, AECF is a parallelogram

Now, area of parallelogram AECF = $AE \times FG$

$\Rightarrow ar(||^{gm}\ AECF) = \frac{1}{3}AB \times FG$ from(1)

$\Rightarrow 3\ ar(||^{gm}\ AECF) = AB \times FG \cdots\cdots (2)$

And $ar(||^{gm}\ ABCD) = AB \times FG \cdots\cdots (3)$

Compare equation 2 and 3

$\Rightarrow 3ar(||^{gm}\ AECF) = ar(||^{gm}\ ABCD)$

$\Rightarrow ar(||^{gm}\ AECF) = \frac{1}{3}ar(||^{gm}\ ABCD)$

Q 18. In a triangle ABC, P and Q are respectively the mid points of AB and BC and R is the mid point of AP. Prove that :

(i) . $ar(\Delta PBQ) = ar(\Delta ARC)$.

(ii) . $ar(\Delta PRQ) = \frac{1}{2}ar(\Delta ARC)$.

(iii) . $ar(\Delta RQC) = \frac{3}{8}ar(\Delta ABC)$.

Solution:

We know that each median of a triangle divides it into two triangles of equal area.

(i) . Since CR is the median of Δ CAP

$\therefore ar(\Delta CRA) = \frac{1}{2}ar(\Delta CAP) \cdots\cdots\cdots (1)$

Also , CP is the median of a Δ CAB

$\therefore ar(\Delta CAP) = ar(\Delta CPB) \cdots\cdots\cdots (2)$

From 1 and 2 , we get

$\therefore ar(\Delta ARC) = \frac{1}{2}ar(\Delta CPB) \cdots\cdots\cdots (3)$

PQ is the median of a Δ PBC

$\therefore ar(\Delta CPB) = 2ar(\Delta PBQ) \cdots\cdots\cdots (4)$

From 3 and 4, we get

$\therefore ar(\Delta ARC) = ar(\Delta PBQ) \cdots\cdots\cdots (5)$

(ii) . Since QP and QR medians of triangles QAB and QAP respectively

$\therefore ar(\Delta QAP) = ar(\Delta QBP) \cdots\cdots\cdots (6)$

And $ar(\Delta QAP) = 2ar(\Delta QRP) \cdots\cdots\cdots (7)$

From 6 and 7, we get

$ar(\Delta PRQ) = \frac{1}{2}ar(\Delta PBQ) \cdots\cdots\cdots (8)$

From 5 and 8, we get

$ar(\Delta PRQ) = \frac{1}{2}ar(\Delta ARC)$

(iii) . Since, LR is a median of Δ CAP

$\therefore$ ar $(\Delta ARC) = \frac{1}{2}$ ar (ΔCAD)

$= \frac{1}{2} \times \frac{1}{2}$ ar (ΔABC)

$= \frac{1}{4}$ ar (ΔABC)

Since RQ is the median of Δ RBC.

$\therefore$ ar $(\Delta RQC) = \frac{1}{2}$ ar (ΔRBC)

$= \frac{1}{2}\{\text{ar}(\Delta ABC) - \text{ar}(\Delta ARC)\}$

$= \frac{1}{2}\{\text{ar}(\Delta ABC) - \frac{1}{4}\text{ar}(\Delta ABC)\}$

$= \frac{3}{8}$ ar (ΔABC)

Q 19. ABCD is a parallelogram. G is a point on AB such that AG = 2GB and E is point on DC such that CE = 2DE and F is the point of BC such that BF = 2FC. Prove that:

(i) . ar $(ADEG) =$ ar $(GBCE)$.

(ii) . ar $(\Delta EGB) = \frac{1}{6}$ ar $(ABCD)$.

(iii) . ar $(\Delta EFC) = \frac{1}{2}$ ar (ΔEBF).

(iv) . ar $(\Delta EGB) = \frac{3}{2} \times$ ar (ΔEFC)

(v) . Find what portion of the area of parallelogram is the area of Δ EFG.

Solution:

Given: ABCD is a parallelogram

AG = 2 GB, CE = 2 DE and BF = 2 FC

To prove:

(i) . ar $(ADEG) =$ ar $(GBCE)$.

(ii) . ar $(\Delta EGB) = \frac{1}{6}$ ar $(ABCD)$.

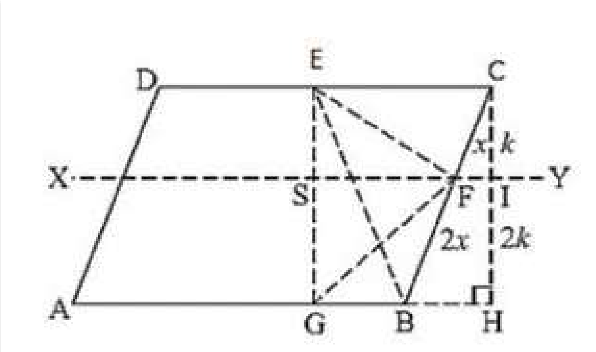
(iii) . ar $(\Delta EFC) = \frac{1}{2}$ ar (ΔEBF).

(iv) . ar $(\Delta EGB) = \frac{3}{2} \times$ ar (ΔEFC)

(v) . Find what portion of the area of parallelogram is the area of Δ EFG.

Construction: Draw a parallel line to AB through point F and a perpendicular line to AB from C

Proof:



(i) .Since ABCD is a parallelogram

So, AB = CD and AD = BC

Consider the two trapezium s ADEG and GBCE

Since AB = DC, EC = 2DE, AG = 2GB

$\Rightarrow \mathrm{ED} = \frac{1}{3}\mathrm{CD} = \frac{1}{3}\mathrm{AB}$ and $\mathrm{EC} = \frac{2}{3}\mathrm{CD} = \frac{2}{3}\mathrm{AB}$

$\Rightarrow \mathrm{AG} = \frac{2}{3}\mathrm{AB}$ and $\mathrm{BG} = \frac{1}{3}\mathrm{AB}$

So , $\mathrm{DE} + \mathrm{AG} = \frac{1}{3}\mathrm{AB} + \frac{2}{3}\mathrm{AB} = \mathrm{AB}$ and $\mathrm{EC} + \mathrm{BG} = \frac{2}{3}\mathrm{AB} + \frac{1}{3}\mathrm{AB} = \mathrm{AB}$

Since the two trapezium ADEG and GBCE have same height and their sum of two parallel sides are equal

Since Area of trapezium = $\frac{\text{sum of parallel sides}}{2} \times \text{height}$

So, $\mathrm{ar}(\mathrm{ADEG}) = \mathrm{ar}(\mathrm{GBCE})$.

(ii) . Since we know from above that

$\mathrm{BG} = \frac{1}{2}\mathrm{AB}$. So

$\mathrm{ar}(\Delta\mathrm{EGB}) = \frac{1}{2} \times \mathrm{GB} \times \text{Height}$

$\mathrm{ar}(\Delta\mathrm{EGB}) = \frac{1}{2} \times \frac{1}{3} \times \mathrm{AB} \times \text{Height}$

$\mathrm{ar}(\Delta\mathrm{EGB}) = \frac{1}{6} \times \mathrm{AB} \times \text{Height}$

$\mathrm{ar}(\Delta\mathrm{EGB})$= $\frac{1}{6}\mathrm{ar}(\mathrm{ABCD})$.

(iii) . Since height if triangle EFC and EBF are equal.So

$\mathrm{ar}(\Delta\mathrm{EFC}) = \frac{1}{2} \times \mathrm{FC} \times \text{Height}$

$\mathrm{ar}(\Delta\mathrm{EFC}) = \frac{1}{2} \times \frac{1}{2} \times \mathrm{FB} \times \text{Height}$

$\mathrm{ar}(\Delta\mathrm{EFC}) = \frac{1}{2}\mathrm{ar}(\mathrm{EBF})$

Hence , $\mathrm{ar}(\Delta\mathrm{EFC})$= $\frac{1}{2}\mathrm{ar}(\Delta\mathrm{EBF})$.

(iv) . Consider the trapezium in which

$\mathrm{ar}(\mathrm{EGBC}) = \mathrm{ar}(\Delta\mathrm{EGB}) + \mathrm{ar}(\Delta\mathrm{EBF}) + \mathrm{ar}(\Delta\mathrm{EFC})$

$\Rightarrow \frac{1}{2}\text{ar}(\text{ABCD}) = \frac{1}{6}\text{ar}(\text{ABCD}) + 2\text{ar}(\Delta\text{EFC}) + \text{ar}(\Delta\text{EFC})$

$\Rightarrow \frac{1}{3}\text{ar}(\text{ABCD}) = 3\text{ar}(\Delta\text{EFC})$

$\Rightarrow \text{ar}(\Delta\text{EFC}) = \frac{1}{9}\text{ar}(\text{ABCD})$

Now from (ii)part we have

$\text{ar}(\Delta\text{EGB}) = \frac{1}{6}\text{ar}(\Delta\text{EFC})$

$\text{ar}(\Delta\text{EGB}) = \frac{3}{2} \times \frac{1}{9}\text{ar}(\text{ABCD})$

$\text{ar}(\Delta\text{EGB}) = \frac{3}{2}\text{ar}(\Delta\text{EFC})$

$\therefore \text{ar}(\Delta\text{EGB}) = \frac{3}{2}\text{ar}(\Delta\text{EFC})$

(v) . In the figure it is given that FB = 2CF .Let CF = x and FB = 2x.

Now consider the two triangles CFI and CBH which are similar triangle.

So by the property of similar triangle CI = k and IH = 2k

Now consider the triangle EGF in which

$\text{ar}(\Delta\text{EFG}) = \text{ar}(\Delta\text{ESF}) + \text{ar}(\Delta\text{SGF})$

$\text{ar}(\Delta\text{EFG}) = \frac{1}{2}\text{SF} \times \text{k} + \frac{1}{2}\text{SF} \times 2\text{k}$

$\text{ar}(\Delta\text{EFG}) = \frac{3}{2}\text{SF} \times \text{k} \cdots\cdots (\text{i})$

Now ,

$\text{ar}(\Delta\text{EGBC}) = \text{ar}(\text{SGBF}) + \text{ar}(\text{ESFC})$

$\text{ar}(\Delta\text{EGBC}) = \frac{1}{2}(\text{SF} + \text{GB}) \times 2\text{k} + \frac{1}{2}(\text{SF} + \text{EC}) \times \text{k}$

$\text{ar}(\Delta\text{EGBC}) = \frac{3}{2}\text{k} \times \text{SF} + (\text{GB} + \frac{1}{2}\text{EC}) \times \text{k}$

$\text{ar}(\Delta\text{EGBC}) = \frac{3}{2}\text{k} \times \text{SF} + (\frac{1}{3}\text{AB} + \frac{1}{2} \times \frac{2}{3}\text{AB}) \times \text{k}$

$\frac{1}{2}\text{ar}(\Delta\text{ABCD}) = \frac{3}{2}\text{k} \times \text{SF} + \frac{2}{3}\text{AB} \times \text{k}$

$\Rightarrow \text{ar}(\Delta\text{ABCD}) = 3\text{k} \times \text{SF} + \frac{4}{3}\text{AB} \times \text{k}$ [Multiply both sides by 2]

$\Rightarrow \text{ar}(\Delta\text{ABCD}) = 3\text{k} \times \text{SF} + \frac{4}{9}\text{ar}(\text{ABCD})$

$\Rightarrow \text{k} \times \text{SF} = \frac{5}{27}\text{ar}(\text{ABCD}) \cdots\cdots\cdots (2)$

From 1 and 2 we have ,

$\text{ar}(\Delta\text{EFG}) = \frac{3}{2} \times \frac{5}{27}\text{ar}(\text{ABCD})$

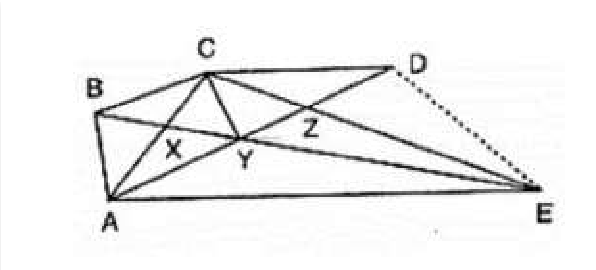
$\text{ar}(\Delta\text{EFG}) = \frac{5}{18}\text{ar}(\text{ABCD})$

Q 20. In figure, CD || AE and CY || BA.

(i) . Name a triangle equal in area of ΔCBX

(ii) . Prove that $ar(\Delta ZDE) = ar(\Delta CZA)$.

(iii) . Prove that $ar(BCZY) = ar(\Delta EDZ)$.



Solution:

Since, triangle BCA and triangle BYA are on the same base BA and between same parallel s BA and CY.

Then $ar(\Delta BCA) = ar(\Delta BYA)$

$\Rightarrow ar(\Delta CBX) + ar(\Delta BXA) = ar(\Delta BXA) + ar(\Delta AXY)$

$\Rightarrow ar(\Delta CBX) = ar(\Delta AXY) \cdots\cdots (1)$

Since, triangles ACE and ADE are on the same base AE and between same parallels CD and AE

Then, $ar(\Delta ACE) = ar(\Delta ADE)$

$ar(\Delta CZA) + ar(\Delta AZE) = ar(\Delta AZE) + ar(\Delta DZE)$

$ar(\Delta CZA) = ar(\Delta DZE) \cdots\cdots (2)$

Adding $ar(\Delta CYG)$ on both sides , we get

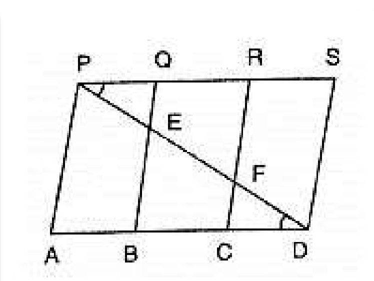
$\Rightarrow ar(\Delta CBX) + ar(\Delta CYZ) = ar(\Delta CAY) + ar(\Delta CYZ)$

$\Rightarrow ar(BCZY) = ar(\Delta CZA) \cdots\cdots (3)$

Compare equation 2 and 3

$\Rightarrow ar(BCZY) = ar(\Delta DZE)$

Q 21. In figure, PSDA is a parallelogram in which PQ = QR =RS and AP || BQ ||CR. Prove that $ar(\Delta PQE) = ar(\Delta CFD)$.



Solution:

Given that PSDA is a parallelogram

Since, AP || BQ || CR ||DS and AD ||PS

Therefore, PQ = CD (equ. 1)

In triangle BED, C is the midpoint of BD and CF || BE

Therefore, F is the midpoint of ED

⇒ EF = PE

Smiliarly,

EF = PE

Therefore, PE = FD (equ . 2)

In triangles PQE and CFD, we have

PE = FD

Therefore, ∠ EPQ = ∠FDC [Alternate angles]

So, by SAS criterion , we have

$$\Delta PQE \cong \Delta DCF$$

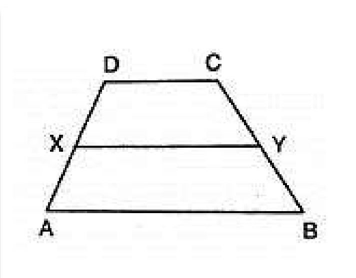
$$\Rightarrow ar(\Delta PQE) = ar(\Delta DCF)$$

Q 22. In figure, ABCD is a trapezium in which AB || DC and DC = 40 cm and AB = 60 cm .If X and Y are , respectively , the mid points of AD and BC , prove that :

(i) . XY = 50 cm

(ii) . DCYX is a trapezium

(iii) . $ar(\text{trap. } DCYX) = \frac{9}{11} ar(XYBA)$.



Solution:

(i) Join DY and produce it to meet AB produced at P.

In triangles BYP and CYD we have,

∠BYP = ∠ CYD [Vertically opposite angles]

∠DCY = ∠ PBY [Since , DC ||AP]

And BY = CY

So, by ASA congruence criterion, we have

$$(\Delta BYP) \cong (\Delta CYD)$$

⇒ DY = Yp and DC = BP

⇒ Y is the midpoint of DP

Also , x is the mid point of AD

Therefore, XY ||AP and XY ||$\frac{1}{2}$AP

$\Rightarrow XY = \frac{1}{2}(AB + BP)$

$\Rightarrow XY = \frac{1}{2}(AB + DC)$

$\Rightarrow XY = \frac{1}{2}(60 + 40)$

= 50 cm

(ii) We have, XY ||AP

⇒ XY || AB and AB ||DC

⇒ XY || DC

⇒ DCYX is a trapezium

(iii) Since x and y are the mid points of Ad and BC respectively.

Therefore, trapezium DCYX and ABYX are of the same height say h cm

Now,

$ar(trap. DCXY) = \frac{1}{2}(DC + XY) \times h$

$\Rightarrow ar(trap. DCXY) = \frac{1}{2}(50 + 40) \times h\ cm^2 = 45h\ cm^2$

$\Rightarrow ar(trap. ABYX) = \frac{1}{2}(AB + XY) \times h$

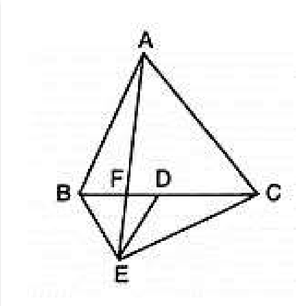
$\Rightarrow ar(trap. ABYX) = \frac{1}{2}(60 + 50) \times h\ cm^2 = 55h\ cm^2$

$\frac{ar(trap.DCYX)}{ar(trap.ABYX)} = \frac{45h}{55h} = \frac{9}{11}$

$\Rightarrow ar(trap. DCYX) = \frac{9}{11} ar(trap. ABYX)$

Q 23. In figure ABC and BDE are two equilateral triangles such that D is the midpoint of BC. AE intersects BC in F. Prove that:

- ***(i)*** . $ar(\Delta BDE) = \frac{1}{4} ar(\Delta ABC)$.
- ***(ii)*** . $ar(\Delta BDE) = \frac{1}{2} ar(\Delta BAE)$.
- ***(iii)*** . $ar(\Delta BFE) = ar(\Delta AFD)$.
- ***(iv)*** . $ar(\Delta ABC) = 2\, ar(\Delta BEC)$.
- ***(v)*** . $ar(\Delta FED) = \frac{1}{8} ar(\Delta AFC)$.
- ***(vi)*** . $ar(\Delta BFE) = 2\, ar(\Delta EFD)$.



Solution:

Given that ABC and BDE are two equilateral triangles.

Let AB = BC = CA = x. Then, BD = $\frac{x}{2}$ = DE = BE

(i) We have,

$ar(\Delta ABC) = \frac{\sqrt{3}}{4}x^2$ and $ar(\Delta BDE) = \frac{\sqrt{3}}{4}(\frac{x}{2})^2 = \frac{1}{4} \times \frac{\sqrt{3}}{4}x^2$

Therefore , $ar(\Delta BDE)$= $\frac{1}{4}ar(\Delta ABC)$.

(ii) . It is given that triangles, ABC and BED are equilateral triangles

∠ ACB = ∠DBE = 60°

⇒ BE || AC (Since , alternative angles are equal)

Trinagles BAF and BEC are on the same base BE and between same parallels BF and AC .

Therefore, $ar(\Delta BAE) = ar(\Delta BEC)$

$\Rightarrow ar(\Delta BAE) = 2ar(\Delta BDE)$ [Since , ED is a median of triangle EBC ; $ar(\Delta BEC) = 2ar(\Delta BDE)$]

$\therefore ar(\Delta BDE) = \frac{1}{2}ar(\Delta BAE)$

(iii) Since, triangles ABC and BDE are equilateral triangles

$\therefore \angle ABC = 60^\circ$ and $\angle BDE = 60^\circ$

$\angle ABC = \angle BDE$

⇒ AB ||DE (since, alternate angles are equal)

Triangles BED and AED are on the same base ED and between same parallels AB and DE.

Therefore, $ar(\Delta BED) = ar(\Delta AED)$

$\Rightarrow ar(\Delta BED) - ar(\Delta EFD) = ar(\Delta AED) - ar(\Delta EFD)$

$\Rightarrow ar(\Delta BEF) = ar(\Delta AFD)$

(iv) Since ED is the median of triangle BEC

Therefore, $ar(\Delta BEC) = 2ar(\Delta BDE)$

$\Rightarrow ar(\Delta BEC) = 2 \times \frac{1}{4}ar(\Delta ABC)$ [From 1 , $ar(\Delta BDE) = \frac{1}{4}ar(\Delta ABC)$]

$\Rightarrow ar(\Delta BEC) = \frac{1}{2}ar(\Delta ABC)$

$\Rightarrow \text{ar}(\Delta ABC) = 2\text{ar}(\Delta BEC)$

(v) $\text{ar}(\Delta AFC) = \text{ar}(\Delta AFD) + \text{ar}(\Delta ADC)$

$\Rightarrow \text{ar}(\Delta BFE) + \frac{1}{2}\text{ar}(\Delta ABC)$ [using part (iii) , and AD is the median of triangle ABC]

$= \text{ar}(\Delta BFE) + \frac{1}{2} \times 4\text{ar}(\Delta BDE)$ (using part (i))

$= \text{ar}(\Delta BFE) = 2\text{ar}(\Delta FED) \cdots\cdot (3)$

$\text{ar}(\Delta BDE) = \text{ar}(\Delta BFE) + \text{ar}(\Delta FED)$

$\Rightarrow 2\text{ar}(\Delta FED) + \text{ar}(\Delta FED)$

$\Rightarrow 3\text{ar}(\Delta FED) \cdots\cdots (4)$

From 2, 3 and 4 we get ,

$\text{ar}(\Delta AFC) = 2\text{ar}(\Delta FED) + 2 \times 3\text{ar}(\Delta FED) = 8\text{ar}(\Delta FED)$

$\text{ar}(\Delta FED) = \frac{1}{8}\text{ar}(\Delta AFC)$

(vi) Let h be the height of vertex E, corresponding to the side BD in triangle BDE.

Let H be the height of vertex A, corresponding to the side BC in triangle ABC

From part (i)

$\text{ar}(\Delta BDE) = \frac{1}{4}\text{ar}(\Delta ABC)$

$\Rightarrow \frac{1}{2} \times BD \times h = \frac{1}{4}(\frac{1}{2} \times BC \times h)$

$\Rightarrow BD \times h = \frac{1}{4}(2BD \times H)$

$\Rightarrow h = \frac{1}{2}H \cdots\cdots\cdot (1)$

From part (iii)

$\text{ar}(\Delta BFE) = \text{ar}(\Delta AFD)$

$\text{ar}(\Delta BFE) = \frac{1}{2} \times FD \times H$

$\text{ar}(\Delta BFE) = \frac{1}{2} \times FD \times 2h$

$\text{ar}(\Delta BFE) = 2(\frac{1}{2} \times FD \times h)$

$\text{ar}(\Delta BFE) = 2\text{ar}(\Delta EFD)$

Q 24. D is the midpoint of side BC of ΔABC and E is the midpoint of BD. If O is the midpoint of AE, Prove that $\text{ar}(\Delta BOE) = \frac{1}{8}\text{ar}(\Delta ABC)$.

Solution:

Given that

D is the midpoint of sides BC of triangle ABC

E is the midpoint of BD and O is the midpoint of AE

Since AD and AE are the medians of triangles, ABC and ABD respectively

$\therefore \operatorname{ar}(\Delta ABD)=\frac{1}{2}\operatorname{ar}(\Delta ABC)\cdots\cdots\cdots(1)$

$\therefore \operatorname{ar}(\Delta ABE)=\frac{1}{2}\operatorname{ar}(\Delta ABD)\cdots\cdots\cdots(2)$

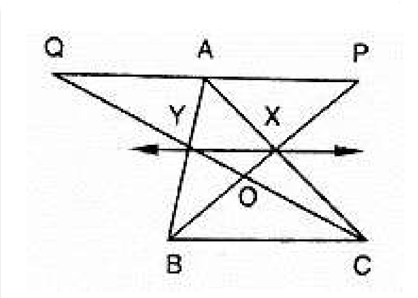
OB is the median of triangle ABE

Therefore, $\therefore \operatorname{ar}(\Delta BOE)=\frac{1}{2}\operatorname{ar}(\Delta ABE)$

From 1, 2 and 3 , we have

$\therefore \operatorname{ar}(\Delta BOE)=\frac{1}{8}\operatorname{ar}(\Delta ABC)$

Q 25. In figure, X and Y are the mid points of AC and AB respectively, QP ||BC and CYQ and BXP are straight lines . Prove that $\operatorname{ar}(\Delta ABP)=\operatorname{ar}(\Delta ACQ)$.



Solution:

Since X and Y are the mid points of AC and AB respectively.

Therefore, XY || BC

Clearly, triangles BYC and BXC are on the same base BC and between the same parallels Xy and BC

$\therefore \operatorname{ar}(\Delta BYC)=\operatorname{ar}(\Delta BXC)$

$\Rightarrow \operatorname{ar}(\Delta BYC)-\operatorname{ar}(\Delta BOC)=\operatorname{ar}(\Delta BXC)-\operatorname{ar}(\Delta BOC)$

$\Rightarrow \operatorname{ar}(\Delta BOY)=\operatorname{ar}(\Delta COX)$

$\Rightarrow \operatorname{ar}(\Delta BOY)+\operatorname{ar}(\Delta XOY)=\operatorname{ar}(\Delta COX)+\operatorname{ar}(\Delta XOY)$

$\Rightarrow \operatorname{ar}(\Delta BXY)=\operatorname{ar}(\Delta CXY)$ (2)

We observed that the quadrilaterals XYAP and XYAQ are on the same base XY and between same parallels XY and PQ.

$\therefore \operatorname{ar}(\text{quad. }XYAP)=\operatorname{ar}(\text{quad}XYQA)\cdots\cdots\cdots(2)$

Adding 1 and 2 , we get

$\therefore \operatorname{ar}(\Delta BXY)+\operatorname{ar}(\text{quad. }XYAP)=\operatorname{ar}(\Delta CXY)+\operatorname{ar}(\text{quad}XYQA)$

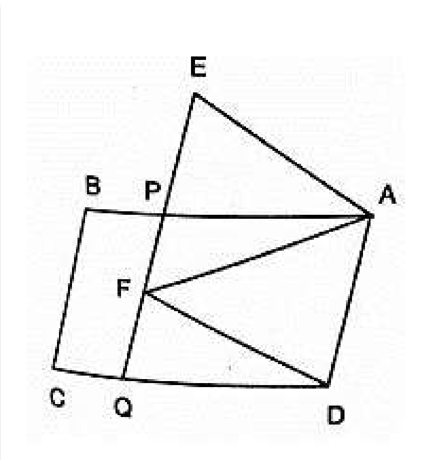
$\Rightarrow \operatorname{ar}(\Delta ABP)=\operatorname{ar}(\Delta ACQ)$

Q 26. In figure, ABCD and AEFD are two parallelograms. Prove that

(i) . PE = FQ

(ii) . $\operatorname{ar}(\Delta APE):\operatorname{ar}(\Delta PFA)=\operatorname{ar}(\Delta QFD):\operatorname{ar}(\Delta PFD)$

(iii) . ar (ΔPEA)= ar (ΔQFD).



Solution:

Given that, ABCD and AEFD are two parallelograms

(i) . In triangles, EPA and FQD

$\angle PEA = \angle QFD$ [corresponding angles]

$\angle EPA = \angle FQD$ [corresponding angles]

PA = QD [opposite sides of parallelogram]

Then, $\Delta EPA \cong \Delta FQD$ [By AAS condition]

Therefore, EP = FQ [C.P.C.T]

(ii) . Since triangles, PEA and QFD stand on equal bases PE and FQ lies between the same parallels EQ and AD

Therefore, ar $(\Delta PEA) = $ ar (ΔQFD) (1)

Since, triangles PEA and PFD stand on the same base PF and between same parallels PF and AD

Therefore, ar $(\Delta PFA) = $ ar (ΔPFD) (2)

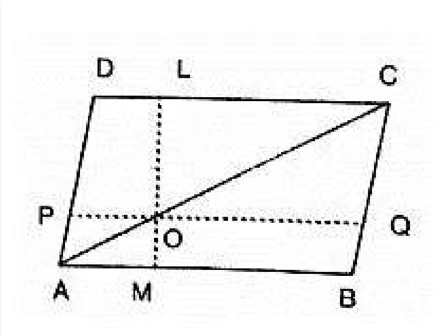
Divide the equation 1 by equation 2

$$\frac{ar(\Delta PEA)}{ar(\Delta PFA)} = \frac{ar(\Delta QFD)}{ar(\Delta PFD)}$$

(iii) . From part (i), $\Delta EPA \cong \Delta FQD$

Then, ar (ΔPEA)= ar (ΔQFD).

Q 27. In figure, ABCD is a parallelogram . O is any point on AC. PQ ||AB and LM || AD. Prove that :
ar $(||^{gm} DLOP) = $ ar $(||^{gm} BMOQ)$.



Solution:

Since a diagonal of a parallelogram divides it into two triangles of equal area

Therefore, $\text{ar}(\Delta ADC) = \text{ar}(\Delta ABC)$

$\Rightarrow \text{ar}(\Delta APO) + \text{ar}(||^{gm} DLOP) + \text{ar}(\Delta OLC)$

$\Rightarrow \text{ar}(\Delta AOM) + \text{ar}(||^{gm} BMOQ) + \text{ar}(\Delta OQC)$ (1)

Since AO and Oc are diagonals of parallelograms AMOP and OQCL respectively.

$\therefore \text{ar}(\Delta APO) = \text{ar}(\Delta AMO)$ (2)

And $\text{ar}(\Delta OLC) = \text{ar}(\Delta OQC)$ (3)

Subtracting 2 and 3 from 1, we get

$\text{ar}(||^{gm} DLOP) = \text{ar}(||^{gm} BMOQ)$.

Q 28. In a triangle ABC, if L and M are points on AB and AC respectively such that LM || BC. Prove that:

(i) . $\text{ar}(\Delta LCM) = \text{ar}(\Delta LBM)$.

(ii) . $\text{ar}(\Delta LBC) = \text{ar}(\Delta MBC)$.

(iii) . $\text{ar}(\Delta ABM) = \text{ar}(\Delta ACL)$.

(iv) . $\text{ar}(\Delta LOB) = \text{ar}(\Delta MOC)$.

Solution:

(i) . Clearly triangles LMB and LMC are on the same base LM and between the same parallels LM and BC.

$\therefore \text{ar}(\Delta LMB) = \text{ar}(\Delta LMC)$ (1)

(ii) . We observe that triangles LBC and MBC are on the same base BC and between same parallels LM and BC.

$\therefore \text{ar}(\Delta LBC) = \text{ar}(\Delta MBC)$ (2)

(iii) . We have,

$\text{ar}(\Delta LMB) = \text{ar}(\Delta LMC)$ [From 1]

$\Rightarrow \text{ar}(\Delta ALM) + \text{ar}(\Delta LMB) = \text{ar}(\Delta ALM) + \text{ar}(\Delta LMC)$

$\Rightarrow \text{ar}(\Delta ABM) = \text{ar}(\Delta ACL)$

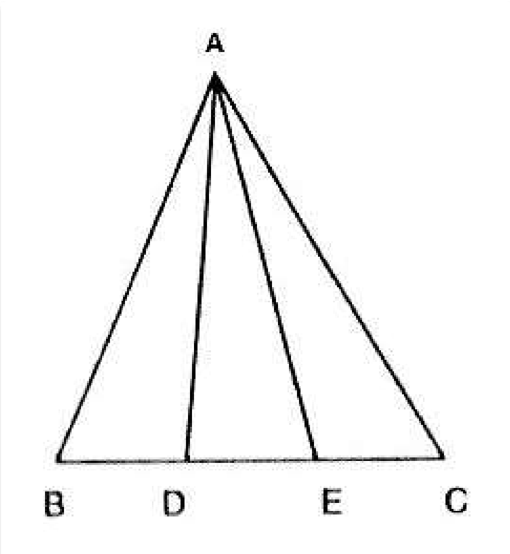
(iv) . We have,

$ar(\Delta LBC) = ar(\Delta MBC)$ [From 1]

$\Rightarrow ar(\Delta LBC) - ar(\Delta BOC) = ar(\Delta MBC) - ar(\Delta BOC)$

$\Rightarrow ar(\Delta LOB) = ar(\Delta MOC)$.

Q 29. In figure, D and E are two points on BC such that BD = DE = EC. Show that $ar(\Delta ABD) = ar(\Delta ADE) = ar(\Delta AEC)$.



Solution:

Draw a line l through A parallel to BC.

Given that, BD = DE = EC

We observed that the triangles ABD and AEC are on the equal bases and between the same parallels l and BC. Therefore, their areas are equal.

Hence, $ar(\Delta ABD) = ar(\Delta ADE) = ar(\Delta AEC)$.

Q 30. In figure, ABC is a right angled triangle at A, BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y. Show that

(i) . $\Delta MBC \cong \Delta ABD$

(ii) . $ar(BYXD) = 2ar(\Delta MBC)$

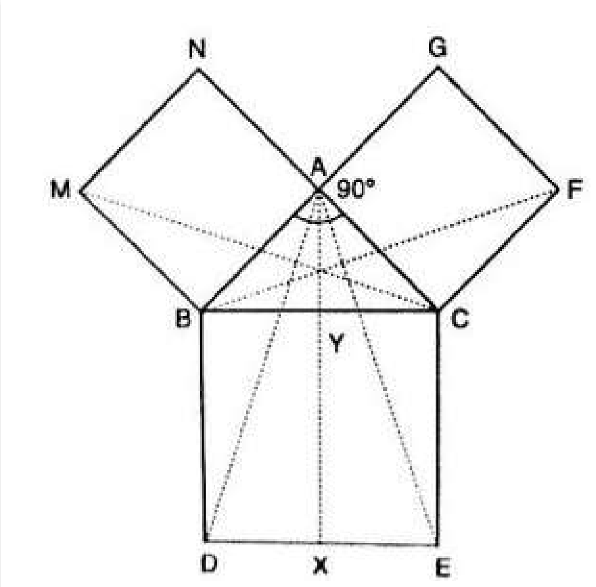
(iii) . $ar(BYXD) = ar(ABMN)$

(iv) . $\Delta FCB \cong \Delta ACE$

(v) . $ar(CYXE) = 2ar(\Delta FCB)$

(vi) . $ar(CYXE) = ar(ACFG)$

(vii) . $ar(BCED) = ar(ABMN) + ar(ACFG)$



Solution:

(i) . In ΔMBC and Δ ABD , we have

MB = AB

BC = BD

And $\angle$ MBC = $\angle$ ABD [since ,$\angle$MBC and $\angle$ABC are obtained by adding $\angle$ABC to a right angle.]

So, by SAS congruence criterion, we have

$$\Delta MBC \cong \Delta ABD$$

$$\Rightarrow ar(\Delta MBC) = ar(\Delta ABD) \cdots\cdots (1)$$

(ii) . Clearly, triangle ABC and rectangle BYXD are on the same base BD and between the same parallels AX and BD .

$$\therefore ar(\Delta ABD) = \frac{1}{2} ar(\text{rect } BYXD)$$

$$\Rightarrow ar(\text{rect } BYXD) = 2ar(\Delta ABD)$$

$$\Rightarrow ar(\text{rect } BYXD) = 2ar(\Delta MBC) \cdots\cdots\cdots (2)$$ [From equ .1]

(iii) . Since triangles MBC and square MBAN are on the same base Mb and between the same parallels MB and NC.

$$\therefore 2ar(\Delta MBC) = ar(MBAN) \cdots\cdots\cdots (3)$$

From equ. 2 and 3, we have

$$ar(\text{sq. } MBAN) = ar(\text{rect} BYXD)$$

(iv) . In triangles FCB and ACE, we have

FC = AC

CB = CE

And , $\angle$FCB = $\angle$ ACE [since ,$\angle$FCB and $\angle$ACE are obtained by adding $\angle$ACB to a right angle.]

So, by SAS congruence criterion, we have

$$\Delta FCB \cong \Delta ACE$$

(v) . We have,

$$\Delta FCB \cong \Delta ACE$$

$$\Rightarrow ar(\Delta FCB) = ar(\Delta ACE)$$

Clearly, triangle ACE and rectangle CYXE are on the same base CE and between same parallels CE and AX.

$$\therefore 2ar(\Delta ACE) = ar(CYXE)$$

$$\Rightarrow 2ar(\Delta FCB) = ar(\Delta CYXE) \cdots\cdots\cdot (4)$$

(vi) . Clearly , triangle FCb and rectangle FCAG are on the same base FC and between the same parallels FC and BG.

$$\therefore 2ar(\Delta FCB) = ar(FCAG) \cdots\cdots (5)$$

From 4 and 5, we get

$$ar(CYXE) = ar(ACFG)$$

(vii) . Applying Pythagoras theorem in triangle ACB, we have

$$BC^2 = AB^2 + AC^2$$

$$\Rightarrow BC \times BD = AB \times MB + AC \times FC$$

$$\Rightarrow ar(BCED) = ar(ABMN) + ar(ACFG)$$