Exercise 9.1: Arithmetic Progressions

1. Write the first terms of each of the following sequences whose n^th term are
   (i) $a_n = 3n + 2$
   (ii) $a_n = \frac{n-2}{3}$
   (iii) $a_n = 3^n$
   (iv) $a_n = \frac{3n-2}{5}$
   (v) $a_n = (-1)^n 2^n$
   (vi) $a_n = \frac{n(n-2)}{2}$
   (vii) $a_n = n^2 - n + 1$
   (viii) $a_n = n^2 - n + 1$
   (ix) $a_n = \frac{2n-3}{6}$

Sol:

We have to write first five terms of given sequences

(i) $a_n = 3n + 2$

Given sequence $a_n = 3n + 2$

To write first five terms of given sequence put n = 1, 2, 3, 4, 5, we get

$a_1 = (3 \times 1) + 2 = 3 + 2 = 5$

$a_2 = (3 \times 2) + 2 = 6 + 2 = 8$

$a_3 = (3 \times 3) + 2 = 9 + 2 = 11$

$a_4 = (3 \times 4) + 2 = 12 + 2 = 14$

$a_5 = (3 \times 5) + 2 = 15 + 2 = 17$

∴ The required first five terms of given sequence $a_n = 3n + 2$ are 5, 8, 11, 14, 17.

(ii) $a_n = \frac{n-2}{3}$

Given sequence $a_n = \frac{n-2}{3}$

To write first five terms of given sequence $a_n = \frac{n-2}{3}$

put n = 1, 2, 3, 4, 5 then we get

$a_1 = \frac{1-2}{3} = \frac{-1}{3}; a_2 = \frac{2-2}{3} = 0$

$a_3 = \frac{3-2}{3} = \frac{1}{3}: a_4 = \frac{4-2}{3} = \frac{2}{3}$

$a_5 = \frac{5-2}{3} = 1$

∴ The required first five terms of given sequence $a_n = \frac{n-2}{3}$ are $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, 1$.

(iii) $a_n = 3^n$

Given sequence $a_n = 3^n$

To write first five terms of given sequence. put n = 1, 2. 3. 4. 5 in given sequence.

Then.

$a_1 = 3^1 = 3; a_2 = 3^2 = 9; a_3 = 27; a_4 = 3^4 = 81; a_5 = 3^5 = 243.$

(iv) $a_n = \frac{3n-2}{5}$

Given sequence. $a_n = \frac{3n-2}{5}$

To write first five terms. put n = 1, 2. 3, 4, 5 in given sequence $a_n = \frac{3n-2}{5}$

Then. we ger

$a_1 = \frac{3\times1-2}{5} = \frac{3-2}{5} = \frac{1}{5}$

$a_2 = \frac{3\times2-2}{5} = \frac{6-2}{5} = \frac{4}{5}$

$a_3 = \frac{3\times3-2}{5} = \frac{9-2}{5} = \frac{7}{5}$

$a_4 = \frac{3\times4-2}{5} = \frac{12-2}{5} = \frac{10}{5}$

$a_5 = \frac{3\times5-2}{5} = \frac{15-2}{5} = \frac{13}{5}$

∴ The required first five terms are $\frac{1}{5}, \frac{4}{5}, \frac{7}{5}, \frac{10}{5}, \frac{13}{5}$

(v) $a_n = (-1)^n 2^n$

Given sequence is $a_n = (-1)^n 2^n$

To get first five terms of given sequence an, put n = 1, 2, 3, 4, 5.

$a_1 = (-1)^1.2^1 = (-1).2 = -2$

$a_2 = (-1)^2.2^2 = (-1).4 = 4$

$a_3 = (-1)^3.2^3 = (-1).8 = -8$

$a_4 = (-1)^4.2^4 = (-1).16 = 16$

$a_5 = (-1)^5.2^5 = (-1).32 = -32$

∴ The first five terms are -2, 4, -8, 16, -32.

(vi) $a_n = \frac{n(n-2)}{2}$

The given sequence is, $a_n = \frac{n(n-2)}{2}$

To write first five terms of given sequence $a_n = \frac{n(n-2)}{2}$

Put n = 1, 2, 3, 4, 5. Then, we get

$a_1 = \frac{1(1-2)}{2} = \frac{1-1}{2} = \frac{-1}{2}$

$a_2 = \frac{2(2-2)}{2} = \frac{2.0}{2} = 0$

$a_3 = \frac{3(3-2)}{2} = \frac{3.1}{2} = \frac{3}{2}$

$a_4 = \frac{4(4-2)}{2} = \frac{4.2}{2} = 4$

$a_5 = \frac{5(5-2)}{2} = \frac{5.3}{2} = \frac{15}{2}$

∴ The required first five terms are $\frac{-1}{2}, 0, \frac{3}{2}, 4, \frac{15}{2}$.

(vii) $a_n = n^2 - n + 1$

The given sequence is, $a_n = n^2 - n + 1$

To write first five terms of given sequence a_{n1} we get put n = 1, 2, 3, 4, 5. Then we get $a_1 = 1^2 - 1 + 1 = 1$

$a_2 = 2^2 - 2 + 1 = 3$

$a_3 = 3^2 - 3 + 1 = 7$

$a_4 = 4^2 - 4 + 1 = 13$

$a_5 = 5^2 - 5 + 1 = 21$

∴The required first five terms of given sequence $a_n = n^2 - n + 1$ are 1, 3, 7, 13, 21

(viii) $a_n = 2n^2 - 3n + 1$

The given sequence is $a_n = 2n^2 - 3n + 1$

To write first five terms of given sequence a_n, we put n = 1, 2, 3, 4, 5. Then we get

$a_1 = 2.1^2 - 3.1 + 1 = 2 - 3 + 1 = 0$

$a_2 = 2.2^2 - 3.2 + 1 = 8 - 6 + 1 = 3$

$a_3 = 2.3^2 - 3.3 + 1 = 18 - 9 + 1 = 10$

$a_4 = 2.4^2 - 3.4 + 1 = 32 - 12 + 1 = 21$

$a_5 = 2.5^2 - 3.5 + 1 = 50 - 15 + 1 = 36$

∴ The required first five terms of given sequence $a_n - 2n^2 - 3n + 1$ are 0, 3, 10, 21, 36

(ix) $a_n = \frac{2n-3}{6}$

Given sequence is, $a_n = \frac{2n-3}{6}$

To write first five terms of given sequence we put n = 1, 2, 3, 4, 5. Then, we get,

$a_1 = \frac{2.1-3}{6} = \frac{2-3}{6} = \frac{-1}{6}$

$a_2 = \frac{2.2-3}{6} = \frac{4-3}{6} = \frac{1}{6}$

$a_3 = \frac{2.4-3}{6} = \frac{8-3}{6} = \frac{5}{6}$

$a_4 = \frac{2.4-3}{6} = \frac{8-3}{6} = \frac{5}{6}$

$a_5 = \frac{2.5-3}{6} = \frac{10-3}{6} = \frac{7}{6}$

∴ The required first five terms of given sequence $a_n = \frac{2_n-3}{6}$ are $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{7}{6}$.

2. Find the indicated terms in each of the following sequences whose nth terms are:

(i) $a_n = 5n - 4; a_{12}\ and\ a_{15}$

(ii) $a_n = \frac{3n-2}{4n+5}; a_7\ and\ a_8$

(iii) $a_n = n(n-1)(n-2); a_5\ and\ a_8$

(iv) $a_n = (n-1)(2-n)(3+n); a_{11}\ a_{21}\ a_3$

(v) $a_n = (-1)^n n; a_3, a_5, a_8$

Sol:

We have to find the required term of a sequence when n^{th} term of that sequence is given.

(i) $a_n = 5n - 4; a_{12}\ and\ a_{15}$

Given n^{th} term of a sequence $a_n = 5n - 4$

To find 12^{th} term, 15^{th} terms of that sequence, we put n = 12, 15 in its n^{th} term.

Then, we get

$a_{12} = 5.12 - 4 = 60 - 4 = 56$

$a_{15} = 5.15 - 4 = 15 - 4 = 71$

∴ The required terms $a_{12} = 56, a_{15} = 71$

(ii) $a_n = \frac{3n-2}{4n+5}; a_7\ and\ a_8$

Given n^{th} term is $(a_n) = \frac{3n-2}{4n+5}$

To find 7^{th}, 8^{th} terms of given sequence, we put n = 7, 8.

$a_7 = \frac{(3.7)-2}{(4.7)+5} = \frac{19}{33}$

$a_8 = \frac{(3.8)-2}{(4.8)+5} = \frac{22}{37}$

∴ The required terms $a_7 = \frac{19}{33}$ and $a_8 = \frac{22}{37}$.

(iii) $a_n = n(n-1)(n-2)$; a_5 and a_8

Given n[th] term is $a_n = n(n-1)(n-2)$

To find 5[th], 8[th] terms of given sequence, put n= 5, 8 in an then, we get

$a_5 = 5(5-1).(5-2) = 5.4.3 = 60$

$a_8 = 8.(8-1).(8-2) = 8.7.6 = 336$

∴ The required terms are $a_5 = 60$ and $a_8 = 336$

(iv) $a_n = (n-1)(2-n)(3+n)$; $a_{11}\ a_{21}\ a_3$

The given n[th] term is $a_n = (n+1)(2-n)(3+n)$

To find a_1, a_2, a_3 of given sequence put n = 1, 2, 3 is an

$a_1 = (1-1)(2-1)(3+1) = 0.1.4 = 0$

$a_2 = (2-1)(2-2)(3+2) = 1.0.5 = 0$

$a_3 = (3-1)(2-3)(3+3) = 2.-1.6 = -12$

∴ The required terms $a_1 = 0$, $a_2 = 0$, $a_3 = -12$

(v) $a_n = (-1)^n n$; a_3, a_5, a_8

The given n[th] term is, $a_n = (-1)^n.n$

To find $a_3.a_5, a_8$ of given sequence put n = 3, 5, 8, in a_n.

$a_3 = (-1)^3.3 = -1.3 = -3$

$a_5 = (-1)^5.5 = -1.5 = -5$

$a_8 = (-1)^8 = 1.8 = 8$

∴ The required terms $a_3 = -3, a_5 = -5, a_8 = 8$

3. Find the next five terms of each of the following sequences given by:

(i) $a_1 = 1, a_n = a_{n-1} + 2,\ n \geq 2$

(ii) $a_1 = a_2 = 2, a_n = a_{n-1} - 3, n > 2$

(iii) $a_1 = -1, a_n = \frac{a_n-1}{n}, n \geq 2$

(iv) $a_1 = 4, a_n = 4\,a_{n-1} + 3, n > 1$

Sol:

We have to find next five terms of following sequences.

(i) $a_1 = 1, a_n = a_{n-1} + 2,\ n \geq 2$

Given, first term (a_1) = 1,

n[th] term $a_n = a_{n-1} + 2, n \geq 2$

To find 2[nd], 3[rd], 4[th], 5[th], 6[th] terms, we use given condition n ≥ 2 for n[th] term $a_n = a_{n-1} + 2$

$a_2 = a_{2-1} + 2 = a_1 + 2 = 1 + 2 = 3\ (\therefore a_1 = 1)$

$a_3 = a_{3-1} + 2 = a_2 + 2 = 3 + 2 = 5$

$a_4 = a_{4-1} + 2 = a_3 + 2 = 5 + 2 = 7$

$a_5 = a_{5-1} + 2 = a_4 + 2 = 7 + 2 = 9$

$a_6 = a_{6-1} + 2 = a_5 + 2 = a + 2 = 11$

∴ The next five terms are,

$a_2 = 3, a_3 = 5, a_4 = 7, a_5 = a, a_6 = 11$

(ii) $a_1 = a_2 = 2, a_n = a_{n-1} - 3, n > 2$

Given.

First term $(a_1) = 2$

Second term $(a_2) = 2$

n^{th} term $(a_n) = a_{n-1} - 3$

To find next five terms i.e.. a_3, a_4, a_5, a_6, a_7 we put n = 3. 4. 5. 6. 7 is a_n

$a_3 = a_{3-1} - 3 = 2 - 3 = -1$

$a_4 = a_{4-1} - 3 = a_3 - 3 = -1 - 3 = -4$

$a_5 = a_{5-1} - 3 = a_4 - 3 = -4 - 3 = -7$

$a_6 = a_{6-1} - 3 = a_5 - 3 = -7 - 3 = -10$

$a_7 = a_{7-1} - 3 = a_6 - 3 = -10 - 3 = -13$

$\therefore$ The next five terms are, $a_3 = -1, a_4 = -4, a_5 = -7, a_6 = -10, a_7 = -13$

(iii) $a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$

Given. first term $(a_1) = -1$

n^{th} term $(a_n) = \frac{a_{n-1}}{n}, n \geq 2$

To find next five terms i.e.. a_2, a_3, a_4, a_5, a_6 we put n = 2. 3. 4. 5. 6 is an

$a_2 = \frac{a_{2-1}}{2} = \frac{a_1}{2} = \frac{-1}{2}$

$a_3 = \frac{a_{3-1}}{3} = \frac{a_2}{3} = \frac{-1/2}{3} = \frac{-1}{6}$

$a_4 = \frac{a_{4-1}}{4} = \frac{a_3}{4} = \frac{-1/6}{4} = \frac{-1}{24}$

$a_5 = \frac{a_{5-1}}{5} = \frac{a_4}{5} = \frac{-1/24}{5} = \frac{-1}{120}$

$\therefore$ The next five terms are.

$a_2 = \frac{-1}{2}, a_3 = \frac{-1}{6}, a_4 = \frac{-1}{24}, a_5 = \frac{-1}{120}, a_6 = \frac{-1}{720}$

(iv) $a_1 = 4, a_n = 4\, a_{n-1} + 3, n > 1$

Given.

First term $(a_1) = 4$

n^{th} term $(a_n) = 4\, a_{n-1} + 3$. n > 1

To find next five terms i.e.. a_2, a_3, a_4, a_5, a_6 we put n= 2. 3. 4. 5. 6 is a_n

Then. we get

$a_2 = 4a_{2-1} + 3 = 4.a_1 + 3 = 4.4 + 3 = 19\ (\because a_1 = 4)$

$a_3 = 4a_{3-1} + 3 = 4.a_2 + 3 = 4(19) + 3 = 79$

$a_4 = 4\, a_{4-1} + 3 = 4.a_3 + 3 = 4(79) + 3 = 319$

$a_5 = 4\, a_{5-1} + 3 = 4.a_4 + 3 = 4(319) + 3 = 1279$

$a_6 = 4.a_{6-1} + 3 = 4.a_5 + 3 = 4(1279) + 3 = 5119$

$\therefore$ The required next five terms are.

$a_2 = 19, a_3 = 79, a_4 = 319, a_5 = 1279, a_6 = 5119$

Exercise 9.2: Arithmetic Progressions

1. For the following arithmetic progressions write the first term a and the common difference d:

   (i) $-5, -1, 3, 7, \ldots\ldots\ldots\ldots$

   (ii) $\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \ldots\ldots\ldots\ldots$

   (iii) 0.3, 0.55, 0.80, 1.05,.........

   (iv) $-1.1, -3.1, -5.1, -7.1, \ldots\ldots$

   Sol:

   We know that if a is the first term and d is the common difference, the arithmetic progression is $a, a+d, a+2d+a+3d, \ldots\ldots\ldots\ldots$

   (i) $-5, -1, 3, 7, \ldots\ldots\ldots\ldots$

   Given arithmetic series is

   $-5, -1, 3, 7 \ldots\ldots\ldots\ldots$

This is in the form of $a, a+d, a+2d+a+3d$.............. by comparing these two

$a=-5, a+d=1, a+2d=3, a+3d=7,$........

First term $(a)=-5$

By subtracting second and first term, we get

$(a+d)-(a)=d$

$-1-(-5)=d$

$4=d$

Common difference $(d)=4$.

(ii) $\frac{1}{5},\frac{3}{5},\frac{5}{5},\frac{7}{5},$.............

Given arithmetic series is.

$\frac{1}{5},\frac{3}{5},\frac{5}{5},\frac{7}{5},$.............

This is in the form of $\frac{1}{5},\frac{2}{5},\frac{5}{5},\frac{7}{5}$.............

$a, a+d, a+2d, a+3d$......

By comparing this two, we get

$a=\frac{1}{5}, a+d=\frac{3}{5}, a+2d=\frac{5}{5}, a+3d=\frac{7}{5}$

$\boxed{\textit{First term } \cos=\frac{1}{5}}$

By subtracting first term from second term, we get

$d=(a+d)-(a)$

$$d=\frac{3}{5}-\frac{1}{5}$$

$$d=\frac{2}{5}$$

$$\boxed{\textit{common difference}(d)=\frac{2}{5}}$$

(iii) 0.3, 0.55, 0.80, 1.05,.........

Given arithmetic series.

0.3, 0.55, 0.80, 1.05,.........

General arithmetic series

$a, a+d, a+2d, a+3d$.........

By comparing,

$a=0.3, a+d=0.55, a+2d=0.80, a+3d=1.05$

First term $(a)=0.3.$

By subtracting first term from second term. We get

$d=(a+d)-(a)$

$d=0.55-0.3$

$d=0.25$

Common difference (d) $=0.25$

(iv) $-1.1,-3.1,-5.1,-7.1,.....$

General series is

$a.a+d.a+2d.a+3d.........$

By comparing this two, we get

$a=-1.1.a+d=-3.1.a+2d=-5.1.a+3d=-71$

First term $(a)=-1.1$

Common difference $(d)=(a+d)-(a)$

$=-3.1-(-1.1)$

Common difference (d) $=-2$

2. Write the arithmetic progressions write first term a and common difference d are as follows:

(i) $a=4,d=-3$

(ii) $a=-1,d=\frac{1}{2}$

(iii) $a=-1.5.d=-0.5$

Sol:

We know that. if first term $(a)=a$ and common difference $=d$, then the arithmetic series is. $a.a+d.a+2d.a+3d.........$

(i) $a=4.d=-3$

Given first term $(a)=4$

Common difference $(d)=-3$

Then arithmetic progression is.

$a.a+d.a+2d.a+3d.........$

$\Rightarrow 4.4-3.a+2(-3).4+3(-3),.........$

$\Rightarrow 4.1.-2.-5.-8.......$

(ii) $a=-1.d=\frac{1}{2}$

Given.

First term $(a) = -1$

Common difference $(d) = \frac{1}{2}$

Then arithmetic progression is.

$\Rightarrow a, a+d, a+2d, a+3d, \ldots\ldots\ldots$

$\Rightarrow -1, -1+\frac{1}{2}, -1+2\frac{1}{2}, -1+3\frac{1}{2}, \ldots\ldots\ldots$

$\Rightarrow -1, \frac{-1}{2}, 0, \frac{1}{2}, \ldots\ldots\ldots$

(iii) $a = -1.5, d = -0.5$

Given

First term $(a) = -1.5$

Common difference $(d) = -0.5$

Then arithmetic progression is

$\Rightarrow a, a+d, a+2d, a+3d, \ldots\ldots$

$\Rightarrow -1.5, -1.5-0.5, -1.5+2(-0.5), -1.5+3(-0.5)$

$\Rightarrow -1.5, -2, -2.5, -3, \ldots\ldots\ldots$

Then required progression is

$-1.5, -2, -2.5, -3, \ldots\ldots\ldots$

3. In which of the following situations, the sequence of numbers formed will form an A.P.?

(i) The cost of digging a well for the first metre is Rs 150 and rises by Rs 20 for each succeeding metre.

(ii) The amount of air present in the cylinder when a vacuum pump removes each time $\frac{1}{4}$ of their remaining in the cylinder.

Sol:

(i) Given.

Cost of digging a well for the first meter $(c_1) = Rs.150$.

Cost rises by $Rs.20$ for each succeeding meter

Then.

Cost of digging for the second meter $(c_2) = Rs.150 + Rs\ 20$

$= Rs\ 170$

Cost of digging for the third meter $(c_3) = Rs.170 + Rs\ 20$

$= Rs\ 210$

Thus, costs of digging a well for different lengths are

150,170,190,210,......

Clearly, this series is in $A \cdot p$.

With first term $(a)=150$, common difference $(d)=20$

(ii) Given

Let the initial volume of air in a cylinder be V liters each time $\frac{3}{4}^{th}$ of air in a remaining i.e.,

$$1-\frac{1}{4}$$

First time, the air in cylinder is $\frac{3}{4}V$.

Second time, the air in cylinder is $\frac{3}{4}V$.

Third time, the air in cylinder is $\left(\frac{3}{4}\right)^2 V$.

Therefore, series is V, $\frac{3}{4}V, \left(\frac{3}{4}\right)^2 V, \left(\frac{3}{4}\right)^3 V$.........

4. Show that the sequence defined by $a_n = 5n - 7$ is an A.P., find its common difference.

Sol:

Given sequence is

$a_n = 5n - 7$

n^{th} term of given sequence $(a_n) = 5n - 7$

$(n+1)^{th}$ term of given sequence $(a_n + 1) - a_n$

$=(5n-2)-(5n-7)$

$= 5$

$\therefore d = 5$

5. Show that the sequence defined by $a_n = 3n^2 - 5$ is not an A.P.

Sol:

Given sequence is.

$a_n = 3n^2 - 5$,

n^{th} term of given sequence $(a_n) = 3n^2 - 5$.

$(n+1)^{th}$ term of given sequence $(a_n + 1) = 3(n+1)^2 - 5$

$= 3(n^2 + 1^2 + 2n.1) - 5$

$= 3n^2 + 6n - 2$

$\therefore$ The common difference $(d) = a_n + 1 - an$

$d = (3n^2 + 6n - 2) - (3n^2 - 5)$

$= 3a^2 + 6n - 2 - 3n^2 + 5$

$= 6n + 3$

Common difference (d) depends on 'n' value

$\therefore$ given sequence is not in $A.p$

Exercise 9.3: Arithmetic Progressions

1. Find:

(i) 10^{th} term of the AP 1,4 , 7, 10….

(ii) 18^{th} term of the AP $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots\ldots\ldots\ldots$

(iii) nth term of the AP $13, 8, 3, -2, \ldots\ldots\ldots\ldots$

(iv) 10^{th} term of the AP $-40, -15, 10, 35, \ldots\ldots\ldots$

(v) 8^{th} term of the AP 11, 104, 91, 78………

(vi) 11^{th} term of the AP 10.0, 10.5, 11.0, 11.2…….

(vii) 9^{th} term of the AP $\frac{3}{4}, \frac{5}{4}, \frac{7}{4} + \frac{9}{4}, \ldots\ldots\ldots\ldots$

Sol:

(i) Given $A.p$ is

$1,4,7,10,..........$

First term $(a)=1$

Common difference (d) = second term first term

$=4-1$

$=3.$

n^{th} term in an $A.p=a+(n-1)d$

10^{th} term in an $1+(10-1)3$

$=1+9.3$

$=1+27$

$=28$

(ii) Given $A.p$ is

$\sqrt{2},3\sqrt{2},5\sqrt{2},............$

First term $(a)=\sqrt{2}$

Common difference = Second term – First term

$=3\sqrt{2}-\sqrt{2}$

$d=2\sqrt{2}$

n^{th} term in an $A.p=a+(n-1)d$

18^{th} term of $A.p=\sqrt{2}+(18-1)2\sqrt{2}$

$=\sqrt{2}+17.2\sqrt{2}$

$=\sqrt{2}(1+34)$

$=35\sqrt{2}$

$\therefore 18^{th}$ term of $A.p$ is $35\sqrt{2}$

(iii) Given $A.p$ is

$13,8,3,-2,...........$

First term $(a)=13$

Common difference $(d)=$ Second term first term

$=8-13$

$=-5$

n^{th} term of an $A.p$ $a_n=a+(n-1)d$

$=13+(n-1)-5$

$=13-5n+5$

$a_n = 18 - 5n$

(iv) Given $A.p$ is

$-40, -15, 10, 35, \dots\dots\dots$

First term $(a) = -40$

Common difference $(d) =$ Second term – first term

$= -15 - (-40)$

$= 40 - 15$

$= 25$

n^{th} term of an $A.p$ $a_n = a + (n-1)d$

10^{th} term of $A.p$ $a_{10} = -40 + (10-1)25$

$= -40 + 9.25$

$= -40 + 225$

$= 185$

(v) Given sequence is

$117.104.91.78 \dots\dots\dots\dots$

First learn can $= 117$

Common difference $(d) =$ Second term – first term

$= 104 - 117$

$= -13$

n^{th} term $a_n = a + (n-1)d$

8^{th} term $a_8 = a + (8-1)d$

$= 117 + 7(-13)$

$= 117 - 91$

$= 26$

(vi) Given $A.p$ is

$10.0, 10.5, 11.0, 11.5, \dots\dots\dots\dots$

First term $(a) = 10.0$

Common difference (d) $=$ Second term – first term

$= 10.5 - 10.0$

$= 0.5$

n^{th} term $a_n = a + (n-1)d$

11^{th} term $a_{11} = 10.0 + (11-1)0.5$

$= 10.0 + 10 \times 0.5$

$= 10.0 + 5$

$= 15.0$

(vii) Given $A.p$ is

$\frac{3}{4}, \frac{5}{4}, \frac{7}{4} + \frac{9}{4}, ...$

First term $(a) = \frac{3}{4}$

Common difference $(d) =$ Second term – first term

$= \frac{5}{4} - \frac{3}{4}$

$= \frac{2}{4}$

n^{th} term $a_n = a + (n-1)d$

9^{th} term $a_9 = a + (9-1)d$

$= \frac{3}{4} + 8.\frac{2}{4}$

$= \frac{3}{4} + \frac{16}{4}$

$= \frac{19}{4}$

2. (i) Which term of the AP 3, 8, 13, …. is 248?
(ii) Which term of the AP 84, 80, 76, … is 0?
(iii) Which term of the AP 4, 9, 14, …. is 254?
(iv) Which term of the AP 21, 42, 63, 84, … is 420?
(v) Which term of the AP 121, 117, 113, … is its first negative term?

Sol:

(i) Given $A.p$ is $3, 8, 13, ...$

First term $(a) = 3$

Common difference $(d) =$ Second term – first term

$= 8 - 3$

$= 5$

n^{th} term $(a_n) = a + (n-1)d$

Given n^{th} term $a_n = 248$

$248 = 3 + (n-1).5$

$248 = -2 + 5n$

$5n = 250$

$n=\frac{250}{5}=50$

50^{th} term is 248.

(ii) Given $A.p$ is $84, 80, 76, ...$

First term $(a)=84$

Common difference $(d)=a_2-a$

$=80-84$

$=-4$

n^{th} term $(a_n)=a+(n-1)d$

Given n^{th} term is 0

$0=84+(n-1)-4$

$+84=+4(n-1)$

$n-1=\frac{\cancel{84}^{21}}{4}=21$

$n=21+1=22$

22^{nd} term is 0.

(iii) Given $A.p$ $4, 9, 14, ...$

First term $(a)=4$

Common difference $(d)=a^2-a$

$=9-4$

$=5$

n^{th} term $(a_n)=a+(n-1)d$

Given n^{th} term is 254

$4+(n-1)5=254$

$(n-1)\cdot 5=250$

$n-1=\frac{250}{5}=50$

$n=51$

$\therefore 51^{st}$ term is 254.

(iv) Given $A.p$

$21, 42, 63, 84, ...$

$a=21, d=a_2-a$

$=42-21$

$=21$

n^{th} term $(a_n) = a + (n-1)d$

Given n^{th} term $= 420$

$21 + (n-1)21 = 420$

$(n-1)21 = 399$

$n - 1 = \frac{399}{21} = 19$

$n = 20$

$\therefore 20^{th}$ term is 420.

(v) Given $A.p$ is $121, 117, 113, ...$

First term $(a) = 121$

Common difference $(d) = 117 - 121$

$= -4$

n^{th} term $(a) = a + (n-1)d$

Given n^{th} term is negative i.e., $a_n < 0$

$121 + (n-1) - 4 < 0$

$121 + 4 - 4n < 0$

$125 - 4n < 0$

$4n > 125$

$n > \frac{125}{4}$

$n > 31.25$

The integer which comes after 31.25 is 32.

$\therefore 32^{nd}$ term is first negative term

Exercise 9.4: Arithmetic Progressions

Question 1. If 12th of an A.P is 82 and 18th term is 124. Then find out the 24th term.

Solution: Given: a_{12} = 82 and a_{18} = 124

we know : a_n = a + (n-1) c.d

=> a_{12} = a + (12-1) c.d

=> 82 = a + 11c.d —(1)

=> 124 = a + (18-1) c.d

=> 124 = a + 17c.d —(2)

Subtracting (2) from (1)

=> (a + 17c.d) – (a + 11c.d) = 124 – 82

=> a +17c.d – a – 11c.d = 42

=> 6c.d = 42

=> c.d = 7

Here we have, Common Difference (c.d) = 7 putting c.d = 7 in equation (1), we get

=> $a + 11 \times 7 = 82$

=> $a = 82 – 77$

=> $a = 5$

Now, we have First Term (a) = 5

we have to find 24th term

$a_{24} = a + (24 -1)$ c.d

$= 5 + 23 \times 7$

$= 5 + 161$

= **166**

Question 2. In an A.P. the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.

Solution: Given

24th term is twice the 10th term

$a_{24} = 2 \times a_{10}$ . . . . . . (1)

let, first term be a

and common difference be d

we know, nth term is $a_n = a + (n – 1)d$

from equation (1), we have

$a + (24-1)d = 2(a + (10-1)d)$

=> $a + 23d = 2 (a + 9d)$

=> $a + 23 d = 2a + 18d$

=> $(23 – 18)d = a$

=> $a = 5d$

we have to prove that,

72^{nd} term is twice the 34^{th} term

=> $a_{72} = 2 \times a_{34}$

=> $a + (72-1)d = 2 [a + (34-1)d]$

=> $a + 71d = 2(a + 33d)$

=> $a + 71d = 2a + 66d$

putting the value of a = 5 in the above equation,

=> $5d + 71d = 2 (5d) + 66d$

=> $76d = 76d$

Hence it is proved…

Question 3. If the $(m+1)^{th}$ term of an A.P. is twice the $(n+1)^{th}$ term of the A.P. Then prove that: $(3m+1)^{th}$ is twice the $(m+n+1)^{th}$ term.

Solution: From the question, we have

$a_{(m+1)} = 2\, a_{(n+1)}$

Let, First term = a

and Common Difference = d

=> $a + (m + 1 – 1)d = 2[a+(n + 1 -1)d]$

=> $a + md = 2a + 2nd$

=> $a = md – 2nd$

=> $a = (m-2n)d$ —(2)

We have to prove, $a_{(3m+1)} = 2\, a_{(m+n+1)}$

=> $a + (3m + 1 – 1)d = 2 [a + (m + n + 1 -1)d]$

$\Rightarrow a + 3md = 2a + 2(m + n)d$

putting the value of $a = (m - 2n)d$, from equation (1)

$\Rightarrow (m - 2n)d + 3md = 2[(m - 2n)d] + 2(m + n)d$

$\Rightarrow m - 2n + 3m = 2m - 4n + 2m + 2n$

$\Rightarrow 4m - 2n = 4m - 2n$

Hence it is proved…

Question 4. If the n^{th} term of the A.P. 9,7,5, … is same as the n^{th} term of the A.P. 15, 12 , 9, … find n.

Solution: we have here,

First sequence is 9,7,5, …

First term (a) = 9,

Common Difference (c.d) = 9 – 7 = -2

n^{th} term = a + (n-1)c.d

$\Rightarrow a_n = 9 + (n-1)(-2)$

$= 9 - 2n + 2$

$= 11 - 2n$

Second sequence is 15, 12, 9, …

here, First term (a) = 15

Common Difference (c.d) = 12 -15 = -3

n^{th} term = a + (n-1)d

$\Rightarrow a'_n = 15 + (n-1)(-3)$

$= 15 - 3n + 3$

$= 18 - 3n$

We are given in the question that the n^{th} term of both the A.P.s are same, So, we can write it as

$a_n = a'_n$

=> $11 - 2n = 18 - 3n$

=> **n = 7**

So, the 7^{th} term of both the A.P.s will be equal.

Question 5. Find the 13^{th} term from the end in the following A.P.

(i). 4, 9, 14, … , 254.

Solution: we have,

First term (a) = 4 and common difference (c.d) = 9 -4 = 5

last term here (l) = 254

n^{th} term from the end is : $l - (n-1)d$

we have to find 13^{th} term from end then : $l - 12d$

$= 254 - 12 \times 5$

$= 254 - 60$

= **194**

(ii). 3, 5, 7, 9, …, 201.

Solution: we have,

First term (a) = 3 and common difference (c.d) = 5 – 3 = 2

last term here (l) = 201

n^{th} term from the end is : $l - (n-1)d$

we have to find 13^{th} term from end then : $l - 12d$

= 201 – 12 X 2

= 201 – 24

= **177**

(iii). 1, 4, 7, 10, … , 88.

Solution: we have,

First term (a) = 1 and common difference (c.d) = 4 -1 = 3

last term here (l) = 88

n^{th} term from the end is : l – (n-1)d

we have to find 13^{th} term from end then : l – 12d

= 88 – 12 X 3 = 88 – 36 = **52**

Question 6. The 4^{th} term of an A.P. is three times the first term and the 7^{th} term exceeds the twice the

third term by 1. Find the A.P.

Solution: Given, 4^{th} term of the A.P = thrice the first term

=> a_4 = 3 first term

Assuming first term to be ‘a’ and the common difference be ‘d’

we have, a+ (4 -1)d = 3 a

=> a + 3d = 3 a

=> a = 32 d $\frac{3}{2}d$ —(1)

and also it is given that,

the 7^{th} term exceeds the twice of the 3^{rd} term by 1

=> a_7 + 1 = 2 X a_3

=> a+ (7-1)d +1 = 2[a + (3-1)d]

=> a + 6d +1 = 2a + 4d

=> a = 2d + 1 —(2)

putting the value of a = 32d$\frac{3}{2}d$ from equation (1) in equation (2)

32d$\frac{3}{2}d$ = 2d + 1

=> 32d$\frac{3}{2}d - $2d = 1

=> 3d–4d2=1 $\frac{3d-4d}{2} = 1$

=> -d = 2

=> d = -2

put d = -2 in a=32d$a = \frac{3}{2}d$

=> a = 32(−2)$\frac{3}{2}(-2)$

=> a = -3

Now, we have a = -3 and d = -2, so the A.P. is -2, -5, -8, -11 , …

Question 7. Calculate the third term and the n^{th} term of an A.P. whose 8^{th} term and 13^{th} term are 48

and 78 respectively.

Solution: Given, $a_8 = 48$ and $a_{13} = 78$

n^{th} term of an A.P. is: $a_n = a + (n-1)d$

so,

$a_8 = a + (8 – 1)d = a + 7d$ —(1)

$a_{13} = a + (13 – 1)d = a + 12d$ —(2)

Equating (1) and (2), we get.

=> a + 12d – (a + 7d) = 78 – 48

=> a +12d – a -7d = 30

=> 5d = 30

=> **d = 6**

Putting the value of d = 6 in equation (1),

a + 7 X 6 = 48

=> a + 42 = 48

=> **a = 4**

Now, we have the first term (a) and the common difference (d) with us,

So, n^{th} term will be: $a_n = a + (n-1)d$

= 4 + (n-1)6

= 4 + 6n – 6

a_n = 6n – 2

and the 3^{rd} term will be

a_3 = 6 X 3 – 2

a_n = 16

Question 8. How many three digit numbers are divisible with 3?

Solution: We know the first three digit number which is divisible by 3 is 102 and the last three digit number which is divisible by 3 is 999

So, here we have

First term (a) = 102

Common Difference (c.d) = 3

last term or n^{th} term (l) = 999

=> a_n = 999

=> a + (n – 1)c.d = 999

=> 102 + (n – 1)3 = 999

=> 102 + 3n – 3 = 999

=> $99 + 3n = 999$

=> $3n = 900$

=> $n = 300$

Therefore, there are 300 terms in the sequence.

Question 9. An A.P. has 50 terms and the first term is 8 and the last term is 155. Find the 41st term from the A.P.

Solution: Given,

First term (a) = 8

Number of terms (n) = 50

Last term (a_n) = 148

=> $a_n = a + (n – 1)d$

=> $155 = 8 + (50 – 1)d$

=> $49 d = 147$

=> $d = 3$

now, 41st term will be: $a + (41-1)d$

=> $8 + 40 X 3$

=> **128**

Question 10. The sum of 4th and 8th term of an A.P. is 24 and the sum of the 6th and 10th terms is 34. Find the first term and the common difference of the A.P.

Solution:

Let’s assume first term be a and common difference be d

Given 4th term + 8th term = 24

=> $a_4 + a_8 = 24$

=> $(a + (4 – 1)d) + (a + (8 -1)d) = 24$

=> $a + 3d + a + 7d = 24$

=> $2a + 10d = 24$ —(1)

And 6^{th} term + 10^{th} term = 34

=> $a_6 + a_{10} = 34$

=> $(a + 5d) + (a + 9d) = 34$

=> $2a + 14d = 34$ —(2)

Subtracting equation (1) from (2),we get

=> $(2a + 14d) – (2a + 10d) = 34 -24$

=> $2a + 14d – 2a – 10d = 10$

=> $4d = 10$

=> d = $52\frac{5}{2}$

Put d = $52\frac{5}{2}$ in equation (1)

=> 2a + 10 X $52\frac{5}{2}$ = 24

=> $2a + 25 = 24$

=> $2a = -1$

=> a = $-12-\frac{1}{2}$

Therefore, we have a = $-12-\frac{1}{2}$ and d = $52\frac{5}{2}$

Question 11. The first term of an A.P. is 7 and its 100^{th} term is -488, Find the 50^{th} term of the same A.P.

Solution: Given,

First term (a) = 7

100^{th} term (a_{100}) = -488

we know, $a_n = a + (n - 1)d$

=> (a_{100}) = a + (100 -1)d

=> 7 + 99d = -488

=> 99d = -495

=> **d = -5**

Now, we have the common difference (d) = 5

We have to find out the 50^{th} term of the A.P.

Then, $a_{50} = a + 49\ d$

= 7 + 49 X (-5)

= 7 – 245

= **– 238**

So, the 50^{th} term of the A.P. is -238

Question 12. Find $a_{40} - a_{30}$ of the following A.P.

(i). 3, 5, 7, 9, . . .

Solution: Provided A.P. is 3, 5, 7, 9, . . .

So, we have first term (a) = 3 and the common difference (d) is 5 – 3 = 2

we have to find $a_{40} - a_{30} = (a + 39d) - (a + 29d)$

= a + 39 d – a -29 d

= 10 d

= 10 X 2

= **20**

(ii). 4, 9, 14, 19, . . .

Solution: Given A.P. is 4, 9, 14, 19, . . .

Common difference (d) = 9 – 4 = 5

we have to find $a_{40} - a_{30} =$ 10 d

= 10 X 5 = 50

Question 13. Write the expression $a_m - a_n$ for the A.P. a, a + d, a + 2d, . . .

Solution: General Arithmetic Progression

a, a + d, a + 2d, . . .

$a_m - a_n = (a + (m - 1)d) - (a + (n - 1)d)$

=> a + md – d – a -nd +d

=> md – kd

=> (m – n) d —(1)

Hence find the common difference of the A.P. for which

(i). 11th term is 5 and 13th term is 79

Solution: Given,

11th term (a_{11}) = 5

and 13th term (a_{13}) = 79

from equation (1),

taking m = 11 and n = 13

=> $a_m - a_n = (13 - 11)\ d$

=> 79 – 5 = 2d

=> 74 = 2d

=> **d = 37**

(ii). $a_{10} - a_5 = 200$

Solution: Given,

here we have the difference between the 10th term and 5th term

Putting the value of m and n in equation (1) as 10 and 5, we have

=> $a_{10} - a_5 = (10 - 5)d$

=> 200 = 5 d

=> **d = 40**

(iii). 20th term is 10 more than the 18th term

Solution: Given,

$a_{20} + 10 = a_{18}$

=> $a_{20} - a_{18} = 10$

from equation (1), we have

$a_m - a_n = (m - n) d$

=> $a_{20} - a_{18} = (20 - 18) d$

=> 10 = 2d

=> **d = 5**

Question 15. Find n if the given value of x is the n term if the given A.P.

(i) 1,2111,3111,4111,....:x=14111 $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \ldots : x = \frac{141}{11}$

(ii) 512,11,1612,22,...:x=550 $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots : x = 550$

(iii) -1 , -3, -5, -7, . . . : x = -151

(iv) 25, 50, 70, 100, . . . : x = 1000

Solution:

(i) Given sequence is

1,2111,3111,4111,....:x=14111 $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \ldots : x = \frac{141}{11}$

first term (a) = 1

Common difference (d) = 2111−1 $\frac{21}{11} - 1$

= 21−1111 $\frac{21-11}{11}$

= 1011 $\frac{10}{11}$

n^{th} term a_n = a + (n-1) X d

=> 17111=1+(n−1).1011 $\frac{171}{11} = 1 + (n-1). \frac{10}{11}$

=> 17111−1=(n−1).1011 $\frac{171}{11} - 1 = (n-1). \frac{10}{11}$

=> 171−1111=(n−1).1011 $\frac{171-11}{11} = (n-1). \frac{10}{11}$

=> 16011=(n−1).1011 $\frac{160}{11} = (n-1). \frac{10}{11}$

=> (n−1)=16011X1110 $(n-1) = \frac{160}{11} X \frac{11}{10}$

=> n = 17

(ii) Given sequence is

512,11,1612,22,...:x=550 $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \ldots : x = 550$

first term (a) = 512=112 $5\frac{1}{2} = \frac{11}{2}$

Common Difference (d) = 11−112=112 $11 - \frac{11}{2} = \frac{11}{2}$

n^{th} term a_n = a + (n-1) X d

=> 550=112+(n–1).112 $550 = \frac{11}{2} + (n-1).\frac{11}{2}$

=> 550=112[1+n–1] $550 = \frac{11}{2}[1 + n-1]$

=> n = 550 X 211 $\frac{2}{11}$

=> 100

(iii) Given sequence is,

-1 , -3, -5, -7, . . . : x = -151

first term (a) = -1

Common Difference (d) = -3 – (-1)

= -3 + 1

= -2

n^{th} term a_n = a + (n-1) X d

=> -151 = -1 + (n – 1) X -2

=> -151 = -1 – 2n + 2

=> – 151 = 1 – 2n

=> 2n = 152

=> n = 76

(iv) Given sequence is,

25, 50, 70, 100, . . . : x = 1000

First term (a) = 25

Common Difference (d) = 50 – 25 = 25

n^{th} term a_n = a + (n-1) X d

we have a_n = 1000

=> 1000 = 25 + (n – 1) 25

=> $975 = (n - 1)25$

=> $n - 1 = 39$

=> $n = 40$

Question 16. If an A.P. consists of n terms with the first term a and n^{th} term 1. Show that the sum of the m^{th} term from the beginning and the m^{th} term from the end is (a + 1).

Solution: First term of the sequence is a

Last term (l) = $a + (n - 1)d$

Total no. of terms = n

Common Difference = d

m^{th} term from the beginning $a_m = a + (n - 1)d$

m^{th} term from the end = $l + (n - 1)(-d)$

=> $a_{(n - m + 1)} = l - (n - 1)d$

=> $a_m + a_{(n - m + 1)} = a + (n - 1)d + (l - (n - 1)d)$

$= a + (n-1)d + l - (n-1)d$

$= a + l$

Question 17. Find the A.P. whose third term is 16 and seventh term exceeds its fifth term by 12.

Solution: Given, $a_3 = 16$

=> $a + (3 - 1)d = 16$

=> $a + 2d = 16$ ... (i)

and $a_7 - 12 = a_5$

=> $a + 6d - 12 = a + 4d$

=> $2d = 12$

=> $d = 6$

Put $d = 6$ in equation (1)

$a + 2 \times 6 = 16$

=> $a + 12 = 16$

=> $a = 4$. So, the sequence is 4, 10, 16, . . .

Question 18. The 7^{th} term of an A.P is 32 and its 13^{th} term is 62. Find the A.P.

Solution: Given

$a_7 = 32$

=> $a + (7 - 1)d = 32$

=> $a + 6d = 32$ … (i)

and $a_{13} = 62$

=> $a + (13 - 1)d = 62$

=> $a + 12d = 62$ …(ii)

equation (ii) – (i), we have

$(a + 12d) - (a + 6d) = 62 - 32$

=> $6d = 30$

=> $d = 5$

Putting $d = 5$ in equation (i)

$a + 6 \times 5 = 32$

=> $a = 32 - 30$

=> $a = 2$

So, the obtained A.P. is

2, 7, 12, 17, . . .

Question 19. Which term of the A.P. 3, 10, 17, . . . will be 84 more than its 13^{th} term?

Solution:

Given A.P. is 3, 10 , 17, . . .

First term (a) = 3

Common Difference (d) = 10 -3 = 7

Let n^{th} term of the A.P. will be 84 more than its 13^{th} term, then

$a_n = 84 + a_{13}$

=> $a + (n – 1)d = a + (13 – 1)d + 84$

=> $(n – 1) X 7 = 12 X 7 + 84$

=> $n – 1 = 24$

=> $n = 25$

Hence, 25^{th} ter, of the given A.P. is 84 more than the 13^{th} term.

Question 20. Two arithmetic progressions have the same common difference. The difference between their 100^{th} terms is 100. What is the difference between their 1000^{th} terms?

Solution:

Let the two A.P. be $a_1, a_2 , a_3 , . . .$ and $b_1, b_2 , b_3, . . .$

$a_n = a1 + (n – 1)d$ and $b_n = a1 + (n – 1)d$

Since common difference of two equation is same and given difference between 100^{th} terms is 100

=> $a_{100} – b_{100} = 100$

=> $a + (100 – 1)d – [b + (100 – 1)d] = 100$

=> $a + 99d - b – 99d = 100$

=> $a + b = 100$ … (1)

Difference between 100th term is

=> $a_{1000} – b_{1000}$

$= a + (1000 – 1)d – [b + (1000 – 1)d]$

$= a + 999d – b – 999d = a – b = 100$ (from equation 1)

Therefore, Difference between 1000th terms of two A.P. is 100.

Question 21. For what value of n, the nth terms of the Arithmetic Progression 63, 65, 67, . . . and , 3, 10, 17, . . . are equal?

Solution:

Given two A.P.s are:

63, 65, 67, . . . and 3, 10, 17, . . .

First term for first A.P. is (a) = 63

Common difference (d) is 65 – 63 = 2

nth term $(a_n) = a + (n – 1)d$

$= 63 + (n – 1) 2$

First term for second A.P. is (a’) = 3

Common Difference (d’) = 10 – 3

= 7

nth term $(a'_n) = a' + (n – 1)d$

$= 3 + (n – 1) 7$

Let nth term of the two sequence be equal then,

=> $63 + (n – 1)2 = 3 + (n – 1)7$

=> $60 = (n – 1).7 – (n – 1).2$

=> $60 = 5(n – 1)$

=> $n – 1 = 12$

=> $n = 13$

Hence, the 13^{th} term of both the A.P.s are same.

Question 22. How many multiple of 4 lie between 10 and 250?

Solution: Multiple of 4 after 10 is 12 and multiple of 4 before 250 is 120/4, remainder is 2, so,

250 – 2 = 248

248 is the last multiple of 4 before 250

the sequence is

12,. . . . . , 248

with first term (a) = 12

Last term (l) = 258

Common Difference (d) = 4

n^{th} term (a_n) = a + (n – 1)d

Here n^{th} term a_n = 248

=> $248 = a + (n – 1)d$

=> $12 + (n – 1)4 = 248$

=> $(n – 1)4 = 236$

=> $n – 1 = 59$

=> $n = 59 + 1$

=> $N = 60$

Therefore, there are 60 terms between 10 and 250 which are multiples of 4

Question 23. How many three digit numbers are divisible by 7 ?

Solution: The three digit numbers are 100, ... , 999

105 us the first 3 digit number which is divisible by 7

and when we divide 999 with 7 remainder is 5, so, 999 – 5 = 994

994 Is the last three digit number which is divisible by 7 .

The sequence here is

105, ... , 994

First term (a) = 105

Last term (l) = 994

Common Difference (d) = 7

Let there are n numbers in the sequence then,

=> $a_n = 994$

=> $a + (n – 1)d = 994$

=> $105 + (n – 1)7 = 994$

=> $(n – 1) X 7 = 889$

=> $n – 1 = 127$

=> $n = 128$

Therefore, there are 128 three digit numbers which are divisible by 7.

Question 24. Which term of the A.P. 8, 14, 20, 26, . . . will be 72 more than its 41^{st} term?

Solution: Given sequence

8, 14, 20, 26, . . .

Let its n term be 72 more than its 41^{st} term

=> $a_n = a_{41} + 72$ ... (1)

For the given sequence,

first term (a) = 8,

Common Difference (d) = 14 – 8 = 6

from equation (1), we have

$a_n = a_{41} + 72$

=> a + (n – 1)d = a + (41 – 1)d + 72

=> 8 + (n – 1)6 = 8 + 40 X 6 + 72

=> (n – 1)6 = 312

=> n – 1 = 52

=> n = 53

Therefore, 53^{rd} term is 72 more than its 41^{st} term.

Question 25. Find the term of the Arithmetic Progression 9, 12, 15, 18, . . . which is 39 more than its 36^{th} term.

Solution: Given A.P. is

9, 12, 15, 18 , . . .

Here we have,

First term (a) = 9

Common Difference (d) = 12 – 9 = 3

Let its nth term is 39 more than its 36th term

So, $a_n = 39 + a_{36}$

=> a + (n – 1)d = 39 + a + (36 – 1)d

=> (n -1)3 = 39 + 35 X 3

=> (n -1)3 = (13 + 35) 3

=> n – 1 = 48

=> n = 49

Therefore, 49^{th} term of the A.P. 39 more than its 36^{th} term.

Question 26. Find the 8^{th} term from the end of the A.P. 7, 10, 13, . . . , 184.

Solution:

Given A.P. is 7, 10, 13, . . . , 184

First term (a) = 7

Common Difference (d) = 10 – 7 = 3

last term (l) = 184

n^{th} term from end = l – (n – 1)d

8^{th} term from end = 184 – (8 – 1)3

= 184 – 7 X 3

= 184 – 21

= 183

Therefore, 8^{th} term from the end is 183

Question 27. Find the 10^{th} term from the end of the A.P. 8, 10, 12, . . . , 126

Solution: Given A.P. is 8, 10, 12, . . . , 126

First term (a) = 8

Common Difference (d) = 10 – 8 = 2

Last term (l) = 126

n^{th} term from end is : l – (n -1)d

So, 10^{th} term from end is : l – (10 – 1)d

= 126 – 9 X 2

= 126 – 18

= 108

Therefore, 109 is the 10^{th} term from the last in the A.P. 8, 10 ,12 , . . 126.

Question 28. The sum of 4^{th} and 8^{th} term of an A.P. is 24 and the sum of 6^{th} and 10^{th} term is 44. Find the Arithmetic Progression.

Solution: Given $a_4 + a_8 = 24$

=> a + (4 – 1)d + a + (8 – 1)d = 24

=> 2a + 3d + 7d = 24

=> 2a + 10d = 24 …(1)

and $a_6 + a_{10} = 44$

=> a + (6 – 1)d + a + (10 – 1)d = 44

=> 2a + 5d + 9 d = 44

=> 2a + 14d = 44 …(2)

equation (2) – equation (1), we get

2a + 14d – (2a + 10d) = 44 -24

=> 4d = 20

=> d = 5

Put d = 5 in equation (1), we get

2a + 10X5 = 24

=> 2a = 24 – 50

=> 2a = -26

=> a = -13

The A.P is -13, – 7, -2, . . .

Question 29: Which term of the A.P. is 3, 15, 27, 39, . . . will be 120 more than its 21st term?

Solution: Given A.P. is 3, 15, 27, 39, . . .

First term (a) = 3

Common Difference (d) = 15 – 3 = 12

Let nth term is 120 more than 21st term

=> $a_n = 120 + a_{21}$

=> a + (n – 1)d = 120 + a + (21 – 1)d

=> (n -1)d = 120 + 20d

=> (n – 1)12 = 120 + 20 X 12

=> n – 1 = 10 + 20

=> n = 31

Therefore, 31st term of the A.P. is 120 more than the 21st term.

Question 30. The 17th term of an A.P. is 5 more than twice its 8th term. If the 11th term of the A.P. is 43. Find the nth term.

Solution: Given

17th term of an A.P is 5 more than twice its 8th term

=> $a_{17} = 5 + 2a_8$

=> a + (17 – 1)d = 5 + 2 [a + (8 – 1)d]

=> a + 16d = 5 + 2a + 14d

=> a + 5 = 2d … (1)

and 11^{th} term of the A>P. is 43

$a_{11} = 43$

=> $a + (11 – 1)d = 43$

=> $a + 10d = 43$

=> $a + 5 X 2d = 43$

from equation (1)

=> $a + 5 X (a + 5) = 43$

=> $a + 5a + 25 = 43$

=> $6a = 18$

=> $a = 3$

Putting the value of a = 3, in equation (1), we get

$3 + 5 = 2d$

=> $2d = 8$

=> $d = 4$

We have to find the n^{th} term $(a_n) = a + (n – 1)d$

$= 3 + (n – 1)4$

$= 3 + 4n – 4$

$= 4n – 1$

Therefore, n^{th} term is $4n – 1$

Question 31. Find the number of all three digit natural number which are divisible by 9.

Solution: First three-digit number that is divisible by 9 is 108.

Next number is 108 + 9 = 117.

And the last three-digit number that is divisible by 9 is 999.

Thus, the progression will be 108, 117, …. , 999.

All are three digit numbers which are divisible by 9, and thus forms an A.P.

having first term (a) : 108

last term (l) = 999

and the common difference (d) as 9

We know that, n^{th} term (a_n) = a + (n – 1)d

According to the question,

999 = 108 + (n – 1)9

=> 999 = 108 + 9n – 9

=> 999 = 99 + 9n

=> 999 = 9n

=> 999 – 99

=> 9n = 900

=>n = 100

Therefore, There are 100 three digit terms which are divisible by 9.

Question 32. The 19^{th} term of an A.P. is equal to three times its 6^{th} term. if its 9^{th} term is 19, find the A.P.

Solution: Let a be the first term

and d be the common difference.

We know that, nth term = an= a + (n – 1)d

According to the question,

$a_{19} = 3a_6$

=> a + (19 – 1)d= 3(a + (6 – 1)d)

=> a + 18d= 3a + 15d

=> 18d- 15d= 3a – a

=>3d= 2a

=> a = 32d ....
(1)

Also, a9 = 19

=> a+(9 – 1)d= 19

=> a+ 8d= 19 ....
(2)

On substituting the values of (1) in (2), we get

=> 32d + 8d= 19

=> 3d+ 16d= 19 x 2

=> 19d= 38

=>d = 2

Now, a = 32x2 [From (1)]

a= 3

Therefore, The A.P. is : 3, 5, 7, 9, . . .

Question 33. The 9th term of an A.P. is equal to 6 times its second term. If its 5th term is 22, find the A.P.

Solution: Let a be the first term

and d be the common difference.

We know that, nth term (a_n) = a + (n – 1)d

According to the question,

a9 = 6a2

=> a + (9 – 1)d = 6(a + (2 – 1)d)

=> a+8d= 6a+6d

=> 8d-6d= 6a-a

=> 2d = 5a

=> a = $25\frac{2}{5}$ .... (1)

Also, a_5 = 22

=> a+(6 – 1)d= 22

=> a + 4d =
22 ....(2)

On substituting the values of (1) in (2), we get

$25\frac{2}{5}$ d + 4d = 22

=> 2d + 20d = 22 X 5

=> 22d = 110

=> d = 5

Now, a = $25\frac{2}{5}$ X 5 [From (1)]

=> a = 2

Thus, the A.P. is : 2, 7, 12, 17, . . .

Question 34. The 24^{th} term of an A.P. is twice its 10^{th} term. Show that its 72^{nd} term is 4 times its 15^{th} term.

Solution: Let a be the first term

and d be the common difference.

We know that,

n^{th} term (a_n) = a + (n – 1)d

According to the question,

a_{24} = 2 a_{10}

=> a + (24 – 1)d = 2(a + (10 – 1)d)

=> a + 23d = 2a + 18d

$\Rightarrow 23d - 18d = 2a - a$

$\Rightarrow 5d = a$

$\Rightarrow a = 5d$ ....
(1)

Also,

$a_{72} = a + (72 - 1) d$

$= 5d + 71d$ [From (1)]

$= 76d$ ....
(2)

and

$a_{15} = a + (15 - 1) d$

$= 5d + 14d$ [From (1)]

$= 19d$ ....
(3)

On comparing (2) and (3), we get

$\Rightarrow 76 d = 4 \times 19 d$

$\Rightarrow a_{72} = 4 \times a_{15}$

Thus, 72^{nd} term of the given A.P. is 4 times its 15^{th} term.

Question 35. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Solution: Since, the number is divisible by both 2 and 5, means it must be divisible by 10.

In the given numbers, first number that is divisible by 10 is 110.

Next number is 110 + 10 = 120.

The last number that is divisible by 10 is 990.

Thus, the progression will be 110, 120, …, 990.

All the terms are divisible by 10,

and thus forms an A.P. having first term as 110

and the common difference as 10.

We know that,

nth term = an= a + (n – 1)d

According to the question,

990 = 110 + (n – 1)10

=> 990 = 110 + 10n – 10

=> 10n = 990 – 100

=> n = 89

Thus, the number of natural numbers between 101 and 999 which are divisible by both 2 and 5 is 89.

Question 36. If the seventh term of an A.P. is 1/9 and its 9th term is 1/7, find the 63rd term.

Solution: Let a be the first term and d be the common difference.

We know that, nth term = an= a + (n – 1)d

According to the question,

$a_7 = 19\frac{1}{9}$

=> a+(7 – 1)d = $19\frac{1}{9}$

=> a+ 6d = $19\frac{1}{9}$ ….(1)

Also, $a_9 = 17\frac{1}{7}$

=> a + (9 – 1)d = $17\frac{1}{7}$

=> a + 8d = 17 $\frac{1}{7}$ ....(2)

On Subtracting (1) from (2), we get

=> 8d – 6d = 17 $\frac{1}{7}$ – 19 $\frac{1}{9}$

=> 2d = 9–763 $\frac{9-7}{63}$

=> 2d = 263 $\frac{2}{63}$

=> d= 163 $\frac{1}{63}$

Put value of d = 163 $\frac{1}{63}$ in equation (1), we get

=> a + 6 X 163 $\frac{1}{63}$ = 19 $\frac{1}{9}$

=> a = 19 $\frac{1}{9}$ – 663 $\frac{6}{63}$

=> a = 7–663 $\frac{7-6}{63}$

=> a = 163 $\frac{1}{63}$

Therefore, a_{63} = a + (63 – 1)d

= 163 $\frac{1}{63}$ + 6263 $\frac{62}{63}$

= 6363 $\frac{63}{63}$

= 1

Thus, 63rd term of the given A.P. is 1.

Question 37. The sum of 5th and 9th terms of an A.P. is 30. If its 25th term is three times its 8th term, Find the A.P.

Solution: Let a be the first term and d be the common difference.

We know that, nth term (a_n) = a + (n – 1)d

According to the question,

$a_5 + a_9 = 30$

=> a + (5 – 1)d + a + (9 – 1)d = 30

=> a + 4d + a + 8d = 30

=> 2a + 12d = 30

=> a + 6d = 15 ....
(1)

Also, $a_{25} = 3(a_8)$

=> a + (25 – 1)d = 3[a + (8 – 1)d]

=> a + 24d = 3a + 21d

=> 3a – a = 24d – 21d

=> 2a = 3d

=> a = 32d $\frac{3}{2}d$ ....(2)

Substituting the value of (2) in (1), we get 32d+6d= 15

=> 32d $\frac{3}{2}d$ + 6d = 15

=> 3d + 12d = 15 x 2

=> 15d = 30

=> d = 2

now, a = 32d $\frac{3}{2}d$ X 2 [From (1)]

=> a = 3

Therefore, the A.P. is 3, 5, 7, 9, . . .

Question 38. Find whether 0 (zero) is a term of the A.P. 40, 37, 34, 31, . . .

Solution: Let a be the first term and d be the common difference.

We know that, nth term = an = a + (n – 1)d

It is given that a = 40, d = -3

and a_n= 0

According to the question,

=> 0 = 40 + (n – 1)(-3)

=> 0 = 40 – 3n + 3

=> 3n = 43

=> n = 433 $\frac{43}{3}$ .... (1)

Here, n is the number of terms, so must be an integer.

Thus, there is no term where 0 (zero) is a term of the A.P. 40, 37, 34, 31,. . .

Question 39. Find the middle term of the A.P. 213, 205, 197, . . . 37.

Solution: Let a be the first term and d be the common difference.

We know that, nth term (a_n) = a + (n – 1)d

It is given that a = 213,

d = -8

and a_n = 37

According to the question,

=> 37 = 213 + (n – 1)(-8)

=> 37 =213 – 8n+ 8

=> 8n = 221 – 37

=> an = 184

=> n=23 ....
(1)

Therefore, total number of terms is 23.

Since, there is odd number of terms.

So, Middle term will be 23 + 12th term, i.e., the 12th term.

a_{12} =213 + (12 – 1)(-8)

a_{12} = 213 – 88

= 125

Thus, the middle term of the A.P. 213, 205, 197, . . . , 37 is 125.

Question 40. If the 5th term of the A.P. is 31 and 25th term is 140 more than the 5th term, find the A.P.

Solution: Let a be the first term and d be the common difference.

We know that, nth term (a_n) = a + (n – 1)d

According to question,

a_6 = 31

=> a + (5 – 1) = 31

=> a + 4d = 31

=> a = 31 – 4d . . . .(1)Also,

a_{25} = 140 + a_5

=> a + (25 – 1) = 140 + 31

=> a + 24d = 171 . . . . (3)

On substituting the values of (1) in (2), we get

31 – 4d +24d = 171

=> 20d = 171 – 31

=> 20d = 140

=> d = 7

=> a = 31 – 4 X 7 [From (1)]

=> a = 3

Thus, the A.P. obtained is 3, 10 , 17, 24, . .

Exercise 9.5: Arithmetic Progressions

1. Find the sum of the following arithmetic progressions:
   (i) 50, 46, 42, ... to 10 terms
   (ii) 1, 3, 5, 7, ... to 12 terms
   (iii) 3, 9/2, 6, 15/2, ... to 25 terms
   (iv) 41, 36, 31, ... to 12 terms
   (v) a + b, a – b, a − 3b, ... to 22 terms
   (vi) $(x - y)^2, (x^2 + y^2), (x + y)^2$, ... to n terms
   (vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots. to\ n\ terms$
   (viii) $-26, -24, -22, \ldots. to\ 36\ terms$

   Sol:

   In an A.P let first term = a, common difference = d, and there are n terms. Then, sum of n terms is,

   $S_n = \frac{n}{2}\{2a + (n-1)d\}$

   (i) Given progression is,
   50, 46, 42, ……to 10 term.
   First term (a) = 50
   Common difference (d) = 46 – 50 = −4
   n^{th} term = 10
   Then $S_{10} = \frac{10}{2}\{2.50 + (10-1) - 4\}$
   $= 5\{100 - 9.4\}$
   $= 5\{100 - 36\}$
   $= 5 \times 64$
   $\therefore S_{10} = 320$

(ii) Given progression is.

1, 3, 5, 7, ...to 12 terms

First term difference (d) = 3 – 1 = 2

n^{th} term = 12

Sum of n^{th} terms $S_{12} = \frac{12}{2} \times \{2.1 + (12 - 1).2\}$

$= 6 \times \{2 + 22\} = 6.24$

$\therefore S_{12} = 144.$

(iii) Given expression is

$3, \frac{9}{2}, 6, \frac{15}{2}, \ldots \ldots to\ 25\ terms$

First term (a) = 3

Common difference (d) $= \frac{9}{2} - 3 = \frac{3}{2}$

Sum of n^{th} terms $S_n, given\ n = 25$

$S_{25} = \frac{n}{2}(2a + (n - 1).d)$

$S_{25} = \frac{25}{2}\left(2.3 + (25 - 1).\frac{3}{2}\right)$

$= \frac{25}{2}\left(6 + 24.\frac{3}{2}\right)$

$= \frac{25}{2}(6 + 36)$

$= \frac{25}{2}(42)$

$\therefore S_{25} = 525$

(iv) Given expression is.

41, 36, 31, ... to 12 terms.

First term (a) = 41

Common difference (d) = 36 – 41 = – 5

Sum of n^{th} terms $S_n, given\ n = 12$

$S_{12} = \frac{n}{2}(2a(n - 1).d)$

$= \frac{12}{6}(2.41 + (12 - 1). -5)$

$= 6(82 + 11.(-5))$

$= 6(27)$

$= 162$

$\therefore S_{12} = 162.$

(v) a + b, a – b, a – 3b, …… to 22 terms

First term (a) = a + b

Common difference (d) = a – b – a – b = –2b

Sum of n^{th} terms $S_n = \frac{n}{2}\{2a(n-1).d\}$

Here n = 22

$S_{22} = \frac{22}{2}\{2.(a+b) + (22-1). -2b\}$

$= 11\{2(a+b) - 22b\}$

= 11 {2a – 20b}

= 22a – 440b

$\therefore S_{22} = 22a - 440b$

(vi) $(x-y)^2, (x^2+y^2), (x+y)^2, \ldots\ldots to\ n\ terms$

First term (a) = $(x-y)^2$

Common difference (d) = $x^2+y^2-(x-y)^2$

$= x^2+y^2-(x^2+y^2-2xy)$

$= x^2+y^2-x^2+y^2+2xy$

= 2xy

Sum of n^{th} terms $S_n = \frac{n}{2}\{2a(n-1).d\}$

$= \frac{n}{2}\{2(x-y)^2 + (n-1).2xy\}$

$= n\{(x-y)^2 + (n-1)xy\}$

$\therefore S_n = n\{(x-y)^2 + (n-1).xy\}$

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots\ldots to\ n\ terms$

$First\ term\ (a) = \frac{x-y}{x+y}$

Common difference (d) = $\frac{3x-2y}{x+y} - \frac{x-y}{x+y}$

$= \frac{3x-2y-x+y}{x+y}$

$= \frac{2x-y}{x+y}$

Sum of n terms $S_n = \frac{n}{2}\{2a+(n-1).d\}$

$= \frac{n}{2}\left\{2.\frac{x-y}{x+y} + (n-1).\frac{2x-y}{x+y}\right\}$

$= \frac{n}{2(x+y)}\{2(x-y) + (n-1)(2x-y)\}$

$= \frac{n}{2(x+y)}\{2x-2y+2nx-ny-2x+y\}$

$= \frac{n}{2(x+y)}\{n(2x-y)-y\}$

$\therefore S_n = \frac{n}{2(x+y)}\{n(2x-y)-y\}$

(viii) Given expression $-26, -24, -22, \ldots\ldots$ To 36 terms

First term (a) = -26

Common difference (d) = $-24-(-26) = -24+26 = 2$

Sum of n terms $S_n = \frac{n}{2}\{2a+(n-1)d\}$

Sum of n terms $S_n = \frac{36}{2}\{2.-26+(36-1)2\}$

$= 18[-52+70]$

$= 18.18$

$= 324$

$\therefore S_n = 324$

2. Find the sum to n term of the A.P. 5, 2, — 1, —4, —7, ...

Sol:

Given AP is 5, 2, -1, -4, -7, ...

$a = 5, d = 2-5 = -3$

$S_n = \frac{n}{2}\{2a+(n-1)d\}$

$= \frac{n}{2}\{2.5+(n-1)-3\}$

$= \frac{n}{2}\{10-3(n-1)\}$

$= \frac{n}{2}\{13-3n\}$

$\therefore S_n = \frac{n}{2}(13-3n)$

3. Find the sum of n terms of an A.P. whose th terms is given by $a_n = 5 - 6n$.

Sol:

Given nth term $a_n = 5-6n$

Put n = 1, $a_1 = 5-6.1 = -1$

We know, first term (a_1) = -1

Last term (a_n) = $5-6n = 1$

Then $S_n = \frac{n}{2}(-1+5-6n)$

$= \frac{n}{2}(4-6n) = \frac{n}{2}(2-3n)$

4. If the sum of a certain number of terms starting from first term of an A.P. is 25, 22, 19, ... is 116. Find the last term.

Sol:

Given AP is 25, 22, 19, ……

First term (a) = 25, d = 22 – 25 = −3.

Given, $S_n = \frac{n}{2}(2a + (n-1)d)$

$116 = \frac{n}{2}(2 \times 25 + (n-1) - 3)$

$232 = n(50 - 3(n-1))$

232 = n(53 – 3n)

232 = 53n – 3n²

$3n^2 - 53n + 232 = 0$

(3n – 29) (n – 8) = 0

∴ n = 8

⟹ a_8 = 25 + (8 – 1) −3

∴ n = 8, a_8 = 4

= 25 – 21 = 4

5. (i) How many terms of the sequence 18, 16, 14, ... should be taken so that their

(ii) How many terms are there in the A.P. whose first and fifth terms are −14 and 2 respectively and the sum of the terms is 40?

(iii) How many terms of the A.P. 9, 17, 25, . . . must be taken so that their sum is 636?

(iv) How many terms of the A.P. 63, 60, 57, . . . must be taken so that their sum is 693?

Sol:

(i) Given sequence, 18, 16, 14, …

a = 18, d = 16 – 18 = −2.

Let, sum of n terms in the sequence is zero

$S_n = 0$

$\frac{n}{2}(2a + (n-1)d) = 0$

$\frac{n}{2}(2.18 + (n-1) - 2) = 0$

$n(18 - (n-1)) = 0$

$n(19 - n) = 0$

n = 0 or n = 19

(ii) ∵ $n = 0$ is not possible. Therefore, sum of 19 numbers in the sequence is zero.

Given, a = -14, a 5 = 2

a + (5 – 1)d = 2

−14 + 4d = 2

$4d = 16 \implies d = 4$

Sequence is −14, −10, -6, −2, 2, ……

Given $S_n = 40$

$40 = \frac{n}{2}\{2(-14) + (n-1)4\}$

$80 = n(-28 + 4n - 4)$

$80 = n(-32 + 4n)$

4(20) = 4n(−8 + n)

$n^2 - 8n - 20 = 0$

$(n - 10)(n + 2) = 0$

$n = 10 \; or \; n = -2$

∴ Sum of 10 numbers is 40 (Since -2 is not a natural number)

(iii) Given AP 9, 17, 25, ...

a = 9, d = 17 – 9 = 8, and $S_n = 636$

$636 = \frac{n}{2}(2.9 + (n-1)8)$

$1272 = n(18 - 8 + 8n)$

1272 = n(10 + 8n)

2 × 636 = 2n(5 + 4n)

$636 = 5n + 4n^2$

$4n^2 + 5n - 636 = 0$

(4n + 53) (n – 12) = 0

∴ n = 12 (Since n $\frac{-53}{4}$ is not a natural number)

Therefore, value of n is 12.

(iv) Given AP, 63, 60, 57, ...

a =63, d = 60 – 63 = −3 $S_n = 693$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$693 = \frac{n}{2}(2.63 + (n-1) - 3)$

1386 = n(126 – 3n + 3)

1386 = (129 – 3n)n

$3n^2 - 129n + 1386 = 0$

$n^2 - 43n + 462 = 0$

n = 21, 22

∴ Sum of 21 or 22 term is 693

Exercise 9.6: Arithmetic Progressions

Question 1. Find the sum of the following Arithmetic Progression.

(i) 50, 46, 42,... To 10 terms

(ii) 1, 3, 4, 7, . . . 26 to 12 terms.

(iii) 3, 92 ,6, 152 ,....$3, \frac{9}{2}, 6, \frac{15}{2}, \ldots.$ **to 25 terms.**

(iv) 41, 36, 31, . . . To 12 terms.

(v) a + b, a – b, a -3b,... To 22 terms.

(vi) $(x-y)^2$ **,** (x^2, y^2)**,** $(x+y)^2$**, . . . to n terms.**

(vii) (x–y)(x+y) , (3x–2y)(x+y) , (5x–3y)(x+y) ,...tonterms$\frac{(x-y)}{(x+y)}, \frac{(3x-2y)}{(x+y)}, \frac{(5x-3y)}{(x+y)}, \ldots tonterms$

(viii) -26, -24, -22, . . . to 36 terms.

Solution:-

In the given problem, we need to find the sum of terms for different A.P.

So, here we use the following formula for the sum of n terms of an A.P.,

S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

Where; a = first term for the given A.P.

d = common difference of the given A.P. n = number of terms

(i) 50, 46, 42,... To 10 terms

Common difference of the A.P. (d)

= $a_2 - a_1$

= 46 – 50

= -4

Number of terms (n) = 10

First term for the given A.P. (a) = 50

So, using the formula we get,

S_{10}=102[2(50)+(10–1)(−4)]$S_{10} = \frac{10}{2}[2(50) + (10-1)\,(-4)]$

= (5) [100 + (9)(-4)]

= (5) [100 – 36]

= (6) [64]

= 320

Therefore, the sum of first 10 terms of the given A.P. is **320**

(ii) 1, 3, 4, 7, . . . 26 to 12 terms.

Common difference of the A.P. (d)

= $a_2 - a_1$

= 3 – 1

= 2

Number of terms (n) = 12

First term (a) = 1

So, using the formula we get,

S_{12}=122[2(1)+(12–1)(2)]$S_{12} = \frac{12}{2}[2(1) + (12-1)(2)]$

= (6) [2 + (11)(2)]

= (6) [2 + 22]

= (6) [24]

= 144

Therefore, the sum of first 10 terms of the given A.P. is **144**

(iii) 3,92,6,152,....$3, \frac{9}{2}, 6, \frac{15}{2}, \ldots$ to 25 terms.

Common difference here is (d): $a_2 - a_1$

= 92–3$\frac{9}{2} - 3$

= 9–62 $\frac{9-6}{2}$

= 32 $\frac{3}{2}$

Number of terms (n) = 25

First term of the A.P. (a) = 3

So, using the formula we get,

S_{25}=252[2(3)+(25–1)(32)]$S_{25} = \frac{25}{2}\left[2(3) + (25-1)\left(\frac{3}{2}\right)\right]$

= (252)[6+(24)(32)]$\left(\frac{25}{2}\right)\left[6 + (24)\left(\frac{3}{2}\right)\right]$

= (252)[6+(722)]$\left(\frac{25}{2}\right)\left[6 + \left(\frac{72}{2}\right)\right]$

= (252)[6+36]$\left(\frac{25}{2}\right)[6 + 36]$

= (252)[42]$\left(\frac{25}{2}\right)$ [42]

= (25) (21)

= 525

Therefore, the sum of first 12 terms for the given A.P. is **162**.

(iv) 41, 36, 31, . . . To 12 terms.

Common Difference of the A.P. (d) = $a_2 - a_1$

= 36 – 41

= -5

Number of terms (n) = 12

First term for the given A.P. (a) = 41

So, using the formula we get,

S_{12}= 122 [2(41)+(12–1)(−5)]$S_{12} = \frac{12}{2}[2(41) + (12-1)(-5)]$

= (6) [82 + (11) (-5)]

= (6) [82 – 55]

= (6) [27]

=162

Therefore, the sum of first 12 terms for the given A.P. **162**

(v) a + b, a – b, a -3b,... To 22 terms.

Common difference of the A.P. (d) = $a_2 - a_1$

= (a – b) – (a + b)

=a – b – a – b

= -2b

Number of terms (n) = 22

First term for the given A.P. (a) = a + b

So, using the formula we get,

S_{22}=222[2(a+b)+(22–1)(−2b)]$S_{22} = \frac{22}{2}[2(a+b)+(22-1)(-2b)]$

= (11)[2(a + b) + (22 – 1)(-2b)]

= (11)[2a + 2b + (21)(-2b)]

= (11)[2a + 2b – 42b]

= (11)[2a – 40b]

= **22a – 40b**

Therefore, the sum of first 22 terms for the given A.P. is: **22a – 40b**

(vi) $(x - y)^2$, (x^2, y^2), $(x + y)^2$, . . . to n terms.

Common difference of the A.P. (d) = $a_2 - a_1$

= $(x^2 - y^2) - (x - y)^2$

= $x^2 + y^2 - (x^2 + y^2 - 2xy)$

= 2xy

Number of terms (n) = n

First term for the given A.P. (a) = $(x - y)^2$

So, using the formula, we get.

S_n=n2[2(x–y)^2+(n–1)2xy]$S_n = \frac{n}{2}\left[2(x-y)^2+(n-1)\,2xy\right]$

Now, taking 2 common from both the terms inside bracket, we get

= n2(2)[(x–y)^2+(n–1)xy]$\frac{n}{2}(2)\left[(x-y)^2+(n-1)\,xy\right]$

= (n)[$(x - y)^2$ + (n – 1)xy]

Therefore, the sum of first n terms of the given A.P. is **(n)[$(x - y)^2$ + (n – 1)xy]**

(vii) (x–y)(x+y) , (3x–2y)(x+y) , (5x–3y)(x+y) ,...tonterms $\frac{(x-y)}{(x+y)}, \frac{(3x-2y)}{(x+y)}, \frac{(5x-3y)}{(x+y)}, \ldots tonterms$

Common difference of the A.P. (d) = $a_2 - a_1$

= (3x–2yx+y)–(x–yx+y) $\left(\frac{3x-2y}{x+y}\right) - \left(\frac{x-y}{x+y}\right)$

= (3x–2y)–(x–y)x+y $\frac{(3x-2y)-(x-y)}{x+y}$

= 3x–2y–x+yx+y $\frac{3x-2y-x+y}{x+y}$

= 2x–yx+y $\frac{2x-y}{x+y}$

So, using the formula we get,

S_n=n2[2(2x–2yx+y)+(n–1)(2x–yx+y)] $S_n = \frac{n}{2}\left[2\left(\frac{2x-2y}{x+y}\right) + (n-1)\left(\frac{2x-y}{x+y}\right)\right]$

= (n2)[(2x–2yx+y)+((n–1)(2x–y)x+y)] $\left(\frac{n}{2}\right)\left[\left(\frac{2x-2y}{x+y}\right) + \left(\frac{(n-1)(2x-y)}{x+y}\right)\right]$

= (n2)[(2x–2yx+y)+(n(2x–y)–1(2x–y)x+y)] $\left(\frac{n}{2}\right)\left[\left(\frac{2x-2y}{x+y}\right) + \left(\frac{n(2x-y)-1(2x-y)}{x+y}\right)\right]$

On further solving, we get

= (n2)(2x–2y+n(2x–y)–2x+yx+y) $\left(\frac{n}{2}\right)\left(\frac{2x-2y+n(2x-y)-2x+y}{x+y}\right)$

= (n2)(n(2x–y)–yx+y) $\left(\frac{n}{2}\right)\left(\frac{n(2x-y)-y}{x+y}\right)$

Therefore, the sum of first n terms for the given A.P. **is (n2)(n(2x–y)–yx+y)** $\left(\frac{n}{2}\right)\left(\frac{n(2x-y)-y}{x+y}\right)$

(viii) -26, -24, -22, . . . to 36 terms.

Common difference of the A.P. (d) = $a_2 - a_1$

= (-24) – (-26)

= -24 + 26

=2

Number of terms (n) = 36

First term for the given A.P. (a) = -26

So, using the formula we get,

$$S_n = \left(\frac{36}{2}\right)\left[2\left(-26\right) + \left(36-1\right)\left(2\right)\right]$$

= (18) [-52 + (35) (2)

= (18) [-52 + 70]

= (18) (18)

= 324

Therefore, the sum of first 36 terms for the given A.P. is **324**

Question 2. Find the sum to n term of the A.P. 5, 2, -1, -4, -7, . . .

Solution: In the given problem, we need to find the sum of the n terms of the given A.P.
5,2,-1,-4,-7,....

So, here we use the following formula for the sum of n terms of an A.P.,

$$S_n = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

and n = number of terms

For the given A.P. (5,2,-1,-4,-7,...),

Common difference of the A.P. (d) = $a_2 - a_1$

= 2 – 5

= -3

Number of terms (n) = n

First term for the given A.P. (a) = 5

So, using the formula we get,

$$S_n = \frac{n}{2}\left[2\left(5\right) + \left(n-1\right)\left(-3\right)\right]$$

= n2[10+(−3n+3)]$\frac{n}{2}[10+(-3n+3)]$

= n2[10–3n+3]$\frac{n}{2}[10–3n+3]$

= n2[13–3n]$\frac{n}{2}[13–3n]$

Therefore, the sum of first n terms for the given A.P. is **n2[13–3n]**$\frac{n}{2}[13–3n]$

Question 3. Find the sum of n terms of an A.P. whose nth terms is given by a_n = 5 – 6n.

Solution: Here, we are given an AP, whose nth term is given by the following expression,

a_n = 5 – 6n

So, here we can find the sum of the n terms of the given A.P., using the formula,

Sn=(n2)(a+l)$S_n=\left(\frac{n}{2}\right)(a+l)$

Where, a = the first term

l = the last term

So, for the given AP,

The first term (a) will be calculated using n = 1 in the given equation for nth term of A.P.

a = 5 – 6(1)

= 5 -6

= -1

Now, the last term (l) or the nth term is given

a_n = 5 – 6n

So, on substituting the values in the formula for the sum of n terms of an AP., we get,

Sn=(n2)[(−1)+5−6n]$S_n=\left(\frac{n}{2}\right)[(-1)+5-6n]$

= (n2)[4–6n]$\left(\frac{n}{2}\right)[4–6n]$

= (n2)(2)[2–3n]$\left(\frac{n}{2}\right)(2)[2-3n]$

= (n) (2 – 3n)

Therefore, the sum of the n terms of the given A.P. is (n) **(2 – 3n)**

Question 5. Find the sum of first 15 term of each of the following sequences having n^{th} term as

(i) $a_n = 3 + 4n$

(ii) $b_n = 5 + 2n$

(iii) $x_n = 6 - n$

(iv) $y_n = 9 - 5n$

Solution:

(i) Here, we are given an A.P. whose nth term is given by the following expression,

$a_n = 3 + 4n$

We need to find the sum of first 15 term & n ,

So, here we can find the sum of the n terms of the given A.P.,

using the formula,

S_n=n2(a+l)$S_n = \frac{n}{2}(a + l)$

Where, a = the first term

l = the last term

So, for the given AP,

The first term (a) will be calculated using n = 1 in the given equation for n^{th} term of A.P.

a = 3 + 4(1)

= 3 + 4

=7

Now, the last term (l) or the nth term is given

$l = a_n = 3 + 4n$

So, on substituting the values in the formula for the sum of n terms of an A.P, we get,

S_n=152(7+3+4(15))$S_n = \frac{15}{2}(7 + 3 + 4(15))$

= 152(10+60)$\frac{15}{2}(10 + 60)$

= 152(70)$\frac{15}{2}(70)$

= (15)(35)

= 525

Therefore, the sum of the 15 terms of the given A.P. is $S_{15} = 525$

(ii) Here, we are given an A.P. whose nth term is given by the following expression,

$b_n = 5 + 2n$

We need to find the sum of first 15 term & n ,

So, here we can find the sum of the n terms of the given A.P.,

using the formula,

S_n=n2(a+l)$S_n = \frac{n}{2}(a + l)$

Where, a = the first term

l = the last term

So, for the given AP,

The first term (a) will be calculated using n = 1 in the given equation for n^{th} term of A.P.

b = 5 + 2(1)

= 5 + 2

= 7

Now, the last term (l) or the n^{th} term is given l = b_n = 5 + 2n

So, on substituting the values in the formula for the sum of n terms of an A.P, we get,

S_n=152(7+5+2(15))$S_n = \frac{15}{2}(7 + 5 + 2(15))$

= 152(12+30)$\frac{15}{2}(12 + 30)$

= 152(42)$\frac{15}{2}(42)$

= (15) (21)

= 315

Therefore, the sum of the 15th term of the given A.P. is 315

(iii) Here, we are given an A.P. whose nth term is given by the following expression,

x_n = 6 – n

We need to find the sum of first 15 term & n ,

So, here we can find the sum of the n terms of the given A.P.,

using the formula,

S_n=n2(a+l)$S_n = \frac{n}{2}(a + l)$

Where, a = the first term

l = the last term

So, for the given AP,

The first term (a) will be calculated using n = 1 in the given equation for nth term of A.P.

b = 6 – 1

= 5

Now, the last term (l) or the nth term is given

l = x_n = 6 – n

So, on substituting the values in the formula for the sum of n terms of an A.P, we get,

S_n=152((5)+6–(15))$S_n = \frac{15}{2}((5) + 6 - (15))$

= 152(11–15)$\frac{15}{2}(11-15)$

= 152(−4)$\frac{15}{2}(-4)$

= (15) (-2)

= -30

Therefore, the sum of the 15 terms of the given A.P. is -30.

(iv) Here, we are given an A.P. whose nth term is given by the following expression,

y_n = 9 – 5n

We need to find the sum of first 15 term & n ,

So, here we can find the sum of the n terms of the given A.P.,

using the formula,

S_n=n2(a+l)$S_n = \frac{n}{2}(a+l)$

Where, a = the first term

l = the last term

So, for the given AP,

The first term (a) will be calculated using n = 1 in the given equation for n^{th} term of A.P.

b = 9 – 5 (1)

= 9 – 5

= 4

Now, the last term (l) or the n^{th} term is given

l = b_n = 9 – 5n

So, on substituting the values in the formula for the sum of n terms of an A.P, we get,

S_n=152((4)+9–5(15))$S_n = \frac{15}{2}((4)+9-5(15))$

= 152(13–75)$\frac{15}{2}(13-75)$

= 152(–62)$\frac{15}{2}(-62)$

= (15) (-31)

= -465

Therefore, the sum of the 15 terms of the given A.P. is -465

Question 6. Find the sum of first 20 terms the sequence whose n^{th} term is a_n= An + B.

Solution: Here, we are given an A.P. whose nth term is given by the following expression

a_n = An + B

We need to find the sum of first 20 terms.

So, here we can find the sum of the n terms of the given A.P.,

using the formula,

S_n=n2(a+l)$S_n = \frac{n}{2}(a+l)$

Where, a = the first term

l = the last term

So, for the given AP,

The first term (a) will be calculated using n = 1 in the given equation for n^{th} term of A.P.

a = A(1) + B

= A + B

Now, the last term (l) or the n^{th} term is given

l = a_n = An + B

So, on substituting the values in the formula for the sum of n terms of an A.P., we get,

S_{20}=202((A+B)+A(20)+B)$S_{20} = \frac{20}{2}((A+B)+A(20)+B)$

= 10[21A + 2B]

= 210A + 20B

Therefore, the sum of the first 20 terms of the given A.P. is 210A+20B

Question 7. Find the sum of first 25 terms of an A.P whose n^{th} term is given by a_n = 2 – 3n.

Solution: Here, we are given an A.P. whose nth term is given by the following expression,

a_n = 2 -3n

We need to find the sum of first 25 terms.

So, here we can find the sum of the n terms of the given AP., using the formula,

S_n=n2(a+l)$S_n = \frac{n}{2}(a+l)$

Where, a = the first term

l = the last term

So, for the given AP,

The first term (a) will be calculated using n= 1 in the given equation for nth term of A.P.

a =2 -3(1)

=2-3

= -1

Now, the last term (1) or the nth term is given l = a_n =2 – 3n

So, on substituting the values in the formula for the sum of n terms of an AP., we get,

S_{25}=252[(−1)+2–3(25)]$S_{25} = \frac{25}{2}[(-1)+2-3(25)]$

= 252[1–75]$\frac{25}{2}[1-75]$

= (25) (-37)

= -925

Therefore, the sum of the 25 terms of the given A.P. is -925

Question 8. Find the sum of first 25 terms of an A.P whose n^{th} term is given by $a_n = 7 - 3n$.

Solution: Here, we are given an AP. whose n^{th} term is given by the following expression,

$a_n = 7 - 3n$.

We need to find the sum of first 25 terms.

So, here we can find the sum of the n terms of the given AP., using the formula,

$$S_n = \frac{n}{2}(a + l)$$

Where, a = the first term

l = the last term

So, for the given AP,

The first term (a) will be calculated using n= 1 in the given equation for nth term of A.P.

a = 7 – 3

= 4

Now, the last term (l) or the n^{th} term is given

$l = a_n = 7 - 3n$

So, on substituting the values in the formula for the sum of n terms of an AP., we get,

$$S_n = \frac{25}{2}[(4) + 7 - 3(25)] = \frac{25}{2}[11 - 75]$$

$$= \frac{25}{2}[-64] = (25)(-32) = -800$$

Therefore, the sum of the 25 terms of the given A.P. is $S_n = -800$

Question 9. If the sum of a certain number of terms starting from first term of an A.P. is 25, 22, 19, . . . , is 116. Find the last term.

Solution:- In the given problem, we have the sum of the certain number of terms of an A.P. and we need to find the last term for that A.P.

So here, let us first find the number of terms whose sum is 116.

For that, we will use the formula,

S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

So for the given A.P(25, 22, 19,…)

The first term (a) = 25

The sum of n terms S_n = 116

Common difference of the A.P. (d) = $a_2 - a_1$

= 22-25

= -3

So, on substituting the values in the formula for the sum of n terms of an A.P., we get,

=> 116=n2[2(25)+(n–1)(–3)]$116 = \frac{n}{2}[2\,(25) + (n-1)\,(-3)]$

=> (n2)[50+(−3+3)]$\left(\frac{n}{2}\right)[50 + (-3 + 3)]$

=> (n2)[53–3n]$\left(\frac{n}{2}\right)[53 - 3n]$

=> 116 X 2 = 53n – $3n^2$

So, we get the following quadratic equation, $3n^2$ – 53n + 232 = 0

On solving by splitting middle term, we get,

=> $3n^2$ – 24n – 29n + 232 = 0

=> 3n(n – 8) – 29 (n – 8) = 0

=> (3n – 29) (n – 8) = 0

Further,

3n – 29 = 0

=> n = 293 $\frac{29}{3}$

Also,

n – 8 = 0

=> n = 8

Since, n cannot be a fraction, so the number of terms is 8.

So, the term is:

$a_8 = a_1 + 7d$

= 25 + 7(-3)

= 25 – 21

= **4**

Therefore, the last term of the given A.P. such that the sum of the terms is 116 is **4**.

Question 10.

(i). How many terms of the sequence 18, 16, 14.... should be taken so that their sum is 0 (Zero).

(ii). How many terms are there in the A.P. whose first and fifth terms are -14 and 2 respectively and the sum of the terms is 40?

(iii). How many terms of the A.P. 9, 17, 25, . . . must be taken so that their sum is 636?

(iv). How many terms of the A.P. 63, 60, 57, . . . must be taken so that their sum is 693?

(v). How many terms of the A.P. is 27, 24, 21. . . should be taken that their sum is zero?

Solution:

(i) AP. is 18,16,14,...

So here, let us find the number of terms whose sum is 0.

For that, we will use the formula,

S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

The first term (a) = 18

The sum of n terms (S_n) = 0

Common difference of the A.P. (d) = $a_2 - a_1$

= 16 – 18

= -2

So, on substituting the values in the formula for the sum of n terms of an AP., we get

=> 0=n2[2(18)+(n–1)(−2)]$0 = \frac{n}{2}[2(18) + (n-1)(-2)]$

=> 0=n2[36+(−2n+2)]$0 = \frac{n}{2}[36 + (-2n + 2)]$

=> 0=n2[38–2n]$0 = \frac{n}{2}[38 - 2n]$

Further,

n2 $\frac{n}{2}$

=> n = 0

Or, 38 – 2n = 0

=> 2n = 38

=> n = 19

Since, the number of terms cannot be zero; the number of terms (n) is 19

(ii) Here, let us take the common difference as d.

So, we are given,

First term (a_1) = -14

Filth term (a_5) = 2

Sum of terms (S_n) = 40

Now,

$a_5 = a_1 + 4d$

=> 2 = -14 + 4d

=> 2 + 14 = 4d

=> 4d = 16

=> d=4

Further, let us find the number of terms whose sum is 40.

For that, we will use the formula,

S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

The first term (a_1) = -14

The sum of n terms (S_n) = 40

Common difference of the A.P. (d) = 4

So, on substituting the values in the formula for the sum of n terms of an A.P., we get,

=> 40=n2[2(−14)+(n–1)(4)]$40 = \frac{n}{2}[2\,(-14) + (n-1)\,(4)]$

=> 40=n2[−28+(4n–4)]$40 = \frac{n}{2}[-28 + (4n-4)]$

=> 40=n2[−32+4n]$40 = \frac{n}{2}[-32 + 4n]$

=> 40 (2) = – 32n + $4n^2$

So, we get the following quadratic equation,

$4n^2 - 32n - 80 = 0$

=> $n^2 - 8n + 20 = 0$

On solving by splitting the middle term, we get

$4n^2 - 10n + 2n + 20 = 0$

=> n(n – 10) + 2(n – 10) = 0

=> (n + 2) (n – 10) = 0

Further,

n + 2 = 0

=> n = -2

Or,

n – 10 = 0

=> n = 10

Since the number of terms cannot be negative.

Therefore, the number of terms (n) is 10.

(iii) AP is 9,17,25,…

So here, let us find the number of terms whose sum is 636.

For that, we will use the formula,

S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

The first term (a) = 9

The sum of n terms (S_n) = 636

Common difference of the A.P. (d) = $a_2 - a_1$

= 17 – 9

= 8

So, on substituting the values in the formula for the sum of n terms of an AP.,

we get,

=> 636=n2[2(9)+(n–1)(8)]$636 = \frac{n}{2}[2\,(9) + (n-1)\,(8)]$

=> 636=n2[18+(8n–8)]$636 = \frac{n}{2}[18 + (8n - 8)]$

=> 636 (2) = (n) [10 + 8n]

=> 1271 = 10n + $8n^2$

So, we get the following quadratic equation,

=> $8n^2$ + 10n – 1272 = 0

=> $4n^2$ + 5n – 636 = 0

On solving by splitting the middle term, we get,

=> $4n^2$ – 48n + 53n – 636 = 0

=> 4n(n – 12) – 53(n – 12) = 0

=> (4n – 53) (n – 12) = 0

Further,

4n – 53 = 0

=> n=534$n = \frac{53}{4}$

Or, n – 12 = 0

=> n = 12

Since, the number of terms cannot be a fraction.

Therefore, the number of terms (n) is 12.

(iv) A.P. is 63,60,57,…

So here. let us find the number of terms whose sum is 693. For that,

we will use the formula.

S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n - 1)\, d]$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

The first term (a) = 63

The sum of n terms (S_n) = 693

Common difference of the A.P. (d) = $a_2 - a_1$

= 60 – 63

= -3

So, on substituting the values in the formula for the sum of n terms of an AP we get.

=> 693=n2[2(63)+(n–1)(−3)]$693 = \frac{n}{2}[2(63) + (n-1)(-3)]$

=> 693=n2[163+(−3n+3)]$693 = \frac{n}{2}[163 + (-3n + 3)]$

=> 693=n2[129–3n]$693 = \frac{n}{2}[129 - 3n]$

=> 693 (2) = 129n – $3n^2$

So. we get the following quadratic equation.

=> $3n^2$ – 129n + 1386 = 0

=> n^2 – 43n + 462

On solving by splitting the middle term, we get.

=> n^2 – 22n – 21n + 462 = 0

=> n(n – 22) -21(n – 22) = 0

=> (n – 22) (n – 21) = 0

Further,

n – 22 = 0

=> n = 22

Or,   n – 21 = 0

=> n = 21

Here, 22^{nd} term will be

$a_{22} = a_1 + 21d$

= 63 + 21(-3)

= 63 – 63

= 0

So, the sum of 22 as well as 21 terms is 693.

Therefore, the number of terms (n) is 21 or 22

(v) A.P. is 27, 24, 21. . .

So here. let us find the number of terms whose sum is 0. For that,

we will use the formula.

S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

The first term (a) = 27

The sum of n terms (S_n) = 0

Common difference of the A.P. (d) = $a_2 - a_1$

= 24 – 27

= -3

So, on substituting the values in the formula for the sum of n terms of an AP we get.

=> 0=n2[2(27)+(n–1)(−3)]$0 = \frac{n}{2}[2\,(27) + (n-1)\,(-3)]$

=> 0 = (n) [54 + (n – 1)(-3)]

=> 0 = (n) [54 – 3n + 3]

=> 0 = n [57 – 3n]

Further we have,

n = 0

Or,

57 – 3n = 0

=> 3n = 57

=> n = 19

The number of terms cannot be zero,

Therefore, the numbers of terms (n) is 19.

Question 11. Find the sum of the first

(i) 11 terms of the A.P. : 2, 6, 10, 14, . . .

(ii) 13 terms of the A.P. : -6, 0, 6, 12, . . .

(iii) 51 terms of the A.P. : whose second term is 2 and fourth term is 8.

Solution: In the given problem,

we need to find the sum of terms for different arithmetic progressions.

So, here we use the following formula for the sum of n terms of an A.P.,

S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

(i) 2,6,10,14,… To 11 terms.

Common difference of the A.P. (d) = $a_2 - a_1$

= 10 – 6

= 4

Number of terms (n) = 11

First term for the given A.P. (a) = 2

So, using the formula we get,

S_{11}=112[2(2)+(11–1)4]$S_{11} = \frac{11}{2}[2(2) + (11-1)4]$

= 112[2(2)+(10)4]$\frac{11}{2}[2(2) + (10)4]$

= 112[4+40]$\frac{11}{2}[4 + 40]$

= 11 X 22

= 242

Therefore, the sum of first 11 terms for the given A.P. is 242

(ii) -6, 0, 6, 12, … to 13 terms.

Common difference of the AR (d) = $a_2 - a_1$

= 6 – 0

= 6

Number of terms (n) = 13

First term for the given AP (a) = -6

So, using the formula we get,

S_{13}=132[2(−6)+(13–1)6]$S_{13} = \frac{13}{2}[2(-6) + (13-1)6]$

= 132[(−12)+(12)6]$\frac{13}{2}[(-12) + (12)6]$

= 132[60]$\frac{13}{2}[60]$

= 390

Therefore, the sum of first 13 terms for the given AR is 390

(iii) 51 terms of an AP whose a_2 = 2 and a_4 = 8

Now,

a_2 = a + d

2 = a + d …(i)

Also,

$a_4 = a + 3$

$8 = a + 3d$ … (2)

Subtracting (1) from (2), we get

$2d = 6$

$d = 3$

Substituting d = 3 in (i), we get

$2 = a + 3$

=> a = -1

Number of terms (n) = 51

First terms for the given A.P.(a) = -1

So, using the formula, we get

S_n=512[2(−1)+(51−1)(3)]$S_n = \frac{51}{2}\left[2\left(-1\right) + \left(51 - 1\right)\left(3\right)\right]$

= 512[−2+150]$\frac{51}{2}\left[-2 + 150\right]$

= 512[158]$\frac{51}{2}\left[158\right]$

= 3774

Therefore, the sum of first 51 terms for the A.P. is 3774.

Question 12. Find the sum of

(i) First 15 multiples of 8

(ii) the first 40 positive integers divisible by (a) 3 (b) 5 (c) 6.

(iii) all 3 – digit natural numbers which are divisible by 13.

Solution: In the given problem,

we need to find the sum of terms for different arithmetic progressions.

So, here we use the following formula for the sum of n terms of an A.P.,

S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

Where: a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

(i) First 15 multiples of 8.

So, we know that the first multiple of 8 is 8 and the last multiple of 8 is 120.

Also, all these terms will form an A.P. with the common difference of 8.

So here,

First term (a) = 8

Number of terms (n) = 15

Common difference (d) = 8

Now, using the formula for the sum of n terms, we get

S_n=152[2(8)+(15–1)8]$S_n = \frac{15}{2}[2\,(8) + (15-1)\,8]$

= 152[16+(14)8]$\frac{15}{2}[16 + (14)8]$

= 152[16+12]$\frac{15}{2}[16 + 12]$

= 152[128]$\frac{15}{2}[128]$

= 960

Therefore, the sum of the first 15 multiples of 8 is 960

(ii)

(a) First 40 positive integers divisible by 3

So, we know that the first multiple of 3 is 3 and the last multiple of 3 is 120.

Also, all these terms will form an A.P. with the common difference of 3.

So here,

First term (a) = 3

Number of terms (n) = 40

Common difference (d) = 3

Now, using the formula for the sum of n terms, we get

S_n=402[2(3)+(40–1)3]$S_n = \frac{40}{2}[2(3) + (40-1)3]$

= 20 [6 + (39)3]

= 20 (6 + 117)

=20 (123)

= 2460

Therefore, the sum of first 40 multiples of 3 is 2460

(b) First 40 positive integers divisible by 5

So, we know that the first multiple of 5 is 5 and the last multiple of 5 is 200.

Also, all these terms will form an A.P. with the common difference of 5.

So here,

First term (a) = 5

Number of terms (n) = 40

Common difference (d) = 5

Now, using the formula for the sum of n terms, we get

S_n=402[2(5)+(40–1)5]$S_n = \frac{40}{2}[2(5) + (40-1)5]$

= 20 [10 + (39)5]

= 20 (10 + 195)

= 20 (205)

= 4100

Therefore, the sum of first 40 multiples of 5 is 4100

(c) First 40 positive integers divisible by 6

So, we know that the first multiple of 6 is 6 and the last multiple of 6 is 240.

Also, all these terms will form an A.P. with the common difference of 6.

So here,

First term (a) = 6

Number of terms (n) = 40

Common difference (d) = 6

Now, using the formula for the sum of n terms, we get

S_n=402[2(6)+(40–1)6]$S_n = \frac{40}{2}[2(6) + (40-1)6]$

= 20[12 + (39) 6]

=20 (12 + 234)

= 20(246)

= 4920

Therefore, the sum of first 40 multiples of 6 is 4920

(ii) All 3 digit natural number which are divisible by 13

So, we know that the first 3 digit multiple of 13 is 104

and the last 3 digit multiple of 13 is 988.

Also, all these terms will form an AR with the common difference of 13.

So here,

First term (a) = 104

Last term (l) = 988

Common difference (d) = 13

So, here the first step is to find the total number of terms.

Let us take the number of terms as n.

Now, as we know,

a_n=a+(n–1)d$a_n = a + (n-1)\, d$

So, for the last term,

988 = 104 + (n – 1)13

=> 988 = 104 + 13n – 13

=> 988 = 91 + 13n

=> 13n = 897

=> n = 69

Now, using the formula for the sum of n terms, we get

S_n=692[2(104)+(69–1)13]$S_n = \frac{69}{2}[2\,(104) + (69-1)\,13]$

= 692[208+884]$\frac{69}{2}[208 + 884]$

= 692[1092]$\frac{69}{2}[1092]$

= 69 (546)

= 37674

Therefore, the sum of all 3 digit multiples of 13 is 37674

Question 13. Find the sum:

(i) 2 + 4 + 6 + . . . + 200

(ii) 3 + 11 + 19 + . . . + 803

(iii) (-5) + (-8) + (-11) + . . . + (-230)

(iv) 1 + 3 + 5 + 7 + . . . + 199

(v) 7+1012+14+...+84$7 + 10\frac{1}{2} + 14 + \ldots + 84$

(vi) 34 + 32 + 30 + . . . + 10

(vii) 25 + 28 + 31 + . . . + 100

Solution: In the given problem, we need to find the sum of terms for different arithmetic progressions. So, here we use the following formula for the sum of n terms of an A.P.,

S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

or

S_n=n2[a+l]$S_n = \frac{n}{2}[a + l]$

Where; a = first term of the given A.P.

d = common difference of the given A.P.

l = last term

n = number of terms

(i) 2 + 4 + 6 + … + 200

Common difference of the A.P. (d) = $a_2 - a_1$

= 6 – 4

= 2

So here,

First term (a) = 2

Last term (l) = 200

Common difference (d) = 2

So, here the first step is to find the total number of terms.

Let us take the number of terms as n.

Now, as we know,
$a_n = a + (n - 1)d$

So, for the last term,

200 = 2 + (n – 1)2

200= 2 + 2n – 2

200 = 2n

Further simplifying,

n = 100

Now using the formula for sum of n terms,

S_{100}=1002[a+l]$S_{100} = \frac{100}{2}[a + l]$

= 50 [2 + 200]

= 50 X 202

= 10100

Therefore, the sum of the A.P is 10100

(ii) 3 + 11 + 19 + . . . + 803

Common difference of the A.P. (d) = $a_2 - a_1$

=19 – 11

= 8

So here,

First term (a) = 3

Last term (l) = 803

Common difference (d) = 8

So, here the first step is to find the total number of terms.

Let us take the number of terms as n.

Now, as we know,

$a_n = a + (n - 1)d$

So, for the last term, Further simplifying,

803=3 + (n – 1)8

=> 803= 3 + 8n – 8

=> 803 + 5 = 8n

=> 808 = 8n

=> n =101

Now, using the formula for the sum of n terms, we get

S_{101}=1012[a+l]$S_{101} = \frac{101}{2}[a + l]$

= 1012[3+803]$\frac{101}{2}[3 + 803]$

= 1012[806]$\frac{101}{2}[806]$

= 101 (403)

= 40703

Therefore, the sum of the A.P. is 40703

(iii) (-5) + (-8) + (-11) + . . . + (-230)

Common difference of the A.P. (d) = $a_2 - a_2$

= -8 – (-5)

= -8 + 5

= -3

So here,

First term (a) = -5

Last term (l) = -230

Common difference (d) = -3

So, here the first step is to find the total number of terms.

Let us take the number of terms as n.

Now, as we know,

$a_n = a + (n – 1)d$

So, for the last term,

-230 = -5 + (n -1)(-3)

=> -230 = -5 -3n +3

=> -230 + 2 = -3n

=> -228 = -3n

=> n = 76

Now, using the formula for the sum of n terms, we get

S_{76}=762[a+l]$S_{76} = \frac{76}{2}[a+l]$

= 38[(- 5) + (-230)]

= 38(-235)

= -8930

Therefore, the sum of the A.P. is -8930

(iv) 1 + 3 + 5 + 7 + . . . + 199

Common difference of the A.P. (d)= $a_2 - a_1$

= 3 – 1

=2

So here,

First term (a) = 1

Last term (l) = 199

Common difference (d) = 2

So, here the first step is to find the total number of terms.

Let us take the number of terms as n.

Now, as we know,

a_n = a + (n – 1)d

So, for the last term,

199 = 1 + (n – 1)2

=> 199= 1 + 2n – 2

=> 199 + 1 = 2n

=> n = 100

Now, using the formula for the sum of n terms, we get

S_{100}=1002[a+l]$S_{100} = \frac{100}{2}[a+l]$

= 50 [1 + 199]

= 50 (200)

= 10000

Therefore, the sum of the A.P. is 10000

(v) 7+1012+14+...+84$7 + 10\frac{1}{2} + 14 + \ldots + 84$

Common difference of the A.P. (d)= $a_2 - a_1$

= 1012−7$10\frac{1}{2} - 7$

= 212−7$\frac{21}{2} - 7$

= 21−142 $\frac{21-14}{2}$

= 72 $\frac{7}{2}$

So here,

First term (a) = 7

Last term (l) = 184

Common difference (d) = 72 $\frac{7}{2}$

So, here the first step is to find the total number of terms.

Let us take the number of terms as n.

Now, as we know,

$a_n = a + (n - 1)d$

So, for the last term,

84=7+(n–1)72 $84 = 7 + (n-1)\frac{7}{2}$ 84=7+7n2–72 $84 = 7 + \frac{7n}{2} - \frac{7}{2}$ 84=14–72+7n2

$84 = \frac{14-7}{2} + \frac{7n}{2}$

84 (2) = 7 + 7n

7n = 161

n = 23

Now, using the formula for sum of n terms, we get

S_n=232[2(7)+(23–1)72] $S_n = \frac{23}{2}\left[2(7) + (23-1)\frac{7}{2}\right]$

= 232[14+(22)72] $\frac{23}{2}\left[14 + (22)\frac{7}{2}\right]$

= 232[14+77] $\frac{23}{2}[14 + 77]$

= 232[91] $\frac{23}{2}[91]$

= 20932 $\frac{2093}{2}$

Therefore, the sum of the A.P. is 20932 $\frac{2093}{2}$

(vi) 34 + 32 + 30 + . . . + 10

Common difference of the A.P. (d) = $a_2 - a_1$

= 32 -34

= -2

So here,

First term (a) = 34

Last term (l) = 10

Common difference (d) = -2

So, here the first step is to find the total number of terms.

Let us take the number of terms as n.

Now, as we know,

$a_n = a + (n – 1)d$

So, for the last term,

=> 10 = 34 + (n – 1)(-2)

=> 10 = 34 – 2n + 2

=> 10 = 36 – 2n

=> 10 – 36 = -2n

Further solving for n,

=> -2n = -26

=> n =13

Now, using the formula for the sum of n terms, we get

S_n=132[a+l]$S_n = \frac{13}{2}[a+l]$

= 132[34+10]$\frac{13}{2}[34+10]$

= 132[44]$\frac{13}{2}[44]$

= 12 (22)

= 286

Therefore, the sum of the A.P. is 286

(vii) 25 + 28 + 31 + . . . + 100

Common difference of the A.P. (d)= $a_2 – a_1$

= 28 – 25

=3

So here,

First term (a) = 25

Last term (l) = 100

Common difference (d) = 3

So, here the first step is to find the total number of terms.

Let us take the number of terms as n.

Now, as we know,

$a_n = a + (n - 1)d$

So, for the last term,

100 = 25 + (n – 1)(3)

100 = 25 + 3n – 3

100 = 22 + 3n

100 – 22 = 3n

Further solving for n,

78 = 3n

n = 26

Now, using the formula for the sum of n terms, we get

S_n=262[a+l]$S_n = \frac{26}{2}[a + l]$

= 13 [25 + 100]

= 13 (125)

= 1625

Therefore, the sum of the given A.P. is 1625

Question 14. The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution :- In the given problem, we have the first and the last term of an A.P.

along with the common difference of the AP Here,

we need to find the number of terms of the AP and the sum of all the terms.

Here,

The first term of the A.P (a) = 17

The last term of the A.P (l) = 350

The common difference of the A.P. = 9

Let the number of terms be n.

So, as we know that,

l = a + (n – 1)d

we get,

350 = 17 + (n- 1) 9

=> 350 =17 + 9n – 9

=> 350 = 8 + 9n

=> 350 – 8 = 9n

Further solving this,

n=38

Using the above values in the formula,

S_n=n2[a+l]$S_n = \frac{n}{2}[a+l]$

=> 382(17+350)$\frac{38}{2}(17+350)$

=> 19 X 367

=> 6973

Therefore, the number of terms is (n) 38 and the sum (S_n) is 6973

Question 15. The third term of an A.P. is 7 and the seventh term exceeds three times the third term by 2. Find the first term, the common difference and the sum of first 20 terms.

Solution: In the given problem, let us take the first term as a

and the common difference as d.

Here, we are given that,

$a_3 = 7$ . . . .(1)

$a_7 = 3a_3 + 2$ . . . .(2)

So, using (1) in (2), we get,

$a_7 = 3(7) + 2$

$=21+2$

$= 23$ . . . .(3)

Also, we know,

$a_n = a + (n - 1)d$

For the 3th term (n = 3),

$a_3 = a + (3 - 1)d$

$\Rightarrow 7 = a + 2d$ (Using 1)

$\Rightarrow a = 7 - 2d$ . . . . (4)

Similarly, for the 7th term (n = 7),

$a_7 = a + (7 - l)\ d$

$24 = a + 6d$ (Using 3)

$a = 24 - 6d$ . . . . (5)

Subtracting (4) from (5), we get,

$a - a = (23 - 6d) - (7 - 2d)$

$\Rightarrow 0 = 23 - 6d - 7 + 2d$

$\Rightarrow 0 = 16 - 4d$

$\Rightarrow 4d = 16$

$\Rightarrow d = 4$

Now, to find a, we substitute the value of d in (4),

$a = 7 - 2(4)$

$\Rightarrow a = 7 - 8$

a = -1

So, for the given A.P, we have d = 4 and a = -1

So, to find the sum of first 20 terms of this A.P.,

we use the following formula for the sum of n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where; a = first term for the given A.P.

d= common difference of the given A.P.

n= number of terms

So, using the formula for n= 20, we get,

$$S_{20} = \frac{20}{2}[2(-1) + (20-1)(4)]$$

=(10)[-2 + (19) (4)]

= (10) [-2 + 76]

= (10) [74]

= 740

Therefore, the sum of first 20 terms for the given A.P. is S_{20} = 740

Question 16. The first term of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. Find the common difference.

Solution: In the given problem, we have the first and the last term of an A.P. along with the sum of all the terms of A.P.

Here, we need to find the common difference of the A.P.

Here,

The first term of the A.P (a) = 2

The last term of the A.P (l) = 50

Sum of all the terms S_n = 442

Let the common difference of the A.P. be d.

So, let us first find the number of the terms (n) using the formula,

$442 = \left(\frac{n}{2}\right)(2+50)$

=> $442 = \left(\frac{n}{2}\right)(52)$

=> 26 n = 442

=> n = 17

Now, to find the common difference of the A.P. we use the following formula,

l = a + (n – 1)d

We get,

50 = 2 + (17 – 1)d

=> 50 = 2 + 16d

=> 16d = 48

=> d = 3

Therefore, the common difference of the A.P. is d = 3

Question 17. If 12^{th} term of an A.P. is -13 and the sum of the first four terms is 24, what is the sum of first 10 terms?

Solution: In the given problem, we need to find the sum of first 10 terms of an A.P.

Let us take the first term a

and the common difference as d

Here, we are given that,

$a_{12} = -13$

$S_4 = 24$

Also, we know,

$a_n = a + (n - 1)d$

For the 12th term (n = 12)

$a_{12} = a + (12 – 1)d$

$-13 = a + 11d$

$a = -13 – 11d$ . . . .(1)

So, as we know the formula for the sum of n terms of an A.P. is given by,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where; a = first term for the given A.P.
d = common difference of the given A.P.

n = number of terms

So, using the formula for n = 4, we get,

$$S_4 = \frac{4}{2}[2(a) + (4-1)d]$$

=> 24 = (2) [2a + (3)(d)]

=> 24 = 4a + 6d

=> 4a = 24 – 6d

=> $a = 6 - \frac{6}{4}d$ . . . .(2)

Subtracting (1) from (2), we get.

=> $a - a = \left(6 - \frac{6}{4}d\right) - (-13 - 11d)$

=> $0 = 6 - \frac{6}{4}d + 13d + 11d$

=> $0 = 19 + \frac{44d - 6d}{4}$ s

On further simplifying for d, we get,

=> $0 = 19 + \frac{38}{4}d$

=> $-19 = \frac{19}{2}d$

=> – 19 X 2 = 19 d

=> d = -2

Now, we have to substitute the value of d in (1),

a = -13 – 11 (-2)

a = -13 + 22

a = 9

Now, using the formula for the sum of n terms of an A.P., for n = 10

we have,

S_{10}= 102 [2(9)+(10–1)(−2)]$S_{10} = \frac{10}{2}[2(9) + (10-1)(-2)]$

= (5)[19 + (9)(-2)]

= (5)(18 – 18)

= 0

Therefore, the sum of first 10 terms for the given A.P. is S_{10} = 0.

Question 18. Find the sum of first 22 terms of an A.P. in which d = 22 and a_{22} = 149.

Solution: In the given problem, we need to find the sum of first 22 terms of an A.P.

Let us take the first term as a.

Here, we are given that,

a_{22} = 149 . . . .(1)

d = 22 . . . .(2)

Also, we know,

a_n = a + (n – 1) d

For the 22nd term (n = 22),

a_{22} = a + (22 – 1) d

149 = a + (21) (22) (Using 1 and 2)

a = 149 – 462

a = – 313 . . . .(3)

So, as we know the formula for the sum of n terms of an A.P. is given by,

S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

So, using the formula for n = 22, we get,

S_{22}=222[2(−313)+(22–1)(22)]$S_{22} = \frac{22}{2}[2\,(-313) + (22-1)\,(22)]$

= (11) [-626 + 462]

= (11) [-164]

= -1804

Therefore, The sum of first 22 terms for the given A.P. is S_{22} = -1804

Question 19. In an A.P., if the first tern is 22, the common difference is -4 and the sum to n terms is 64, find n.

Solution: In the given problem,

we need to find the number of terms of an A.P.

Let us take the number of terms as n.

Here, we are given that,

a = 22

d = -4

S„ = 64

So, as we know the formula for the sum of n terms of an A.P. is given by,

S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

So, using the formula we get,

=> S_n=n2[2(22)+(n–1)(−4)]$S_n = \frac{n}{2}[2(22) + (n-1)(-4)]$

=> 64=n2[2(22)+(n–1)(−4)]$64 = \frac{n}{2}[2(22) + (n-1)(-4)]$

=> 64(2) = n(48 – 4n)

=> 128 = 48n – $4n^2$

Further rearranging the terms, we get a quadratic equation,

$4n^2 - 48n + 128 = 0$

On taking 4 common, we get,

$n^2 - 12n + 32 = 0$

Further, on solving the equation for n by splitting the middle term, we get,

$n^2 - 12n + 32 = 0$

$n^2 - 8n - 4n + 32 = 0$

n (n – 8) – 4 (n – 8) = 0

(n – 8) (n – 4) = 0

So, we get

n – 8 = 0

=> n = 8

Also,

n – 4 = 0

=> n = 4

Therefore, n = 4 or 8.

Question 20. In an A.P., if the 5th and 12th terms are 30 and 65 respectively, what is the sum of first 20 terms ?

Solution: In the given problem, let us take the first term as a

and the common difference d

Here, we are given that,

$a_5 = 30$ . . . .(1)

$a_{12} = 65$ . . . .(2)

Also, we know,

$a_n = a + (n - 1)d$

For the 5th term (n = 5),

$a_5 = a + (5 - 1)d$

$30 = a + 4d$ (Using 1)

$a = 30 - 4d$ . . . .(3)

Similarly, for the 12th term (n = 12),

$a_{12} = a + (12 - 1) d$

$65 = a + 11d$ (Using 2)

$a = 65 - 11d$ . . . .(4)

Subtracting (3) from (4), we get,

$a - a = (65 - 11d) - (30 - 4d)$

$0 = 65 - 11d - 30 + 4d$

$0 = 35 - 7d$

$7d = 35$

$d = 5$

Now, to find a, we substitute the value of d in (4).

$a = 30 - 4(5)$

$a = 30 - 20$

$a = 10$

So, for the given A.P. d = 5 and a = 10

So, to find the sum of first 20 terms of this A.P.,

we use the following formula for the sum of n terms of an A.P.,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where; a = first term of the given A.P.

d = common difference of the given A.P.

n = number of terms

So, using the formula for n = 20, we get

$$S_{20} = \frac{20}{2}[2(10) + (20-1)(5)]$$

= (10)[20 + (19)(5)]

= (10)[20 + 95]

= (10)[115]

= 1150

Therefore, the sum of first 20 terms for the given A.P. is 1150

Question 21. Find the sum of first 51 terms of an A.P. whose second and third terms are 14 and 18 respectively.

Solution: In the given problem,

let us take the first term as a

and the common difference as d.

Here, we are given that,

$a_2 = 14$ .... (1)

$a_3 = 18$ .... (2)

Also, we know,

$a_n = a + (n - 1)d$

For the 2nd term (n = 2),

=> $a_2 = a + (2 – 1)d$

=> 14 = a + d (Using 1)

=> a = 14 – d . . . . (3)

Similarly, for the 3rd term (n = 3),

=> $a_3 = a + (3 – 1)d$

=> 18 = a + 2d (Using 2)

=> a = 18 – 2d . . . . (4)

Subtracting (3) from (4),

we get, a – a = (18 – 2d) – (14 – d)

0 = 18 – 2d – 14 + d

0 = 4 – d

d = 4

Now, to find a, we substitute the value of d in (4),

a = 14 – 4

a = 10

So, for the given A.P. d = 4 and a = 10

So, to find the sum of first 51 terms of this A.P.,

we use the following formula for the sum of n terms of an A.P.,

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Where, a = the first term of the A.P.

d = common difference of the A.P.

n = number of terms

So, using the formula for n = 51, we get

$$S_{51} = \frac{51}{2}[2(10) + (51-1)(4)]$$

= 512[20+(40)4]$\frac{51}{2}[20 + (40)\,4]$

= 512[220]$\frac{51}{2}[220]$

= 51 (110)

= 5610

Therefore, the sum of the first 51 terms of the given A.P. is 5610

Question 23. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Solution: Let a be the first term and d be the common difference.

We know that, sum of first n terms is

S_n=n2(2a+(n–1)d)$S_n = \frac{n}{2}(2a + (n{-}1)d)$

Where, a = the first term of the A.P.

d = common difference of the A.P.

n = number of terms

Also, nth term = a_n = a + (n – 1)

According to the question,

First term (a) = 5,

last term (a_n)= 45

and sum of n terms (S_n) = 400

Now,

a_n = a + (n – 1)d

=> 45 = 5 + (n – 1)d

=> 40 = nd – d

=> nd – d = 40 . . . . (1)

Also,

S_n=n2(2(a)+(n–1)d)$S_n = \frac{n}{2}(2(a) + (n–1)d)$ 400=n2(2(5)+(n–1)d)
$400 = \frac{n}{2}(2(5) + (n–1)d)$

800 = n (10 + nd – d)

800 = n (10 + 40) from (1)

n = 16 . . . . (2)

On substituting (2) in (1), we get

nd – d = 40

16d – d = 40

15d = 40

d = 83 $\frac{8}{3}$

Thus, common difference of the given A.P. is 83.

Question 24. In an A.P. the first term is 8, n^th term is 33 and the sum of first n term is 123. Find n and the d, the common difference.

Solution: In the given problem,

we have the first and the nth term of an A.P. along with the sum of the n terms of A.P.

Here, we need to find the number of terms

and the common difference of the A.P

Here,

The first term of the A.P (a) = 8 The

nth term of the A.P (l)= 33

Sum of all the terms S_n = 123

Let the common difference of the A.P. be d.

So, let us first find the number of the terms (n) using the formula, 123=(n2)(8+33)$123 = \left(\frac{n}{2}\right)(8+33)$ 123=(n2)(41)$123 = \left(\frac{n}{2}\right)(41)$ n=(123)(2)41

$n = \frac{(123)(2)}{41}$ n=24641$n = \frac{246}{41}$

n = 6

Now, to find the common difference of the A.P. we use the following formula,

l = a + (n – 1)d

we get

33 = 8 + (6 – 1)d

33 = 8 + 5d

5d = 25

d = 5

Therefore, the number of terms is n = 6 and the common difference of the A.P. is d = 5.

Question 25. In an A.P. the first term is 22, n^{th} term is -11 and the sum of first n term is 66. Find n and the d, the common difference.

Solution: In the given problem,

we have the first and the nth term of an A.P. along with the sum of the n terms of A.P.

Here, we need to find the number of terms and the common difference of the A.P.

Here,

The first term of the A.P (a) = 22

The nth term of the A.P (l) = -11

Sum of all the terms S„ = 66

Let the common difference of the A.P. be d.

So, let us first find the number of the terms (n) using the formula,

66=(n2)[22+(−11)]$66 = \left(\frac{n}{2}\right)[22 + (-11)]$ 66=(n2)[22–11]$66 = \left(\frac{n}{2}\right)[22 – 11]$

(66)(2) = n(11)

6 X 2 = n

n = 12

Now, to find the common difference of the A.P. we use the following formula,

l = a + (n – 1)d

we get,

-11 = 22 + (12 – 1)d

-11 = 22 + 11d

11d = -33

d = -3

Therefore, the number of terms is n = 12 and the common difference d = -3

Question 26. The first and the last terms of an A.P. are 7 and 49 respectively. If sum of all its terms is 420, find the common difference.

Solution: Let a be the first term and d be the common difference.

We know that, sum of first n terms is:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Also, n^{th} term $(a_n) = a + (n – 1)d$

According to question,

first term (a) = 7

last term $(a_n) = 49$

and sum of n terms $(S_n) = 420$

Now,

$a_n = a + (n – 1)d$

=> 49 = 7 + (n – 1)d

=> 43 = nd – d

=> nd – d = 42 . . . . .(1)

Also,

$$S_n = \frac{n}{2}(2(7) + (n-1)d)$$

=> 840 = n [14 + nd – d]

=> 840 = n [14 + 42] [from (1)]

=> 840 = 54 n

=> n = 15 . . . . (2)

on substituting (2) in (1), we get

nd – d = 42

=> 15d – d = 42

=> 14d = 42

=> d = 3

Thus, the common difference of the given A.P. is 3.

Question 28. The sum of first q terms of an A.P. is 162. The ratio of its 6^{th} term to its 13^{th} term is 1 : 2. Find the first and 15^{th} term of the A.P.

Solution: Let a be the first term and d be the common difference.

We know that, sum of first n terms is:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Also, nth term = $a_n = a + (n – 1) d$

According to the question,

$S_q = 162$

and $a_6 : a_{13} = 1 : 2$

Now, $2a_6 = a_{13}$

=> 2 [a + (6 – 1d)] = a + (13 – 1)d

=> 2a + 10d = a + 12d

=> a = 2d . . . . (1)

Also, S_9 = 162

=> S_9=92(2a+(9–1)d)$S_9 = \frac{9}{2}(2a + (9 – 1)d)$

=> 162 = 92(2a+8d)$\frac{9}{2}(2a + 8d)$

=> 162 X 2 = 9 [4d + 8d] [from (1)]

=> 324 = 9 X 12d

=> d = 3

=> a =2d [from (1)]

=> a = 6

Thus, the first term of the A.P. is 6

Now, a_{15} = a + 14d = 6 + 14 X 3 = 6 + 42

a_{15} = 48

Therefore, 15^{th} term of the A.P. is 48

Question 29. If the 10^{th} term of an A.P. is 21 and the sum of its first 10 terms is 120, find its n^{th} term.

Solution: Let a be the first term and d be the common difference.

We know that, sum of first n terms is :

S_n=n2(2a+(n–1)d)$S_n = \frac{n}{2}(2a + (n – 1)d)$

and n^{th} term is given by:

$a_n = a + (n – 1)d$

Now,

given in question,

S_{10} = 120

=> 120=102(2a+(10–1)d)$120 = \frac{10}{2}(2a + (10-1)d)$

=> 120 = 5 (2a + 9d)

=> 24 = 2a + 9d . . . .(1)

Also,

a_{10} = 21

=> 21 = a + (10 – 1)d

=> 21 = a + 9d . . . . (2)

Subtracting (2) from (1), we get

24 – 21 = 2a + 9d – a – 9d

a = 3

Putting a = 3 in equation (2), we have

3 + 9d = 21

9d = 18

d = 2

So, we have now

first term = 3

common difference = 2

Therefore, the n^{th} term can be calculated by:

a_n = a + (n – 1)d

= 3 + (n – 1) 2

= 3 + 2n -2

= 2n + 1

Therefore, the n^{th} term of the A.P is (a_n) = 2n + 1

Question 30. The sum of first 7 terms of an A.P. is 63 and the sum of its next 7 terms is 161. Find the 28^{th} term of this A.P.

Solution: Let a be the first term and d be the common difference.

We know that, sum of first n terms

S_n=n2(2a+(n–1)d)$S_n = \frac{n}{2}(2a + (n-1)d)$

It is given that sum of the first 7 terms of an A.P. is 63.

And sum of next 7 terms is 161.

Sum of first 14 terms = Sum of first 7 terms + sum of next 7 terms

= 63 + 161 = 224

Now, S_7=72(2a+(7–1)d)$S_7 = \frac{7}{2}(2a + (7-1)d)$

=> 63 (2) = 7 (2a + 6d)

=> 9 X 2 = 2a + 6d

=> 2a + 6d = 18 . . . . (1)

Also, S_{14}=142(2a+(14–1)d)$S_{14} = \frac{14}{2}(2a + (14-1)d)$

=> 224 = 7 (2a + 13d)

=> 32 = 2a + 13d . . . . (2)

On subtracting (1) from (2), we get

=> 13d – 6d = 32 – 18

=> 7d = 14

=> d = 2

From (1)

$2a + 6(2) = 18$

$2a = 18 – 12$

$a = 3$

Also, n^{th} term = $a_n = a + (n – 1)d$

=> $a_{28} = a + (28 – 1)d$

$= 3 + 27 (2)$

$= 3 + 54$

$= 57$

Thus, the 28^{th} term is 57.

Question 31. The sum of first seven terms of an A.P. is 182. If its 4^{th} and 17^{th} terms are in ratio 1 : 5, find the A.P.

Solution: In the given problem,

let us take the first term as a

and the common difference as d.

Here, we are given that,

$S_{17} = 182$

We know that, sum of first term is:

S_n=n2(2a+(n–1)d)$S_n = \frac{n}{2}(2a + (n-1)d)$

So, from question

S_7=72(2a+(7–1)d)$S_7 = \frac{7}{2}(2a + (7-1)d)$

182 X 2 = 7 (2a + 6d)

$364 = 14a + 42d$

$26 = a + 3d$

$a = 26 - 3d$ . . . (1)

Also,

we are given that 4^{th} term and 17^{th} term are in a ratio 0f 1 : 5

Therefore,

=> $5\ (a_4) = 1\ (a_{17})$

=> $5\ (a + 3d) = 1\ (a + 16d)$

=> $5a + 15d = a + 16d$

=> $4a = d$ . . . . (2)

On substituting (2) in (1), we get

=> $4\ (26 - 3d) = d$

=> $104 - 12d = d$

=> $104 = 13d$

=> $d = 8$

from (2), we get

=> $4a = d$

=> $4a = 8$

=> $a = 2$

Thus we get, first term $a = 2$ and the common difference $d = 8$.

The required A.P. is 2, 10, 18, 26, . . .

Question 33. In an A.P. the sum of first ten terms is -150 and the sum of its next 10 term is -550. Find the A.P.

Solution: Here, we are given Sn, = -150 and sum of the next ten terms is -550.

Let us take the first term of the A.P. as a

and the common difference as d.

So, let us first find S10.

For the sum of first 10 terms of this A.P,

First term = a

Last term = a_{10}

So, we know,

$a_n = a + (n - 1)d$

For the 10th term (n = 10),

$a_n = a + (10 - 1)d$

= a + 9d

So, here we can find the sum of the n terms of the given A.P., using the formula,

S_n=(n2)(a+l)$S_n = \left(\frac{n}{2}\right)(a + l)$

Where, a = the first term

l = the last term

So, for the given A.P,

S_{10}=(102)(a+a+9d)$S_{10} = \left(\frac{10}{2}\right)(a + a + 9d)$

-150 = 5 (2a + 9d)

-150 = 10a + 45 d

a=150–45d10$a = \frac{150 - 45d}{10}$ . . . . (1)

Similarly, for the sum of next 10 terms (S_{10}),

First term = a_{11}

Last term = a_{20}

For the 11th term (n = 11),

$a_{11} = a + (11 - 1)\, d$

= a + 10d

For the 20th term (n = 20),

$a_{20} = a + (20 - 1)\, d$

$= a + 19d$

So, for the given AP,

S10=(102)(a+10d+a+19d)$S_{10} = \left(\frac{10}{2}\right)(a + 10d + a + 19d)$

$-550 = 5\,(2a + 29d)$

$-550 = 10a + 145d$

a=−550–145d10 $a = \frac{-550 – 145d}{10}$ . . . . (2)

Now subtracting (1) from (2),

a – a = (−550–145d10)–(−150–45d10)$\left(\frac{-550–145d}{10}\right) - \left(\frac{-150–45d}{10}\right)$

$0 = -550 - 145d + 150 + 45d$

$0 = -400 - 100d$

$100d = -400$

$d = -4$

Substituting the value of d in (1)

a=(−150–45(−4)10)$a = \left(\frac{-150–45(-4)}{10}\right)$ a=(−150+18010)$a = \left(\frac{-150+180}{10}\right)$

$= 3$

So, the A.P. is 3, -1, -5, -9,. . . with a = 3, d= -4

Question 35. In an A.P. , the first term is 2, the last term is 29 and the sum of the terms is 155, find the common difference of the A.P.

Solution: In the given problem,

we have the first and the last term of an A.P. along with the sum of all the terms of A.P.

Here, we need to find the common difference of the A.P.

Here,

The first term of the A.P (a) = 2

The last term of the AP (l) = 29

Sum of all the terms (S_n) = 155

Let the common difference of the A.P. be d.

So, let us first find the number of the terms (n) using the formula,

155=n2(2+29)$155 = \frac{n}{2}(2 + 29)$

155 (2) = n (31)

31n =310

n = 10

Now, to find the common difference of the A.P. we use the following formula,

l = a + (n – 1) d

We get,

29 = 2 + (10 – 1)d

29 = 2 + (9)d

29-2 = 9d

9d = 27

d = 3

Therefore, the common difference of the A.P. is d = 3

Question 37. Find the number of terms of the A.P. -12, -9, -6, . . . , 21. If 1 is added to each term of this A.P., then find the sum of all terms of the A.P. thus obtained.

Solution: First term, a1 = -12

Common difference, d = a2 – a1 = -9 – (– 12)

= – 9 + 12 = 3

n^{th} term = $a_n = a + (n – 1)d$

=> 21 = -12 + (n – 1)3

=> 21 = -12 + 3n – 3

=> 21 = 3n – 15

=> 36 = 3n

=> n = 12

Therefore, the number of terms is 12

Now, when 1 is added top each of the 12 terms, the sum will increase by 12.

So, the sum of all the terms of the A.P. thus obtained

=> $S_{12} + 12 = 122[a+l]+12 = \frac{12}{2}[a+l]+12$

= 6 [-12 + 21] + 12

= 6 X 9 + 12

= 66

Therefore, the sum after adding 1 to each of the term we get 66

Question 38. The sum of first n terms of an A.P. is $3n^2 + 6n$. Find the n^{th} term of this A.P.

Solution: In the given problem,

let us take the first term as a

and the common difference as d.

we know that n^{th} term is given by:

$a_n = S_n - S_{n-1}$

we have given here

$S_n = 3n^2 + 6n$

So, using this to find the n^{th} term,

=> $a_n = [3n^2 + 6n] - [3(n-1)^2 + 6(n-1)]$

$= [3n^2 + 6n] – [3 (n^2 + 1^2 – 6n) + 6n – 6]$

$= 3n^2 + 6n – 3n^2 – 3 + 6n – 6n + 6$

$= 6n + 3$

Therefore, the n^{th} term of this A.P. is 6n + 3

Question 39. The sum of n terms of an A.P. is 5n – n^2. Find the n^{th} term of this A.P.

Solution: Let a be the first term and d be the common difference.

We know that, sum of first n terms is :

S_n=n2(2a+(n–1)d)$S_n = \frac{n}{2}(2a + (n-1)d)$

It is given that sum of the first n terms of an A.P. is 5n – n^2.

First term = a = S_1 = 5 (1) – $(1)^2$ = 4.

Sum of first two terms = S_2= 5 (2) – $(2)^2$ = 6.

Second term = $S_2 – S_1$ = 6 – 4 = 2.

Common difference = d = Second term – First term

= 2 – 4 = -2

Also, nth term = a_n = a + (n – 1) d

=> a_n = 4 + (n – 1)(-2)

=> a_n = 4 – 2n + 2

=> a_n = 6 – 2n

Thus, nth term of this A.P. is 6 – 2n.

Question 41. The sum of first n terms of an A.P. is $3n^2$+ 4n. Find the 25^{th} term of this A.P.

Solution: In the given problem,

we have sum of n terms as

$S_n = 5n^2 + 3n$

we know,

$a_n = S_n - S_{n-1}$

We have to find out 25th term, so n = 25

=> $a_{25} = S_{25} - S_{24}$

$= [3 (25)^2 + 4 (25)] - [3 (24)^2 + 4 (24)]$

$= (3 \text{ X } 625 + 100) - (3 \text{ X } 576 + 96)$

$= 1975 - 1824$

$= 151$

Therefore, its 25th term is 151

Question 42. The sum of first n terms of an A.P. is $5n^2 + 3n$. If its mth term is 168, find the value of m. Also find the 20th term of this A.P.

Solution: Here, we are given the Sum of the A.P. as $S_n = 5n^2 + 3n$.

and its mth term is $a_m = 168$

Let us assume its first term as a,

and the common difference as d

We know,

$a_n = S_n - S_{n-1}$

So, here

=> $a_n = (5n^2 + 3n) - [5(n-1)^2 + 3 (n-1)]$

$= 5n^2 + 3n - [5(n^2 + 1 - 2n) + 3n - 3]$

$= 5n^2 + 3n – 5n^2 -5 + 10n – 3n + 3$

$= 10n – 2$

We are given,

$a_m = 168$

Putting m in place of n , we get

=> $a_m = 10m – 2$

= > $168 = 10m – 2$

=> $10m = 170$

=> $m = 17$

and

$a_{20} = S_{20} – S_{19}$

$= [5 (20)^2 + 3 (20)] – [5 (19)^2 + 3 (19)]$

$= [2000 + 60] – [1805 + 57]$

$= 2060 – 1862$

$= 198$

Therefore, in the given A.P. m = 17 and the 20^{th} term is a_{20} = 198

Question 45. If the sum of first n terms of an A.P. is $4n – n^2$, what is the first term? What is the sum of first two terms? What is the second term? Similarly find the third, the tenth and the n^{th} term.

Solution: In the given problem,

the sum of n terms of an A.P. is given by the expression, $S_n = 4n – n^2$

So here, we can find the first term by substituting n = 1,

$S_n = 4n – n^2$

$= 4(1)—1^2$

$= 4 – 1$

$= 3$

Similarly, the sum of first two terms can be given by,

$S_2 = 4(2) – (2)2$

$= 8 – 4$

$= 4$

Now, as we know,

$a_n = S_n – S_{n-1}$

So,

$a_2 = S_2 – S_1$

$= 4 – 3$

$= 1$

Now, using the same method we have to find the third, tenth and nth term of the A.P.

So, for the third term,

$a_3 = S_3 – S_2$

$=[4 (3) – (3)^2] – [4(2) – (2)^2]$

$= (12 – 9) – (8 – 4)$

$= 3 – 4$

$= -1$

Also, for the tenth term.

$a_{10} = S_{10} – S_9$

$= [44(10) – (10)^2] – [4(9) – (9)^2]$

$=(40 – 100) – (36 – 81)$

$= – 60 + 45$

$= -15$

So, for the nth term,

$a_n = S_n – S_{n-1}$

=[4(n) – (12)2] – [4(n – 1) – (n – 1)2]

= (4n – n^2) – (4n – 4 – n^2 – 1 + 2n)

= 4n – n2 – 4n + 4 + n^2 + 1 – 2n

= 5 – 2n

Therefore, a = 3, S_2 = 4, a_2 = 1, a_3 = -1, a_{10} = -15

Question 46. If the sum of first n terms of an A.P. is 12(3n^2+7n)$\frac{1}{2}(3n^2 + 7n)$, then find its nth term. Hence write the 20th term.

Solution: Let a be the first term and d be the common difference.

We know that, sum of first n terms is:

S_n=n2(2a+(n–1)d)$S_n = \frac{n}{2}(2a + (n–1)d)$

It is given that the sum of the first n terms of an A.P. is:

12(3n^2+7n)$\frac{1}{2}(3n^2 + 7n)$

Therefore, first term (a) = S_1=12(3(1)^2+7(1))$S_1 = \frac{1}{2}(3(1)^2 + 7(1))$

= 12(3X1+7)$\frac{1}{2}(3X1 + 7)$

= 12(10)$\frac{1}{2}(10)$

= 5

Sum of first two terms = S_2=12(3(2)^2+7(2))$S_2 = \frac{1}{2}(3(2)^2 + 7(2))$

= 12(3X4+14)$\frac{1}{2}(3X4 + 14)$

= 12(26)$\frac{1}{2}(26)$

= 13

Therefore, second term = $S_2 - S_1$

= 13 – 5 = 8

Common difference = d = second term – first term

= 8 – 5 = 3

Also, n^{th} term of the A.P. is : a + (n – 1)d

= 5 + (n – 1)3

= 5 + 3n – 3

= 3n + 2

Thus, n^{th} term of this A.P. is 3n + 2.

Now, we have to find the 20^{th} term, so

Putting n = 2o in the above equation, we get

a_{20} = 3 (20) + 2

= 60 + 2

= 62

Thus, 20^{th} term of this A.P. is 62.

Question 47. In an A.P. the sum of first n terms is $3n^22+132n\frac{3n^2}{2}+\frac{13}{2}n$. Find its 25^{th} term.

Solution: Here the sum of first n terms is given by the expression,

$S_n = 3n^22+132n S_n = \frac{3n^2}{2}+\frac{13}{2}n$

We need to find the 25^{th} term of the A.P.

So, we know that the n^{th} term of an A.P. is given by,

$a_n = S_n - S_{n-1}$

So, $a_{25} = S_{25} - S_{24}$ . . . . (1)

So, using the expression for the sum of n terms,

we find the sum of 25 terms (S_{25}) and the sum of 24 terms (S_{25}), we get,

S_n=3(25)²2+132(25)$S_n = \frac{3(25)^2}{2} + \frac{13}{2}(25)$

= 3(625)2+3252 $\frac{3(625)}{2} + \frac{325}{2}$

= 22002 $\frac{2200}{2}$

= 1100

Similarly,

S_n=3(24)²2+132(24)$S_n = \frac{3(24)^2}{2} + \frac{13}{2}(24)$

= 3(576)2+3122 $\frac{3(576)}{2} + \frac{312}{2}$

= 20402 $\frac{2040}{2}$

= 1020

Now, using the above values in (1),

$a_{21} = S_{25} - S_{24}$

= 1100 – 1020

= 80

Therefore, a_{25} = 80

Question 48. Find the sum of all natural numbers between 1 and 100, which are divisible by 3.

Solution: In this problem,

we need to find the sum of all the multiples of 3 lying between 1 and 100.

So, we know that the first multiple of 3 after 1 is 3 and the last multiple of 3 before 100 is 99.

Also, all these terms will form an A.P. with the common difference of 3.

So here,

First term (a) = 3

Last term (l) = 99

Common difference (d) = 3

So, here the first step is to find the total number of terms.

Let us take the number of terms as n.

Now, as we know,

$a_n = a + (n - 1)d$

So, for the last term,

99 = 3 + (n – 1)3

=> 99 = 3 + 3n – 3

=> 99 = 3n

Further simplifying,

=> n = 33

Now, using the formula for the sum of n terms,

i.e. S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

we get,

=> S_{33}=332[2(3)+(33–1)3]$S_{33} = \frac{33}{2}[2\,(3) + (33-1)\,3]$

= S_{33}=332[6+(32)3]$S_{33} = \frac{33}{2}[6 + (32)\,3]$

= S_{33}=332[6+96]$S_{33} = \frac{33}{2}[6 + 96]$

= S_{33}=33(102)2$S_{33} = \frac{33(102)}{2}$

= 33 (51)

= 1683

Therefore, the sum of all the mulstiples of 3 lying between 1 and 100 is S_n = 1683

Question 50. Find the sum of all odd numbers between (i) 0 and 50 (ii) 100 and 200.

Solution:

(i) In this problem, we need to find the sum of all odd numbers lying between 0 and 50.

So, we know that the first odd number after 0 is 1

and the last odd number before 50 is 49.

Also, all these terms will form an AP. with the common difference of 2.

So here,

First term (a) = 1

Last term (0 = 49

Common difference (d)= 2

So, here the first step is to find the total number of terms.

Let us take the number of terms as n. Now, as we know,

$a_n = a + (n - 1)d$

So, for the last term,

=> 49 = 1 + (n – 1)d

=> 49 = 1 + 2n – 2

=> 49 = 2n – 2

=> 49 + 1 = 2n

Further simplifying,

=> 50 = 2n

=> n = 25

Now, using the formula for the sum of n terms,

=> S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

for n = 25, we get

=> S_{25}=252[2(1)+(25–1)2]$S_{25} = \frac{25}{2}[2(1) + (25-1)\,2]$

= 252[2+24×2]$\frac{25}{2}[2 + 24 \times 2]$

= 25 X 25

= 625

Therefore, the sum of all the odd numbers lying between 0 and 50 is 625.

(ii) In this problem,

we need to find the sum of all odd numbers lying between 100 and 200.

So, we know that the first odd number after 0 is 101

and the last odd number before 200 is 199.

Also, all these terms will form an AR. with the common difference of 2.

So here,

First term (a) = 101

Last term (a_n) = 199

Common difference (d)= 2

So, here the first step is to find the total number of term.

Let us take the number of terms as n.

Now, as we know,

$a_n = a + (n - 1)d$

So, for the last term,

=> 199 = 101 + (n – 1)2

=> 199 = 101 + 2n – 2

=> 199 = 99 + 2n

=> 199 – 99 = 2n

Further simplifying,

=> 100 = 2n

=> n = 50

Now, using the formula for the sum of n terms,

$$S_n = \frac{n}{2}[2(a) + (n-1)d]$$

For n = 50, we get

=> $S_{50} = \frac{50}{2}[2(101) + (50-1)2]$

= 25 [202 + (49) 2]

= 25(202 + 98)

= 25 (300)

= 7500

Therefore, the sum of all the odd numbers lying between 100 and 200 is 7500

Question 52. Find the sum of all integers between 84 and 719, which are multiples of 5.

Solution: In this problem,

we need to find the sum of all the multiples of 5 lying between 84 and 719.

So, we know that the first multiple of 5 after 84 is 85

and the last multiple of 5 before 719 is 715.

Also, all these terms will form an A.P.

with the common difference of 5.

So here,

First term (a) = 85

Last term (l) = 715

Common difference (d) = 5

So, here the first step is to find the total number of terms.

Let us take the number of terms as n.

Now, as we know,

$a = a + (n - 1)d$

So, for the last term.

$715 = 85 + (n - 1)5$

$715 = 85 + 5n - 5$

$715 = 80 + 5n$

$715 - 80 = 5n$

Further simplifying,

$635 = 5n$

$n = 127$

Now, using the formula for the sum of n terms,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

For n = 127,

$$S_{127} = \frac{127}{2}[2(85) + (127-1)5]$$

$$= \frac{127}{2}[170 + 630]$$

$$= \frac{127(800)}{2}$$

$= 50800$

Therefore, the sum of all the multiples of 5 lying between 84 and 719 is 50800.

Question 53. Find the sum of all integer between 50 and 500, which are divisible by 7.

Solution: In this problem,

we need to find the sum of all the multiples of 7 lying between 50 and 500.

So, we know that the first multiple of 7 after 50 is 56

and the last multiple of 7 before 500 is 497.

Also, all these terms will form an A.P. with the common difference of 7.

So here,

First term (a) = 56

Last term (l) = 497

Common difference (d) = 7

So, here the first step is to find the total number of terms.

Let us take the number of terms as n.

Now, as we know,

$a_n = a + (n - 1)d$

So, for the last term.

497 = 56 + (n – 1)7

=> 497 = 56 + 7n -7

=> 497 = 49 + 7n

=> 497 – 49 = 7n

Further simplifying,

448 = 7n

n = 64

Now, using the formula for the sum of n terms,

$S_n = \frac{n}{2}[2a + (n-1)d]$

for n = 64, we get

$S_{64} = \frac{64}{2}[2(56) + (64-1)7]$

= 32 [112 + (63)7]

= 32 [112 + 441]

=32 (553)

= 17696

Therefore, the sum of all the multiples of 7 lying between 50 and 500 is 17696

Question 54. Find the sum of all even integers between 101 and 999.

Solution: In this problem,

we need to find the sum of all the even numbers lying between 101 and 999.

So, we know that the first even number after 101 is 102

and the last even number before 999 is 998.

Also, all these terms will form an A.P. with the common difference of 2.

So here,

First term (a) = 102

Last term (l) = 998

Common difference (d) = 2

So, here the first step is to find the total number of terms.

Let us take the number of terms as n.

Now, as we know,

$a_n = a + (n - 1)d$

So, for the last term,

=> 998 = 102 + (n – 1)2

=> 998 = 102 + 2n – 2

=> 998 = 100 + 2n

=> 998 – 100 = 2n

Further simplifying,

=> 898 = 2n

=> n = 449

Now, using the formula for the sum of n terms,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

For n = 449, we get

$$S_{449} = \frac{449}{2}[2(102) + (449-1)2]$$

$$= \frac{449}{2}[204 + (448)2]$$

$$= \frac{449}{2}[204 + 896]$$

$$= \frac{449}{2}[1100]$$

= 449 (550)

= 246950

Therefore, the sum of all even numbers lying between 101 and 999 is 246950

Question 55.
(i) Find the sum of all integers between 100 and 550, which are divisible by 9.

Solution: In this problem,

we need to find the sum of all the multiples of 9 lying between 100 and 550.

So, we know that the first multiple of 9 after 100 is 108

and the last multiple of 9 before 550 is 549.

Also, all these terms will form an A.P. with the common difference of 9.

So here,

First term (a) = 108

Last term (l) = 549

Common difference (d) = 9

So, here the first step is to find the total number of terms.

Let us take the number of terms as n.

Now, as we know,

$a_n = a + (n - 1)d$

So, for the last term.

=> 549 = 108 + (n – 1)d

=> 549 = 108 + 9n – 9

=> 549 = 99 + 9n

=> 549 – 99 = 9n

Further simplifying

=> 9n = 450

=> n = 50

Now, using the formula for the sum of n terms,

S_n=n2[2a+(n–1)d]$S_n = \frac{n}{2}[2a + (n-1)\,d]$

We get,

S_n=502[2(108)+(50–1)9]$S_n = \frac{50}{2}[2\,(108) + (50-1)\,9]$

= 25 [216 + (49)9]

= 25 (216 + 441)

= 25 (657)

= 16425

Therefore, the sum of all the multiples of 9 lying between 100 and 550 is 16425

Question 56. Let there be an A.P. with first term ‘a’, common difference ‘d’. If a_n denotes its n^{th} term and S_n the sum of first n terms, find.

(i) n and S_n, if a = 5 , d = 3 , and a_n = 50.

(ii) **n and a, if $a_n = 4$, $d = 2$ and $S_n = -14$.**

(iii) **d, if $a = 3$, $n = 8$ and $S_n = 192$.**

(iv) **a, if $a_n = 28$, $S_n = 144$ and $n = 9$.**

(v) **n and d, if $a = 8$, $a_n = 62$ and $S_n = 120$.**

(vi) **n and a_n , if $a = 2$, $d = 8$ and $S_n = 90$.**

Solution:

(i) Here, we have an A.P. whose nth term (a_n), first term (a) and common difference (d) are given. We need to find the number of terms (n) and the sum of first n terms (S_n).

Here,

First term (a) = 5

Last term (a_n) = 50

Common difference (d) = 3

So here we will find the value of n using the formula, $a_n = a + (n – 1) d$

So, substituting the values in the above mentioned formula

=> 50 = 5 + (n – 1) 3

=> 50 = 5 + 3n – 3

=> 50 = 2 + 3n

=> 3n = 50 – 2

Further simplifying for n,

3n = 48

n =16

Now, here we can find the sum of the n terms of the given A.P., using the formula,

S_n=(n2)(a+l)$S_n = \left(\frac{n}{2}\right)(a+l)$

Where, a = the first term

l = the last term

So, for the given A.P,

on substituting the values in the formula for the sum of n terms of an A.P., we get,

S_{16}=(162)(5+50)$S_{16} = \left(\frac{16}{2}\right)(5+50)$

= 8 (55)

= 440

Therefore, for the given A.P. we have, n = 16 and S_{16} = 440

(ii) Here, we have an A.P. whose nth term (an), sum of first n terms (S0) and common difference (d) are given. We need to find the number of terms (n) and the first term (a).

Here,

Last term (l) = 4

Common difference (d) = 2

Sum of n terms (S_n) = -14

So here we will find the value of n using the formula, a_n = a + (n -1) d

So, substituting the values in the above mentioned formula

=> 4 = a + (n – 1) 2

=> 4=a+2n-2

=> 4 + 2 = a + 2n

=> n = 6–a2 $\frac{6-a}{2}$ . . . . (1)

Now, here the sum of the n terms is given by the formula,

S_n=(n2)(a+l)$S_n = \left(\frac{n}{2}\right)(a+l)$

Where, a = the first term

l = the last term

So, for the given A.P,

on substituting the values in the formula for the sum of n terms of an A.P., we get,

=> −14=n2(a+4)$-14 = \frac{n}{2}(a+4)$

=> 14 (2) = n (a + 4)

=> n=−28a+4$n = \frac{-28}{a+4}$ . . . . (2)

Equating (1) and (2), we get,

6–a2=−28a+4 $\frac{6-a}{2} = \frac{-28}{a+4}$

(6 – a)(a + 4) = -28(2)

6a – a^2 + 24 – 4a = -56

-a^2 + 2a + 24 + 56 = 0

So, we get the following quadratic equation,

-a^2 + 2a + 80 = 0

a^2 – 2a – 80 = 0

Further solving it for a by splitting the middle term,

a^2 – 2a – 80 = 0

a^2 – 10a + 8a – 80 = 0

a(a – 10) + 8(a – 10) = 0

(a – 10)(a + 8) = 0

So, we get,

a – 10 = 0

a = 10

or,

a + 8 = 10

a = -8

Substituting, a = 10 in (1)

n=6–102$n = \frac{6-10}{2}$ n=−42$n = \frac{-4}{2}$

n = -2

Here, we get n as negative, which is not possible. SO, we take a = -8

n=6–(−8)2$n = \frac{6-(-8)}{2}$ n=6+82$n = \frac{6+8}{2}$ n=142$n = \frac{14}{2}$

n = 7

Therefore, for the given A.P. n = 7 and a = -8

(iii) Here, we have an A.P. whose first term (a), sum of first n terms (S0) and the number of terms (n) are given. We need to find common difference (d).

Here,

First term (a) = 3

Sum of n terms (S_n) = 192

Number of terms (n) = 8

So here we will find the value of n using the formula, a_n = a + (n -1) d

So, to find the common difference of this A.P.,

we use the following formula for the sum of n terms of an A.P

S_n=(n2)[2a+(n–1)d]$S_n = \left(\frac{n}{2}\right)[2a+(n-1)d]$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

So, using the formula for n = 8, we get,

S_8=(82)[2(3)+(8–1)d]$S_8 = \left(\frac{8}{2}\right)[2(3)+(8-1)d]$

192 = 4 [6 + 7d]

192 = 24 + 28d

28d = 192 – 24

28 d = 168

$d = 6$

Therefore, the common difference of the given A.P. is d = 6

(iv) Here, we have an A.P. whose nth term (an), sum of first n terms (S_n) and the number of terms (n) are given. We need to find first term (a).

Here,

Last term (a_9) = 28

Sum of n terms (S_n)= 144

Number of terms (n) = 9

Now,

$a_9 = a + 8d$

$28 = a + 8d$ . . . . (1)

Also, using the following formula for the sum of n terms of an AP

S_n=(n2)[2a+(n–1)d]$S_n = \left(\frac{n}{2}\right)[2a + (n-1)\,d]$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

So, using the formula for n = 9, we get,

S_9=(92)[2a+(9–1)d]$S_9 = \left(\frac{9}{2}\right)[2a + (9-1)\,d]$

$144\,(2) = 9\,[2a + 8d]$

$288 = 18a + 72d$ . . . . (2)

Multiplying (1) by 9, we get

$9a + 72d = 252$ . . . . (3)

Further, subtracting (3) from (2), we get

$9\,a = 36$

$a = 4$

Therefore, the first term of the given A.P. is a = 4

(v) Here, we have an A.P. whose nth term (an), sum of first n terms (S_n) and first term (a) are given. We need to find the number of terms (n) and the common difference (d).

Here,

First term (a) = 8

Last term (a_n) = 62

Sum of n terms (S_n) = 210

Now, here the sum of the n terms is given by the formula,

$$S_n = \left(\frac{n}{2}\right)(a + l)$$

Where, a = the first term

l = the last term

So, for the given A.P,

on substituting the values in the formula for the sum of n terms of an A.P., we get,

$$210 = \frac{n}{2}[8 + 62]$$

210 (2) = n (70)

$$n = \frac{420}{70}$$

n = 7

Also, here we will find the value of d using the formula,

$a_n = a + (n – 1) d$

So, substituting the values in the above mentioned formula

62= 8 + (6 – 1)d

5d = 54

$$d = \frac{54}{5}$$

Therefore, for the given A.P. n = 6 and d=545 $d = \frac{54}{5}$

(vi) Here, we have an A.P. whose first term (a), common difference (d) and sum of first n terms are given. We need to find the number of terms (n) and the nth term (a_n).

Here,

First term (a) = 2

Sum of first nth terms (S_n) = 90

Common difference (d) = 8

So, to find the number of terms (n) of this A.P.,

we use the following formula for the sum of n terms of an A.P

S_n=(n2)[2a+(n–1)d] $S_n = \left(\frac{n}{2}\right)\left[2a + (n-1)\,d\right]$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

So, using the formula for d = 8, we get,

S_n=(n2)[2(2)+(n–1)8] $S_n = \left(\frac{n}{2}\right)\left[2(2) + (n-1)\,8\right]$ 90=(n2)[4+8n–8]
$90 = \left(\frac{n}{2}\right)\left[4 + 8n - 8\right]$

90 (2) = n [8n – 4]

180 = $8n^2$ – 4n

Further solving the above quadratic equation,

$8n^2$ -4n -180 = 0

$2n^2$ – n – 45 =0

Further solving for n,

$2n^2$ – 10n + 9n – 45 = 0

2n(n – 5) + 9(n – 5) = 0

(2n – 9)(n – 5) = 0

Now,

2n + 9 = 0

n=-92 $n = \frac{-9}{2}$

Also,

n – 5 = 0

n = 5

Since, n cannot be a fraction.

Thus, n = 5

Also, we will find the value of n^{th} term (a_n), using the formula,

a_n = a + (n – 1) d

So, substituting the values in the above formula,

a_n = 2 + (5 – 1) 8

a_n = 2 + 4 (8)

a_n = 2 + 32

a_n = 34

Therefore, for the given A.P., n = 5 and a_n = 34