Question 13.1:

(a) Two stable isotopes of lithium $^{6}_{3}\text{Li}$ and $^{7}_{3}\text{Li}$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.

(b) Boron has two stable isotopes, $^{10}_{5}\text{B}$ and $^{11}_{5}\text{B}$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of $^{10}_{5}\text{B}$ and $^{11}_{5}\text{B}$.

Answer

(a) Mass of lithium isotope $^{6}_{3}\text{Li}$, m_1 = 6.01512 u

Mass of lithium isotope $^{7}_{3}\text{Li}$, m_2 = 7.01600 u

Abundance of $^{6}_{3}\text{Li}$, η_1= 7.5%

Abundance of $^{7}_{3}\text{Li}$, η_2= 92.5%

The atomic mass of lithium atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$= \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{92.5 + 7.5}$$

$$= 6.940934 \text{ u}$$

(b) Mass of boron isotope $^{10}_{5}\text{B}$, m_1 = 10.01294 u

Mass of boron isotope $^{11}_{5}\text{B}$, m_2 = 11.00931 u

Abundance of $^{10}_{5}\text{B}$, η_1 = x%

Abundance of $^{11}_{5}\text{B}$, η_2= (100 − x)%

Atomic mass of boron, m = 10.811 u

The atomic mass of boron atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$10.811 = \frac{10.01294 \times x + 11.00931 \times (100 - x)}{x + 100 - x}$$

$$1081.11 = 10.01294x + 1100.931 - 11.00931x$$

$$\therefore x = \frac{19.821}{0.99637} = 19.89\%$$

And 100 − x = 80.11%

Hence, the abundance of ${}^{10}_{5}B$ is 19.89% and that of ${}^{11}_{5}B$ is 80.11%.

Question 13.2:

The three stable isotopes of neon: ${}^{20}_{10}Ne$, ${}^{21}_{10}Ne$ and ${}^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

Answer

Atomic mass of ${}^{20}_{10}Ne$, m_1= 19.99 u

Abundance of ${}^{20}_{10}Ne$, η_1 = 90.51%

Atomic mass of ${}^{21}_{10}Ne$, m_2 = 20.99 u

Abundance of ${}^{21}_{10}Ne$, η_2 = 0.27%

Atomic mass of ${}^{22}_{10}Ne$, m_3 = 21.99 u

Abundance of ${}^{22}_{10}Ne$, η_3 = 9.22%

The average atomic mass of neon is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$= \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{90.51 + 0.27 + 9.22}$$

$$= 20.1771 \text{ u}$$

Question 13.3:

Obtain the binding energy (in MeV) of a nitrogen nucleus $\left({}_{7}^{14}\text{N}\right)$, given $m\left({}_{7}^{14}\text{N}\right)$=14.00307 u

Answer

Atomic mass of nitrogen $\left({}_{7}\text{N}^{14}\right)$, m = 14.00307 u

A nucleus of nitrogen ${}_{7}\text{N}^{14}$ contains 7 protons and 7 neutrons.

Hence, the mass defect of this nucleus, $\Delta m = 7m_H + 7m_n - m$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n= 1.008665 u

$\therefore \Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307$

= 7.054775 + 7.06055 − 14.00307

= 0.11236 u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 0.11236 \times 931.5$ MeV/c^2

Hence, the binding energy of the nucleus is given as:

$E_b = \Delta mc^2$

Where,

c = Speed of light

$\therefore E_b = 0.11236 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

= 104.66334 MeV

Hence, the binding energy of a nitrogen nucleus is 104.66334 MeV.

Question 13.4:

Obtain the binding energy of the nuclei $^{56}_{26}\text{Fe}$ and $^{209}_{83}\text{Bi}$ in units of MeV from the following data:

$m\left(^{56}_{26}\text{Fe}\right)$ = 55.934939 u $m\left(^{209}_{83}\text{Bi}\right)$ = 208.980388 u

Answer

Atomic mass of $^{56}_{26}\text{Fe}$, m_1 = 55.934939 u

$^{56}_{26}\text{Fe}$ nucleus has 26 protons and (56 − 26) = 30 neutrons

Hence, the mass defect of the nucleus, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\therefore \Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$

= 26.20345 + 30.25995 − 55.934939

= 0.528461 u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 0.528461 \times 931.5\ \text{MeV}/c^2$

The binding energy of this nucleus is given as:

$E_{b1} = \Delta mc^2$

Where,

c = Speed of light

$\therefore E_{b1} = 0.528461 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

= 492.26 MeV

Average binding energy per nucleon $= \frac{492.26}{56} = 8.79\ \text{MeV}$

Atomic mass of ${}^{209}_{83}\text{Bi}$, m_2 = 208.980388 u

${}^{209}_{83}\text{Bi}$ nucleus has 83 protons and (209 − 83) 126 neutrons.

Hence, the mass defect of this nucleus is given as:

$\Delta m' = 83 \times m_H + 126 \times m_n - m_2$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\therefore \Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$

= 83.649475 + 127.091790 − 208.980388

= 1.760877 u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m' = 1.760877 \times 931.5$ MeV/c^2

Hence, the binding energy of this nucleus is given as:

$E_{b2} = \Delta m'c^2$

$= 1.760877 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

= 1640.26 MeV

Average bindingenergy per nucleon = $\frac{1640.26}{209} = 7.848$ MeV

Question 13.5:

A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}\text{Cu}$ atoms (of mass 62.92960 u).

Answer

Mass of a copper coin, m' = 3 g

Atomic mass of $_{29}\text{Cu}^{63}$ atom, m = 62.92960 u

The total number of $_{29}\text{Cu}^{63}$ atoms in the coin, $N = \frac{N_A \times m'}{\text{Mass number}}$

Where,

N_A = Avogadro's number = 6.023×10^{23} atoms /g

Mass number = 63 g

$$\therefore N = \frac{6.023 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22} \text{ atoms}$$

$_{29}\text{Cu}^{63}$ nucleus has 29 protons and (63 − 29) 34 neutrons

$\therefore$Mass defect of this nucleus, $\Delta m' = 29 \times m_H + 34 \times m_n - m$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\therefore \Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296$

= 0.591935 u

Mass defect of all the atoms present in the coin, $\Delta m = 0.591935 \times 2.868 \times 10^{22}$

$= 1.69766958 \times 10^{22}$ u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 1.69766958 \times 10^{22} \times 931.5$ MeV/c^2

Hence, the binding energy of the nuclei of the coin is given as:

$E_b = \Delta m c^2$

$= 1.69766958 \times 10^{22} \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

$= 1.581 \times 10^{25}$ MeV

But 1 MeV = 1.6×10^{-13} J

$E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13}$

$= 2.5296 \times 10^{12}$ J

This much energy is required to separate all the neutrons and protons from the given coin.

Question 13.6:

Write nuclear reaction equations for

(i) *α*-decay of ${}^{226}_{88}\text{Ra}$ **(ii)** *α*-decay of ${}^{242}_{94}\text{Pu}$

(iii) β^--decay of ${}^{32}_{15}\text{P}$ **(iv)** β^--decay of ${}^{210}_{83}\text{Bi}$

(v) β^+-decay of ${}^{11}_{6}\text{C}$ **(vi)** β^+-decay of ${}^{97}_{43}\text{Tc}$

(vii) Electron capture of ${}^{120}_{54}\text{Xe}$

Answer

α is a nucleus of helium $\left({}_2\text{He}^4\right)$ and β is an electron (e^- for β^- and e^+ for β^+). In every *α*-decay, there is a loss of 2 protons and 4 neutrons. In every β^+-decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^--decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

(i) $${}_{88}\text{Ra}^{226} \longrightarrow {}_{86}\text{Rn}^{222} + {}_2\text{He}^4$$

(ii) $${}^{242}_{94}\text{Pu} \longrightarrow {}^{238}_{92}\text{U} + {}^{4}_{2}\text{He}$$

(iii) $${}^{32}_{15}\text{P} \longrightarrow {}^{32}_{16}\text{S} + e^- + \overline{\nu}$$

(iv) $${}^{210}_{83}\text{B} \longrightarrow {}^{210}_{84}\text{PO} + e^- + \overline{\nu}$$

(v) $${}^{11}_{6}\text{C} \longrightarrow {}^{11}_{5}\text{B} + e^+ + \nu$$

(vi) $${}^{97}_{43}\text{Tc} \longrightarrow {}^{97}_{42}\text{MO} + e^+ + \nu$$

(vii) $${}^{120}_{54}\text{Xe} + e^+ \longrightarrow {}^{120}_{53}\text{I} + \nu$$

Question 13.7:

A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

Answer

Half-life of the radioactive isotope = T years

Original amount of the radioactive isotope = N_0

(a) After decay, the amount of the radioactive isotope = N

It is given that only 3.125% of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32}$$

$$\text{But } \frac{N}{N_0} = e^{-\lambda t}$$

Where,

λ = Decay constant

t = Time

$$\therefore -\lambda t = \frac{1}{32}$$

$$-\lambda t = \ln 1 - \ln 32$$
$$-\lambda t = 0 - 3.4657$$

$$t = \frac{3.4657}{\lambda}$$

$$\text{Since } \lambda = \frac{0.693}{T}$$

$$\therefore t = \frac{3.466}{\frac{0.693}{T}} \approx 5T \text{ years}$$

Hence, the isotope will take about 5T years to reduce to 3.125% of its original value.

(b) After decay, the amount of the radioactive isotope = N

It is given that only 1% of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 1\% = \frac{1}{100}$$

But $\frac{N}{N_0} = e^{-\lambda t}$

$$\therefore e^{-\lambda t} = \frac{1}{100}$$

$$-\lambda t = \ln 1 - \ln 100$$

$$-\lambda t = 0 - 4.6052$$

$$t = \frac{4.6052}{\lambda}$$

Since, $\lambda = 0.693/T$

$$\therefore t = \frac{4.6052}{\frac{0.693}{T}} = 6.645T \text{ years}$$

Hence, the isotope will take about 6.645*T* years to reduce to 1% of its original value.

Question 13.8:

The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive $^{14}_{6}C$ present with the stable carbon isotope $^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of $^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of $^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Answer

Decay rate of living carbon-containing matter, R = 15 decay/min

Let N be the number of radioactive atoms present in a normal carbon- containing matter.

Half life of $^{14}_{6}C$, $T_{1/2}$ = 5730 years

The decay rate of the specimen obtained from the Mohenjodaro site:

R' = 9 decays/min

Let N' be the number of radioactive atoms present in the specimen during the Mohenjodaro period.

Therefore, we can relate the decay constant, $\lambda$and time, t as:

$$\frac{N}{N'} = \frac{R}{R'} = e^{-\lambda t}$$

$$e^{-\lambda t} = \frac{9}{15} = \frac{3}{5}$$

$$-\lambda t = \log_e \frac{3}{5} = -0.5108$$

$$\therefore t = \frac{0.5108}{\lambda}$$

$$\text{But } \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730}$$

$$\therefore t = \frac{0.5108}{\frac{0.693}{5730}} = 4223.5 \text{ years}$$

Hence, the approximate age of the Indus-Valley civilisation is 4223.5 years.

Question 13.9:

Obtain the amount of $^{60}_{27}\text{Co}$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}\text{Co}$ is 5.3 years.

Answer

The strength of the radioactive source is given as:

$$\frac{dN}{dt} = 8.0 \text{ mCi}$$

$$= 8 \times 10^{-3} \times 3.7 \times 10^{10}$$

$$= 29.6 \times 10^{7} \text{ decay/s}$$

Where,

N = Required number of atoms

Half-life of $^{60}_{27}\text{Co}$, $T_{1/2}$ = 5.3 years

$= 5.3 \times 365 \times 24 \times 60 \times 60$

$= 1.67 \times 10^8$ s

For decay constant λ, we have the rate of decay as:

$$\frac{dN}{dt} = \lambda N$$

Where, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.67 \times 10^8}\ s^{-1}$

$$\therefore N = \frac{1}{\lambda}\frac{dN}{dt}$$

$$= \frac{29.6 \times 10^7}{\frac{0.693}{1.67 \times 10^8}} = 7.133 \times 10^{16} \text{ atoms}$$

For ${}_{27}Co^{60}$:

Mass of 6.023×10^{23} (Avogadro's number) atoms = 60 g

$\therefore$Mass of 7.133×10^{16} atoms $= \frac{60 \times 7.133 \times 10^{16}}{6.023 \times 10^{23}} = 7.106 \times 10^{-6}$ g

Hence, the amount of ${}_{27}Co^{60}$ necessary for the purpose is 7.106×10^{-6} g.

Question 13.10:

The half-life of ${}^{90}_{38}Sr$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

Answer

Half life of ${}^{90}_{38}Sr$, $t_{1/2}$ = 28 years

$= 28 \times 365 \times 24 \times 60 \times 60$

$= 8.83 \times 10^8$ s

Mass of the isotope, m = 15 mg

90 g of ${}^{90}_{38}Sr$ atom contains 6.023×10^{23} (Avogadro's number) atoms.

Therefore, 15 mg of ${}^{90}_{38}\text{Sr}$ contains:

$$\frac{6.023 \times 10^{23} \times 15 \times 10^{-3}}{90}, \text{ i.e., } 1.0038 \times 10^{20} \text{ number of atoms}$$

Rate of disintegration, $\frac{dN}{dt} = \lambda N$

Where,

λ = Decay constant $= \frac{0.693}{8.83 \times 10^{8}} \text{ s}^{-1}$

$$\therefore \frac{dN}{dt} = \frac{0.693 \times 1.0038 \times 10^{20}}{8.83 \times 10^{8}} = 7.878 \times 10^{10} \text{ atoms/s}$$

Hence, the disintegration rate of 15 mg of the given isotope is
7.878×10^{10} atoms/s.

Question 13.11:

Obtain approximately the ratio of the nuclear radii of the gold isotope ${}^{197}_{79}\text{Au}$ and the silver isotope ${}^{107}_{47}\text{Ag}$.

Answer

Nuclear radius of the gold isotope ${}_{79}\text{Au}^{197}$ = R_{Au}

Nuclear radius of the silver isotope ${}_{47}\text{Ag}^{107}$ = R_{Ag}

Mass number of gold, A_{Au} = 197

Mass number of silver, A_{Ag} = 107

The ratio of the radii of the two nuclei is related with their mass numbers as:

$$\frac{R_{Au}}{R_{Ag}} = \left(\frac{R_{Au}}{R_{Ag}}\right)^{\frac{1}{3}}$$

$$= \left(\frac{197}{107}\right)^{\frac{1}{3}} = 1.2256$$

Hence, the ratio of the nuclear radii of the gold and silver isotopes is about 1.23.

Question 13.12:

Find the Q-value and the kinetic energy of the emitted *α*-particle in the α-decay of (a) ${}^{226}_{88}\mathrm{Ra}$ and (b) ${}^{220}_{86}\mathrm{Rn}$.

Given $m\left({}^{226}_{88}\mathrm{Ra}\right)$ = 226.02540 u, $m\left({}^{222}_{86}\mathrm{Rn}\right)$ = 222.01750 u,

$m\left({}^{220}_{86}\mathrm{Rn}\right)$ = 220.01137 u, $m\left({}^{216}_{84}\mathrm{Po}\right)$ = 216.00189 u.

Answer

(a) Alpha particle decay of ${}^{226}_{88}\mathrm{Ra}$ emits a helium nucleus. As a result, its mass number reduces to (226 − 4) 222 and its atomic number reduces to (88 − 2) 86. This is shown in the following nuclear reaction.

$${}^{226}_{88}\mathrm{Ra} \longrightarrow {}^{222}_{86}\mathrm{Ra} + {}^{4}_{2}\mathrm{He}$$

Q-value of

emitted *α*-particle = (Sum of initial mass − Sum of final mass) c^2

Where,

c = Speed of light

It is given that:

$$m\left({}^{226}_{88}\mathrm{Ra}\right) = 226.02540 \text{ u}$$

$$m\left({}^{222}_{86}\mathrm{Rn}\right) = 222.01750 \text{ u}$$

$$m\left({}^{4}_{2}\mathrm{He}\right) = 4.002603 \text{ u}$$

Q-value = [226.02540 − (222.01750 + 4.002603)] u c^2

= 0.005297 u c^2

But 1 u = 931.5 MeV/c^2

∴Q = 0.005297 × 931.5 ≈ 4.94 MeV

Kinetic energy of the *α*-particle $= \left(\dfrac{\text{Mass number after decay}}{\text{Mass number before decay}}\right) \times Q$

$$= \frac{222}{226} \times 4.94 = 4.85 \text{ MeV}$$

(b) Alpha particle decay of $\left({}^{220}_{86}\text{Rn}\right)$ is shown by the following nuclear reaction.

$${}^{220}_{86}\text{Rn} \longrightarrow {}^{216}_{84}\text{Po} + {}^{4}_{2}\text{He}$$

It is given that:

Mass of $\left({}^{220}_{86}\text{Rn}\right)$ = 220.01137 u

Mass of $\left({}^{216}_{84}\text{Po}\right)$ = 216.00189 u

$\therefore Q$-value = $\left[220.01137 - \left(216.00189 + 4.00260\right)\right] \times 931.5$

≈ 641 MeV

Kinetic energy of the α-particle $= \left(\frac{220-4}{220}\right) \times 6.41$

= 6.29 MeV

Question 13.13:

The radionuclide ^{11}C decays according to

$${}^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + e^{+} + \nu: \quad \text{T}_{1/2} = 20.3 \text{ min}$$

The maximum energy of the emitted positron is 0.960 MeV.

Given the mass values:

$$m\left({}^{11}_{6}\text{C}\right) = 11.011434 \text{ u and } m\left({}^{11}_{6}\text{B}\right) = 11.009305 \text{ u,}$$

calculate Q and compare it with the maximum energy of the positron emitted

Answer

The given nuclear reaction is:

$${}^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + e^{+} + \nu$$

Half life of ${}^{11}_{6}\text{C}$ nuclei, $T_{1/2} = 20.3$ min

Atomic mass of

Maximum energy possessed by the emitted positron = 0.960 MeV

The change in the Q-value (ΔQ) of the nuclear masses of the $^{11}_{6}C$ nucleus is given as:

$$\Delta Q = \left[m'\left({}_{6}C^{11}\right) - \left[m'\left({}^{11}_{5}B\right) + m_e \right] \right] c^2 \qquad \ldots (1)$$

Where,

m_e = Mass of an electron or positron = 0.000548 u

c = Speed of light

m' = Respective nuclear masses

If atomic masses are used instead of nuclear masses, then we have to add 6 m_e in the case of ^{11}C and 5 m_e in the case of ^{11}B.

Hence, equation (1) reduces to:

$$\Delta Q = \left[m\left({}_{6}C^{11}\right) - m\left({}^{11}_{5}B\right) - 2m_e \right] c^2$$

Here, $m\left({}_{6}C^{11}\right)$ and $m\left({}^{11}_{5}B\right)$ are the atomic masses.

$\therefore \Delta Q = [11.011434 - 11.009305 - 2 \times 0.000548]\ c^2$

$= (0.001033\ c^2)$ u

But 1 u = 931.5 Mev/c^2

$\therefore \Delta Q = 0.001033 \times 931.5 \approx 0.962$ MeV

The value of Q is almost comparable to the maximum energy of the emitted positron.

Question 13.14:

The nucleus $^{23}_{10}\text{Ne}$ decays by β^- emission. Write down the β decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$m\left(^{23}_{10}\text{Ne}\right)$ = 22.994466 u

$m\left(^{23}_{11}\text{Na}\right)$ = 22.989770 u.

Answer

In β^- emission, the number of protons increases by 1, and one electron and an antineutrino are emitted from the parent nucleus.

β^- emission of the nucleus $^{23}_{10}\text{Ne}$ is given as:

$$^{23}_{10}\text{Ne} \rightarrow {}^{23}_{11}\text{Na} + e^- + \bar{\nu} + Q$$

It is given that:

Atomic mass of $m\left(^{23}_{10}\text{Ne}\right)$ = 22.994466 u

Atomic mass of $m\left(^{23}_{11}\text{Na}\right)$ = 22.989770 u

Mass of an electron, m_e = 0.000548 u

Q-value of the given reaction is given as:

$$Q = \left[m\left(^{23}_{10}\text{Ne}\right) - \left[m\left(^{23}_{11}\text{Na}\right) + m_e\right]\right]c^2$$

There are 10 electrons in $^{23}_{10}\text{Ne}$ and 11 electrons in $^{23}_{11}\text{Na}$. Hence, the mass of the electron is cancelled in the Q-value equation.

$$\therefore Q = \left[22.994466 - 22.989770\right]c^2$$
$$= \left(0.004696\ c^2\right)\ \text{u}$$

But $1\ \text{u} = 931.5\ \text{MeV}/c^2$

$$\therefore Q = 0.004696 \times 931.5 = 4.374\ \text{MeV}$$

The daughter nucleus is too heavy as compared to e^- and $\bar{\nu}$. Hence, it carries negligible energy. The kinetic energy of the antineutrino is nearly zero. Hence, the maximum kinetic energy of the emitted electrons is almost equal to the *Q*-value, i.e., 4.374 MeV.

Question 13.15:

The *Q* value of a nuclear reaction $A + b \rightarrow C + d$ is defined by

$Q = [\, m_A + m_b - m_C - m_d]c^2$ where the masses refer to the respective nuclei. Determine from the given data the *Q*-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) ${}_1^1\text{H} + {}_1^3\text{H} \rightarrow {}_1^2\text{H} + {}_1^2\text{H}$

(ii) ${}_6^{12}\text{C} + {}_6^{12}\text{C} \rightarrow {}_{10}^{20}\text{Ne} + {}_2^4\text{He}$

Atomic masses are given to be

$m\left({}_1^2\text{H}\right) = 2.014102 \text{ u}$

$m\left({}_1^3\text{H}\right) = 3.016049 \text{ u}$

$m\left({}_6^{12}\text{C}\right) = 12.000000 \text{ u}$

$m\left({}_{10}^{20}\text{Ne}\right) = 19.992439 \text{ u}$

Answer

(i) The given nuclear reaction is:

${}_1\text{H}^1 + {}_1^3\text{H} \rightarrow {}_1^2\text{H} + {}_1^2\text{H}$

It is given that:

Atomic mass $m\left({}_1^1\text{H}\right) = 1.007825 \text{ u}$

Atomic mass $m\left({}_1^3\text{H}\right) = 3.016049 \text{ u}$

Atomic mass $m\left({}_1^2\text{H}\right) = 2.014102 \text{ u}$

According to the question, the *Q*-value of the reaction can be written as:

$$Q = \left[m\left({}_1^1\text{H}\right) + m\left({}_1^3\text{H}\right) - 2m\left({}_1^2\text{H}\right)\right]c^2$$

$$= \left[1.007825 + 3.016049 - 2 \times 2.014102\right]c^2$$

$$Q = \left(-0.00433\ c^2\right)\ \text{u}$$

$$\text{But}\ \ 1\ \text{u} = 931.5\ \text{MeV}/c^2$$

$$\therefore Q = -0.00433 \times 931.5 = -4.0334\ \text{MeV}$$

The negativeQ-value of the reaction shows that the reaction is endothermic.

(ii) The given nuclear reaction is:

$${}_6^{12}\text{C} + {}_6^{12}\text{C} \rightarrow {}_{10}^{20}\text{Ne} + {}_2^4\text{He}$$

It is given that:

Atomic mass of $m\left({}_6^{12}\text{C}\right) = 12.0\ \text{u}$

Atomic mass of $m\left({}_{10}^{20}\text{Ne}\right) = 19.992439\ \text{u}$

Atomic mass of $m\left({}_2^4\text{He}\right) = 4.002603\ \text{u}$

The *Q*-value of this reaction is given as:

$$Q = \left[2m\left({}_6^{12}\text{C}\right) - m\left({}_{10}^{20}\text{Ne}\right) - m\left({}_2^4\text{He}\right)\right]c^2$$

$$= \left[2 \times 12.0 - 19.992439 - 4.002603\right]c^2$$

$$= \left(0.004958\ c^2\right)\ \text{u}$$

$$= 0.004958 \times 931.5 = 4.618377\ \text{MeV}$$

The positive *Q*-value of the reaction shows that the reaction is exothermic.

Question 13.16:

Suppose, we think of fission of a ${}^{56}_{26}\mathrm{Fe}$ nucleus into two equal fragments, ${}^{28}_{13}\mathrm{Al}$. Is the fission energetically possible? Argue by working out Q of the process. Given $m\left({}^{56}_{26}\mathrm{Fe}\right) = 55.93494\ \mathrm{u}$ and $m\left({}^{28}_{13}\mathrm{Al}\right) = 27.98191\,\mathrm{u}$.

Answer

The fission of ${}^{56}_{26}\mathrm{Fe}$ can be given as:

$${}^{56}_{13}\mathrm{Fe} \longrightarrow 2\,{}^{28}_{13}\mathrm{Al}$$

It is given that:

Atomic mass of $m\left({}^{56}_{26}\mathrm{Fe}\right)$ = 55.93494 u

Atomic mass of $m\left({}^{28}_{13}\mathrm{Al}\right) = 27.98191\ \mathrm{u}$

The Q-value of this nuclear reaction is given as:

$$Q = \left[m\left({}^{56}_{26}\mathrm{Fe}\right) - 2m\left({}^{28}_{13}\mathrm{Al}\right)\right]c^2$$
$$= \left[55.93494 - 2 \times 27.98191\right]c^2$$
$$= \left(-0.02888\ c^2\right)\ \mathrm{u}$$

But $1\,\mathrm{u} = 931.5\ \mathrm{MeV}/c^2$

$\therefore Q = -0.02888 \times 931.5 = -26.902\ \mathrm{MeV}$

The Q-value of the fission is negative. Therefore, the fission is not possible energetically. For an energetically-possible fission reaction, the Q-value must be positive.

Question 13.17:

The fission properties of ${}^{239}_{94}\mathrm{Pu}$ are very similar to those of ${}^{235}_{92}\mathrm{U}$.

The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure ${}^{239}_{94}\mathrm{Pu}$ undergo fission?

Answer

Average energy released per fission of ${}^{239}_{94}\mathrm{Pu}$, $E_{av} = 180\ \mathrm{MeV}$

Amount of pure ${}_{94}\mathrm{Pu}^{239}$, m = 1 kg = 1000 g

N_A= Avogadro number = 6.023 × 10^{23}

Mass number of ${}^{239}_{94}\mathrm{Pu}$ = 239 g

1 mole of ${}_{94}\mathrm{Pu}^{239}$ contains N_A atoms.

∴*m* g of ${}_{94}\mathrm{Pu}^{239}$ contains $\left(\frac{N_A}{\text{Mass number}} \times m\right)$ atoms

$$= \frac{6.023 \times 10^{23}}{239} \times 1000 = 2.52 \times 10^{24} \text{ atoms}$$

∴Total energy released during the fission of 1 kg of ${}^{239}_{94}\mathrm{Pu}$ is calculated as:

$$E = E_{av} \times 2.52 \times 10^{24}$$
$$= 180 \times 2.52 \times 10^{24} = 4.536 \times 10^{26} \text{ MeV}$$

Hence, 4.536×10^{26} MeV is released if all the atoms in 1 kg of pure ${}_{94}\mathrm{Pu}^{239}$ undergo fission.

Question 13.18:

A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much ${}^{235}_{92}\mathrm{U}$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of ${}^{235}_{92}\mathrm{U}$ and that this nuclide is consumed only by the fission process.

Answer

Half life of the fuel of the fission reactor, $t_{\frac{1}{2}} = 5$ years

= 5 × 365 × 24 × 60 × 60 s

We know that in the fission of 1 g of ${}^{235}_{92}U$ nucleus, the energy released is equal to 200 MeV.

1 mole, i.e., 235 g of ${}^{235}_{92}U$ contains 6.023 × 10^{23} atoms.

∴1 g ${}^{235}_{92}U$ contains $\frac{6.023 \times 10^{23}}{235}$ atoms

The total energy generated per gram of ${}^{235}_{92}U$ is calculated as:

$$E = \frac{6.023 \times 10^{23}}{235} \times 200 \text{ MeV/g}$$
$$= \frac{200 \times 6.023 \times 10^{23} \times 1.6 \times 10^{-19} \times 10^{6}}{235} = 8.20 \times 10^{10} \text{ J/g}$$

The reactor operates only 80% of the time.

Hence, the amount of ${}^{235}_{92}U$ consumed in 5 years by the 1000 MW fission reactor is calculated as:

$$= \frac{5 \times 80 \times 60 \times 60 \times 365 \times 24 \times 1000 \times 10^{6}}{100 \times 8.20 \times 10^{10}} \text{ g}$$
$$\approx 1538 \text{ kg}$$

∴Initial amount of ${}^{235}_{92}U$ = 2 × 1538 = 3076 kg

Question 13.19:

How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

$${}^{2}_{1}H + {}^{2}_{1}H \longrightarrow {}^{3}_{1}He + n + 3.27 \text{ MeV}$$

Answer

The given fusion reaction is:

$${}_{1}^{2}H + {}_{1}^{2}H \longrightarrow {}_{2}^{3}He + n + 3.27 \text{ MeV}$$

Amount of deuterium, m = 2 kg

1 mole, i.e., 2 g of deuterium contains 6.023×10^{23} atoms.

∴2.0 kg of deuterium contains $= \frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$ atoms

It can be inferred from the given reaction that when two atoms of deuterium fuse, 3.27 MeV energy is released.

∴Total energy per nucleus released in the fusion reaction:

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} \text{ MeV}$$

$$= \frac{3.27}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6}$$

$$= 1.576 \times 10^{14} \text{ J}$$

Power of the electric lamp, P = 100 W = 100 J/s

Hence, the energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is calculated as:

$$\frac{1.576 \times 10^{14}}{100} \text{ s}$$

$$\frac{1.576 \times 10^{14}}{100 \times 60 \times 60 \times 24 \times 365} \approx 4.9 \times 10^{4} \text{ years}$$

Question 13.20:

Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Answer

When two deuterons collide head-on, the distance between their centres, *d* is given as:

Radius of 1st deuteron + Radius of 2nd deuteron

Radius of a deuteron nucleus = 2 fm = 2×10^{-15} m

$\therefore d = 2 \times 10^{-15} + 2 \times 10^{-15} = 4 \times 10^{-15}$ m

Charge on a deuteron nucleus = Charge on an electron = $e = 1.6 \times 10^{-19}$ C

Potential energy of the two-deuteron system:

$$V = \frac{e^2}{4\pi \in_0 d}$$

Where,

$\in_0$ = Permittivity of free space

$$\frac{1}{4\pi \in_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15}} \text{ J}$$

$$= \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15} \times \left(1.6 \times 10^{-19}\right)} \text{ eV}$$

$$= 360 \text{ keV}$$

Hence, the height of the potential barrier of the two-deuteron system is

360 keV.

Question 13.21:

From the relation $R = R_0A^1/^3$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Answer

We have the expression for nuclear radius as:

$R = R_0A^1/^3$

Where,

R_0 = Constant.

A = Mass number of the nucleus

Nuclear matter density, $\rho = \frac{\text{Mass of the nucleus}}{\text{Volume of the nucleus}}$

Let m be the average mass of the nucleus.

Hence, mass of the nucleus = mA

$$\therefore \rho = \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3mA}{4\pi\left(R_0 A^{\frac{1}{3}}\right)^3} = \frac{3mA}{4\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Hence, the nuclear matter density is independent of A. It is nearly constant.

Question 13.22:

For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K−shell, is captured by the nucleus and a neutrino is emitted).

$$e^+ + {}_Z^A X \rightarrow {}_{Z-1}^{A} Y + \nu$$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice−versa.

Answer

Let the amount of energy released during the electron capture process be Q_1. The nuclear reaction can be written as:

$$e^{+} + {}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + \nu + Q_1 \qquad \ldots (1)$$

Let the amount of energy released during the positron capture process be Q_2. The nuclear reaction can be written as:

$${}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + e^{+} + \nu + Q_2 \qquad \ldots (2)$$

$m_N\left({}_{Z}^{A}X\right)$ = Nuclear mass of ${}_{Z}^{A}X$

$m_N\left({}_{Z-1}^{A}Y\right)$ = Nuclear mass of ${}_{Z-1}^{A}Y$

$m\left({}_{Z}^{A}X\right)$ = Atomic mass of ${}_{Z}^{A}X$

$m\left({}_{Z-1}^{A}Y\right)$ = Atomic mass of ${}_{Z-1}^{A}Y$

m_e = Mass of an electron

c = Speed of light

Q-value of the electron capture reaction is given as:

$$Q_1 = \left[m_N\left({}_{Z}^{A}X\right) + m_e - m_N\left({}_{Z-1}^{A}Y\right)\right]c^2$$

$$= \left[m\left({}_{Z}^{A}X\right) - Zm_e + m_e - m\left({}_{Z-1}^{A}Y\right) + (Z-1)m_e\right]c^2$$

$$= \left[m\left({}_{Z}^{A}X\right) - m\left({}_{Z-1}^{A}Y\right)\right]c^2 \qquad \ldots (3)$$

Q-value of the positron capture reaction is given as:

$$Q_2 = \left[m_N\left({}_{Z}^{A}X\right) - m_N\left({}_{Z-1}^{A}Y\right) - m_e\right]c^2$$

$$= \left[m\left({}_{Z}^{A}X\right) - Zm_e - m\left({}_{Z-1}^{A}Y\right) + (Z-1)m_e - m_e\right]c^2$$

$$= \left[m\left({}_{Z}^{A}X\right) - m\left({}_{Z-1}^{A}Y\right) - 2m_e\right]c^2 \qquad \ldots (4)$$

It can be inferred that if $Q_2 > 0$, then $Q_1 > 0$; Also, if $Q_1 > 0$, it does not necessarily mean that $Q_2 > 0$.

In other words, this means that if β^+ emission is energetically allowed, then the electron capture process is necessarily allowed, but not vice-versa. This is because the *Q*-value must be positive for an energetically-allowed nuclear reaction.

Question 13.23:

In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are $^{24}_{12}\text{Mg}$ (23.98504u), $^{25}_{12}\text{Mg}$ (24.98584u) and $^{26}_{12}\text{Mg}$ (25.98259u). The natural abundance of $^{24}_{12}\text{Mg}$ is 78.99% by mass. Calculate the abundances of other two isotopes.

Answer

Average atomic mass of magnesium, m = 24.312 u

Mass of magnesium isotope $^{24}_{12}\text{Mg}$, m_1 = 23.98504 u

Mass of magnesium isotope $^{25}_{12}\text{Mg}$, m_2 = 24.98584 u

Mass of magnesium isotope $^{26}_{12}\text{Mg}$, m_3 = 25.98259 u

Abundance of $^{24}_{12}\text{Mg}$, η_1= 78.99%

Abundance of $^{25}_{12}\text{Mg}$, $\eta_2 = x\%$

Hence, abundance of $^{26}_{12}\text{Mg}$, $\eta_3 = 100 - x - 78.99\% = (21.01 - x)\%$

We have the relation for the average atomic mass as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$24.312 = \frac{23.98504 \times 78.99 + 24.98584 \times x + 25.98259 \times (21.01 - x)}{100}$$

$$2431.2 = 1894.5783096 + 24.98584x + 545.8942159 - 25.98259x$$

$$0.99675x = 9.2725255$$

$$\therefore x \approx 9.3\%$$

$$\text{And } 21.01 - x = 11.71\%$$

Hence, the abundance of $^{25}_{12}\text{Mg}$ is 9.3% and that of $^{26}_{12}\text{Mg}$ is 11.71%.

Question 13.24:

The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei $^{41}_{20}\text{Ca}$ and $^{27}_{13}\text{Al}$ from the following data:

$m\left(^{40}_{20}\text{Ca}\right)$ = 39.962591 u

$m\left(^{41}_{20}\text{Ca}\right)$) = 40.962278 u

$m\left(^{26}_{13}\text{Al}\right)$ = 25.986895 u

$m\left(^{27}_{13}\text{Al}\right)$) = 26.981541 u

Answer

For $^{41}_{20}\text{Ca: Separation energy} = 8.363007\text{ MeV}$

For $^{27}_{13}\text{Al: Separation energy} = 13.059\text{ MeV}$

A neutron $\left({}_0n^1\right)$ is removed from a $^{41}_{20}\text{Ca}$ nucleus. The corresponding nuclear reaction can be written as:

$$^{41}_{20}\text{Ca} \longrightarrow {}^{40}_{20}\text{Ca} + {}^{1}_{0}\text{n}$$

It is given that:

Mass $m\left(^{40}_{20}\text{Ca}\right)$ = 39.962591 u

Mass $m\left(^{41}_{20}\text{Ca}\right)$) = 40.962278 u

Mass $m\left({}_0n^1\right)$ = 1.008665 u

The mass defect of this reaction is given as:

Δm = $m\left(^{40}_{20}\text{Ca}\right) + \left(^{1}_{0}\text{n}\right) - m\left(^{41}_{20}\text{Ca}\right)$

$$= 39.962591 + 1.008665 - 40.962278 = 0.008978\text{ u}$$

$$\text{But } 1\text{ u} = 931.5\text{ MeV}/c^2$$

$\therefore \Delta m = 0.008978 \times 931.5\ \text{MeV}/c^2$

Hence, the energy required for neutron removal is calculated as:

$$E = \Delta mc^2$$
$$= 0.008978 \times 931.5 = 8.363007\ \text{MeV}$$

For ${}^{27}_{13}\text{Al}$, the neutron removal reaction can be written as:

$${}^{27}_{13}\text{Al} \longrightarrow {}^{26}_{13}\text{Al} + {}^{1}_{0}\text{n}$$

It is given that:

Mass $m\left({}^{27}_{13}\text{Al}\right)$ = 26.981541 u

Mass $m\left({}^{26}_{13}\text{Al}\right)$ = 25.986895 u

The mass defect of this reaction is given as:

$$\Delta m = m\left({}^{26}_{13}\text{Al}\right) + m\left({}^{1}_{0}\text{n}\right) - m\left({}^{27}_{13}\text{Al}\right)$$

$$= 25.986895 + 1.008665 - 26.981541$$
$$= 0.014019\ \text{u}$$
$$= 0.014019 \times 931.5\ \text{MeV}/c^2$$

Hence, the energy required for neutron removal is calculated as:

$$E = \Delta mc^2$$
$$= 0.014019 \times 931.5 = 13.059\ \text{MeV}$$

Question 13.25:

A source contains two phosphorous radio nuclides ${}^{32}_{15}\text{P}$ ($T_{1/2}$ = 14.3d) and ${}^{33}_{15}\text{P}$ ($T_{1/2}$ = 25.3d). Initially, 10% of the decays come from ${}^{33}_{15}\text{P}$. How long one must wait until 90% do so?

Answer

Half life of $^{32}_{15}P$, $T_{1/2} =$ 14.3 days

Half life of $^{33}_{15}P$, $T'_{1/2}$ = 25.3 days

$^{33}_{15}P$ nucleus decay is 10% of the total amount of decay.

The source has initially 10% of $^{33}_{15}P$ nucleus and 90% of $^{32}_{15}P$ nucleus.

Suppose after *t* days, the source has 10% of $^{32}_{15}P$ nucleus and 90% of $^{33}_{15}P$ nucleus.

Initially:

Number of $^{33}_{15}P$ nucleus = *N*

Number of $^{32}_{15}P$ nucleus = 9 *N*

Finally:

Number of $^{33}_{15}P$ nucleus $= 9\ N'$

Number of $^{32}_{15}P$ nucleus $= N'$

For $^{32}_{15}P$ nucleus, we can write the number ratio as:

$$\frac{N'}{9N} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$N' = 9N(2)^{\frac{-t}{14.3}} \qquad \dots (1)$$

For $^{33}_{15}P$, we can write the number ratio as:

$$\frac{9N'}{N} = \left(\frac{1}{2}\right)^{\frac{t}{T'_{1/2}}}$$

$$9N' = N(2)^{\frac{-t}{25.3}} \qquad \dots (2)$$

On dividing equation (1) by equation (2), we get:

$$\frac{1}{9} = 9 \times 2^{\left(\frac{t}{25.3} - \frac{t}{14.3}\right)}$$

$$\frac{1}{81} = 2^{-\left(\frac{11t}{25.3 \times 14.3}\right)}$$

$$\log 1 - \log 81 = \frac{-11t}{25.3 \times 14.3} \log 2$$

$$\frac{-11t}{25.3 \times 14.3} = \frac{0 - 1.908}{0.301}$$

$$t = \frac{25.3 \times 14.3 \times 1.908}{11 \times 0.301} \approx 208.5 \text{ days}$$

Hence, it will take about 208.5 days for 90% decay of ${}_{15}P^{33}$.

Question 13.26:

Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

$${}^{223}_{88}\text{Ra} \longrightarrow {}^{209}_{82}\text{Pb} + {}^{14}_{6}\text{C}$$

$${}^{223}_{88}\text{Ra} \longrightarrow {}^{219}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

Answer

Take a ${}^{14}_{6}\text{C}$ emission nuclear reaction:

$${}^{223}_{88}\text{Ra} \longrightarrow {}^{209}_{82}\text{Pb} + {}^{14}_{6}\text{C}$$

We know that:

Mass of ${}^{223}_{88}\text{Ra}$, m_1 = 223.01850 u

Mass of ${}^{209}_{82}\text{Pb}$, m_2 = 208.98107 u

Mass of ${}^{14}_{6}\text{C}$, m_3 = 14.00324 u

Hence, the Q-value of the reaction is given as:

$Q = (m_1 - m_2 - m_3)\, c^2$

$= (223.01850 - 208.98107 - 14.00324)\ c^2$

$= (0.03419\ c^2)$ u

But 1 u = 931.5 MeV/c^2

$\therefore Q = 0.03419 \times 931.5$

= 31.848 MeV

Hence, the Q-value of the nuclear reaction is 31.848 MeV. Since the value is positive, the reaction is energetically allowed.

Now take a $^{4}_{2}\text{He}$ emission nuclear reaction:

$^{223}_{88}\text{Ra} \longrightarrow ^{229}_{86}\text{Rn} + ^{4}_{2}\text{He}$

We know that:

Mass of $^{223}_{88}\text{Ra},$ $m_1 = 223.01850$

Mass of $^{219}_{82}\text{Rn},$ $m_2 = 219.00948$

Mass of $^{4}_{2}\text{He}$, $m_3 = 4.00260$

Q-value of this nuclear reaction is given as:

$Q = (m_1 - m_2 - m_3)\ c^2$

$= (223.01850 - 219.00948 - 4.00260)\ C^2$

$= (0.00642\ c^2)$ u

$= 0.00642 \times 931.5 = 5.98$ MeV

Hence, the Q value of the second nuclear reaction is 5.98 MeV. Since the value is positive, the reaction is energetically allowed.

Question 13.27:

Consider the fission of $^{238}_{92}\text{U}$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are

$^{140}_{58}Ce$ and $^{99}_{44}Ru$. Calculate Q for this fission process. The relevant atomic and particle masses are

$m\left(^{238}_{92}U\right)$ =238.05079 u

$m\left(^{140}_{58}Ce\right)$ =139.90543 u

$m\left(^{99}_{44}Ru\right)$ = 98.90594 u

Answer

In the fission of $^{238}_{92}U$, 10 β^- particles decay from the parent nucleus. The nuclear reaction can be written as:

$$^{238}_{92}U + ^{1}_{0}n \longrightarrow ^{140}_{58}Ce + ^{99}_{44}Ru + 10\,^{0}_{-1}e$$

It is given that:

Mass of a nucleus $^{238}_{92}U$, m_1 = 238.05079 u

Mass of a nucleus $^{140}_{58}Ce$, m_2 = 139.90543 u

Mass of a nucleus $^{99}_{44}Ru$, m_3 = 98.90594 u

Mass of a neutron $^{1}_{0}n$, m_4 = 1.008665 u

Q-value of the above equation,

$$Q = \left[m'\left(^{238}_{92}U\right) + m\left(^{1}_{0}n\right) - m'\left(^{140}_{58}Ce\right) - m'\left(^{99}_{44}Ru\right) - 10m_e\right]c^2$$

Where,

m' = Represents the corresponding atomic masses of the nuclei

$m'\left(^{238}_{92}U\right) = m_1 - 92m_e$

$m'\left(^{140}_{58}Ce\right) = m_2 - 58m_e$

$m'\left(^{99}_{44}Ru\right) = m_3 - 44m_e$

$m\left(^{1}_{0}n\right) = m_4$

$$Q = [m_1 - 92m_e + m_4 - m_2 + 58m_e - m_3 + 44m_e - 10m_e]c^2$$
$$= [m_1 + m_4 - m_2 - m_3]c^2$$
$$= [238.0507 + 1.008665 - 139.90543 - 98.90594]c^2$$
$$= [0.247995\ c^2]\ \text{u}$$

But $1\ \text{u} = 931.5\ \text{MeV}/c^2$

$\therefore Q = 0.247995 \times 931.5 = 231.007\ \text{MeV}$

Hence, the *Q*-value of the fission process is 231.007 MeV.

Question 13.28:

Consider the D−T reaction (deuterium−tritium fusion)

$${}^2_1\text{H} + {}^3_1\text{H} \longrightarrow {}^4_2\text{He} + \text{n}$$

(a) Calculate the energy released in MeV in this reaction from the data:

$m({}^2_1\text{H})$ = 2.014102 u

$m({}^3_1\text{H})$ = 3.016049 u

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction? (Hint: Kinetic energy required for one fusion event =average thermal kinetic energy available with the interacting particles = $2(3kT/2)$; k = Boltzman's constant, T = absolute temperature.)

Answer

(a) Take the D-T nuclear reaction: ${}^2_1\text{H} + {}^3_1\text{H} \longrightarrow {}^4_2\text{He} + \text{n}$

It is given that:

Mass of ${}^2_1\text{H}$, m_1= 2.014102 u

Mass of ${}^3_1\text{H}$, m_2 = 3.016049 u

Mass of ${}^4_2\text{He}$ m_3 = 4.002603 u

Mass of ${}^1_0\text{n}$, m_4 = 1.008665 u

Q-value of the given D-T reaction is:

$Q = [m_1 + m_2 - m_3 - m_4]\, c^2$

$= [2.014102 + 3.016049 - 4.002603 - 1.008665]\, c^2$

$= [0.018883\, c^2]$ u

But 1 u = 931.5 MeV/c^2

$\therefore Q = 0.018883 \times 931.5 = 17.59$ MeV

(b) Radius of deuterium and tritium, $r \approx 2.0$ fm $= 2 \times 10^{-15}$ m

Distance between the two nuclei at the moment when they touch each other, $d = r + r = 4 \times 10^{-15}$ m

Charge on the deuterium nucleus = e

Charge on the tritium nucleus = e

Hence, the repulsive potential energy between the two nuclei is given as:

$$V = \frac{e^2}{4\pi \epsilon_0 (d)}$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15}} = 5.76 \times 10^{-14} \text{ J}$$

$$= \frac{5.76 \times 10^{-14}}{1.6 \times 10^{-19}} = 3.6 \times 10^5 \text{ eV} = 360 \text{ keV}$$

Hence, 5.76×10^{-14} J or 360 keV of kinetic energy (KE) is needed to overcome the Coulomb repulsion between the two nuclei.

However, it is given that:

KE $= 2 \times \frac{3}{2} kT$

Where,

k = Boltzmann constant = 1.38×10^{-23} m^2 kg s^{-2} K^{-1}

T = Temperature required for triggering the reaction

$$\therefore T = \frac{\text{KE}}{3K}$$

$$= \frac{5.76 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 1.39 \times 10^{9} \text{ K}$$

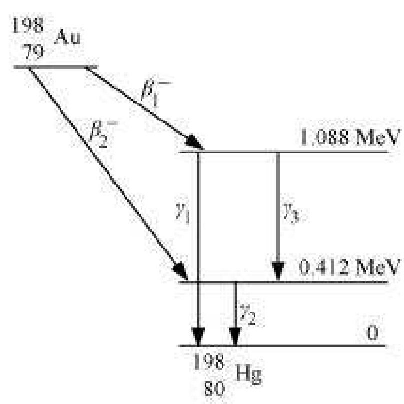
Hence, the gas must be heated to a temperature of 1.39×10^9 K to initiate the reaction.

Question 13.29:

Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme shown in Fig. 13.6. You are given that

m (^{198}Au) = 197.968233 u

m (^{198}Hg) =197.966760 u



Answer

It can be observed from the given γ-decay diagram that γ_1 decays from the 1.088 MeV energy level to the 0 MeV energy level.

Hence, the energy corresponding to γ_1-decay is given as:

E_1 = 1.088 − 0 = 1.088 MeV

$h\nu_1 = 1.088 \times 1.6 \times 10^{-19} \times 10^{6}$ J

Where,

h = Planck's constant = 6.6×10^{-34} Js

v_1 = Frequency of radiation radiated by γ_1-decay

$$\therefore v_1 = \frac{E_1}{h}$$

$$= \frac{1.088 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 2.637 \times 10^{20} \text{ Hz}$$

It can be observed from the given γ-decay diagram that γ_2 decays from the 0.412 MeV energy level to the 0 MeV energy level.

Hence, the energy corresponding to γ_2-decay is given as:

$E_2 = 0.412 - 0 = 0.412$ MeV

$hv_2 = 0.412 \times 1.6 \times 10^{-19} \times 10^6$ J

Where,

v_2 = Frequency of radiation radiated by γ_2-decay

$$\therefore v_2 = \frac{E_2}{h}$$

$$= \frac{0.412 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 9.988 \times 10^{19} \text{ Hz}$$

It can be observed from the given γ-decay diagram that γ_3 decays from the 1.088 MeV energy level to the 0.412 MeV energy level.

Hence, the energy corresponding to γ_3-decay is given as:

$E_3 = 1.088 - 0.412 = 0.676$ MeV

$hv_3 = 0.676 \times 10^{-19} \times 10^6$ J

Where,

v_3 = Frequency of radiation radiated by γ_3-decay

$$\therefore v_3 = \frac{E_3}{h}$$

$$= \frac{0.676 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 1.639 \times 10^{20} \text{ Hz}$$

Mass of $m\left({}^{198}_{78}\text{Au}\right)$ = 197.968233 u

Mass of $m\left({}^{198}_{80}\text{Hg}\right)$ = 197.966760 u

1 u = 931.5 MeV/c^2

Energy of the highest level is given as:

$$E = \left[m\left({}^{198}_{78}\text{Au} \right) - m\left({}^{190}_{80}\text{Hg} \right) \right]$$

$$= 197.968233 - 197.966760 = 0.001473 \text{ u}$$
$$= 0.001473 \times 931.5 = 1.3720995 \text{ MeV}$$

β_1 decays from the 1.3720995 MeV level to the 1.088 MeV level

∴Maximum kinetic energy of the β_1 particle = 1.3720995 − 1.088

= 0.2840995 MeV

β_2 decays from the 1.3720995 MeV level to the 0.412 MeV level

∴Maximum kinetic energy of the β_2 particle = 1.3720995 − 0.412

= 0.9600995 MeV

Question 13.30:

Calculate and compare the energy released by a) fusion of 1.0 kg of hydrogen deep within Sun and b) the fission of 1.0 kg of ^{235}U in a fission reactor.

Answer

(a) Amount of hydrogen, *m* = 1 kg = 1000 g

1 mole, i.e., 1 g of hydrogen (${}^{1}_{1}\text{H}$) contains 6.023 × 10^{23} atoms.

∴1000 g of ${}^{1}_{1}\text{H}$ contains 6.023 × 10^{23} × 1000 atoms.

Within the sun, four $^{1}_{1}H$ nuclei combine and form one $^{4}_{2}He$ nucleus. In this process 26 MeV of energy is released.

Hence, the energy released from the fusion of 1 kg $^{1}_{1}H$ is:

$$E_1 = \frac{6.023 \times 10^{23} \times 26 \times 10^3}{4}$$
$$= 39.1495 \times 10^{26} \text{ MeV}$$

(b) Amount of $^{235}_{92}U$ = 1 kg = 1000 g

1 mole, i.e., 235 g of $^{235}_{92}U$ contains 6.023 × 10^{23} atoms.

∴1000 g of $^{235}_{92}U$ contains $\frac{6.023 \times 10^{23} \times 1000}{235}$ atoms

It is known that the amount of energy released in the fission of one atom of $^{235}_{92}U$ is 200 MeV.

Hence, energy released from the fission of 1 kg of $^{235}_{92}U$ is:

$$E_2 = \frac{6 \times 10^{23} \times 1000 \times 200}{235}$$
$$= 5.106 \times 10^{26} \text{ MeV}$$

$$\therefore \frac{E_1}{E_1} = \frac{39.1495 \times 10^{26}}{5.106 \times 10^{26}} = 7.67 \approx 8$$

Therefore, the energy released in the fusion of 1 kg of hydrogen is nearly 8 times the energy released in the fission of 1 kg of uranium.

Question 13.31:

Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ^{235}U to be about 200MeV.

Answer

Amount of electric power to be generated, $P = 2 \times 10^5$ MW

10% of this amount has to be obtained from nuclear power plants.

∴Amount of nuclear power, $P_1 = \frac{10}{100} \times 2 \times 10^5$

$= 2 \times 10^4$ MW

$= 2 \times 10^4 \times 10^6$ J/s

$= 2 \times 10^{10} \times 60 \times 60 \times 24 \times 365$ J/y

Heat energy released per fission of a ^{235}U nucleus, E = 200 MeV

Efficiency of a reactor = 25%

Hence, the amount of energy converted into the electrical energy per fission is calculated as:

$$\frac{25}{100} \times 200 = 50 \text{ MeV}$$

$$= 50 \times 1.6 \times 10^{-19} \times 10^6 = 8 \times 10^{-12} \text{ J}$$

Number of atoms required for fission per year:

$$\frac{2 \times 10^{10} \times 60 \times 60 \times 24 \times 365}{8 \times 10^{-12}} = 78840 \times 10^{24} \text{ atoms}$$

1 mole, i.e., 235 g of U^{235} contains 6.023×10^{23} atoms.

∴Mass of 6.023×10^{23} atoms of U^{235} = 235 g = 235×10^{-3} kg

∴Mass of 78840×10^{24} atoms of U^{235}

$$= \frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times 78840 \times 10^{24}$$

$$= 3.076 \times 10^{4} \text{ kg}$$

Hence, the mass of uranium needed per year is 3.076×10^{4} kg.