Exercise 8.1: Quadratic Equations

1. (i) x^2 – 3x + 2 = 0 , x = 2 , x = -1

Here LHS = x^2-3x+2

RHS = 0

Now, substitute x = 2 in LHS

We get,

$(2)^2 -3(2)+2 = 4-6+2 = 6-6 = 0$ = RHS

Since, LHS =RHS

Therefore, x – 2 is a solution of the given equation.

Similarly, substituting x= -1 in LHS

We get,

$(-1)^2 -3(-1)+2 = 1+3+2 = 6 \neq$ RHS

Since, LHS ≠ RHS = x= -1 is not the solution of the given equation.

(ii) $x^2 + x + 1 = 0$, $x = 0$, $x = 1$

Here, LHS = $x^2 + x + 1$ and RHS =0

Now, substituting x= 0 and x= 1 in LHS

= 0^2+0+1 = $(1)^2+1+1$ = 1 =3

LHS ≠ RHS

Both x=0 and x=1 are not solutions of the given equation.

(iii) $x^2-3\sqrt{3}x+6=0$ $x^2 - 3\sqrt{3}x + 6 = 0$, $x=\sqrt{3}$ and $x=-2\sqrt{3}$ $x = \sqrt{3}\, and\, x = -2\sqrt{3}$

Here,

LHS = $x^2-3\sqrt{3}x+6=0$ $x^2 - 3\sqrt{3}x + 6 = 0$ and RHS =0

Substituting the value of $x=\sqrt{3}$ and $x=-2\sqrt{3}$ $x = \sqrt{3}\, and\, x = -2\sqrt{3}$ in LHS

$\sqrt{3}^2-3\sqrt{3}\times\sqrt{3}+6$ $\sqrt{3}^2 - 3\sqrt{3} \times \sqrt{3} + 6$

= 3-9+6

= 0

= RHS

$-2\sqrt{3}^2-3\sqrt{3}\times-2\sqrt{3}+6$ $-2\sqrt{3}^2 - 3\sqrt{3} \times -2\sqrt{3} + 6$

= 12+18+6

= 36

≠ RHS

$x=\sqrt{3}$ $x = \sqrt{3}$ is a solution of the above mentioned equation

Whereas, $x=-2\sqrt{3}$ $x = -2\sqrt{3}$ is not a solution of the above mentioned equation .

(iv) $x + 1x = 136$ $x + \frac{1}{x} = \frac{13}{6}$

where $x = 56$ and $x = 43$ $where\, x = \frac{5}{6}\, and\, x = \frac{4}{3}$

Here, LHS = x+ 1x=136 $x + \frac{1}{x} = \frac{13}{6}$ and RHS = 136 $\frac{13}{6}$

Substituting wherex=56andx=43 $where\, x = \frac{5}{6}\, and\, x = \frac{4}{3}$ in the LHS

= 56 + 156 $\frac{5}{6} + \frac{1}{\frac{5}{6}}$

= 56 + 65 $\frac{5}{6} + \frac{6}{5}$

= 25+3630 $\frac{25+36}{30}$

= 6130 $\frac{61}{30}$

≠ RHS

= 43 + 143 $\frac{4}{3} + \frac{1}{\frac{4}{3}}$

= 43 + 34 $\frac{4}{3} + \frac{3}{4}$

= 16+912 $\frac{16+9}{12}$

=2512 $\frac{25}{12}$

≠RHS

wherex=56andx=43 $where\, x = \frac{5}{6}\, and\, x = \frac{4}{3}$ are not the solutions of the given equation.

(v) $2x^2$-x+9 = x^2+4x+3 , x = 2 and x = 3

= $2x^2$-x+9 – x^2+4x+3

= x^2 -5x+6 = 0

Here, LHS = x^2 -5x+6 and RHS = 0

Substituting x = 2 and x = 3

= x^2 -5x+6

= $(2)^2$ – 5(2) +6

=10-10

=0

= RHS

= x^2 -5x+6

= $(3)^2$ – 5(3) +6

= 9-15+6

=15-15

=0

= RHS

x = 2 and x = 3 both are the solutions of the given quadratic equation.

(vi) $x^2-\sqrt{2}x-4=0$ $x^2 - \sqrt{2}x - 4 = 0$

$x=-\sqrt{2}$ and $x=-2\sqrt{2}$ $x = -\sqrt{2}\, and\, x = -2\sqrt{2}$

Here, LHS = $x^2-\sqrt{2}x-4=0$ $x^2 - \sqrt{2}x - 4 = 0$

And RHS = 0

Substituting the value $x=-\sqrt{2}$ and $x=-2\sqrt{2}$ $x = -\sqrt{2}\, and\, x = -2\sqrt{2}$ in LHS

= $(-\sqrt{2})^2-\sqrt{2}\times\sqrt{2}-4$ $(-\sqrt{2})^2 - \sqrt{2} \times \sqrt{2} - 4$

= 2 – 2 – 4

= -4

≠RHS

= $(-2\sqrt{2})^2-\sqrt{2}\times2\sqrt{2}-4$ $(-2\sqrt{2})^2 - \sqrt{2} \times 2\sqrt{2} - 4$

= 8 -4 -4

=8-8

=0

= RHS

x=−2√2$x = -2\sqrt{2}$ is the solution of the above mentioned quadratic equation .

(vii) a^2x^2-3abx+$2b^2$ = 0

x=ab andx=ba $x = \frac{a}{b}\ and\ x = \frac{b}{a}$

Here, LHS = a^2x^2 -3abx+$2b^2$ and RHS = 0

Substituting the x=ab andx=ba $x = \frac{a}{b}\ and\ x = \frac{b}{a}$ in LHS

= a^2(ab)^2−3ab(ab)+$2b^2$$a^2(\frac{a}{b})^2 - 3ab(\frac{a}{b}) + 2b^2$

= a⁴b² −3a^2+$2b^2$$\frac{a^4}{b^2} - 3a^2 + 2b^2$

≠ RHS

= a^2(ba)^2−3ab(ba)+$2b^2$$a^2(\frac{b}{a})^2 - 3ab(\frac{b}{a}) + 2b^2$

= b^2-$3b^2$ +$2b^2$ = 0 = RHS

x=ba $x = \frac{b}{a}$ is the solution of the above mentioned quadratic equation .

3.

(i) Given that 23 $\frac{2}{3}$ is a root of the given equation.

The equation is $7x^2$+kx-3 = 0

According to the question 23 $\frac{2}{3}$ satisfies the equation.

= 7(23)^2+k(23)−3$7(\frac{2}{3})^2 + k(\frac{2}{3}) - 3$

= 7(49)+ 2k3 −3$7(\frac{4}{9}) + \frac{2k}{3} - 3$

= 2k3 =27−289 $\frac{2k}{3} = \frac{27-28}{9}$

= 2k3 =−19 $\frac{2k}{3} = \frac{-1}{9}$

= k=−16 $k = \frac{-1}{6}$

(ii) Given that x=a is a root of the given equation x^2-x(a+b)+k =0

= x=a satisfies the equation

= a^2-a(a+b)+k=0

= $a^2 - a^2$-ab+k =0

K = ab

(iii) Given that x=√$2x = \sqrt{2}$ is a root of the given equation kx^2+√2x−$4kx^2 + \sqrt{2}x - 4$

x=√$2x = \sqrt{2}$ satisfies the given quadratic equation.

=k$\sqrt{2}^2$+√2√2−$4k\sqrt{2}^2 + \sqrt{2}\sqrt{2} - 4$

=2k+2-4 =0

= 2k-2=0

K = 1

(iv) Given that x= -a is the root of the given equation x^2+3ax+k = 0

Therefore,

= $(-a)^2$+3a(-a)+k = 0

= a^2+3a^2+k =0

= k = $4a^2$= -a satisfies the equation

(v) Given that x=√$2x = \sqrt{2}$ is a root of the given equation kx^2+√2x−$4kx^2 + \sqrt{2}x - 4$

x=√$2x = \sqrt{2}$ satisfies the given quadratic equation.

=k$\sqrt{2}^2$+√2√2−$4k\sqrt{2}^2 + \sqrt{2}\sqrt{2} - 4$

=2k+2-4 =0

= 2k-2=0

K = 1

4. Given to check whether 3 is a root of the equation $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+16}$
$\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+16}$

LHS = $\sqrt{x^2-4x+3}+\sqrt{x^2-9}\sqrt{x^2-4x+3}+\sqrt{x^2-9}$

RHS= $\sqrt{4x^2-14x+16}\sqrt{4x^2-14x+16}$

Substituting x=3 in LHS

$\sqrt{3^2-4\times3+3}\sqrt{3^2-4\times3+3}$ + $\sqrt{3^2-9}\sqrt{3^2-9}$

$\sqrt{9-12+3}\sqrt{9-12+3}$ + $\sqrt{9-9}\sqrt{9-9}$

$\sqrt{12-12}\sqrt{12-12}$ + $\sqrt{9-9}\sqrt{9-9}$

= 0

Similarly putting x=3 in RHS

Extra open brace or missing close brace Extra open brace or missing close brace
$\sqrt{4(3)^2-14(3)+16}\sqrt{4(3)^2-14(3)+16}$ $\sqrt{52-42}\sqrt{52-42}$ $\sqrt{10}\sqrt{10}$

≠ RHS

X=3 is not the solution the given quadratic equation.

Exercise 8.2: Quadratic Equations

Question 1: The product of two consecutive positive integers is 306. Form the quadratic equation to find the integers, if x denotes the smaller integer.

Solution:

Given that the smallest integer of 2 consecutive integers is denoted by x

The two integers be x and x+1

According to the question, the product of the integers is 306

Now,

$X(x+1) = 306 = x^2+x-306=0$

The required quadratic equation of the equation is $x^2+x-306=0$

Question 2: John and Jivani together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 128. Form the quadratic equation to find how many marbles they to start with if John had x marbles.

Solution:

Given that John and Jilani are having the total of 45 marbles.

Let us consider John is having x marbles

Jivani is having (45-x) marbles.

Number of marbles john had after losing 5 marbles = x-5

Number of marbles jivani had after losing 5 marbles = (45-x)-5 = 40-x

According to the question the product of the marbles that they are having now is 128

Now, (x-5)(40-x) = 128

= $40x-x^2-200 = 128$

= $x^2 -45x+128+200 =0$

= $x^2 -45x+328 = 0$

The required quadratic equation is $x^2 -45x+328 = 0$.

Question 3: A cottage industry produces a certain number of toys in a day. The cost of production of each toy was found to be 55 minutes the number of articles produced in a day. On a particular day , the total cost of production was Rs. 750. If x denotes the number of toys produced that day, form the quadratic equation to find x.

Solution:

Given

(y) Denotes the number of toys produced in a day.

The cost of production of each toy = (55 – y)

Total cost of production is nothing but the product of number of toys produced in a day and cost of production of each toy = y (55-y)

According to the question

The total cost of production is Rs.750

= y (55-y) = 750

$= 55y - y^2 = 750$

$= y^2 - 55y + 750 = 0$

The required quadratic equation of the given data is $y^2 - 55y + 750 = 0$.

Question 4: The height of the right triangle is 7 cm less than its base. If the hypotenuse is 13cm, form the quadratic equation to find the base of the triangle.

Solution:

According to the question

The hypotenuse of the triangle = 13 cm

Let the base of the triangle = x cm

So, the height of the triangle = (x-7) cm

Applying Pythagoras theorem in the right angled triangle, we get,

$(\text{Base})^2 + (\text{height})^2 = (\text{hypotenuse})^2$

$X^2 + (x-7)^2 = (13)^2$

$X^2 + x^2 + 49 - 14x = 169$

$2x^2 - 14x - 120 = 0$

$2(x^2 - 7x - 60) = 0$

$x^2 - 7x - 60 = 0$

The required quadratic equation is $x^2 - 7x - 60 = 0$

Question 5: The average speed of the express train is 11 km/hr more than that of the passenger train. The total distance covered by the train is 132 km. Also, time taken by the express train is 1 hour is less than that of the passenger train. Find the quadratic equation of this problem.

Solution:

Let the average speed of the express train be = x km /hr

Given, the average speed of the express train is 10 km/ hr less than that of passenger train = (x-11) km/hr

We know that:

Time taken for travel = distance travelled / average speed

Time taken for express train = distance travelled / average speed of the express train

= 132x $\frac{132}{x}$

Hence time taken by the passenger train = 132x−11 $\frac{132}{x-11}$

According to the question,

Time taken by the express train is 1 hour less than that of passenger train

Time taken by passenger train – time taken by express train = 1 hour

132x−11 $\frac{132}{x-11}$ −132x $\frac{132}{x}$ = 1

132(1x−11 − 1x)$132(\frac{1}{x-11} - \frac{1}{x})$ =1

132(x−(x−11)x×(x−11))$132(\frac{x-(x-11)}{x\times(x-11)})$ =1

132(x−x+11x2−11x)$132(\frac{x-x+11}{x^2-11x})$ = 1

132(11x2−11x)$132(\frac{11}{x^2-11x})$ = 1

$1452 = x^2 - 11x$

$X^2 - 11x - 1452 = 0$

The quadratic equation of the given problem is $X^2 - 11x - 1452 = 0$.

Question 6: A train travels 360 km at a uniform speed. If the speed had been 5 km/ hr more, it would have taken 1 hour less for the same journey. Form the quadratic equation to find the speed of the train.

Solution:

Let the speed of the train be = x km /hr

Distance travelled by the train = 360 km

We know that,

Time taken for travel = distance travelled ÷ speed of the train

= 360x $\frac{360}{x}$

If the speed of the train is increased by 4 km /hr then time taken = 360x+5 $\frac{360}{x+5}$

According to the question,

The time of travel is reduced by 1 hour when the speed of the train is increased by 5 km /hr

360x $\frac{360}{x}$ − 360x+5 $\frac{360}{x+5}$ = 1

360(1x − 1x+5)$360(\frac{1}{x} - \frac{1}{x+5})$ =1

360(x+5−xx(x+5))$360(\frac{x+5-x}{x(x+5)})$ =1

360(5x(x+5))$360(\frac{5}{x(x+5)})$ =1

360(5x2+5x)$360(\frac{5}{x^2+5x})$ =1

$X^2 + 5x = 1800$

The required quadratic equation is $X^2 + 5x - 1800=0$.

Exercise 8.3: Quadratic Equations

Question 1: Find the roots of the equation (x – 4) (x + 2) = 0

Sol:

The given equation is (x – 4) (x + 2) = 0

Either x – 4 = 0 therefore x = 4

Or, x + 2 = 0 therefore x = -2

The roots of the above mentioned quadratic equation are 4 and -2 respectively.

Question 2: Find the roots of the equation (2x + 3) (3x – 7) = 0

Sol:

The given equation is (2x + 3) (3x – 7) = 0.

Either 2x+3 =0, therefore x=−32 $x = \frac{-3}{2}$

Or, 3x-7 =0, therefore x=73 $x = \frac{7}{3}$

The roots of the above mentioned quadratic equation are X=−32 $x = \frac{-3}{2}$ and X=73 $x = \frac{7}{3}$ respectively.

Question 3: Find the roots of the quadratic equation $3x^2$-14x-5 = 0

Sol:

The given equation is $3x^2$-14x-5 = 0

= $3x^2$-14x-5 = 0

= $3x^2$-15x+x-5 = 0

= 3x(x-5)+1(x-5) =0

= (3x+1)(x-5) = 0

Either 3x+1 =0 therefore X=−13 $x = \frac{-1}{3}$

Or, x-5 =0 therefore x=5

The roots of the given quadratic equation are 5 and X=−13 $x = \frac{-1}{3}$ respectively.

Question 4: Find the roots of the equation $9x^2$-3x-2=0.

Sol:

The given equation is $9x^2$-3x-2 =0.

= $9x^2$-3x-2 =0.

= $9x^2$ -6x+3x-2 =0

= 3x (3x-2)+1(3x-2) =0

= (3x-2)(3x+1) = 0

Either, 3x-2 =0 therefore X=23 $x = \frac{2}{3}$

Or, 3x+1 = 0 therefore X=−13 $x = \frac{-1}{3}$

The roots of the above mentioned quadratic equation are X=23 $x = \frac{2}{3}$ and X=−13 $x = \frac{-1}{3}$ respectively.

Question 5: Find the roots of the quadratic equation 1x−1−1x+5=67 $\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}$

Sol:

The given equation is 1x−1−1x+5=67 $\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}$

= 1x−1−1x+5=67 $\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}$

= x+5−x+1(x−1)(x+5)=67 $\frac{x+5-x+1}{(x-1)(x+5)} = \frac{6}{7}$

= 6x2+4x−5=67 $\frac{6}{x^2+4x-5} = \frac{6}{7}$

Cancelling out the like terms on both the sides of the numerator. We get,

= 1x2+4x−5=17 $\frac{1}{x^2+4x-5} = \frac{1}{7}$

= $x^2+4x-5 = 7$

= $x^2+4x-12 = 0$

= $x^2+6x-2x-12 = 0$

= x(x+6)-2(x-6) =0

=(x+6)(x-2) =0

Either x+6 =0

Therefore x= -6

Or, x-2 =0

Therefore x=2

The roots of the above mentioned quadratic equation are 2 and -6 respectively.

Question 6: Find the roots of the equation $6x^2$+11x+3=0.

Sol:

The given equation is $6x^2$+11x+3 =0.

= $6x^2$+11x+3 =0.

= $6x^2$ +9x+2x+3 =0

= 3x(2x+3) +1(2x+3) =0

= (2x+3)(3x+1) =0

Either, 2x+3 =0 therefore x=−32 $x = \frac{-3}{2}$

Or, 3x+1 =0 therefore x=−13 $x = \frac{-1}{3}$

The roots of the above mentioned quadratic equation are x=−32 $x = \frac{-3}{2}$ and x=−13 $x = \frac{-1}{3}$ respectively .

Question 7: Find the roots of the equation $5x^2$-3x-2=0

Sol:

The given equation is $5x^2$-3x-2=0.

= $5x^2$-3x-2=0.

= $5x^2$ -5x+2x-2 = 0

= 5x(x-1) +2(x-1) =0

= (5x+2) (x-1) =0

Either 5x+2 =0 therefore x=−25 $x = \frac{-2}{5}$

Or, x-1 =0 therefore x= 1

The roots of the above mentioned quadratic equation are 1 and x=−25 $x = \frac{-2}{5}$ respectively.

Question 8: Find the roots of the equation $48x^2$-13x-1=0

Sol:

The given equation is $48x^2$-13x-1=0.

= $48x^2$-13x-1=0.

= $48x^2$-16x+3x-1=0.

= 16x (3x-1) +1(3x-1) =0

= (16x+1)(3x-1) =0

Either 16x+1 =0 therefore x=−116 $x = \frac{-1}{16}$

Or, 3x-1 =0 therefore x=13 $x = \frac{1}{3}$

The roots of the above mentioned quadratic equation are x=−116 $x = \frac{-1}{16}$

And x=13 $x = \frac{1}{3}$ respectively.

Question 9: Find the roots of the equation $3x^2$=-11x-10

Sol:

The given equation is $3x^2$=-11x-10

= $3x^2$=-11x-10

= $3x^2$+11x+10 = 0

= $3x^2$+6x+5x+10 =0

= 3x(x+2) +5(x+2) =0

= (3x+2)(x+2) =0

Either 3x+2 =0 therefore x=−23 $x = \frac{-2}{3}$

Or, x+2 =0 therefore x= -2

The roots of the above mentioned quadratic equation are X=-23 $x = \frac{-2}{3}$ and -2 respectively.

Question 10: Find the roots of the equation 25x(x+1) =-4

Sol:

The given equation is 25x(x+1) =4

= 25x(x+1) =-4

= $25x^2$+25x +4 = 0

= $25x^2$+20x+5x+4 =0

= 5x (5x+4)+1(5x+4)=0

= (5x+4)(5x+1) =0

Either 5x+4 = 0 therefore X=-45 $x = \frac{-4}{5}$

Or, 5x+1 =0 therefore X=-15 $x = \frac{-1}{5}$

The roots of the quadratic equation are X=-45 $x = \frac{-4}{5}$ and X=-15 $x = \frac{-1}{5}$ respectively.

Question 12: Find the roots of the quadratic equation 1x−1x-2=3 $\frac{1}{x} - \frac{1}{x-2} = 3$

Sol:

The given equation is 1x−1x-2=3 $\frac{1}{x} - \frac{1}{x-2} = 3$

= 1x−1x-2=3 $\frac{1}{x} - \frac{1}{x-2} = 3$

= x-2−xx(x-2)=3 $\frac{x-2-x}{x(x-2)} = 3$

$= 2x(x-2) = 3 \frac{2}{x(x-2)} = 3$

Cross multiplying both the sides. We get,

$=2 = 3x(x-2)$

$= 2 = 3x^2-6x$

$= 3x^2-6x-2 = 0$

$= 3x^2-3x-3x-2 = 0$

$=3x^2-(3+\sqrt{3})x-(3-\sqrt{3})x+[(\sqrt{3}^2)-1^2] 3x^2-(3+\sqrt{3})x-(3-\sqrt{3})x+[(\sqrt{3}^2)-1^2]$

$= 3x^2-(3+\sqrt{3})x-(3-\sqrt{3})x+[(\sqrt{3}^2)-1^2][(\sqrt{3}^2)-1^2]$
$3x^2-(3+\sqrt{3})x-(3-\sqrt{3})x+[(\sqrt{3}^2)-1^2][(\sqrt{3}^2)-1^2]$

$= \sqrt{3}^2x^2-\sqrt{3}(\sqrt{3}+1)x-\sqrt{3}(\sqrt{3}-1)x+(\sqrt{3}+1)(\sqrt{3}-1)$
$\sqrt{3}^2x^2-\sqrt{3}(\sqrt{3}+1)x-\sqrt{3}(\sqrt{3}-1)x+(\sqrt{3}+1)(\sqrt{3}-1)$

$= \sqrt{3}x(\sqrt{3}+1)x-(\sqrt{3}x-(\sqrt{3}+1))(\sqrt{3}-1)\sqrt{3}x(\sqrt{3}+1)x-(\sqrt{3}x-(\sqrt{3}+1))(\sqrt{3}-1)$

$= (\sqrt{3}x-\sqrt{3}-1)(\sqrt{3}x-\sqrt{3}+1)(\sqrt{3}-1)(\sqrt{3}x-\sqrt{3}-1)(\sqrt{3}x-\sqrt{3}+1)(\sqrt{3}-1)$

Either $= (\sqrt{3}x-\sqrt{3}-1)(\sqrt{3}x-\sqrt{3}-1)$

Therefore $x=\sqrt{3}+1\sqrt{3}x = \frac{\sqrt{3}+1}{\sqrt{3}}$

Or, $(\sqrt{3}x-\sqrt{3}+1)(\sqrt{3}-1)(\sqrt{3}x-\sqrt{3}+1)(\sqrt{3}-1)$

Therefore, $x=\sqrt{3}-1\sqrt{3}x = \frac{\sqrt{3}-1}{\sqrt{3}}$

The roots of the above mentioned quadratic equation are $x=\sqrt{3}-1\sqrt{3}x = \frac{\sqrt{3}-1}{\sqrt{3}}$ and $x=\sqrt{3}+1\sqrt{3}x = \frac{\sqrt{3}+1}{\sqrt{3}}$ respectively.

Question 13: Find the roots of the quadratic equation $x-1x=3x-\frac{1}{x}=3$

Sol:

The given equation is x−1x=3 $x - \frac{1}{x} = 3$

= x−1x=3 $x - \frac{1}{x} = 3$

= x^2−1x=3 $\frac{x^2-1}{x} = 3$

= x^2-1 = 3x

= x^2-1 -3x=0

= x^2−(32+32)x−1=0 $x^2 - (\frac{3}{2} + \frac{3}{2})x - 1 = 0$

= x^2−3+√32x−3−√32x−1=0 $x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - 1 = 0$

= x^2−3+√32x−3−√32x−−44=0 $x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - \frac{-4}{4} = 0$

= x^2−3+√32x−3−√32x−9−134=0 $x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - \frac{9-13}{4} = 0$

= x^2−3+√32x−3−√32x−(3)²−(√13²)(2)²=0 $x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - \frac{(3)^2-(\sqrt{13^2})}{(2)^2} = 0$

= x^2−3+√32x−3−√32x+(3+√132)(3−√132)=0
$x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x + (\frac{3+\sqrt{13}}{2})(\frac{3-\sqrt{13}}{2}) = 0$

= (x−3+√132)(x−3−√132)=0 $(x - \frac{3+\sqrt{13}}{2})(x - \frac{3-\sqrt{13}}{2}) = 0$

Either (x−3+√132)=0 $(x - \frac{3+\sqrt{13}}{2}) = 0$

Therefore 3+√132 $\frac{3+\sqrt{13}}{2}$

Or, (x−3−√132)=0 $(x - \frac{3-\sqrt{13}}{2}) = 0$

Therefore 3−√132 $\frac{3-\sqrt{13}}{2}$

The roots of the above mentioned quadratic equation are 3+√132 $\frac{3+\sqrt{13}}{2}$ and 3−√132 $\frac{3-\sqrt{13}}{2}$ respectively.

Question 14: Find the roots of the quadratic equation 1x+4−1x−7=1130

$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

Sol:

The given equation is 1x+4−1x−7=1130 $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

= 1x+4−1x−7=1130 $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

= x−7−x−4(x+4)(x−7)=1130 $\frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$

= −11(x+4)(x−7)=1130 $\frac{-11}{(x+4)(x-7)} = \frac{11}{30}$

Cancelling out the like numbers on both the sides of the equation

= −1(x+4)(x−7)=130 $\frac{-1}{(x+4)(x-7)} = \frac{1}{30}$

= x^2-3x-28 =-30

= x^2-3x-2 =0

= x^2-2x-x-2 =0

= x(x-2)-1(x-2) =0

= (x-2)(x-1) =0

Either x-2 = 0

Therefore x= 2

Or, x-1 = 0

Therefore x= 1

The roots of the above mentioned quadratic equation are 1 and 2 respectively.

Question 15: Find the roots of the quadratic equation a^2x^2-3abx+$2b^2$=0

Sol:

The given equation is a^2x^2-3abx+$2b^2$=0

= a^2x^2-3abx+$2b^2$=0

= a^2x^2-abx-2abx+$2b^2$=0

= ax(ax-b)-2b(ax-b) =0

= (ax-b)(ax-2b) =0

Either ax-b=0 therefore X=ba $x = \frac{b}{a}$

Or, ax-2b =0 therefore X=2ba $x = \frac{2b}{a}$

The roots of the quadratic equation are X=2ba $x = \frac{2b}{a}$ and X=ba $x = \frac{b}{a}$ respectively.

Question 16: Find the roots of the $4x^2$+4bx-(a^2-b^2) =0

Sol:

-4(a^2-b^2) = -4(a-b)(a+b)

= -2(a-b) * 2(a+b)

=2(b-a) * 2(b+a)

= $4x^2$+ (2(b-a) + 2(b+a)) – (a-b)(a+b) = 0

= $4x^2$ + 2(b-a)x++ 2(b+a)x+(b-a)(a+b) =0

= 2x(2x+(b-a)) +(a+b)(2x+(b-a)) = 0

= (2x+(b-a))(2x+b+a) = 0

Either, (2x+(b-a)) = 0

Therefore X=a−b2 $x = \frac{a-b}{2}$

Or, (2x+b+a) =0

Therefore X=−a−b2 $x = \frac{-a-b}{2}$

The roots of the above mentioned quadratic equation are X=−a−b2 $x=\frac{-a-b}{2}$ and X=a−b2 $x=\frac{a-b}{2}$ respectively.

Question 17: Find the roots of the equation $ax^2+(4a^2-3b)x-12ab=0$

Sol:

The given equation is $ax^2+(4a^2-3b)x-12ab=0$

= $ax^2+(4a^2-3b)x-12ab=0$

= $ax^2+4a^2x-3bx-12ab=0$

= ax(x-4a) – 3b(x-4a) =0

= (x-4a)(ax-4b) = 0

Either x-4a =0

Therefore x= 4a

Or, ax-4b = 0

Therefore X=4ba $x=\frac{4b}{a}$

The roots of the above mentioned quadratic equation are X=4ba $x=\frac{4b}{a}$ and 4a respectively.

Question 18: Find the roots of x+3x+2=3x−72x−3 $\frac{x+3}{x+2}=\frac{3x-7}{2x-3}$

Sol:

The given equation is x+3x+2=3x−72x−3 $\frac{x+3}{x+2}=\frac{3x-7}{2x-3}$

= =(x+3)(2x-3)=(x+2){3x-7)

$=2x^2-3x+6x-9=3x^2-x-14$

$=2x^2+3x-9=3x^2-x-14$

$=x^2-3x-x-14+9=0$

$= x^2-5x+x-5 = 0$

=x(x-5)+1(x-5)=0

=(x-5) (x+l)-0

Either x-5-0 or x+1=0

x=5 and x=-1

The roots of the above mentioned quadratic equation are 5 and -1 respectively.

Question 19: Find the roots of the equation 2xx−4+2x−5x−3=253 $\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$

Sol:

The given equation is 2xx−4+2x−5x−3=253 $\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$

= 2x(x−3)+(2x−5)(x−4)(x−4)(x−3)=253 $\frac{2x(x-3)+(2x-5)(x-4)}{(x-4)(x-3)}=\frac{25}{3}$

= $2x^2-6x+2x^2-5x-8x+20x^2-4x-3x+12$=253 $\frac{2x^2-6x+2x^2-5x-8x+20}{x^2-4x-3x+12}=\frac{25}{3}$

= $4x^2-19x+20x^2-7x+12$=253 $\frac{4x^2-19x+20}{x^2-7x+12}=\frac{25}{3}$

$= 3(4x^2-19x+20) = 25(x^2-7x+12)$

$= 12x^2-57x+60 = 25x^2 - 175x+300$

$= 13x^2-78x-40x+240=0$

$= 13x^2-118x+240=0$

$= 13x^2-78x-40x+240=0$

= 13x(x-6)-40(x-6) =0

= (x-6)(13x-40) =0

Either x-6 = 0 therefore x= 6

Or , 13x-40 = 0 therefore x = 4013 $\frac{40}{13}$

The roots of the above mentioned quadratic equation are 6 and 4013 $\frac{40}{13}$ respectively.

Question 20: Find the roots of the quadratic equation x+3x−2−1−xx=174

$\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$

Sol:

The given equation is x+3x−2−1−xx=174 $\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$

= x(x+3)−(x−2)(1−x)x(x−2)=174 $\frac{x(x+3)-(x-2)(1-x)}{x(x-2)} = \frac{17}{4}$

= x2+3x−x+x2+2−2xx2−2x=174 $\frac{x^2+3x-x+x^2+2-2x}{x^2-2x} = \frac{17}{4}$

= 2x2+2x2−2x=174 $\frac{2x^2+2}{x^2-2x} = \frac{17}{4}$

= $4(2x^2+2) = 17(x^2-2x)$

= $8x^2+8 = 17x^2-34x$

= $9x^2-34x-8 = 0$

= $9x^2-36x+2x-8 = 0$

= 9x(x-4)+2(x-4) =0

= (9x+2)(x-4) =0

Either 9x+2 = 0 therefore X=−29 $x = \frac{-2}{9}$

Or, x-4 = 0 therefore x= 4

The roots of the above mentioned quadratic equation are X=−29 $x = \frac{-2}{9}$ and 4 respectively.

Question 21: Find the roots of the quadratic equation 1x−2+2x−1=6x $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$

Sol:

The equation is 1x−2+2x−1=6x $\frac{1}{x-2}+\frac{2}{x-1}=\frac{6}{x}$

= (x−1)+2(x−2)(x−2)(x−1)=6x $\frac{(x-1)+2(x-2)}{(x-2)(x-1)}=\frac{6}{x}$

= (x−1)+2x−4(x2−2x−x+2)=6x $\frac{(x-1)+2x-4}{(x^2-2x-x+2)}=\frac{6}{x}$

= 3x−5(x2−3x+2)=6x $\frac{3x-5}{(x^2-3x+2)}=\frac{6}{x}$

= $x(3x-5) = 6(x^2-3x+2)$

= $3x^2-5x= 6x^2-18x+12$

= $3x^2-13x+12 =0$

= $3x^2-9x-4x+12 =0$

= $3x(x-3)-4(x-3) =0$

= $(x-3)(3x-4) = 0$

Either x-3 = 0 therefore x= 3

Or, 3x-4 = 0 therefore 43 $\frac{4}{3}$

The roots of the above mentioned quadratic equation are 3 and 43 $\frac{4}{3}$ respectively.

Question 22: Find the roots of the quadratic equation x+1x−1−x−1x+1=56

$\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{5}{6}$

Sol:

The equation is x+1x−1−x−1x+1=56 $\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{5}{6}$

= (x+1)2−(x−1)2x2−1=56 $\frac{(x+1)^2-(x-1)^2}{x^2-1}=\frac{5}{6}$

= 4xx2−1=56 $\frac{4x}{x^2-1}=\frac{5}{6}$

= 6(4x) = 5(x^2-1)

= 24x= 5x^2-5

= 5x^2-24x-5 =0

= 5x^2-25x+x-5 =0

= 5x(x-5)+1(x-5) =0

= (5x+1)(x-5) =0

Either x-5 =0

Therefore x= 5

Or, 5x+1 = 0

Therefore X=−15 $x = \frac{-1}{5}$

The roots of the above mentioned quadratic equation are X=−15 $x = \frac{-1}{5}$ and 5 respectively.

Question 23: Find the roots of the quadratic equation x−12x+1 + 2x+1x−1 = 52

$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = \frac{5}{2}$

Sol:

The equation is x−12x+1 + 2x+1x−1 = 52 $\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = \frac{5}{2}$

= (x−1)2+(2x+1)22x2−2x+x−1 = 52 $\frac{(x-1)^2+(2x+1)^2}{2x^2-2x+x-1} = \frac{5}{2}$

= x2−2x+1+4x2+4x+12x2−x−1 = 52 $\frac{x^2-2x+1+4x^2+4x+1}{2x^2-x-1} = \frac{5}{2}$

= 5x2+2x+22x2−x−1 = 52 $\frac{5x^2+2x+2}{2x^2-x-1} = \frac{5}{2}$

= 2(5x^2+2x+2) = 5(2x^2-x-1)

= 10x^2+4x+4 = 10x^2-5x-5

Cancelling out the equal terms on both sides of the equation. We get,

= 4x+5x+4+5=0

= 9x+9=0

= 9x = -9

X = -1

X = -1 is the only root of the given equation.

Question 24: Find the roots of the quadratic equation mnx^2+nm=1−2x
$\frac{m}{n}x^2+\frac{n}{m}=1-2x$

The given equation is mnx^2+nm=1−2x$\frac{m}{n}x^2+\frac{n}{m}=1-2x$

= mnx^2+nm=1−2x$\frac{m}{n}x^2+\frac{n}{m}=1-2x$

= $m^2x^2+n^2$mn=1−2x$\frac{m^2x^2+n^2}{mn}=1-2x$

= $m^2x^2+2mnx+(n^2-mn)=0$

Now we solve the above quadratic equation using factorization method

Therefore

= (m^2x^2+mnx+m√mnx)+(mnx−m√mnx(n+√mn)(n−√mn))=0
$(m^2x^2+mnx+m\sqrt{mnx})+(mnx-m\sqrt{mnx}(n+\sqrt{mn})(n-\sqrt{mn}))=0$

= (m^2x^2+mnx+m√mnx)+(mx(n−√mn)+(n+√mn)(n−√mn))=0
$(m^2x^2+mnx+m\sqrt{mnx})+(mx(n-\sqrt{mn})+(n+\sqrt{mn})(n-\sqrt{mn}))=0$

= mx(mx+n+√mn)+(n−√mn)(mx+n+√mn)=0
$mx(mx+n+\sqrt{mn})+(n-\sqrt{mn})(mx+n+\sqrt{mn})=0$

= (mx+n+√mn)(mx+n−√mn)=0$(mx+n+\sqrt{mn})(mx+n-\sqrt{mn})=0$

Now, one of the products must be equal to zero for the whole product to be zero for the whole product to be zero. Hence, we equate both the products to zero in order to find the value of x .

Therefore,

(mx+n+√mn)=0$(mx+n+\sqrt{mn})=0$ mx=−n−√mn$mx=-n-\sqrt{mn}$
x=−n−√mnm$x=\frac{-n-\sqrt{mn}}{m}$

Or

(mx+n−√mn)=0$(mx+n-\sqrt{mn})=0$ x=−n+√mnm $x=\frac{-n+\sqrt{mn}}{m}$ x=−n+√mnm $x=\frac{-n+\sqrt{mn}}{m}$

The roots of the above mentioned quadratic equation are x=−n+√mnm $x=\frac{-n+\sqrt{mn}}{m}$ and x=−n−√mnm $x=\frac{-n-\sqrt{mn}}{m}$ respectively.

Question 25: Find the roots of the quadratic equation x−ax−b+x−bx−a=ab+ba

$\frac{x-a}{x-b}+\frac{x-b}{x-a}=\frac{a}{b}+\frac{b}{a}$

Sol:

The given equation is x−ax−b+x−bx−a=ab+ba $\frac{x-a}{x-b}+\frac{x-b}{x-a}=\frac{a}{b}+\frac{b}{a}$

= x−ax−b+x−bx−a=ab+ba $\frac{x-a}{x-b}+\frac{x-b}{x-a}=\frac{a}{b}+\frac{b}{a}$

= (x−a)²+(x−b)²(x−a)(x−b)=ab+ba $\frac{(x-a)^2+(x-b)^2}{(x-a)(x-b)}=\frac{a}{b}+\frac{b}{a}$

= x²−2ax+a²+x²−2bx+b²x²+ab−bx−ax=a²+b²ab $\frac{x^2-2ax+a^2+x^2-2bx+b^2}{x^2+ab-bx-ax}=\frac{a^2+b^2}{ab}$

= $(2x^2-2x(a+b)+a^2+b^2)ab = (a^2+b^2)(x^2-(a+b)x+ab)$

= $(2abx^2-2abx(a+b)+ab(a^2+b^2)) = (a^2+b^2)(x^2-(a^2+b^2)(a+b)x+(a^2+b^2)(ab)$

= $(a^2+b^2-2ab)x-(a+b)(a^2+b^2-2ab)x=0$

= $(a-b)^2x^2-(a+b)(a+b)^2x^2=0$

= $x(a-b)^2(x-(a+b))=0$

= $x(x-(a+b))=0$

Either x= 0

Or, (x-(a+b))=0

Therefore x= a+b

The roots of the above mentioned quadratic equation are 0 and a+b respectively.

Question 26: Find the roots of the quadratic equation 1(x−1)(x−2)+1(x−2)(x−3)+1(x−3)(x−4)=16 $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{1}{6}$

Sol:

The given equation is 1(x−1)(x−2)+1(x−2)(x−3)+1(x−3)(x−4)=16

$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{1}{6}$

= 1(x−1)(x−2)+1(x−2)(x−3)+1(x−3)(x−4)=16 $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{1}{6}$

= (x−3)(x−4)+(x−1)(x−4)+(x−1)(x−2)(x−1)(x−2)(x−3)(x−3)(x−4)=16

$\frac{(x-3)(x-4)+(x-1)(x-4)+(x-1)(x-2)}{(x-1)(x-2)(x-3)(x-3)(x-4)}=\frac{1}{6}$

= (x−3)(x−4)+(x−1)[(x−4)+(x−2)](x−1)(x−2)(x−3)(x−3)(x−4)=16 $\frac{(x-3)(x-4)+(x-1)[(x-4)+(x-2)]}{(x-1)(x-2)(x-3)(x-3)(x-4)}=\frac{1}{6}$

= (x−3)(x−4)+(x−1)(2x−6)(x−1)(x−2)(x−3)(x−3)(x−4)=16 $\frac{(x-3)(x-4)+(x-1)(2x-6)}{(x-1)(x-2)(x-3)(x-3)(x-4)}=\frac{1}{6}$

= (x−3)(x−4)+(x−1)2(x−3)(x−1)(x−2)(x−3)(x−3)(x−4)=16 $\frac{(x-3)(x-4)+(x-1)2(x-3)}{(x-1)(x-2)(x-3)(x-3)(x-4)}=\frac{1}{6}$

= (x−3)[(x−4)+(2x−2)](x−1)(x−2)(x−3)(x−3)(x−4)=16 $\frac{(x-3)[(x-4)+(2x-2)]}{(x-1)(x-2)(x-3)(x-3)(x-4)}=\frac{1}{6}$

= (x−3)(3x−6)(x−1)(x−2)(x−3)(x−3)(x−4)=16 $\frac{(x-3)(3x-6)}{(x-1)(x-2)(x-3)(x-3)(x-4)}=\frac{1}{6}$

= 3(x−3)(x−2)(x−1)(x−2)(x−3)(x−3)(x−4)=16 $\frac{3(x-3)(x-2)}{(x-1)(x-2)(x-3)(x-3)(x-4)}=\frac{1}{6}$

Cancelling out the like terms on both the sides of numerator and denominator. We get,

= 3(x−1)(x−2)(x−4)=16 $\frac{3}{(x-1)(x-2)(x-4)}=\frac{1}{6}$

= (x-1)(x-4) = 18

= x^2-4x-x+4 =18

= x^2-5x-14=0

$= x^2-7x+2x-14=0$

$= x(x-7)+2(x-7)=0$

$= (x-7)(x+2)=0$

Either $x-7 = 0$

Therefore $x=7$

Or, $x+2 =0$

Therefore $x= -2$

The roots of the above mentioned quadratic equation are 7 and -2 respectively.

Question 27: Find the roots of the quadratic equation ax−a+bx−b=2cx−c

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$$

Sol:

The given equation is ax−a+bx−b=2cx−c $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$

= ax−a+bx−b=2cx−c $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$

= a(x−b)+b(x−a)(x−b)(x−a)=2cx−c $\frac{a(x-b)+b(x-a)}{(x-b)(x-a)} = \frac{2c}{x-c}$

= ax−ab+bx−ab(x2−bx−ax+ab)=2cx−c $\frac{ax-ab+bx-ab}{(x^2-bx-ax+ab)} = \frac{2c}{x-c}$

$= (x-c)(ax-2ab+bx) = 2c(x^2-bx-ax+ab)$

$= (a+b)x^2-2abx-(a+b)cx+2abc= 2cx^2-2c(a+b)x+2abc$

Question 28: Find the roots of the Question $x^2+2ab=(2a+b)x$

Sol:

The given equation is $x^2+2ab=(2a+b)x$

$= x^2+2ab = (2a+b)x$

$= x^2-(2a+b)x+2ab = 0$

$= x^2-2ax-bx+2ab = 0$

$= x(x-2a)-b(x-2a) =0$

$= (x-2a)(x-b) =0$

Either $x-2a = 0$

Therefore $x= 2a$

Or, $x-b =0$

Therefore $x= b$

The roots of the above mentioned quadratic equation are 2a and b respectively.

Question 29: Find the roots of the quadratic equation $(a+b)^2x^2-4abx-(a-b)^2 =0$

Sol:

The given equation is $(a+b)^2x^2-4abx-(a-b)^2 =0$

$= (a+b)^2x^2-4abx-(a-b)^2 =0$

$= (a+b)^2x^2-((a+b)^2 -(a-b)^2)x-(a-b)^2 =0$

$= (a+b)^2x^2-(a+b)^2 x +(a-b)^2 x-(a-b)^2 =0$

$= (a+b)^2x(x-1) (a+b)^2 (x-1)=0$

$= (x-1) (a+b)^2x+(a+b)^2) =0$

Either $x-1 =0$

Therefore $x= 1$

Or, $(a+b)^2x+(a+b)^2) =0$

Therefore $-(\text{a-ba+b})^2-(\frac{a-b}{a+b})^2$

The roots of the above mentioned quadratic equation are −(a−ba+b)2$-(\frac{a-b}{a+b})^2$ and 1 respectively .

Question 30: Find the roots of the quadratic equation $a(x^2+1)-x(a^2+1)= 0$

Sol:

The given equation is $a(x^2+1)-x(a^2+1)= 0$

$= a(x^2+1)-x(a^2+1)= 0$

$= ax^2+a-a^2x-x= 0$

$= ax(x-a)-1(x-a) =0$

$= (x-a)(ax-1) =0$

Either $x-a =0$

Therefore $x= a$

Or, $ax-1 =0$

Therefore X=1a$x=\frac{1}{a}$

The roots of the above mentioned quadratic equation are (a) and X=1a$x=\frac{1}{a}$ respectively.

Question 31: Find the roots of the quadratic equation x2+(a+1a)x+1=0 $x^2+(a+\frac{1}{a})x+1=0$.

Sol:

The given equation is x2+(a+1a)x+1=0$x^2+(a+\frac{1}{a})x+1=0$

= x2+(a+1a)x+1=0$x^2+(a+\frac{1}{a})x+1=0$

= x2+ax+xa+(a×1a)=0$x^2+ax+\frac{x}{a}+(a\times\frac{1}{a})=0$

= x(x+a)+1a(x+a)=0$x(x+a)+\frac{1}{a}(x+a)=0$

= (x+a)(x+1a)=0$(x+a)(x+\frac{1}{a})=0$

Either x+a = 0

Therefore x= -a

Or , (x+1a)=0$(x+\frac{1}{a})=0$

Therefore X=1a$x=\frac{1}{a}$

The roots of the above mentioned quadratic equation are X=1a$x=\frac{1}{a}$ and –a respectively.

Question 32: Find the roots of the quadratic equation $abx^2+(b^2-ac)x-bc=0$

Sol:

The given equation is $abx^2+(b^2-ac)x-bc=0$

= $abx^2+(b^2-ac)x-bc=0$

= $abx^2+b^2x-acx-bc=0$

= bx (ax+b)-c (ax+b)=0

= (ax+b)(bx-c) = 0

Either, ax+b = 0

Therefore X=−ba$x=\frac{-b}{a}$

Or, bx-c = 0

Therefore X=cb$x=\frac{c}{b}$

The roots of the above mentioned quadratic equation are X=cb$x=\frac{c}{b}$ and X=−ba$x=\frac{-b}{a}$ respectively.

Question 33: Find the roots of the quadratic equation $a^2b^2x^2+b^2x-a^2x-1=0$

Sol:

The given equation is $a^2b^2x^2+b^2x-a^2x-1=0$

= $a^2b^2x^2+b^2x-a^2x-1=0$

= $b^2x(a^2x+1)-1(a^2x+1)$

= $(a^2x+1)(b^2x-1)=0$

Either $(a^2x+1)=0$

Therefore X=−1a2 $x = \frac{-1}{a^2}$

Or, $(b^2x-1)=0$

Therefore X=1b2 $x = \frac{1}{b^2}$

The roots of the above mentioned quadratic equation are X=1b2 $x = \frac{1}{b^2}$ and X=−1a2 $x = \frac{-1}{a^2}$ respectively.

Exercise 8.4: Quadratic Equations

By using the method of completing the square, find the roots of the following quadratic equations: (1) $\mathbf{x^2 - 4\sqrt{2x} + 6 = 0}$ $x^2 - 4\sqrt{2x} + 6 = 0$

Soln:

$x^2 - 4\sqrt{2x} + 6 = 0$ $x^2 - 4\sqrt{2x} + 6 = 0$

i.e. $x^2 - 2 \times x \times 2\sqrt{2} + (2\sqrt{2})^2 - (2\sqrt{2})^2 + 6 = 0$ $x^2 - 2 \times x \times 2\sqrt{2} + (2\sqrt{2})^2 - (2\sqrt{2})^2 + 6 = 0$

$(x - 2\sqrt{2})^2 = (2\sqrt{2})^2 - 6$ $(x - 2\sqrt{2})^2 = (2\sqrt{2})^2 - 6$

$= (x - 2\sqrt{2})^2 = (4 \times 2) - 6$ $(x - 2\sqrt{2})^2 = (4 \times 2) - 6$

$= (x - 2\sqrt{2})^2 = 8 - 6$ $(x - 2\sqrt{2})^2 = 8 - 6$

$= (x - 2\sqrt{2})^2 = 2$ $(x - 2\sqrt{2})^2 = 2$

$= (x - 2\sqrt{2}) = \pm\sqrt{2}$ $\pm\sqrt{2}$

$= (x - 2\sqrt{2}) = \sqrt{2}$ $\sqrt{2}$ or $(x - 2\sqrt{2}) = -\sqrt{2}$ $\sqrt{2}$

$x = \sqrt{2} + 2\sqrt{2}$ $x = \sqrt{2} + 2\sqrt{2}$ or $x = -\sqrt{2} + 2\sqrt{2}$ $x = -\sqrt{2} + 2\sqrt{2}$

= x=3√2$x = 3\sqrt{2}$ or x=√2$x = \sqrt{2}$

So, the roots for the given equation are :

x=3√2$x = 3\sqrt{2}$ or x=√2$x = \sqrt{2}$

(2) 2x²–7x+3=0$2x^2 - 7x + 3 = 0$

Soln:

2x²–7x+3=0$2x^2 - 7x + 3 = 0$ 2(x²–7x2+32)=0$2(x^2 - \frac{7x}{2} + \frac{3}{2}) = 0$ x²–2×72×12×x+32=0$x^2 - 2 \times \frac{7}{2} \times \frac{1}{2} \times x + \frac{3}{2} = 0$ x²–2×74×x+(74)²–(74)²+32=0
$x^2 - 2 \times \frac{7}{4} \times x + (\frac{7}{4})^2 - (\frac{7}{4})^2 + \frac{3}{2} = 0$ x²–2×74×x+(74)²–(4916)+32=0
$x^2 - 2 \times \frac{7}{4} \times x + (\frac{7}{4})^2 - (\frac{49}{16}) + \frac{3}{2} = 0$ (x–74)²–4916+32=0$(x - \frac{7}{4})^2 - \frac{49}{16} + \frac{3}{2} = 0$ (x–74)²=4916–32$(x - \frac{7}{4})^2 = \frac{49}{16} - \frac{3}{2}$ (x–74)²=49-2616$(x - \frac{7}{4})^2 = \frac{49-26}{16}$ (x–74)²=2516
$(x - \frac{7}{4})^2 = \frac{25}{16}$ (x–74)²=(54)²$(x - \frac{7}{4})^2 = (\frac{5}{4})^2$ x–74=±54$x - \frac{7}{4} = \pm\frac{5}{4}$

x–74=54$x - \frac{7}{4} = \frac{5}{4}$ or x–74=–54$x - \frac{7}{4} = -\frac{5}{4}$

x=74+54$x = \frac{7}{4} + \frac{5}{4}$ or x=74–54$x = \frac{7}{4} - \frac{5}{4}$

x=124$x = \frac{12}{4}$ or x=24$x = \frac{2}{4}$

x = 3 or x = 1/2

(3) 3x²+11x+10=0$3x^2 + 11x + 10 = 0$

Soln: 3x²+11x+10=0$3x^2 + 11x + 10 = 0$

x²+11x3+103=0$x^2 + \frac{11x}{3} + \frac{10}{3} = 0$ x²+2×12×11x3+103=0
$x^2 + 2 \times \frac{1}{2} \times \frac{11x}{3} + \frac{10}{3} = 0$ x²+2×11x6+(116)²–(116)²+103=0
$x^2 + 2 \times \frac{11x}{6} + (\frac{11}{6})^2 - (\frac{11}{6})^2 + \frac{10}{3} = 0$ (x+116)²=(116)²–103$(x + \frac{11}{6})^2 = (\frac{11}{6})^2 - \frac{10}{3}$
(x+116)²=12136–103$(x + \frac{11}{6})^2 = \frac{121}{36} - \frac{10}{3}$ (x+116)²=121–12036$(x + \frac{11}{6})^2 = \frac{121-120}{36}$

(x+ 116)^2= 136 $(x+\frac{11}{6})^2=\frac{1}{36}$ (x+ 116)^2=(16)^2 $(x+\frac{11}{6})^2=(\frac{1}{6})^2$ x+ 116 =± 16
$x+\frac{11}{6}=\pm\frac{1}{6}$

x+ 116 = 16 $x+\frac{11}{6}=\frac{1}{6}$ or x+ 116 =– 16 $x+\frac{11}{6}=-\frac{1}{6}$

x= 16 – 116 $x=\frac{1}{6}-\frac{11}{6}$ or x= -16 – 116 $x=\frac{-1}{6}-\frac{11}{6}$

x= -106 $x=\frac{-10}{6}$ or x= -126 $x=\frac{-12}{6}$ = – 2

x = – 5/3 or x = – 2

(4) 2x^2+x–4=0$2x^2+x-4=0$

Soln: 2x^2+x–4=0$2x^2+x-4=0$

2(x^2+ x2 – 42)=0$2(x^2+\frac{x}{2}-\frac{4}{2})=0$ x^2+2× 12 × 12 ×x–2=0$x^2+2\times\frac{1}{2}\times\frac{1}{2}\times x-2=0$
x^2+2× 14 ×x+(14)^2–(14)^2–2=0$x^2+2\times\frac{1}{4}\times x+(\frac{1}{4})^2-(\frac{1}{4})^2-2=0$ (x+ 14)^2=
(14)^2+2$(x+\frac{1}{4})^2=(\frac{1}{4})^2+2$

(x+ 14)^2$(x+\frac{1}{4})^2$ = (14)^2+2$(\frac{1}{4})^2+2$

(x+ 14)^2$(x+\frac{1}{4})^2$ = 116 +2$\frac{1}{16}+2$

(x+ 14)^2$(x+\frac{1}{4})^2$ = 1+2×1616 $\frac{1+2\times16}{16}$

(x+ 14)^2$(x+\frac{1}{4})^2$ = 1+3216 $\frac{1+32}{16}$

(x+ 14)^2$(x+\frac{1}{4})^2$ = 3316 $\frac{33}{16}$

(x+ 14)$(x+\frac{1}{4})$ = ±√3316 $\pm\sqrt{\frac{33}{16}}$

(x+ 14)$(x+\frac{1}{4})$ = √3316 $\sqrt{\frac{33}{16}}$ or (x+ 14)$(x+\frac{1}{4})$ = – √3316 $\sqrt{\frac{33}{16}}$

x= √334 – 14 $x=\frac{\sqrt{33}}{4}-\frac{1}{4}$ or x=– √334 – 14 $x=-\frac{\sqrt{33}}{4}-\frac{1}{4}$

x= √33–14 $x=\frac{\sqrt{33}-1}{4}$ or x= -√33–14 $x=\frac{-\sqrt{33}-1}{4}$

So, x=√33–14 $x = \frac{\sqrt{33}-1}{4}$ or x=–√33–14 $x = \frac{-\sqrt{33}-1}{4}$

Are the two roots of the given equation.

(5) 2x²+x+4=0$2x^2 + x + 4 = 0$

Soln: 2x²+x+4=0$2x^2 + x + 4 = 0$

x²+x2+2=0$x^2 + \frac{x}{2} + 2 = 0$ x²+2×12×12×x+2=0$x^2 + 2 \times \frac{1}{2} \times \frac{1}{2} \times x + 2 = 0$

x²+2×14×x+(14)²–(14)²+2=0$x^2 + 2 \times \frac{1}{4} \times x + (\frac{1}{4})^2 - (\frac{1}{4})^2 + 2 = 0$ x²+2×14×x+

(14)²=(14)²–2$x^2 + 2 \times \frac{1}{4} \times x + (\frac{1}{4})^2 = (\frac{1}{4})^2 - 2$ (x+14)²=116–2$(x + \frac{1}{4})^2 = \frac{1}{16} - 2$

(x+14)²$(x + \frac{1}{4})^2$ = 1−3216 $\frac{1-32}{16}$

(x+14)²$(x + \frac{1}{4})^2$ = −3116 $\frac{-31}{16}$

(x+14 $(x + \frac{1}{4}$ = ±√−3116$\pm\sqrt{\frac{-31}{16}}$

(x+14 $(x + \frac{1}{4}$ = √−314 $\frac{\sqrt{-31}}{4}$ or

(x+14 $(x + \frac{1}{4}$ = −√−314 $\frac{-\sqrt{-31}}{4}$

x=√−31−14 $x = \frac{\sqrt{-31}-1}{4}$ or x=−√−31−14 $x = \frac{-\sqrt{-31}-1}{4}$

Since, √−31$\sqrt{-31}$ is not a real number,

Therefore, the equation doesn't have real roots.

(6) 4x²+4√3x+3=0$4x^2 + 4\sqrt{3}x + 3 = 0$

Soln: 4x²+4√3x+3=0$4x^2 + 4\sqrt{3}x + 3 = 0$

x2+4√3x4+34=0$x^2+\frac{4\sqrt{3}x}{4}+\frac{3}{4}=0$ x2+2×12×√3×x+34=0

$x^2+2\times\frac{1}{2}\times\sqrt{3}\times x+\frac{3}{4}=0$ x2+2×√32×x+(√32)2–(√32)2+34=0

$x^2+2\times\frac{\sqrt{3}}{2}\times x+(\frac{\sqrt{3}}{2})^2-(\frac{\sqrt{3}}{2})^2+\frac{3}{4}=0$ (x+√32)2–34+34=0

$(x+\frac{\sqrt{3}}{2})^2-\frac{3}{4}+\frac{3}{4}=0$

(x+√32)2$(x+\frac{\sqrt{3}}{2})^2$ = 0

(x+√32)$(x+\frac{\sqrt{3}}{2})$ = 0 and (x+√32)$(x+\frac{\sqrt{3}}{2})$ = 0

x=-√32$x=\frac{-\sqrt{3}}{2}$ and x=-√32$x=\frac{-\sqrt{3}}{2}$

Therefore, x=-√32$x=\frac{-\sqrt{3}}{2}$ and x=-√32$x=\frac{-\sqrt{3}}{2}$

Are the real roots of the given equation.

(7) √2x2–3x–2√2=0$\sqrt{2}x^2-3x-2\sqrt{2}=0$

Soln: √2x2–3x–2√2=0$\sqrt{2}x^2-3x-2\sqrt{2}=0$

x2–3x√2–2√2√2=0$x^2-\frac{3x}{\sqrt{2}}-\frac{2\sqrt{2}}{\sqrt{2}}=0$ x2–3x√2–2=0$x^2-\frac{3x}{\sqrt{2}}-2=0$ x2–2×12×3x√2–2=0

$x^2-2\times\frac{1}{2}\times\frac{3x}{\sqrt{2}}-2=0$ x2–2×3x2√2+(32√2)2–(32√2)2–2=0

$x^2-2\times\frac{3x}{2\sqrt{2}}+(\frac{3}{2\sqrt{2}})^2-(\frac{3}{2\sqrt{2}})^2-2=0$ (x–32√2)2=98+2$(x-\frac{3}{2\sqrt{2}})^2=\frac{9}{8}+2$

(x–32√2)2=9+168$(x-\frac{3}{2\sqrt{2}})^2=\frac{9+16}{8}$ (x–32√2)2=258$(x-\frac{3}{2\sqrt{2}})^2=\frac{25}{8}$

(8) √3x2+10x+7√3=0$\sqrt{3}x^2+10x+7\sqrt{3}=0$

Soln: √3x2+10x+7√3=0$\sqrt{3}x^2+10x+7\sqrt{3}=0$

x2+10x√3+7√3√3=0$x^2+\frac{10x}{\sqrt{3}}+\frac{7\sqrt{3}}{\sqrt{3}}=0$ x2+2×12×10x√3+7=0

$x^2+2\times\frac{1}{2}\times\frac{10x}{\sqrt{3}}+7=0$ (x+5√3)2=253–7$(x+\frac{5}{\sqrt{3}})^2=\frac{25}{3}-7$ (x+5√3)2=25–213

$(x+\frac{5}{\sqrt{3}})^2=\frac{25-21}{3}$ (x+5√3)²=43 $(x+\frac{5}{\sqrt{3}})^2=\frac{4}{3}$ x+5√3=±√43 $x+\frac{5}{\sqrt{3}}=\pm\sqrt{\frac{4}{3}}$

x+5√3=+2√3 or x+5√3=−2√3 $x+\frac{5}{\sqrt{3}}=+\frac{2}{\sqrt{3}} or x+\frac{5}{\sqrt{3}}=-\frac{2}{\sqrt{3}}$ x=-3√3 or x=-7√3

$x=\frac{-3}{\sqrt{3}} or x=\frac{-7}{\sqrt{3}}$ x=−√3 and x=-7√3 $x=-\sqrt{3} and x=\frac{-7}{\sqrt{3}}$

(9) x²–(√2+1)x+√2=0 $x^2-(\sqrt{2}+1)x+\sqrt{2}=0$

Soln: x²–(√2+1)x+√2=0 $x^2-(\sqrt{2}+1)x+\sqrt{2}=0$

x²–2×12(√2+1)x+√2=0 $x^2-2\times\frac{1}{2}(\sqrt{2}+1)x+\sqrt{2}=0$ x²–2×√2+12x+(√2+12)²–(√2+12)²+√2=0 $x^2-2\times\frac{\sqrt{2}+1}{2}x+(\frac{\sqrt{2}+1}{2})^2-(\frac{\sqrt{2}+1}{2})^2+\sqrt{2}=0$ (x–√2+12)²=(√2+12)²–√2 $(x-\frac{\sqrt{2}+1}{2})^2=(\frac{\sqrt{2}+1}{2})^2-\sqrt{2}$

(x–√2+12)² $(x-\frac{\sqrt{2}+1}{2})^2$ = (2+2√2+1)4–√2 $\frac{(2+2\sqrt{2}+1)}{4}-\sqrt{2}$

(x–√2+12)² $(x-\frac{\sqrt{2}+1}{2})^2$ = (3+2√2–4√2)4 $\frac{(3+2\sqrt{2}-4\sqrt{2})}{4}$

(x–√2+12)² $(x-\frac{\sqrt{2}+1}{2})^2$ = (3–2√2)4 $\frac{(3-2\sqrt{2})}{4}$

(x–√2+12)² $(x-\frac{\sqrt{2}+1}{2})^2$ = (2+1–2√2)4 $\frac{(2+1-2\sqrt{2})}{4}$

(x–√2+12)² $(x-\frac{\sqrt{2}+1}{2})^2$ = (√2)²+1–2√2(2)² $\frac{(\sqrt{2})^2+1-2\sqrt{2}}{(2)^2}$

(x–√2+12)² $(x-\frac{\sqrt{2}+1}{2})^2$ = (√2–1)²(2)² $\frac{(\sqrt{2}-1)^2}{(2)^2}$

(x–√2+12)² $(x-\frac{\sqrt{2}+1}{2})^2$ = (√2–12)² $(\frac{\sqrt{2}-1}{2})^2$

x–√2+12 $x-\frac{\sqrt{2}+1}{2}$ = ±$\pm$ (√2–12) $(\frac{\sqrt{2}-1}{2})$

x–√2+12 $x-\frac{\sqrt{2}+1}{2}$ = (√2–12) $(\frac{\sqrt{2}-1}{2})$

Or x–√2+12 $x-\frac{\sqrt{2}+1}{2}$ = (-√2–12) $(\frac{-\sqrt{2}-1}{2})$

x=√2+12$x = \frac{\sqrt{2}+1}{2}$ + (√2−12)($\frac{\sqrt{2}-1}{2}$)

Or x=√2+12$x = \frac{\sqrt{2}+1}{2}$ + (−√2−12)($\frac{-\sqrt{2}-1}{2}$)

x=2√22orx=22$x = \frac{2\sqrt{2}}{2} or x = \frac{2}{2}$

x=√2$x = \sqrt{2}$ or x = 1.

(10) $x^2 - 4ax + 4a^2 - b^2 = 0$

Soln: $x^2 - 4ax + 4a^2 - b^2 = 0$

$x^2 - 2(2a).x + (2a)^2 - b^2 = 0$

$(x - 2a)^2 = b^2$

x – 2a = ±$\pm$ b

x – 2a = b or x – 2a = – b

x = 2a + b or x = 2a – b

Therefore, x = 2a + b or x = 2a – b are the two roots of the given equation.

Exercise 8.5: Quadratic Equations

Q.1: Find the discriminant of the following quadratic equations :

1: $2x^2 - 5x + 3 = 0$

Soln: $2x^2 - 5x + 3 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

Here, a = 2 , b = -5 and c = 3

The discriminant, $D = b^2 - 4ac$

$D = (-5)^2 - 4 \times 2 \times 3$

$D = 25 - 24 = 1$

Therefore, the discriminant of the following quadratic equation is 1.

2) $x^2 + 2x + 4 = 0$

Soln: $x^2 + 2x + 4 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

Here, a = 1 , b = 2 and c = 4

The discriminant is :-

$D = (2)^2 – 4 \times 1 \times 4$

$D = 4 – 16 = – 12$

The discriminant of the following quadratic equation is = – 12.

3) (x -1) (2x -1) = 0

Soln: (x -1) (2x -1) = 0

The provided equation is (x -1) (2x -1) = 0

By solving it, we get $2x^2 – 3x + 1 = 0$

Now this equation is in the form of $ax^2 + bx + c = 0$

Here, a = 2 , b = -3 , c = 1

The discriminant is :-

$D = (-3)^2 – 4 \times 2 \times 1$

$D = 9 – 8 = 1$

The discriminant of the following quadratic equation is = 1.

4) $x^2 – 2x + k = 0$

Soln: $x^2 – 2x + k = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

Here, a = 1 , b = -2 , and c = k

$D = b^2 - 4ac$

$D = (-2)^2 - 4(1)(k)$

$= 4 - 4k$

Therefore, the discriminant, D of the equation is $(4 - 4k)$

5) $\sqrt{3}x^2+2\sqrt{2}x-2\sqrt{3}=0$ $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

Soln: $\sqrt{3}x^2+2\sqrt{2}x-2\sqrt{3}=0$ $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

here $a=\sqrt{3}$, $b=2\sqrt{2}$ x and $c=-2\sqrt{3}$ $here a = \sqrt{3}, b = 2\sqrt{2} x and c = -2\sqrt{3}$

The discriminant is, $D = b^2 - 4ac$

$(2\sqrt{2})^2-(4\times\sqrt{3}\times-2\sqrt{3})$ $(2\sqrt{2})^2 - (4 \times \sqrt{3} \times -2\sqrt{3})$

$D = 8 + 24 = 32$

The discriminant, D of the following equation is 32.

6) $x^2 - x + 1 = 0$

Soln: $x^2 - x + 1 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$

Here, $a = 1$, $b = -1$ and $c = 1$

The discriminant is $D = b^2 - 4ac$

$(-1)^2 - 4 \times 1 \times 1$

$1 - 4 = -3$

Therefore, The discriminant D of the following equation is -3.

Q.2: 1) $16x^2 = 24x + 1$

Soln: $16x^2 - 24x - 1 = 0$

The given equation can be written in the form of, $ax^2 + bx + c = 0$

the discriminant is given by the following equation, $D = b^2 - 4ac$

here, a = 16 , b = -24 and c = – 1

Therefore, the discriminant is given as,

$D = (-24)^2 - 4(16)(-1)$

= 576 + 64

= 640

For a quadratic equation to have real roots, D ≥$\geq$ 0.

Here it can be seen that the equation satisfies this condition, hence it has real roots.

the roots of an equation can be found out by using,

x=−b±√D2a $x = \frac{-b\pm\sqrt{D}}{2a}$

Therefore, the roots of the following equation are as follows,

x=−(−24)±√6402(16) $x = \frac{-(-24)\pm\sqrt{640}}{2(16)}$ x=24±8√1032 $x = \frac{24\pm8\sqrt{10}}{32}$ x=3±√104 $x = \frac{3\pm\sqrt{10}}{4}$

The values of x for both the cases will be :

x=3+√104 $x = \frac{3+\sqrt{10}}{4}$ and,

x=3−√104 $x = \frac{3-\sqrt{10}}{4}$

2) $x^2 + x + 2 = 0$

Soln: $x^2 + x + 2 = 0$

The given equation can be written in the form of, $ax^2 + bx + c = 0$

the discriminant is given by the following equation, $D = b^2 - 4ac$

here, a = 1, b = 1 and c = 2.

Therefore, the discriminant is given as,

$D = (1)^2 - 4(1)(2)$

$= 1 - 8$

$= -7$

For a quadratic equation to have real roots, $D \geq\geq 0$.

Here we find that the equation does not satisfy this condition, hence it does not have real roots.

3) $\mathbf{\sqrt{3}x^2+10x-8\sqrt{3}=0}\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Soln: $\sqrt{3}x^2+10x-8\sqrt{3}=0\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

The given equation can be written in the form of, $ax^2 + bx + c = 0$

the discriminant is given by the following equation, $D = b^2 - 4ac$

here, $a = \sqrt{3}\sqrt{3}$, b = 10 and $c = -8\sqrt{3}-8\sqrt{3}$

Therefore, the discriminant is given as,

$D = (10)^2 - 4(\sqrt{3}\sqrt{3})(-8\sqrt{3}-8\sqrt{3}) = 100 + 96 = 196$

For a quadratic equation to have real roots, $D \geq\geq 0$

Here it can be seen that the equation satisfies this condition, hence it has real roots.

the roots of an equation can be found out by using,

$x = -b\pm\sqrt{D}2a\, x = \frac{-b\pm\sqrt{D}}{2a}$

Therefore, the roots of the equation are given as follows,

$x = -10\pm\sqrt{196}2\sqrt{3}\, x = \frac{-10\pm\sqrt{196}}{2\sqrt{3}}$ $x = -10\pm142\sqrt{3}\, x = \frac{-10\pm14}{2\sqrt{3}}$ $x = -5\pm7\sqrt{3}\, x = \frac{-5\pm7}{\sqrt{3}}$

The values of x for both the cases will be :

x=-5+7√3 $x = \frac{-5+7}{\sqrt{3}}$

x=2√3 $x = \frac{2}{\sqrt{3}}$ and,

x=-5-7√3 $x = \frac{-5-7}{\sqrt{3}}$ x=-4√3 $x = -4\sqrt{3}$

4) $3x^2 - 2x + 2 = 0$

Soln: $3x^2 - 2x + 2 = 0$

The given equation can be written in the form of, $ax^2 + bx + c = 0$

the discriminant is given by the following equation, $D = b^2 - 4ac$

here, a = 3 , b = -2 and c = 2.

Therefore, the discriminant is given as,

$D = (-2)^2 - 4(3)(2)$

$= 4 - 24 = -20$

For a quadratic equation to have real roots, D ≥ $\geq$ 0

Here it can be seen that the equation does not satisfy this condition, hence it has no real roots.

5) 2x²–2√6x+3=0 $2x^2 - 2\sqrt{6}x + 3 = 0$

Soln: 2x²–2√6x+3=0 $2x^2 - 2\sqrt{6}x + 3 = 0$

The given equation can be written in the form of, $ax^2 + bx + c = 0$

the discriminant is given by the following equation, $D = b^2 - 4ac$

here, a = 2 , b = −2√6 $-2\sqrt{6}$ and c =3.

Therefore, the discriminant is given as, D = $(-2\sqrt{6})^2 - 4(2)(3)$ $(-2\sqrt{6})^2 - 4(2)(3)$

= 24 – 24 = 0

For a quadratic equation to have real roots, D ≥ $\geq$ 0

Here it can be seen that the equation satisfies this condition, hence it has real roots.

the roots of an equation can be found out by using,

x=-b±√D2a $x = \frac{-b\pm\sqrt{D}}{2a}$ x=-(2√6)±02(2) $x = \frac{-(2\sqrt{6})\pm 0}{2(2)}$ x=-(√6)2 $x = \frac{-(\sqrt{6})}{2}$ x=−√32

$x = -\sqrt{\frac{3}{2}}$

6) $3a^2x^2 + 8abx + 4b^2 = 0$

Soln: $3a^2x^2 + 8abx + 4b^2 = 0$

The given equation can be written in the form of, $ax^2 + bx + c = 0$

the discriminant is given by the following equation, $D = b^2 - 4ac$

here, $a = 3a^2$, $b = 8ab$ and $c = 4b^2$

Therefore, the discriminant is given as,

$D = (8ab)^2 - 4(3a^2)(4b^2)$

$= 64a^2b^2 - 48a^2b^2 = 16a^2b^2$

For a quadratic equation to have real roots, D ≥ $\geq$ 0

Here it can be seen that the equation satisfies this condition, hence it has real roots.

the roots of an equation can be found out by using,

x=-b±√D2a $x = \frac{-b\pm\sqrt{D}}{2a}$

Therefore, the roots of the equation are given as follows,

x=-(8ab)±√16a²b²2(3a²) $x = \frac{-(8ab)\pm\sqrt{16a^2b^2}}{2(3a^2)}$ x=-(8ab)±4ab6a² $x = \frac{-(8ab)\pm 4ab}{6a^2}$ x=-(4b)±2b3a

$x = \frac{-(4b)\pm 2b}{3a}$

The values of x for both the cases will be :

x=−(4b)+2b3a $x = \frac{-(4b)+2b}{3a}$

x=−(2b)3a $x = \frac{-(2b)}{3a}$ and

x=−(4b)−2b3a $x = \frac{-(4b)-2b}{3a}$ x=−2ba $x = \frac{-2b}{a}$

7.) 3x²+2√5x–5=0 $3x^2 + 2\sqrt{5}x - 5 = 0$

Soln.: 3x²+2√5x–5=0 $3x^2 + 2\sqrt{5}x - 5 = 0$

The given equation can be written in the form of, $ax^2 + bx + c = 0$

the discriminant is given by the following equation, $D = b^2 - 4ac$

here, a = 3, b = 2√5 $2\sqrt{5}$ and c = – 5.

Therefore, the discriminant is given as,

D = (2√5)²–4(3)(−5) $(2\sqrt{5})^2 - 4(3)(-5)$

= 20 + 60

= 80

For a quadratic equation to have real roots, D ≥ $\geq$ 0

Here it can be seen that the equation satisfies this condition, hence it has real roots.

the roots of an equation can be found out by using,

x=−b±√D2a $x = \frac{-b \pm \sqrt{D}}{2a}$ x=−(2√5)±√802(3) $x = \frac{-(2\sqrt{5}) \pm \sqrt{80}}{2(3)}$ x=−(2√5)±4√52(3) $x = \frac{-(2\sqrt{5}) \pm 4\sqrt{5}}{2(3)}$ x=−sqrt5±2√53 $x = \frac{-sqrt5 \pm 2\sqrt{5}}{3}$

The values of x for both the cases will be :

x=−sqrt5 +2√53 $x = \frac{-sqrt5 + 2\sqrt{5}}{3}$ x=sqrt53 $x = \frac{sqrt5}{3}$

And,

X=-sqrt5 -2√53 $x = \frac{-sqrt5 - 2\sqrt{5}}{3}$

x = – √5 $\sqrt{5}$

8.) $x^2 - 2x + 1 = 0$

Soln.: $x^2 - 2x + 1 = 0$

The given equation can be written in the form of, $ax^2 + bx + c = 0$

the discriminant is given by the following equation, $D = b^2 - 4ac$

here, a =1, b = – 2 and c = 1

Therefore, the discriminant is given as,

$D = (-2)^2 - 4(1)(1)$

$= 4 - 4$

$= 0$

For a quadratic equation to have real roots, D ≥ $\geq$ 0

Here it can be seen that the equation satisfies this condition, hence it has real roots.

the roots of an equation can be found out by using,

X=-b±√D2a $x = \frac{-b \pm \sqrt{D}}{2a}$ X=-(-2)±√02(1) $x = \frac{-(-2) \pm \sqrt{0}}{2(1)}$

x = 2/2

x = 1

Therefore, the equation real roots and its value is 1

9.) 2x²+5√3x+6=0 $2x^2 + 5\sqrt{3}x + 6 = 0$

Soln.: 2x²+5√3x+6=0 $2x^2 + 5\sqrt{3}x + 6 = 0$

The given equation can be written in the form of, $ax^2 + bx + c = 0$

the discriminant is given by the following equation, $D = b^2 - 4ac$

here, $a = 2$, $b = 5\sqrt{3}5\sqrt{3}$ and $c = 6$.

Therefore, the discriminant is given as,

$D = (5\sqrt{3})^2(5\sqrt{3})^2 - 4(2)(6)$

$= 75 - 48$

$= 27$

For a quadratic equation to have real roots, $D \geq \geq 0$

Here it can be seen that the equation satisfies this condition, hence it has real roots.

the roots of an equation can be found out by using,

$x = -b\pm\sqrt{D}2a \quad x = \frac{-b\pm\sqrt{D}}{2a}$

Therefore, the roots of the equation are given as follows

$x = -(5\sqrt{3})\pm\sqrt{27}2(2) \quad x = \frac{-(5\sqrt{3})\pm\sqrt{27}}{2(2)} \quad x = -(5\sqrt{3})\pm3\sqrt{3}4 \quad x = \frac{-(5\sqrt{3})\pm3\sqrt{3}}{4}$

The values of x for both the cases will be :

$x = -(5\sqrt{3})+3\sqrt{3}4 \quad x = \frac{-(5\sqrt{3})+3\sqrt{3}}{4} \quad x = -\sqrt{3}2 \quad x = \frac{-\sqrt{3}}{2}$

And,

$x = -(5\sqrt{3})-3\sqrt{3}4 \quad x = \frac{-(5\sqrt{3})-3\sqrt{3}}{4} \quad x = -2\sqrt{3} \quad x = -2\sqrt{3}$

10.) $\mathbf{\sqrt{2}x^2+7x+5\sqrt{2}=0} \quad \sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Soln.: $\sqrt{2}x^2+7x+5\sqrt{2}=0 \quad \sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

The given equation can be written in the form of, $ax^2 + bx + c = 0$

the discriminant is given by the following equation, $D = b^2 - 4ac$

here, a = √2 $\sqrt{2}$, b = 7 , c = 5√2 $5\sqrt{2}$

Therefore, the discriminant is given as,

D = (7)²–4(√2)(5√2) $(7)^2 - 4(\sqrt{2})(5\sqrt{2})$

D = 49 – 40

D = 9

For a quadratic equation to have real roots, D ≥ $\geq$ 0

Here it can be seen that the equation satisfies this condition, hence it has real roots.

the roots of an equation can be found out by using,

x=−b±√D2a $x = \frac{-b \pm \sqrt{D}}{2a}$

Therefore, the roots of the equation are given as follows,

x=−(7)±√92(√2) $x = \frac{-(7) \pm \sqrt{9}}{2(\sqrt{2})}$ x=−7±32(√2) $x = \frac{-7 \pm 3}{2(\sqrt{2})}$

The values of x for both the cases will be :

x=−7+32(√2) $x = \frac{-7+3}{2(\sqrt{2})}$ x=−√2 $x = -\sqrt{2}$

And

x=−7–32(√2) $x = \frac{-7-3}{2(\sqrt{2})}$ x=−5√2 $x = -\frac{5}{\sqrt{2}}$

11.) 2x²–2√2x+1=0 $2x^2 - 2\sqrt{2}x + 1 = 0$

Soln.: 2x²–2√2x+1=0 $2x^2 - 2\sqrt{2}x + 1 = 0$

The given equation can be written in the form of, $ax^2 + bx + c = 0$

the discriminant is given by the following equation, $D = b^2 - 4ac$

here, a = 2 , b = –2√2 $-2\sqrt{2}$, c = 1

Therefore, the discriminant is given as,

$D = (-2\sqrt{2})^2 - 4(2)(1)(-2\sqrt{2})^2 - 4(2)(1)$

= 8 – 8

= 0

For a quadratic equation to have real roots, D ≥$\geq$ 0

Here it can be seen that the equation satisfies this condition, hence it has real roots.

the roots of an equation can be found out by using,

x=-b±√D2a $x = \frac{-b \pm \sqrt{D}}{2a}$

Therefore, the roots of the equation are given as follows,

x=-(-2√2)±√02(2) $x = \frac{-(-2\sqrt{2}) \pm \sqrt{0}}{2(2)}$ x=2√24 $x = \frac{2\sqrt{2}}{4}$ x=1√2 $x = \frac{1}{\sqrt{2}}$

12.) $3x^2 - 5x + 2 = 0$

Soln.: $3x^2 - 5x + 2 = 0$

The given equation can be written in the form of, $ax^2 + bx + c = 0$

the discriminant is given by the following equation, $D = b^2 - 4ac$

here, a = 3, b = -5 and c = 2.

Therefore, the discriminant is given as,

$D = (-5)^2 - 4(3)(2)$

= 25 – 24

= 1

For a quadratic equation to have real roots, D ≥$\geq$ 0

Here it can be seen that the equation satisfies this condition, hence it has real roots.

the roots of an equation can be found out by using,

x=-b±√D2a $x = \frac{-b \pm \sqrt{D}}{2a}$

Therefore, the roots of the equation are given as follows,

X=-(-5)±√12(3) $x = \frac{-(-5)\pm\sqrt{1}}{2(3)}$ X=5±16 $x = \frac{5\pm1}{6}$

The values of x for both the cases will be :

X=5+16 $x = \frac{5+1}{6}$

x = 1

And,

X=5–16 $x = \frac{5-1}{6}$

x = 2/3

Q.3) Solve for x : 1.) **x-1x-2+x-3x-4=3 13 ,x≠2,4** $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}, x \neq 2, 4.$

Soln.: x-1x-2+x-3x-4=3 13 ,x≠2,4 $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}, x \neq 2, 4$

The above equation can be solved as follows:

(x−1)(x−4)+(x−3)(x−2)(x−2)(x−4)=103 $\frac{(x-1)(x-4)+(x-3)(x-2)}{(x-2)(x-4)} = \frac{10}{3}$ x2–5x+4+x2–5x+6x2–6x+8=103 $\frac{x^2-5x+4+x^2-5x+6}{x^2-6x+8} = \frac{10}{3}$

$6x^2 - 30x + 30 = 10x^2 - 60x + 80$

$4x^2 - 30x + 50 = 0$

$2x^2 - 15x + 25 = 0$

The above equation is in the form of $ax^2 + bx + c = 0$

The discriminant is given by the equation, $D = b^2 - 4ac$

Here, a = 2 , b = -15 , c = 25

$D = (-15)^2 - 4(2)(25)$

= 225 – 200

= 25

the roots of an equation can be found out by using,

x=-b±√D2a $x = \frac{-b\pm\sqrt{D}}{2a}$

Therefore, the roots of the equation are given as follows,

x=-(-15)±√252(2) $x = \frac{-(-15)\pm\sqrt{25}}{2(2)}$ x=15±54 $x = \frac{15\pm5}{4}$

The values of x for both the cases will be :

x=15+54 $x = \frac{15+5}{4}$

X = 5

Also,

x=15–54 $x = \frac{15-5}{4}$ x=52 $x = \frac{5}{2}$

2) x+1x=3,x≠0 $x + \frac{1}{x} = 3, x \neq 0$

Soln.: x+1x=3,x≠0 $x + \frac{1}{x} = 3, x \neq 0$

The above equation can be solved as follows:

x2+1x=3 $\frac{x^2+1}{x} = 3$

$X^2 + 1 = 3x$

$X^2 - 3x + 1 = 0$

The above equation is in the form of $ax^2 + bx + c = 0$

The discriminant is given by the equation, $D = b^2 - 4ac$

Here, a = 1 , b = – 3 , c = 1

$D = (-3)^2 - 4(1)(1)$

D = 9 – 4

D = 5

the roots of an equation can be found out by using,

x=-b±√D2a $x = \frac{-b \pm \sqrt{D}}{2a}$ x=-(-3)±√52(1) $x = \frac{-(-3) \pm \sqrt{5}}{2(1)}$ x=3±√52 $x = \frac{3 \pm \sqrt{5}}{2}$

The values of x for both the cases will be :

x=3 +√52 $x = \frac{3 + \sqrt{5}}{2}$

And, x=3 –√52 $x = \frac{3 - \sqrt{5}}{2}$

3.) **16x–1=15x+1 ,x≠0,–1** $\frac{16}{x} - 1 = \frac{15}{x+1}, x \neq 0, -1$

Soln. : 16x–1=15x+1 ,x≠0,–1 $\frac{16}{x} - 1 = \frac{15}{x+1}, x \neq 0, -1$

The above equation can be solved as follows:

16–xx=15x+1 $\frac{16-x}{x} = \frac{15}{x+1}$

$(16 - x)(x + 1) = 15x$

$16x + 16 - x^2 - x = 15x$

$15x + 16 - x^2 - 15x = 0$

$16 - x^2 = 0$

$X^2 - 16 = 0$

The above equation is in the form of $ax^2 + bx + c = 0$

The discriminant is given by the equation, $D = b^2 - 4ac$

Here, a = 1 , b = 0 , c = -16

$D = (0)^2 - 4(1)(-16)$

D = 64

the roots of an equation can be found out by using,

x=-b±√D2a $x = \frac{-b \pm \sqrt{D}}{2a}$

Therefore, the roots of the equation are given as follows,

x=−0±√642(1) $x = \frac{-0\pm\sqrt{64}}{2(1)}$ x=±82 $x = \frac{\pm 8}{2}$ x=±4 $x = \pm 4$

Exercise 8.6: Quadratic Equations

Question 1: Determine the nature of the roots of the following quadratic equations.

Solution: (i) $2x^2 -3x + 5 =0$

The given quadratic equation is in the form of $ax^2 +bx +c =0$

So a= 2, b= -3, c= 5

We know, determinant (D) = $b^2 – 4ac$

= $(-3)^2 -4(2)(5)$

= 9 – 40

= -31<0

Since D<0, the determinant of the equation is negative, so the expression does not having any real roots.

(ii) $2x^2 -6x + 3=0$

The given quadratic equation is in the form of $ax^2 + bx + c = 0$

So a= 2, b= -6, c= 3

We know, determinant (D) = $b^2 - 4ac$

$= (-6)^2 - 4(2)(3)$

$= 36 - 24$

$= 12 < 0$

Since D>0, the determinant of the equation is positive, so the expression does having any real and distinct roots.

(iii) For what value of k $(4-k)x^2 + (2k+4)x + (8k+1) = 0$ is a perfect square.

The given equation is $(4-k)x^2 + (2k+4)x + (8k+1) = 0$

Here, a= 4-k, b= 2k+4, c= 8k+1

The discriminate (D) = $b^{2-}4ac$

$= (2k+4)^2 - 4(4-k)(8k+1)$

$= (4k^2 + 16 + 16k) - 4(32k + 4 - 8k^2 - k)$

$= 4(k^2 + 8k^2 + 4k - 31k + 4 - 4)$

$= 4(9k^2 - 27k)$

$D = 4(9k^2 - 27k)$

The given equation is a perfect square

D= 0

$4(9k^2 - 27k) = 0$

$9k^2 - 27k = 0$

Taking out common of of 3 from both sides and cross multiplying

= k^2 -3k =0

= K (k-3) =0

Either k=0

Or k =3

The value of k is to be 0 or 3 in order to be a perfect square.

(iv) Find the least positive value of k for which the equation x^2+kx+4=0 has real roots.

The given equation is x^2+kx+4=0 has real roots

Here, a= 1, b= k, c= 4

The discriminate (D) = $b^2 - 4ac \geq 0$

= $k^2 - 16 \geq 0$

= $k \leq 4$,$k \geq -4$

The least positive value of k =4 for the given equation to have real roots.

(v) Find the value of k for which the given quadratic equation has real roots and distinct roots.

Kx^2+ 2x+1 =0

The given equation is Kx^2+ 2x+1 =0

Here, a= k, b= 2, c= 1

The discriminate (D) = $b^2 - 4ac \geq 0$

= $4 - 4k \geq 0 = 4k \leq 4$

$K \leq 1$

The value of $k \leq 1$ for which the quadratic equation is having real and equal roots.

(vi) Kx^2 +6x+1 = 0

The given equation is $Kx^2 + 6x+1 = 0$

Here, a= k, b= 6, c= 1

The discriminate (D) = $b^2 - 4ac \geq 0$

= $36 - 4k \geq 0$

= $4k \leq 36$

$K \leq 9$

The value of $k \leq 9$ for which the quadratic equation is having real and equal roots.

(vii) $x^2 - kx + 9 = 0$

The given equation is $X^2 - kx + 9 = 0$

Here, a= 1, b= -k, c= 9

Given that the equation is having real and distinct roots.

Hence, the discriminate (D) = $b^2 - 4ac \geq 0$

= $k^2 - 4(1)(9) \geq 0$

= $k^2 - 36 \geq 0$

= $k \geq -6$ and $k \leq 6$

The value of k lies between -6 and 6 respectively to have the real and distinct roots.

Question 2: Find the value of k.

(i) $Kx^2 + 4x + 1 = 0$

The given equation $Kx^2 + 4x + 1 = 0$ is in the form of $ax^2 + bx + c = 0$ where a= k, b= 4, c= 1

Given that, the equation has real and equal roots

D= b^2-4ac=0

= 4^2-4(k)(1)=0

= 16-4k=0

= k = 4

The value of k is 4

(ii) $kx^2-2\sqrt{5}x+4=0$ $kx^2 - 2\sqrt{5}x + 4 = 0$

The given equation $kx^2-2\sqrt{5}x+4=0$ $kx^2 - 2\sqrt{5}x + 4 = 0$ is in the form of ax^2+bx+c=0 where

a= k , b= $-2\sqrt{5}$ $-2\sqrt{5}$, c= 4

Given that, the equation has real and equal roots

D= b^2-4ac=0

= $-2\sqrt{5}^2-4\times k\times 4=0$ $-2\sqrt{5}^2 - 4 \times k \times 4 = 0$

= 20-16k =0

= k=54 $k = \frac{5}{4}$

The value of k is k=54 $k = \frac{5}{4}$

(iii) $3x^2$-5x+2k =0

The given equation $3x^2$-5x+2k=0 is in the form of ax^2+bx+c=0 where a= 3, b= -5, c= 2k

Given that, the equation has real and equal roots

D= b^2-4ac=0

= $(-5)^2$-4(3)(2k)=0

= 25-24k =0

K = k=2524 $k = \frac{25}{24}$

The value of the k is k=2524 $k = \frac{25}{24}$

(iv) $4x^2$+kx+9=0

The given equation $4x^2$+kx+9=0 is in the form of ax^2+bx+c=0 where a= 4, b= k, c= 9

Given that, the equation has real and equal roots

D= b^2-4ac=0

= k^2-4(4)(9)=0

= k^2 -144=0

= k = 12

The value of k is 12

(v) $2kx^2$-40x+25=0

The given equation $2kx^2$-40x+25=0 is in the form of ax^2+bx+c=0 where a= 2k, b= -40, c= 25

Given that, the equation has real and equal roots

D= b^2-4ac=0

= $(-40)^2$-4(2k)(25)=0

= 1600-200k=0

= k = 8

The value of k is 8

(vi) $9x^2$-24x+k = 0

The given equation $9x^2$-24x+k=0 is in the form of ax^2+bx+c=0 where a= 9, b= -24, c= k

Given that, the equation has real and equal roots

D= b^2-4ac=0

= $(-24)^2$-4(9)(k)=0

= 576-36k = 0 = k = 16

The value of k is 16

(vii) $4x^2$-3kx+1 =0

The given equation $4x^2$-3kx+1=0 is in the form of ax^2+bx+c=0 where a= 4, b= -3k, c= 1

Given that, the equation has real and equal roots D= b^2-4ac=0

= $(-3k)^2$-4(4)(1)=0

= $9k^2$-16=0

K = 43 $\frac{4}{3}$

The value of k is 43 $\frac{4}{3}$

(viii) x^2-2(5+2k)x+3(7+10k) =0

The given equation X^2-2(5+2k)x+3(7+10k) =0 is in the form of ax^2+bx+c=0 where a= 1, b=+2(52k) , c= 3(7+10k)

Given that, the nature of the roots of the equation are real and equal roots

D= b^2-4ac=0

= $(+2(52k))^2$-4(1)(3(7+10k))=0

= $4(5+2k)^2$ -12(7+10k)=0

$= 25+4k^2+20k-21-30k=0$

$= 4k^2-10k+4=0$

Simplifying the above equation. We get,

$= 2k^2-5k+2=0$

$= 2k^2-4k-k+2=0$

$=2k(k-2)-1(k-2) =0$

$= (k-2)(2k-1) =0$K=2 and k = 12 $\frac{1}{2}$

The value of k can either be 2 or 12 $\frac{1}{2}$

(ix) $(3k+1)x^2+2(k+1)x+k =0$

The given equation $(3k+1)x^2+2(k+1)x+k =0$is in the form of $ax^2+bx+c=0$ where a= 3k+1, b=+2(k+1) , c= (k)

Given that, the nature of the roots of the equation are real and equal roots

$D= b^2-4ac=0$

$= [2(k+1)]^2 -4(3k+1)(k) =0$

$= (k+1)^2-k(3k+1)=0$

$= -2k^2+k+1 =0$

This equation can also be written as $2k^2-k-1=0$

The value of k can be obtained by

K = **k=** 1+√94 $k = \frac{1+\sqrt{9}}{4}$ = 1

Or , **k=** 1−√94 $k = \frac{1-\sqrt{9}}{4}$ = −12 $\frac{-1}{2}$

The value of k are 1 and −12 $\frac{-1}{2}$ respectively.

(x) $Kx^2+kx+1 = -4x^2-x$

Bringing all the x components on one side we get ,

$x^2(4+k)+x(k+1)+1 =0$

The given equation $Kx^2+kx+1 = -4x^2-x$ is in the form of $ax^2+bx+c=0$ where a=4+k,b=+k+1 , c= 1

Given that, the nature of the roots of the equation are real and equal roots

$D= b^2-4ac=0$

$= (k+1)^2-4(4+k)(1) =0$

$= k^2-2k-10 =0$

The equation is also in the form $ax^2+bx+c=0$

The value of k is obtained by a=1 , b= -2 , c = -15

k=−b±√b2−4ac2a $k = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Putting the respective values in the above formula we will obtain the value of k

The value of k are 5 and -3 for different given quadratic equation.

(xi) $(k+1)x^2+2(k+3)x+k+8 =0$

The given equation $(k+1) x^2+2(k+3)x+k+8 =0$ is in the form of $ax^2+bx+c=0$ where a=k+1,b=2(k+3), c= k+8

Given the nature of the roots of the equation are real and equal .

$D= b^2-4ac =0$

$= [2(k+30]^2-4(k+1)(k+8)=0$

$= 4(k+3)^2-4(k+1)(k+8) =0$

Taking out 4 as common from the LHS of the equation and dividing the same on the RHS

= $(k+3)^2-(k+1)(k+8) = 0$

= $k^2+9+6k - (k^2+9k+18) = 0$

Cancelling out the like terms on the LHS side

= 9+6k-9k-8 =0

= -3k+1 =0

= 3k = 1

k= 13 $k = \frac{1}{3}$

The value of k of the given equation is k= 13 $k = \frac{1}{3}$

(xii) x^2-2kx+7k-12=0

The given equation is $X^2-2kx+7k-12=0$

The given equation is in the form of $ax^2+bx+c=0$ where a=1,b=-2k, c= 7k-12

Given the nature of the roots of the equation are real and equal .

D= $b^2-4ac = 0$

= $(2k)^2-4(1)(7k-12) = 0$

= $4k^2-28k+48 = 0$

= $k^2-7k+12 = 0$

The value of k can be obtained by

k= −b±√b2−4ac2a $k = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Here a = 1 , b = -7k , c = 12

By calculating the value of k is 7±√12 $\frac{7 \pm \sqrt{1}}{2}$ = 4 , 3

The value of k for the given equation is 4 and 3 respectively.

(xiii) $(k+1)x^2-2(3k+1)x+8k+1 =0$

The given equation is $(k+1)x^2-2(3k+1)x+8k+1 =0$

The given equation is in the form of $ax^2+bx+c=0$ where $a=k+1, b=-2(k+1)$, $c= 8k+1$

Given the nature of the roots of the equation are real and equal.

$D= b^2-4ac =0$

$= (-2(k+1))^2-4(k+1)(8k+1) =0$

$= 4(3k+1)^2-4(k+1)(8k+1) =0$

Taking out 4 as common from the LHS of the equation and dividing the same on the RHS

$= (3k+1)^2-(k+1)(8k+1) =0$

$= 9k^2+6k+1 -(8k^2+9k+1) =0$

$= 9k^2+6k+1 - 8k^2-9k-1 =0$

$= k^2-3k =0$

$= k(k-3) =0$

Either $k =0$

Or, $k-3 =0 = k=3$

The value of k for the given equation is 0 and 3 respectively.

(xiv) $5x^2-4x+2 +k(4x^2-2x+1) =0$

The given equation $5x^2-4x+2 +k(4x^2-2x+1) =0$ can be written as $x^2(5+4k)-x(4+2k)+2-k =0$

The given equation is in the form of $ax^2+bx+c=0$ where $a=5+4k$, $b=-(4+2k)$, $c= 2-k$

Given the nature of the roots of the equation are real and equal.

$D= b^2-4ac =0$

$= [-(4+2k)]^2-4(5+4k)(2-k)=0$

= $16+4k^2+16-4(10-5k+8k-4k^2] = 0$

= $16+4k^2+16-40+20k-32k+16k^2 = 0$

= $20k^2 -4k-24 = 0$

Taking out 4 as common from the LHS of the equation and dividing the same on the RHS

= $5k^2-k-6 = 0$

The value of k can be obtained by equation

k= −b±√b2−4ac2a $k = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Here a = 5 , b= -1 , c =-6

k= 1±√−12−4(5)(−6)2(5) $k = \frac{1\pm\sqrt{-1^2-4(5)(-6)}}{2(5)}$ k=65and−1 $k = \frac{6}{5} and - 1$

The value of k for the given equation are k=65and−1 $k = \frac{6}{5} and - 1$ respectively.

(xv) $(4-k)x^2+(2k+4)x+(8k+1) = 0$

The given equation is $(4-k)x^2+(2k+4)x+(8k+1) = 0$

The given equation is in the form of $ax^2+bx+c=0$ where a=4-k, b=(2k+4), c= 8k+1

Given the nature of the roots of the equation are real and equal.

D= $b^2-4ac = 0$

= $(2k+4)^2-4(4-k)(8k+1)=0$

= $4k^2+16k+16 -4(-8k^2+32k+4-k)=0$

= $4k^2+16k+16 + 32k^2-124k-16 = 0$

Cancelling out the like and opposite terms. We get,

= $36k^2-108k = 0$

Taking out 4 as common from the LHS of the equation and dividing the same on the RHS

= $9k^2-27k = 0$

= 9k(k-3) =0

Either 9k =0

K =0

Or, k-3 =0

K= 3

The value of k for the given equation is 0 and 3 respectively.

(xvi) $(2k+1)x^2+2(k+3)x+(k+5) = 0$

The given equation is $(2k+1)x^2+2(k+3)x+(k+5) = 0$

The given equation is in the form of $ax^2+bx+c=0$ where a=2k+1, b=2(k+3), c= k+5

Given the nature of the roots of the equation are real and equal.

D= b^2-4ac =0

= $[2(k+3)]^2-4(2k+1)(k+5) =0$

Taking out 4 as common from the LHS of the equation and dividing the same on the RHS

= $[(k+3)]^2-(2k+1)(k+5) =0$

= $K^2+9+6k-(2k^2+11k+5) =0$

= $-k^2- 5k+4 =0$

= $k^2+5k-4 =0$

The value of k can be obtained by k=65and−1 $k = \frac{6}{5} and - 1$ respectively.

Here a= 1 , b = 5 , c= -4

Now k=−5±√5²−4(1)(−4)2×1 $k = \frac{-5\pm\sqrt{5^2-4(1)(-4)}}{2\times1}$

K = k=−5±√412 $k = \frac{-5\pm\sqrt{41}}{2}$

The value of k for the given equation is k=−5±√412 $k = \frac{-5\pm\sqrt{41}}{2}$

(xvii) $4x^2$-2(k+1)x+(k+4) =0

The given equation is $4x^2$-2(k+1)x+(k+4) = 0

The given equation is in the form of $ax^2+bx+c=0$ where a=4, b=-2(k+1), c= k+4

Given the nature of the roots of the equation are real and equal.

D= b^2-4ac =0

= $[-2(k+1)]^2$-4(4)(k+4)=0

Taking out 4 as common from the LHS of the equation and dividing the same on the RHS

= $(k+1)^2$-4(k+4)=0

= k^2+1+2k-4k-16 =0

= k^2-2k-15 =0

The value of k can be obtained by k=65and−1$k = \frac{6}{5} and - 1$ respectively.

Here a= 1 , b = -2 , c= -15

K = k=2±√692$k = \frac{2 \pm \sqrt{69}}{2}$

The value of k for the given equation is k=2±√692$k = \frac{2 \pm \sqrt{69}}{2}$

Question 3: In the following, determine the set of values of k for which the given quadratic equation has real roots:

Solution:

(i) $2x^2$+3x+k = 0

The given equation is $2x^2$+3x+k = 0

The given quadratic equation has equal and real roots

$D = b^2 - 4ac \geq 0$

The given equation is in the form of $ax^2+bx+c = 0$ so , a= 2 , b= 3 , c =k

$= 9 - 4(2)(k) \geq 0$

$= 9 - 8k \geq 0$

= **k≤**98 $k \leq \frac{9}{8}$

The value of k does not exceed **k≤**98 $k \leq \frac{9}{8}$ to have a real root.

(ii) $2x^2+kx+3 = 0$

The given equation is $2x^2+kx+3 = 0$

The given quadratic equation has equal and real roots

$D = b^2 - 4ac \geq 0$

The given equation is in the form of $ax^2+bx+c = 0$ so, a= 2 , b= k , c =3

$= k^2 - 4(2)(3) \geq 0$

$= k^2 - 24 \geq 0$

$= k^2 \geq 24$

k≥√24 $k \geq \sqrt{24}$ k≥√24 $k \geq \sqrt{24}$

The value of k should not exceed k≥√24 $k \geq \sqrt{24}$ in order to obtain real roots .

(iii) $2x^2-5x-k = 0$

The given equation is $2x^2-5x-k = 0$

The given quadratic equation has equal and real roots

$D = b^2 - 4ac \geq 0$

The given equation is in the form of $ax^2+bx+c=0$ so, a= 2 , b= -5 , c =-k

= 25 -4(2)(-k) ≥ 0

= 25-8k ≥ 0

= **k≤** 258 $k \leq \frac{25}{8}$

The value of k should not exceed **k≤** 258 $k \leq \frac{25}{8}$

(iv) $Kx^2+6x+1=0$

The given equation is $Kx^2+6x+1 = 0$

The given quadratic equation has equal and real roots

$D = b^2-4ac \geq 0$

The given equation is in the form of $ax^2+bx+c=0$ so, a= k , b= 6 , c =1

= 36 -4(k)(1) ≥ 0

= 36-4k ≥ 0

= k ≤ 9

The value of k for the given equation is k ≤ 9

(v) $x^2-kx+9=0$

The given equation is $X^2-kx+9 = 0$

The given quadratic equation has equal and real roots

$D = b^2-4ac \geq 0$

The given equation is in the form of $ax^2+bx+c=0$ so, a= 1 , b= -k , c =9

$= k^2-4(1)(-9) \geq 0$

$= k^2-36 \geq 0$

$= k^2 \geq 36$

k≥√36$k \geq \sqrt{36}$

$K \geq 6$ and $k \leq -6$

The value of k should in between $K \geq 6$ and $k \leq -6$ in order to maintain real roots.

Question 4: Determine the nature of the roots of the following quadratic equations.

Solution:

(i) $2x^2 - 3x + 5 = 0$

The given quadratic equation is in the form of $ax^2 + bx + c = 0$

So a= 2, b= -3, c= 5

We know, determinant (D) = $b^2 - 4ac$

$= (-3)^2 - 4(2)(5)$

$= 9 - 40$

$= -31 < 0$

Since D<0, the determinant of the equation is negative, so the expression does not having any real roots.

(ii) $2x^2 - 6x + 3 = 0$

The given quadratic equation is in the form of $ax^2 + bx + c = 0$

So a= 2, b= -6, c= 3

We know, determinant (D) = $b^2 - 4ac$

$= (-6)^2 - 4(2)(3)$

$= 36 - 24$

$= 12 < 0$

Since D>0, the determinant of the equation is positive, so the expression does having any real and distinct roots.

(iii) For what value of k $(4-k)x^2 + (2k+4)x + (8k+1) = 0$ is a perfect square

The given equation is $(4-k)x^2 + (2k+4)x + (8k+1) = 0$

Here, a= 4-k, b= 2k+4, c= 8k+1

The discriminate (D) = $b^2 - 4ac$

$= (2k+4)^2 - 4(4-k)(8k+1)$

$= (4k^2+16+16k) - 4(32k+4-8k^2-k)$

$= 4(k^2 + 8k^2+4k-31k+4-4)$

$=4(9k^2-27k)$

$D = 4(9k^2-27k)$

The given equation is a perfect square

D= 0

$4(9k^2-27k) = 0$

$9k^2-27k=0$

Taking out common of of 3 from both sides and cross multiplying

$K^2 - 3k = 0$

K (k-3) =0

Either k=0

Or k =3

The value of k is to be 0 or 3 in order to be a perfect square.

(iv) Find the least positive value of k for which the equation $x^2+kx+4=0$ has real roots.

The given equation is $x^2+kx+4=0$ has real roots

Here, a= 1, b= k, c= 4

The discriminate (D) = $b^2 - 4ac \geq 0$

= $k^2 - 16 \geq 0$

= $k \leq 4$,$k \geq -4$

The least positive value of k =4 for the given equation to have real roots.

(v) Find the value of k for which the given quadratic equation has real roots and distinct roots.

$Kx^2 + 2x + 1 = 0$

The given equation is $Kx^2 + 2x + 1 = 0$

Here, a= k, b= 2, c= 1

The discriminate (D) = $b^2 - 4ac \geq 0$

= $4 - 4k \geq 0$

= $4k \leq 4$

$K \leq 1$

The value of $k \leq 1$ for which the quadratic equation is having real and equal roots.

(vi) $Kx^2 + 6x + 1 = 0$

The given equation is $Kx^2 + 6x + 1 = 0$

Here, a= k, b= 6, c= 1

The discriminate (D) = $b^2 - 4ac \geq 0$

= $36 - 4k \geq 0$

= $4k \leq 36$

= $K \leq 9$

The value of $k \leq 9$ for which the quadratic equation is having real and equal roots.

(vii) $x^2 - kx + 9 = 0$

The given equation is $X^2 - kx + 9 = 0$

Here, a= 1, b= -k, c= 9

Given that the equation is having real and distinct roots.

Hence, the discriminate (D) = $b^2 - 4ac \geq 0$

$= k^2 - 4(1)(9) \geq 0$

$= k^2 - 36 \geq 0$

$= k \geq -6$ and $k \leq 6$

The value of k lies between -6 and 6 respectively to have the real and distinct roots.

Question 5: Find the values of k for which the given quadratic equation has real and distinct roots.

Solution:

(i) $Kx^2 + 2x + 1 = 0$

The given equation is $Kx^2 + 2x + 1 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$ so, a= k, b= 2 , c =1

$D = b^2 - 4ac \geq 0$

$= 4 - 4(1)(k) \geq 0$

$= 4k \leq 4$

$= k \leq 1$

The value of k for the given equation is $k \leq 1$

(ii) $Kx^2 + 6x + 1 = 0$

The given equation is $Kx^2 + 6x + 1 = 0$

The given equation is in the form of $ax^2 + bx + c = 0$ so, a= k, b= 6 , c =1

$D = b^2 - 4ac \geq 0$

$= 36-4(1)(k) \geq 0$

$= 4k \leq 36$

$= k \leq 9$

The value of k for the given equation is $k \leq 9$

Question 6: For what value of k , $(4-k)x^2+(2k+4)x+(8k+1)=0$, is a perfect square.

Solution:

The given equation is $(4-k)x^2+(2k+4)x+(8k+1)=0$

The given equation is in the form of $ax^2+bx+c=0$ so, a= 4-k , b= 2k+4 , c =8k+1

$D = b^2-4ac$

$= (2k+4)^2-4(4-k)(8k+1)$

$= 4k^2 +16+4k-4(32+4-8k^2-k)$

$= 4(k^2 +4+k-32-4+8k^2+k)$

$=4(9k^2-27k)$

Since the given equation is a perfect square

Therefore D =0

$=4(9k^2-27k) =0$

$= (9k^2-27k) =0$

$= 3k (k-3) =0$

Therefore 3k =0

K = 0

Or, k-3 =0

K =3

The value of k should be 0 or 3 to be perfect square.

Question 7: If the roots of the equation $(b-c)x^2 +(c-a)x+(a-b)=0$ are equal , then prove that $2b = a + c$.

Solution:

The given equation is $(b-c)x^2 +(c-a)x+(a-b)=0$.

The given equation is the form of $ax^2 + bx + c =0$. So,

$a= (b-c)$, $b= (c-a)$, $c= (a-b)$

According to question the equation is having real and equal roots.

Hence discriminant(D) = $b^2 -2ac =0$

$= (c-a)^2 -4(b-c)(a-b) =0$

$= c^2+a^2 -2ac – 4(ab-b^2-ac+cb) =0$

$= c^2+a^2 -2ac – 4ab+4b^2+4ac-4cb =0$

$= c^2+a^2 +2ac – 4ab+4b^2-4cb =0$

$= (a+c)^2 -4ab +4b^2 -4cb =0$

$= (c+a-2b)^2 = 0$

$= (c+a-2b) = 0$

$= c+a = 2b$

Hence it is proved that $c+a = 2b$.

Question 8: If the roots of the equation $(a^2 +b^2) x^2– 2(ac+bd)x+(c^2+ d^2)= 0$ are equal. Prove that $a÷b = c÷ d$.

Solution:

The given equation is $(a^2 +b^2)x^2 – 2(ac+bd)x+(c^2+ d^2)= 0$.

The equation is in the form of $ax^2 + bx= c =0$

Hence, a= (a^2+b^2) ,b= – 2(ac+bd) , c=(c^2+d^2) .

The given equation is having real and equal roots.

Discriminant(D) = $b^2 - 4ac = 0$

= $[-2(ac+bd)]^2 - 4(a^2+b^2)(c^2+d^2) = 0$

= $(ac+bd)^2 - (a^2+b^2)(c^2+d^2) = 0$

= $a^2c^2 + b^2d^2 + 2abcd - (a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) = 0$

Cancelling out the equal and opposite terms. We get,

= $2abcd - a^2d^2 - b^2c^2 = 0$

= $abcd + abcd - a^2d^2 - b^2c^2 = 0$

= ad(bc-ad) +bc(ad-bc) =0

= ad(bc-ad) -bc(bc-ad) =0

= (ad-bc)(bc-ad) =0

= ad –bc =0

= (a÷b) = (c÷ d)

Hence, it is proved.

Question 9: If the roots of the equation $ax^2+2bx+c=0$ and $bx^2-2\sqrt{ca}x+b=0$ $bx^2 - 2\sqrt{ca}\,x + b = 0$ are simultaneously real , then prove that $b^2-ac=0$.

Solution:

The given equations are $ax^2+2bx+c=0$ and $bx^2-2\sqrt{ca}x+b=0$$bx^2 - 2\sqrt{ca}\,x + b = 0$

These two equations are of the form $ax^2+bx+c=0$.

Given that the roots of the two equations are real. Hence, D ≥ 0 that is $b^2-4ac \geq 0$

Let us assume that $ax^2+2bx+c=0$ be equation (i)

And $bx^2-2\sqrt{ca}x+b=0$$bx^2 - 2\sqrt{ca}\,x + b = 0$ be (ii)

From equation (i) b^2-4ac ≥ 0

= 4 b^2-4ac ≥ 0 .... (iii)

From equation (ii) b^2-4ac ≥ 0

= $(2\sqrt{ca})^2 - 4b^2 \geq 0 (2\sqrt{ca})^2 - 4b^2 \geq 0$................................. (iv)

Given, that the roots of equation (i) and (ii) are simultaneously real and hence equation (iii) = equation (iv).

= $4b^2$-4ac = 4ac -4 b^2

= 8ac = $8b^2$

= b^2-ac =0.

Hence it is proved that b^2-ac =0.

Question 10: If p, q are the real roots and p ≠q . Then show that the roots of the equation (p-q)x^2 +5(p+q)x -2(p-q) = 0 are real and equal.

Solution:

The given equation is (p-q)x^2 +5(p+q)x -2(p-q) =0

Given, p , q are real and p ≠ q.

Then, Discriminant (D) = b^2 –ac

= $[5(p+q)]^2$ -4(p-q)(-2(p-q))

= $25(p+q)^2 + (p-q)^2$

We know that the square of any integer is always positive that is , greater than zero.

Hence, (D) = b^2 –ac ≥ 0

As given, p , q are real and p ≠ q.

Therefore,

= $25(p+q)^2 + (p-q)^2 > 0$ = D > 0

Therefore, the roots of this equation are real and unequal.

Question 11: If the roots of the equation $(c^2-ab)x^2 -2(a^2-bc)x +b^2-ac =0$ are equal , then prove that either a=0 or $a^3+b^3+c^3 = 3abc$.

Solution:

The given equation is $(c^2-ab)x^2 -2(a^2-bc)x +b^2-ac =0$

This equation is in the form of $ax^2 +bx +c =0$

So, $a = (c^2-ab)$, $b = -2(a^2-bc)$, $c = b^2-ac$.

According to the question, the roots of the given question are equal.

Hence, D= 0 , $b^2-4ac =0$

$= [-2(a^2-bc)]^2 -4(c^2-ab)(b^2-ac) =0$

$= 4(a^2-bc)^2 – 4(c^2-ab)(b^2-ac) =0$

$= 4a (a^3+b^3+c^3- 3abc) =0$

Either 4a =0 therefore, a =0

Or, $(a^3+b^3+c^3- 3abc) =0$

$= (a^3+b^3+c^3) =3abc$

Hence its is proved.

Question 12: Show that the equation $2(a^2+b^2)x^2+2(a+b)x+1 =0$ has no real roots , when $a\neq b$.

Solution:

The given equation is $2(a^2+b^2)x^2+2(a+b)x-1 =0$

This equation is in the form of $ax^2+bx+c =0$

Here, $a= 2(a^2+b^2)$, $b= 2(a+b)$, c = +1.

Given, $a \neq b$

The discriminant(D) = b^2 -4ac

=$[2(a+b)]^2$ -4 $(2(a^2+b^2))(1)$

= $4(a+b)^2$ -$8(a^2+b^2)$

= $4(a^2+b^2+2ab)$ –$8a^2$-$8b^2$

= =+2ab –$4a^2$-$4b^2$

According to the question a ≠ b, as the discriminant D has negative squares so the value of D will be less than zero.

Hence, D < 0, when a ≠ b.

Question 13: Prove that both of the roots of the equation (x-a)(x-b) +(x-c)(x-b)+(x-c)(x-a) =0 are real but they are equal only when a=b=c.

Solution:

The given equation is (x-a)(x-b) +(x-c)(x-b)+(x-c)(x-a) =0

By solving the equation, we get it as,

$3x^2$-2x(a+b+c)+(ab+bc+ca) =0

This equation is in the form of $ax^2+bx+c = 0$

Here , a =3 , b = 2(a+b+c) , c = (ab+bc+ca)

The discriminat (D) = b^2-4ac

= $[-2(a+b+c)]^2$ -4(3)(ab+bc+ca)

=$4(a+b+c)^2$-12(ab+bc+ca)

=$4[(a+b+c)^2$ -3(ab+bc+ca)]

= 4[$a^2+b^2+c^2$ –ab-bc-ca]

= 2[$2a^2+2b^2+2c^2$ –2ab-2bc-2ca]

= $2[(a-b)^2+(b-c)^2+(c-a)^2]$

Here clearly D ≥ 0 , if D = 0 then ,

$[(a-b)^2+(b-c)^2+(c-a)^2] = 0$

a –b = 0

b – c = 0

c – a = 0

Hence, a=b=c=0

Hence, it is proved.

Question 14: If a, b, c are real numbers such that ac ≠ 0, then, show that at least one of the equations $ax^2 +bx+c =0$ and $-ax^2+bx+c =0$ has real roots.

Solution:

The given equation are $ax^2+ bx+c =0$ .... (i)

And- $ax^2+bx+c =0$ ....(ii)

Given, equations are in the form of $ax^2+ bx+c =0$ also given that a ,b, c are real numbers and ac ≠ 0.

The Discriminant(D) = $b^2 -4ac$

For equation (i) = $b^2 -4ac$ ....(iii)

For equation (ii) = $b^2-4(-a)(c)$

= $b^2 +4ac$ ....(iv)

As a , b , c are real and given that ac ≠ 0 , hence $b^2 -4ac > 0$ and $b^2 +4ac > 0$

Therefore, D > 0

Hence proved.

Question 15: If the equation $(1+m^2)x+2mcx+(c^2-a^2)=0$ has real and equal roots , prove that $c^2= a^2(1+m^2)$.

Solution:

The given equation is $(1+m^2)x^2+2mcx+(c^2-a^2)=0$

The above equation is in the form of $ax^2+bx+c=0$.

Here $a = (1+m^2)$, $b= 2mc$, $c= +(c^2-a^2)$

Given, that the nature of the roots of this equation is equal and hence $D=0$, $b^2-4ac=0$

$= (2mc)^2 -4(1+m^2)(c^2-a^2) =0$

$= 4m^2c^2 - 4(c^2+m^2c^2-a^2 -a^2m^2) =0$

$= 4(m^2c^2- c^2+m^2c^2+a^2 +a^2m^2) =0$

$= m^2c^2- c^2+m^2c^2+a^2 +a^2m^2 =0$

Now cancelling out the equal and opposite terms ,

$= a^2 +a^2m^2 - c^2 =0$

$= a^2 (1+ m^2) - c^2 =0$

Therefore, $c^2 = a^2 (1+ m^2)$

Hence it is proved that as $D= 0$, then the roots are equal of $c^2 = a^2 (1+ m^2)$.

Exercise 8.7: Quadratic Equations

Question 1: Find the consecutive numbers whose squares have the same sum of 85.

Solution:

Let the two consecutive two natural numbers be (x) and (x+1) respectively.

Given,

That the sum of their squares is 85.

Then, by hypothesis, we get,

$= x^2 + (x+1)^2 = 85$

$= x^2 + x^2 + 2x + 1 = 85$

$= 2x^2 + 2x + 1 - 85 = 0$

$= 2x^2 + 2x + -84 = 0$

$= 2(x^2 + x + -42) = 0$

Now applying factorization method, we get,

$= x^2 + 7x - 6x - 42 = 0$

$= x(x+7) -6(x+7) =0$

$= (x-6)(x+7) =0$

Either,

$x-6 =0$ therefore , $x=6$

$x+7 =0$ therefore $x= -7$

Hence the consecutive numbers whose sum of squares is 85 are 6 and -7 respectively.

Question 2: Divide 29 into two parts so that the sum of the squares of the parts is 425.

Solution:

Let the two parts be (x) and (29-x) respectively.

According to the question, the sum of the two parts is 425.

Then by hypothesis,

$= x^2 +(29-x)^2 = 425$

$= x^2+x^2+841+-58x = 425$

$= 2x^2-58x+841 -425 =0$

$= 2x^2-58x+416 =0$

$= x^2-29x +208 =0$

Now, applying the factorization method

$= x^2 -13x-16x+208 =0$

$= x(x-13) -16(x-13) =0$

$= (x-13)(x-16) =0$

Either $x-13 =0$ therefore $x=13$

Or, $x-16 =0$ therefore $x=16$

The two parts whose sum of the squares is 425 are 13 and 16 respectively.

Question 3: Two squares have sides x cm and (x+4) cm. The sum of their areas is 656 cm^2.find the sides of the squares.

Solution:

Given,

The sum of the sides of the squares are = x cm and (x+4) cm respectively.

The sum of the areas = 656 cm^2

We know that,

Area of the square = side * side

Area of the square = x(x+4) cm^2

Given that the sum of the areas is 656 cm^2

Hence by hypothesis,

= x(x+4) +x(x+4) = 656

= 2x(x+4) = 656

= x^2 +4x =328

Now by applying factorization method,

= x^2 +20x-16x-328 =0

= x(x+20)-16(x+20) =0

= (x+20)(x16) =0

Either x+20 =0 therefore x=-20

Or, x-16 =0 therefore x= 16

No negative value is considered as the value of the side of the square can never be negative.

Therefore, the side of the square is 16.

Therefore, x+4 =16+4 =20 cm

Hence, the side of the square is 20cm.

Question 4: The sum of two numbers is 48 and their product is 432. Find the numbers.

Solution:

Given the sum of two numbers is 48.

Let the two numbers be x and 48-x also the sum of their product is 432.

According to the question

$=x(48-x) = 432$

$= 48x-x^2=432$

$= x^2-48x+432=0$

$= x^2-36x-12x+432=0$

$= x(x-36)-12(x-36) =0$

$=(x-36)(x-12) =0$

Either $x-36=0$ therefore $x = 36$

Or, $x-12=0$ therefore $x =12$

The two numbers are 12 and 36 respectively.

Question 5: If an integer is added to its square, the sum is 90. Find the integer with the help of quadratic equation.

Solution:

Let the integer be x

Given that if an integer is added to its square , the sum is 90

$= x+ x^2 = 90$

$= x^2 +x- 90=0$

$= x^2 +10x-9x- 90=0$

= x(x+10)-9(x+10) =0

= (x+10)(x-9) =0

Either x+10 = 0

Therefore x= -10

Or, x-9 =0

Therefore x =9

The values of the integer are 9 and -10 respectively.

Question 6: Find the whole numbers which when decreased by 20 is equal to69 times the reciprocal of the numbers.

Solution:

Let the whole number be x cm

As it is decreased by 20 = (x-20) = 69x $\frac{69}{x}$

x-20 = 69x $\frac{69}{x}$

x(x-20) =69

x^2 -20x -69 =0

Now by applying factorization method ,

x^2 -23x+3x-69 =0

x(x-23) +3(x-23)=0

(x-23)(x+3) =0

Either, x= 23

Or, x= -3

As the whole numbers are always positive x= -3 is not considered.

The whole number is 23.

Question 7: Find the consecutive natural numbers whose product is 20

Solution:

Let the two consecutive natural number be x and x+1 respectively.

Given that the product of natural numbers is 20

$= x(x+1) = 20$

$= x^2+x-20 = 0$

$= x^2+5x-4x-20 = 0$

$= x(x+5)-4(x+5) = 0$

$= (x+5)(x-4) = 0$

Either $x+5 = 0$

Therefore $x = -5$

Considering the positive value of x.

Or, $x-4 = 0$

Therefore $x = 4$

The two consecutive natural numbers are 4 and 5 respectively.

Question 8: The sum of the squares of two consecutive odd positive integers is 394. Find the two numbers?

Solution:

Let the consecutive odd positive integer are 2x-1 and 2x+1 respectively.

Given, that the sum of the squares is 394.

According to the question,

$(2x-1)^2 + (2x+1)^2 = 394$

$4x^2 + 1 - 4x + 4x^2 + 1 + 4x = 394$

Now cancelling out the equal and opposite terms ,

$8x^2 + 2 = 394$

$8x^2 = 392$

$X^2 = 49$

X= 7 and -7

Since the value of the edge of the square cannot be negative so considering only the positive value.

That is 7

Now, 2x-1 = 14-1 = 13

2x+1 = 14 +1 = 15

The consecutive odd positive numbers are 13 and 15 respectively.

Question 9: The sum of two numbers is 8 and 15 times the sum of the reciprocal is also 8 . Find the numbers.

Solution:

Let the numbers be x and 8-x respectively.

Given that the sum of the numbers is 8 and 15 times the sum of their reciprocals.

According to the question,

= 15(1x+ 18−x)=815$(\frac{1}{x}+\frac{1}{8-x})=8$

= 15 8−x+xx(8−x) =815$\frac{8-x+x}{x(8-x)}=8$

= 15× 88x−x2 =815 $\times\ \frac{8}{8x-x^2}=8$

$= 120=8(8x-x^2)$

$= 120 = 64x-8x^2$

$=8x^2-64x+120=0$

$= 8(x^2-8x+15)=0$

$= x^2-8x+15=0$

$= x^2-5x-3x+15=0$

$=x(x-5)-3(x-5) =0$

$= (x-5)(x-3) =0$

Either x-5 = 0 therefore x =5

Or, x-3 =0 therefore x =3

The two numbers are 5 and 3 respectively.

Question 10: The sum of a number and its positive square root is $625\frac{6}{25}$. Find the numbers.

Solution:

Let the number be x

By the hypothesis, we have

x+√x=625 $x+\sqrt{x}=\frac{6}{25}$

Let us assume that $x=y^2$, we get

$y+y^2$=625 $y+y^2=\frac{6}{25}$

$= 25y^2+25y-6 =0$

The value of y can be determined by:

y=−b±√b²−4ac2a $y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Where a = 25, b = 25 , c =-6

y=−25±√625+60050 $y=\frac{-25\pm\sqrt{625+600}}{50}$ y=−25±3550 $y=\frac{-25\pm 35}{50}$ y=15 andy=−1110

$y=\frac{1}{5}\ and\ y=\frac{-11}{10}$

= x=y^2= 15^2=125 $\frac{1}{5}^2 = \frac{1}{25}$

The number x is 125 $\frac{1}{25}$

Question 11: There are three consecutive integers such that the square of the first increased by the product of the other two integers gives 154. What are the integers?

Solution:

Let the three consecutive numbers be x, x+1, x+2 respectively.

X^2+(x+1)(x+2) =154

= x^2+x^2+3x+2 =154

= $3x^2$+3x-152=0

The value of x can be obtained by the formula

X=−b±√b²−4ac2a $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here a =3 , b =3 , c =152

x= X=−3±√9−12166 $x = \frac{-3 \pm \sqrt{9-1216}}{6}$

x=8andx=−192 $x = 8\, and\, x = \frac{-19}{2}$

Considering the value of x

If x=8

x+1 =9

x+2 = 10

The three consecutive numbers are 8 , 9 , 10 respectively.

Question 12: The product of two successive integral multiples of 5 is 300. Determine the multiples.

Solution:

Given that the product of two successive integral multiples of 5 is 300

Let the integers be 5x and 5(x+1)

According to the question,

$5x[5(x+1)] = 300$

$= 25x(x+1) = 300$

$= x^2+x = 12$

$= x^2+x-12=0$

$= x^2+4x-3x-12=0$

$= x(x+4)-3(x+4) = 0$

$=(x+4)(x-3) = 0$

Either $x+4 = 0$

Therefore $x=-4$

Or, $x-3 = 0$

Therefore $x = 3$

$x = -4$

$5x = -20$

$5(x+1) = -15$

$x=3$

$5x = 15$

$5(x+1) = 20$

The two successive integral multiples are 15,20 and -15 and -20 respectively.

Question 13: The sum of the squares of two numbers is 233 and one of the numbers is 3 less than the other number. Find the numbers.

Solution:

Let the number is x

Then the other number is 2x-3

According to the question:

$x^2+(2x-3)^2 = 233$

$= x^2+4x^2+9-12x = 233$

$= 5x^2-12x-224 = 0$

The value of x can be obtained by X=−b±√b^2−4ac2a $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Here a= 5 , b= -12 , c =-224

x = X=12±√144+20(224)2(5) $x = \frac{12\pm\sqrt{144+20(224)}}{2(5)}$

x=8andx=−285 $x = 8\, and\, x = \frac{-28}{5}$

Considering the value of x =8

2x-3 = 15

The two numbers are 8 and 15 respectively.

Question 14: The difference of two number is 4 . If the difference of the reciprocal is 421 $\frac{4}{21}$. find the numbers.

Solution:

Lethe two numbers be x and x-4 respectively.

Given, that the difference of two numbers is 4 .

By the given hypothesis we have,

=1x−4−1x=421 $\frac{1}{x-4} - \frac{1}{x} = \frac{4}{21}$

=x−x+4x(x−4)=421 $\frac{x-x+4}{x(x-4)} = \frac{4}{21}$

= 84 = 4x(x-4)

= x^2-4x-21 =0

Applying factorization theorem,

= x^2 -7x+3x-21 =0

=(x-7)(x+3) =0

Either x-7 =0 therefore x= 7

Or, x+3 = 0 therefore x = -3

Hence the required numbers are -3 and 7 respectively.

Question 15: Let us find two natural numbers which differ by 3 and whose squares have the sum 117.

Solution:

Let the numbers be x and x-3

According to the question

$x^2+(x-3)^2=117$

= $x^2+x^2+9-6x-117$ =0

= $2x^2$-6x-108 =0

= x^2-3x-54 =0

= x^2-9x+6x-54 =0

= x(x-9)+6(x-9) =0

=(x-9)(x+6) =0

Either x-9 =0 therefore x=9

Or ,x+6 =0 therefore x=-6

Considering the positive value of x that is 9

x=9

$x-3 = 6$

The two numbers are 6 and 9 respectively.

Question 16: The sum of the squares of these consecutive natural numbers is 149. Find the numbers.

Solution:

Let the numbers be x , x+1, and x+2 respectively.

According to given hypothesis

$X^2 + (x+1)^2 + (x+2)^2 = 149$

$X^2 + X^2 + X^2 + 1 + 2x + 4 + 4x = 149$

$3x^2 + 6x - 144 = 0$

$X^2 + 2x - 48 = 0$

Now applying factorization method,

$X^2 + 8x - 6x - 48 = 0$

$X(x+8) - 6(x+8) = 0$

$(x+8)(x-6) = 0$

Either $x+8 = 0$ therefore $x = -8$

Or, $x-6 = 0$ therefore $x = 6$

Considering only the positive value of x that is 6 and discarding the negative value.

$x=6$

$x+1 = 7$

$x+2 = 8$

The three consecutive numbers are 6 , 7 , and 8 respectively.

Question 17: Sum of two numbers is 16. The sum of their reciprocal is $13\frac{1}{3}$.find the numbers.

Solution:

Given that the sum of the two natural numbers is 16

Let the two natural numbers be x and 16-x respectively

According to the question

= 1x+116−x=13 $\frac{1}{x} + \frac{1}{16-x} = \frac{1}{3}$

= 16−x+xx(16−x)=13 $\frac{16--x+x}{x(16-x)} = \frac{1}{3}$

= 16x(16−x)=13 $\frac{16}{x(16-x)} = \frac{1}{3}$

= $16x-x^2 = 48$

= $-16x+x^2+48 = 0$

= $+x^2-16x+48=0$

= $+x^2-12x-4x+48=0$

= x(x-12)-4(x-12) =0

= (x-12)(x-4) =0

Either x-12 =0 therefore x= 12

Or , x-4 =0 therefore x= 4

The two numbers are 4 and 12 respectively.

Question 18: Determine the two consecutive multiples of 3 whose product is 270

Solution:

Let the consecutive multiples of 3 are 3xand 3x+3

According to the question

$3x(3x+3) = 270$

$= x(3x+3) = 90$

$= 3x^2+3x = 90$

$= 3x^2+3x - 90=0$

$= x^2+x - 30=0$

$= x^2+6x-5x - 30=0$

$= x(x+6)-5(x+6) = 0$

$= (x+6)(x-5) = 0$

Either $x+6 = 0$ therefore $x=-6$

Or , $x-5 = 0$ therefore $x=5$

Considering the positive value of x

$x=5$

$3x = 15$

$3x+3 = 18$

The two consecutive multiples of 3 are 15 and 18 respectively.

Question 19: The sum of a number and its reciprocal is 174 $\frac{17}{4}$. find the numbers.

Solution:

Lethe number be x

According to the question

$x^2+1x = 174\ \frac{x^2+1}{x} = \frac{17}{4}$

$= 4(x^2+1)=17x$

$= 4x^2+4-17x=0$

$= 4x^2+4-16x-x=0$

= 4x(x-4)-1(x-4) =0

=(4x-1)(x-4) =0

Either x-4 =0 therefore x=4

Or, 4x-1 =0 therefore x=14 $x = \frac{1}{4}$

The value of x is 4

Question 20: A two digit is such that the products of its digits is 8when 18 is subtracted from the number, the digits interchange their places. Find the number?

Solution:

Let the digits be x and x-2 respectively.

The product of the digits is 8

According to the question

x(x-2) = 8

= x^2-2x-8 =0

= x^2-4x+2x-8 =0

= x(x-4)+2(x-4) =0

Either x-4 =0 therefore x=4

Or , x+2 =0 therefore x= -2

Considering the positive value of x = 4

x-2 = 2

The two digit number is 42.

Question 21: A two digit number is such that the product of the digits is 12, when 36 is added to the number, the digits interchange their places .find the number.

Solution:

Let the tens digit be x

Then, the unit digit = 12x $\frac{12}{x}$

Therefore the number = 10x+12x $10x + \frac{12}{x}$

And, the number obtained by interchanging the digits = x+120x $x + \frac{120}{x}$

= 10x+12x+36=x+120x $10x + \frac{12}{x} + 36 = x + \frac{120}{x}$

= 9x+12−120x+36=0 $9x + \frac{12-120}{x} + 36 = 0$

= $9x^2$+12−120x+36xx=0 $\frac{9x^2+12-120x+36x}{x} = 0$

= $9x^2$+−108x+36xx=0 $\frac{9x^2+-108x+36x}{x} = 0$

= $9(x^2+4x-12)=0$

= $(x^2+4x-12)=0$

= $x^2+6x-2x-12=0$

= x(x+6)-2(x+6) =0

=(x-2)(x+6) =0

Either x-2 = 0 therefore x=2

Or, x+6 =0 therefore x= -6

Since a digit can never be negative. So x=2

The number is 26.

Question 22: A two digit number is such that the product of the digits is 16 when 54 is subtracted from the number, the digits are interchanged. Find the number.

Solution:

Let the two digits be:

Tens digit be x

Units digit be 16x $\frac{16}{x}$

Numbers = 10x+16x $10x + \frac{16}{x}$ ....(i)

Number obtained by interchanging = 10(10x+16x)$10(10x + \frac{16}{x})$

10x+16x $10x + \frac{16}{x}$ – 10(10x+16x)$10(10x + \frac{16}{x})$ =54

= $10x^2$+16-160+x^2 = 54

= $9x^2$-54x-144= 0

= x^2-6x-16 =0

= x^2-8x+2x-16 =0

= x(x-8)+2(x-8) =0

=(x-8)(x+2)=0

Either x-8 =0 therefore x=8

Or, x+2 =0 therefore x =-2

A digit can never be negative so x = 8

Hence by putting the value of x in the above equation (i) the number is 82.

Question 23: Two numbers differ by 3 and their product is 504. Find the numbers.

Solution:

Let the numbers be x and x-3 respectively.

According to the question

= x(x-3) =504

=x^2-3x-504 =0

= x^2-24x+21x-504 =0

= x(x-24)+21 (x-24) =0

=(x-24)(x+21) =0

Either x-24 = 0 therefore x =24

Or , x+21 =0 , therefore x =-21

x = 24 and x= -21

x-3 = 21 and x-3 = -24

The two numbers are 21 a nd 24 and -21 and -24 respectively.

Question 24: Two numbers differ by 4 and their product is 192. Find the numbers.

Solution:

Let the two numbers be x and x-4 respectively

Given that the product of the numbers is 192

According to the question

= x(x-4) = 192

= x^2-4x -192 =0

= x^2-16x+12x -192 =0

= x(x-16) +12(x-16) =0

= (x-16) (x+12) =0

Either x-16 =0 therefore x= 16

Or, x+12 =0 therefore x= -12

Considering only the positive value of x

x=16S

x-4 = 12

The two numbers are 12 and 16 respectively.

Question 25: A two digit number is 4 times the sum of its digits and twice the product of its digits. Find the numbers.

Solution:

Let the digit in the tens and the units place be x and y respectively.

Then it is represented by 10x+y

According to the question,

10x+y = 4(sum of the digits) and 2xy

10x+y = 4(x+y) and 10x+y = 2xy

10x+y = 4x+4y and 10x+y = 2xy

6x-3y =0 and 10x+y-2xy =0

y= 2x and 10x + 2x -2x(2x) = 0

12x= $4x^2$

4x(x-3) =0

Either 4x=0 therefore x= 0

Or, x-3 =0 therefore x= 3

We have y = 2x

When x= 3 , y= 6

Question 26: The sum of the squares of two positive integers is 208. If the square of the large number is 18 times the smaller. Find the numbers.

Solution:

Let the smaller number be x

Then, square of the large number be = 18x

Also, square of the smaller number be = x^2

It is given that the sum of the square of the integer is 208.

Therefore,

= $x^2 + 18x = 208$

= $x^2 + 18x - 208 = 0$

Applying factorization theorem,

= $x^2 + 26x - 8x - 208 = 0$

= $x(x+26) - 8(x+26) = 0$

= $(x+26)(x-8) = 0$

Either x+26=0 therefore x=-26

Or, x-8 =0 therefore x= 8

Considering the positive number, therefore x= 8.

Square of the largest number =18x = 18*8 = 144

Largest number = $\sqrt{144} = 12 \sqrt{144} = 12$

Hence the numbers are 8 and 12 respectively.

Question 27: The sum of two numbers is 18. The sum of their reciprocal is $14\frac{1}{4}$ .find the numbers.

Solution:

Let the numbers be x and (18-x) respectively.

According to the given hypothesis,

$1x - 118-x = 14 \frac{1}{x} - \frac{1}{18-x} = \frac{1}{4}$ $18-x+xx(18-x) = 14 \frac{18-x+x}{x(18-x)} = \frac{1}{4}$ $18-x^2+18x = 14 \frac{18}{-x^2+18x} = \frac{1}{4}$

= $72 = 18x - x^2$

= $x^2 - 18x + 72 = 0$

Applying factorization theorem, we get,

= $x^2 - 6x - 12x + 72 = 0$

= x(x-6)-12(x-6) =0

= (x-6)(x-12) =0

Either, x= 6

Or, x=12

The two numbers are 6 and 12 respectively.

Question 28: The sum of two numbers a and b is 15 and the sum of their reciprocals 1a $\frac{1}{a}$ and 1b $\frac{1}{b}$ is 310 $\frac{3}{10}$. Find the numbers a and b.

Solution:

Let us assume a number x such that

1x−115−x=310 $\frac{1}{x} - \frac{1}{15-x} = \frac{3}{10}$ 15−x+xx(15−x)=310 $\frac{15-x+x}{x(15-x)} = \frac{3}{10}$ 1515x−x^2=310 $\frac{15}{15x-x^2} = \frac{3}{10}$

= $3x^2$-45x+150=0

= x^2 -15 x+50 = 0

Applying factorization theorem,

= x^2– 10x-5x+50=0

= x(x-10)-5(x-10) =0

= (x-10)(x-5) =0

Either, x-10 =0 therefore x=10

Or, x-5=0 therefore x=5

Case (i)

If x = a , a=5 and b= 15-x , b= 10

Case (ii)

If x = 15-a = 15-10 = 5 ,

x=a=10 , b= 15-10 =5

Hence when a=5 , b=10

a=10 , b= 5

Question 29: The sum of two numbers is 9. The sum of their reciprocal is $12\frac{1}{2}$.find the numbers.

Solution:

Given that the sum of the two numbers is 9

Let the two number be x and 9-x respectively

According to the question

1x+19−x=12 $\frac{1}{x} + \frac{1}{9-x} = \frac{1}{2}$

= 9−x+xx(9−x)=12 $\frac{9-x+x}{x(9-x)} = \frac{1}{2}$

= 99x−x2=12 $\frac{9}{9x-x^2} = \frac{1}{2}$

= $9x-x^2= 18$

= $x^2-9x+18 =0$

= $x^2-6x-3x+18 =0$

= x(x-6)-3(x-6) =0

= (x-6)(x-3)=0

Either x-6 =0 therefore x= 6

Or x-3 =0 therefore x=3

The two numbers are 3 and 6 respectively

Question 30: Three consecutive positive integers are such that the sum of the squares of the first and the product of the other two is 46. Find the integers.

Solution:

Let the consecutive positive integers be x , x+1, x+2 respectively

According to the question

X^2+(x+1)(x+2) = 46

= $x^2+x^2+3x+2 = 46$

=2 $x^2+3x+2 = 46$

= 2 $x^2+3x+2 -46=0$

= 2 $x^2-8x+11x+ -44=0$

= 2x(x-4)+11(x-4) =0

= (x-4)(2x+11) =0

Either x-4 =0 therefore x=4

Or, 2x+11 =0 therefore x=−112 $x = \frac{-11}{2}$

Considering the positive value of x that is x= 4

The three consecutive numbers are 4 , 5 and 6 respectively

Question 31: The difference of squares of two numbers is 88. If the large number is 5 less than the twice of the smaller, then find the two numbers

Solution:

Let the smaller number be x and larger number is 2x-5

It is given that the difference of the squares of the number is 88

According to the question

$(2x-5)^2-x^2= 88$

= $4x^2+25-20x- x^2 =88$

= $3x^2-20x-63 =0$

= $3x^2$-27x+7x-63 =0

= 3x(x-9)+7(x-9) =0

= (x-9)(3x+7)=0

Either x-9 =0 therefore x=9

Or, 3x+7 =0 therefore x=-73 $x = \frac{-7}{3}$

Since a digit can never be negative so x= 9

Hence the number is 2x-5 = 13

The required numbers are 9 and 13 respectively

Question 32: The difference of squares of two numbers is180. The square of the smaller number is 8 times the larger number. Find the two numbers

Solution:

Let the number be x

According to the question

X^2-8x = 180

X^2-8x-180 =0

= X^2+10x-18x-180 =0

= x(x+10)-18(x-10) =0

= (x-18)(x+10) =0

Either x-18 = 0 therefore x= 18

Or, x+10 =0 therefore x=-10

Case (i)

X=18

8x= 144

Larger number = √144=12 $\sqrt{144} = 12$

Case (ii)

X= -10

Square of the larger number 8x= -80

Here in this case no perfect square exist

Hence the numbers are 18 and 12 respectively .

Exercise 8.8: Quadratic Equations

Q.1: The speed of a boat in still water is 8km/hr. It can go 15 km upstream and 22 km downstream in 5 hours. Find the speed of the stream.

Sol: Let the speed of stream be x km/hr,

Then, speed downstream = (8 + x) km/hr

Therefore, Speed upstream = (8 – x) km/hr

Time taken by the boat to go 15km upstream 15/(8 – x)hr

Time taken by the boat to return 22km downstream = 22/(8 + x)hr

Acc. To the question, the boat returns to the same point in 5 hr so,

15(8–x)+22(8+x)=5 $\frac{15}{(8-x)} + \frac{22}{(8+x)} = 5$ 15(8+x)+22(8–x)(8–x)(8+x)=5 $\frac{15(8+x)+22(8-x)}{(8-x)(8+x)} = 5$

120+15x+176–22x64–x²=5 $\frac{120+15x+176-22x}{64-x^2} = 5$ 296–7x64–x²=5 $\frac{296-7x}{64-x^2} = 5$

$5x^2 - 7x + 296 - 320 = 0$

$5x^2 - 7x - 24 = 0$

$5x^2 - 15x + 8x - 24 = 0$

5x(x – 3) + 8(x – 3) = 0

(x – 3)(5x + 8) = 0

X = 3, x = -8/5

Since the speed of the stream can never be negative,hence the speed of the stream is 3 km/hr.

Q.2: A train traveling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/hr more. Find the original speed of the train.

Sol: let the usual speed of train be x km/hr

Then, increased speed of the train = (x + 10) km/hr

Time taken by the train under usual speed to cover 360 km = 360/x hr.

Time taken by the train under increased speed to cover 360 km = 360/(x+5) hr.

360x—360(x+5) $\frac{360}{x} - \frac{360}{(x+5)} = 4860\frac{48}{60}$

360(x+5)–360xx(x+5) $\frac{360(x+5)-360x}{x(x+5)} = 45\frac{4}{5}$

360x+1800–360xx2+5x $\frac{360x+1800-360x}{x_2+5x} = 45\frac{4}{5}$

$1800(5) = 4(x^2 + 5x)$

$9000 = 4x^2 + 20x$

$4x^2 + 20x – 9000 = 0$

$x^2 + 5x – 2250 = 0$

$x^2 + 50x – 45x – 2250 = 0$

x(x + 50) – 45(x + 50) = 0

(x + 50)(x – 45) = 0

x = – 50 or x = 45

Since, the speed of the train can never be negative. Hence, original speed of train is 45 km/hr.

Q.3: A fast train takes one hour less than a slow train for a journey of 200km. If the speed of the slow train is 10km/hr less than that of the fast train, find the speed of the two train.

Sol: Let the speed of the fast train be x km/hr

Then, the speed of the slow train be = (x-10) km/hr

Time taken by the fast train to cover 200 km = 200/x hr

Time taken by the slow train to cover 200 km = 200/(x – 10) hr

200x—200(x−10)=1 $\frac{200}{x} - \frac{200}{(x-10)} = 1$ (200(x–10)–200x)x(x−10)=1 $\frac{(200(x-10)-200x)}{x(x-10)} = 1$ 200x–2000–200xx²–10x=1 $\frac{200x-2000-200x}{x^2-10x} = 1$

$x^2 - 10x = -2000$

$x^2 - 10x + 2000 = 0$

$x^2 - 50x + 40x + 2000 = 0$

x(x – 50) + 40(x – 50) = 0

(x – 50)(x + 40) = 0

x = 50 or x = – 40

Since, the speed of train can never be negative.

Therefore, x = 50

So, speed of the fast train is 50 km/hr

And speed of slow train

(50 – 10) = 40 km/hr (given speed of slow train is 10km/hr less than fast train)

Q.4: A passenger train takes one hour less for a journey of 150 km if its speed is increased 5 km/hr from its usual speed. Find the usual speed of the train.

Sol: Let the usual speed of train be x km/hr

Then, increased speed of the train = (x + 5)km/hr

Time taken by the train under usual speed speed to cover 150 km = 150/x hr

Time taken by the train under increased speed to cover 150km = 150(x +5)hr

So,

150x−150(x+5)=1 $\frac{150}{x} - \frac{150}{(x+5)} = 1$ 150(x+5)–150xx(x+5)=1 $\frac{150(x+5)-150x}{x(x+5)} = 1$ 150x+750–150xx²+5x=1 $\frac{150x+750-150x}{x^2+5x} = 1$

$750 = x^2 + 5x$

$x^2 + 5x - 750 = 0$

$x^2 - 25x + 30x - 750 = 0$

$x(x - 25) + 30(x - 25) = 0$

$(x - 25)(x + 30) = 0$

x = 25 or x = -30

Since, the speed of the train can never be negative

Therefore, the usual speed of the train is x = 25km/hr

Q.5: The time taken by a person to cover 150 km was 2.5 hrs more than the time taken on the return journey. If he returned at the speed of 10 km/hr more than the speed of going, what was the speed per hour in each direction?

Sol: let the ongoing speed of person be x km/hr,

Then the returning speed of the person is = (x + 10) km/hr

Time taken by the person in going direction to cover 150 km = 150/x hr

Time taken by the person in returning direction to cover 150 km = 150/(x + 10)hr

Therefore,

150x−150(x+10)=52 $\frac{150}{x}-\frac{150}{(x+10)}=\frac{5}{2}$ 150(x+10)–150xx(x+10)=52 $\frac{150(x+10)-150x}{x(x+10)}=\frac{5}{2}$

150x+1500–150xx²+10x=52 $\frac{150x+1500-150x}{x^2+10x}=\frac{5}{2}$ 1500x²+10x=52 $\frac{1500}{x^2+10x}=\frac{5}{2}$

$3000 = 5x^2 + 50x$

$5x^2 + 50x – 3000 = 0$

$5(x^2 + 10x – 600) = 0$

$x^2 + 10x – 600 = 0$

$x^2 – 20x + 30x – 600 = 0$

$x(x – 20) + 30(x – 20) = 0$

$(x – 20)(x + 30) = 0$

x = 20 or x = -30

Since the speed of train can never be negative

Therefore, x = 20

Then, (x + 10)

(20 + 10) = 30

Hence, ongoing speed of person is 20km/hr.

And returning speed of the person is 30 km/hr.

Q.6: A plane left 40 minutes late due to bad weather and in order to reach the destination, 1600 km away in time, it had to increase its speed by 400 km/hr from its usual speed.. Find the usual speed of the plane.

Sol: let the usual speed of plane be x km/hr,

Then the increased speed of the plane is = (x + 4000) km/hr

Time taken by the plane under usual speed to cover 1600km = 1600/x hr

Time taken by the plane under increased speed to cover 1600 km = 1600/(x + 400)hr

Therefore,

1600x−1600(x+400)=4060 $\frac{1600}{x} - \frac{1600}{(x+400)} = \frac{40}{60}$ 1600(x+400)–1600xx(x+400)=23

$\frac{1600(x+400)-1600x}{x(x+400)} = \frac{2}{3}$ 1600x+640000–1600xx^2+400x=23 $\frac{1600x+640000-1600x}{x^2+400x} = \frac{2}{3}$

$1920000 = 2x^2 + 800x$

$2x^2 + 800x - 1920000 = 0$

$2(x^2 + 400x - 960000) = 0$

$x^2 + 400x - 960000 = 0$

$x^2 - 800x + 1200x - 960000 = 0$

$x(x - 800) + 1200(x - 800) = 0$

$(x - 800)(x + 1200) = 0$

x = 800 or x = -1200

Since the speed of the train can never be negative

Therefore, the usual speed of the train is 800 km/hr.

Q.7: An aeroplane takes 1 hour less for a journey of 1200km if its speed is increased by 100 km/hr from its usual speed of the plane. Find its usual speed.

Sol: let the usual speed of plane be x km/hr,

Then the increased speed of the plane is = (x + 100) km/hr

Time taken by the plane under usual speed to cover 1200km = 1200/x hr

Time taken by the plane under increased speed to cover 1200 km = 1200/(x + 100)hr

Therefore,

1200x−1200(x+100)=1 $\frac{1200}{x} - \frac{1200}{(x+100)} = 1$ 1200(x+100)–1200xx(x+100)=1 $\frac{1200(x+100)-1200x}{x(x+100)} = 1$

1200x+120000–1200xx^2+100x=1 $\frac{1200x+120000-1200x}{x^2+100x} = 1$

$120000 = x^2 + 100x$

$x^2 + 100x - 120000 = 0$

$x^2 - 300x + 400x - 120000 = 0$

$x(x - 300) + 400(x - 300) = 0$

$x = 300$ or $x = -400$

Since, the speed of the aeroplane can never be negative

Hence, the usual speed of train is 300 km/hr

Q.9: A train covers a distance of 90 km at a uniform speed . had the speed been 15 km/hr more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

Sol: let the usual speed of train be x km/hr,

Then the increased speed of the train is = (x + 15) km/hr

Time taken by the train under usual speed to cover 90km = 90/x hr

Time taken by the train under increased speed to cover 90 km = 90/(x + 15)hr

Therefore,

90x—90(x+15)=3060 $\frac{90}{x} - \frac{90}{(x+15)} = \frac{30}{60}$ 90(x+15)–90xx(x+15)=12 $\frac{90(x+15)-90x}{x(x+15)} = \frac{1}{2}$ 90x+1350–90xx²+15x=12 $\frac{90x+1350-90x}{x^2+15x} = \frac{1}{2}$

$2700 = x^2 + 15x$

$x^2 + 15x - 2700 = 0$

$x^2 - 45x + 60x - 2700 = 0$

$x(x - 45) + 60(x - 45) = 0$

$(x - 45)(x + 60) = 0$

$x = 45$ or $x = -60$

Since, the speed of the train can never be negative

Therefore, the original speed of the train is 45 km/hr.

Q.10: A train travels 360 km at a uniform speed. If the speed had ben 5 km/hr more, it would have taken 1 hr less for the same journey. Find the speed of the train.

Sol: let the usual speed of train be x km/hr,

Then the increased speed of the train is = (x + 5) km/hr

Time taken by the train under usual speed to cover 360km = 360/x hr

Time taken by the train under increased speed to cover 360 km = 360/(x + 5)hr

Therefore,

360x−360(x+5)=1 $\frac{360}{x} - \frac{360}{(x+5)} = 1$ 360(x+5)–360xx(x+5)=1 $\frac{360(x+5)-360x}{x(x+5)} = 1$ 360x+1800–360xx²+5x=1 $\frac{360x+1800-360x}{x^2+5x} = 1$

$1800 = x^2 + 5x$

$x^2 + 5x - 1800 = 0$

$x^2 - 40x + 45x - 1800 = 0$

$x(x - 40) + 45(x - 40) = 0$

$(x + 45)(x - 40) = 0$

x = 40 or x = – 45

Since the speed of the train can never be negative

Hence, the original speed of the train is 40 km/hr.

Q.11: An express train takes 1 hour less than a passenger train to travel 132 km between mysore and Bangalore. If the average speed of the express train is 11 km/hr more than that of the passenger train, find the average speed of the two trains.

Sol: let the usual speed of train be x km/hr,

Then the increased speed of the train is = (x + 11) km/hr

Time taken by the passenger train to cover 132 km = 132/x hr

Time taken by the express train to cover 132 km = 132/(x + 11)hr

Therefore,

132x−132(x+11)=1 $\frac{132}{x} - \frac{132}{(x+11)} = 1$ 132(x+11)–132xx(x+11)=1 $\frac{132(x+11)-132x}{x(x+11)} = 1$ 132x+1452–132xx²+11x=1 $\frac{132x+1452-132x}{x^2+11x} = 1$

$1452 = x^2 + 11x$

$x^2 + 11x - 1452 = 0$

$x^2 - 33x + 44x - 1452 = 0$

$x(x - 33) + 44(x - 33) = 0$

$(x - 33)(x + 44) = 0$

x = 33 or x = – 44

Since, speed of train can never be negative

Therefore, speed of passenger train = 33 km/hr.

And speed of express train, = (x + 11) = (33 + 11) = 44 km/hr.

Q.12: An aeroplane left 50 minutes later than its scheduled time, and in order to reach the destination, 1250 km away , in time, it had to increase its speed by 250 km/hr from its usual speed. Find its usual speed.

Sol: let the usual speed of aeroplane be x km/hr,

Then the increased speed of the aeroplane is = (x + 250) km/hr

Time taken by the aeroplane under usual speed to cover 1250 km = 1250/x hr

Time taken by the aeroplane under increased speed to cover 1250 km = 1250/(x + 250)hr

Therefore,

1250x−1250(x+250)=5060 $\frac{1250}{x} - \frac{1250}{(x+250)} = \frac{50}{60}$ 1250(x+250)–1250xx(x+250)=56 $\frac{1250(x+250)-1250x}{x(x+250)} = \frac{5}{6}$ 1250x+312500–1250xx²+250x=56 $\frac{1250x+312500-1250x}{x^2+250x} = \frac{5}{6}$

$1875000 = 5x^2 + 1250x$

$5x^2 + 1250x - 1875000 = 0$

$5(x^2 + 1250x – 375000) = 0$

$X^2 + 250x – 375000 = 0$

$X^2 – 500x + 750x – 375000 = 0$

$X(x – 500) + 750(x – 500) = 0$

$(x – 500)(x + 750) = 0$

$X = 500$ or $x = – 750$

Since, the speed of the aeroplane can never be negative,

Therefore, the usual speed of the train = 500 km/hr.

Q.13: While boarding an aeroplane, a passenger got hurt. The pilot showing promptness and concern, made arrangements to hospitalise the injured and so the plane started late by 30 minutes to reach the destination, 1500 km away in time, the pilot increased the speed by 100 km/hr. Find the original speed /hour of the plane.

Sol: let the usual speed of aeroplane be x km/hr,

Then the increased speed of the aeroplane is = (x + 100) km/hr

Distance to be travelled = 1500 km.

Time taken to reach the destination at original speed, t1 = 1500/x hr

Time taken to reach the destination at increasing speed, t2 = 1500/(x + 100)hr

Acc. To the question,

t1 – t2 = 30 min.

1500x—1500(x+100)=3060 $\frac{1500}{x} - \frac{1500}{(x+100)} = \frac{30}{60}$ 1500(x+100)–1500xx(x+100)=12

$\frac{1500(x+100)-1500x}{x(x+100)} = \frac{1}{2}$ 1500x+150000–1500xx²+100x=12 $\frac{1500x+150000-1500x}{x^2+100x} = \frac{1}{2}$

150000x²+100x=12 $\frac{150000}{x^2+100x} = \frac{1}{2}$

$300000 = x^2 + 100x$

$x^2 + 100x – 300000 = 0$

$x^2 + 600x – 500x – 300000 = 0$

$x(x + 600) – 500(x + 600) = 0$

$(x – 500)(x + 600) = 0$

$x = 500$ or $x = – 600$

Since, the speed of plane can never be negative

Therefore, the original speed of the plane is 500 km/hr.

Q.14: A motor boat whose speed in still water 18 km/hr takes 1 hour more to go 24 km up stream than to return downstream to the same point. Find the speed of the stream.

Soln: let the usual speed of stream be x km/hr,

Speed of the boat in still water is = 18 km/hr

Distance to be travelled = 24 km.

Speed of theboat upstream = speed of the boat in still water – speed of the stream = (18 – x) km/hr

Speed of the boat downstream = speed of the boat in still water + speed of the stream = (18 + x) km/hr.

Time of upstream journey, t1 = 24/(18 – x) km/hr

Time of downstream journey, t2 = 24/(18 + x) km/hr

Acc. To the question, t1 – t2 = 1hr.

2418–x—24(18+x)=1 $\frac{24}{18-x} - \frac{24}{(18+x)} = 1$ 24(18+x–18+x(18–x)(18+x)=1 $\frac{24(18+x-18+x}{(18-x)(18+x)} = 1$

24(2x)18²–x²=1 $\frac{24(2x)}{18^2-x^2} = 1$

$48x = 324 – x^2$

$x^2 + 48x – 324 = 0$

$x^2 + 54x – 6x – 324 = 0$

$x(x + 54) – 6(x + 54) = 0$

$(x - 6)(x + 54) = 0$

$x = 6$ or $x = -54$

Since the speed can never be negative

Therefore the speed of stream is 6 km/hr.

Exercise 8.9: Quadratic Equations

Q.1: Ashu is x years old while his mother Mrs. Veena is x^2 years old. Five years hence Mrs. Veena will be three times old as Ashu. Find their present ages.

Sol:

Given that Ashu's present age is x years and his mother Mrs. Veena is x^2 years

Then, acc. to question,

Five years later, Ashu is (x + 5) years

And his mother Mrs. Veena is $(x^2 + 5)$ years

So,

$x^2 + 5 = 3(x + 5)$

$x^2 + 5 = 3x + 15$

$x^2 + 5 – 3x – 15 =0$

$x^2 – 5x + 2x + 10 = 0$

$x(x – 5) + 2(x – 5) = 0$

$(x – 5)(x + 2) = 0$

$X = 5$ or $x = – 2$

Since, the age can never be negative

Therefore, ashu's present age is 5 years and his mother's age is 25 years.

Q.2: The sum of the ages of a man and his son is 45 years. Five years ago, the product of their ages was four times the man's age at the time. Find their present ages.

Sol:

Let the present age of the man be x years

Then, present age of his son is = (45 – x) years

Five years ago, man's age = (x – 5) years

And his son's age = (45 – x – 5) = (40 – x) years

Then, acc. To question,

$(x – 5)(40 – x) = 4(x – 5)$

$40x – x^2 + 5x – 200 = 4x – 20$

$-X^2 + 45x – 200 = 4x – 20$

$-x^2 + 45x – 200 – 4x + 20 = 0$

$-x^2 + 41x – 180 = 0$

$x^2 – 36x – 5x + 180 = 0$

$x(x – 36) – 5(x – 36) = 0$

$(x – 36)(x – 5) = 0$

$x = 36$ or $x = 5$

But, the father's age can never be 5 years

Therefore, when $x = 36$,

$45 – x = 45 – 36 = 9$

Hence, man's present age is 36 years and his son's age is 9 years.

Q.3: The product of Shikha's age five years ago and her age 8 years later is 30, her age at both times being given in years. Find her present age.

Sol:

let the present age of shikha be x years

Then, 8 years later, age of her = (x + 8) years

Five years ago, her age = (x – 5) years

Then, acc. To question,

$(x – 5)(x +8) = 30$

$x^2 + 8x – 5x – 40 = 30$

$x^2 + 3x – 40 – 30 = 0$

$x^2 + 3x – 70 = 0$

$x(x – 7) + 10(x – 7) = 0$

$(x – 7)(x + 10) = 0$

$x = 7$ or $x = -10$

Since, the age can never be negative

Heence, the present age of shikha is = 7 years.

Q.4: The product of Ramu's age (in years) five years ago and his age (in years) nine years later is 15. Determine Ramu's present age.

Sol:

let the present age of ramu be x years

Then, 9 years later, age of her = (x + 9) years

Five years ago, her age = (x – 9) years

Then, acc. to question,

$(x - 5)(x + 5) = 15$

$x^2 + 9x - 5x - 45 = 15$

$x^2 + 4x - 45 - 15 = 0$

$x^2 + 4x - 60 = 0$

$x^2 - 6x + 10x - 60 = 0$

$x(x - 6) + 10(x - 6) = 0$

$(x - 6)(x + 10) = 0$

$x = 6$ or $x = -10$

Since, the age can be never be negative

Therefore, the present age of ramu is = 6 years

Q.5: Is the following situation possible? if so, determine their present ages.

The sum of the ages of two friends is 20 years. four years ago, the product of their ages in years was 48.

Sol:

let the present age of two friends be x years and (20 – x) years respectively

Then, 4 years later, the age of two friends will be (x – 4) years and (20 – x – 4) years

Then, acc. To the question

$(x - 4)(20 - x - 4) = 48$

$(x - 4)(16 - x) = 48$

$16x - x^2 - 64 + 4x = 48$

$-x^2 + 20x - 64 - 48 = 0$

$x^2 - 20x + 112 = 0$

Let the discriminant of the above quadratic eqn.

$D = b^2 – 4ac$

Here, a = 1, b = -20, c = 112

$D = (-20)^2 – (4x1x112) = 400 – 448 = – 48$

Since, D < 0

The above question does not have real roots.

Hence, the given situation is not possible.

Q.6: A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

Sol:

let the present age of girl be x years then, age of her sister (x/2) years

Then, 4 years later, age of girl = (x + 4) years and her sister's age be (x2+4)$(\frac{x}{2}+4)$ years

Then, acc. to the question,

(x+4)(x2+4)=160$(x+4)(\frac{x}{2}+4)=160$

(x + 4) (x + 8) = 160×2

$x^2 + 8x + 4x + 32 = 320$

$x^2 + 12x – 288 = 0$

$x^2 – 12x + 24x – 288 = 0$

x(x – 12) + 24(x – 12) = 0

(x – 12) (x + 24) = 0

x = 12 or x = – 24

Since, the can never be negative,

Therefore, the present age of the girl is = 12 years.

And her sister's age will be,

x2=122 $\frac{x}{2}=\frac{12}{2}$ = 6 years.

Q.7: The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is 1/3. Find his present age.

Sol: let the present age of Rehman be x years

Then, 8 years late, age of her = (x + 5) years

Five years ago, her age = (x – 3) years

Acc. To question,

1(x–3)+1(x+5)=13 $\frac{1}{(x-3)}+\frac{1}{(x+5)}=\frac{1}{3}$ x+5+x–3(x–3)(x+5)=13 $\frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$ 2x+2x2+5x–3x–15=13 $\frac{2x+2}{x^2+5x-3x-15}=\frac{1}{3}$

$x^2 + 2x - 15 = 6x + 6$

$x^2 + 2x - 15 - 6x - 6 = 0$

$x^2 - 4x - 21 = 0$

$x^2 - 7x + 3x - 21 = 0$

x(x – 7) + 3(x – 7) = 0

(x – 7)(x + 3) = 0

x = 7 or x = -3

Since, the age can never be negative

Therefore, the present age of Rehman be = 7 years.

Exercise 8.10: Quadratic Equations

Q.1) The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.

Sol: let the length of one side of the right triangle be x cm then,

the other side be = (x + 5) cm

and given that hypotenuse = 25 cm

By using Pythagoras Theorem,

$x^2 + (x + 5)^2 = 25^2$

$x^2 + x^2 + 10x + 25 = 625$

$2x^2 + 10x + 25 - 625 = 0$

$2x^2 + 10x - 600 = 0$

$x^2 + 5x - 300 = 0$

$x^2 - 15x + 20x - 300 = 0$

$x(x - 15) + 20(x - 15) = 0$

$(x - 15)(x + 20) = 0$

$x = 15$ or $x = -20$

Since, the side of triangle can never be negative

Therefore, when, $x = 15$

And, $x + 5 = 15 + 5 = 20$

Therefore, length of side of right triangle is = 15 cm and other side is = 20 cm

Q.2: The diagonal of a rectangular field is 60 meters more than the shorter side. If the longer side is 30 meters more than the shorter side, find the sides of the field.

Sol:

let the length of smaller side of rectangle be x metres then, the larger side be (x + 30) metres and

diagonal be = (x + 60) metres

By using Pythagoras theorem,

$x^2 + (x + 30)^2 = (x + 60)^2$

$x^2 + x^2 + 60x + 900 = x^2 + 120x + 3600$

$2x^2 + 60x + 900 - x^2 - 120x - 3600 = 0$

$x^2 - 60x - 2700 = 0$

$x^2 - 90x + 30x - 2700 = 0$

$x(x - 90) + 30(x - 90) = 0$

$(x - 90)(x + 30) = 0$

$x = 90$ or $x = -30$

Since, the side of rectangle can never be negative

Therefore, $x = 90$

$x + 30 = 90 + 30 = 120$

Therefore, the length of smaller side of rectangle is = 90 metres and larger side is = 120 metres.

Q.3: The hypotenuse of a right triangle is $3\sqrt{10}3\sqrt{10}$ cm. If the smaller leg is tripled and the longer leg doubled, new hypotenuse will be $9\sqrt{5}9\sqrt{5}$ cm. How long are the legs of the triangle?

Sol:

let the length of smaller side of right triangle be = x cm then large side be = y cm

By using Pythagoras theorem,

$x^2+y^2=(3\sqrt{10})^2 x^2 + y^2 = (3\sqrt{10})^2$

$x^2 + y^2 = 90$ ….eqn.(1)

If the smaller side is triple and the larger side is doubled, the new hypotenuse is $9\sqrt{5}9\sqrt{5}$ cm

Therefore,

$(3x)^2 + (2y)^2 = (9\sqrt{5})^2(9\sqrt{5})^2$

$9x^2 + 4y^2 = 405$ ….eqn.(2)

From equation (1) we get,

$y^2 = 90 – x^2$

Now putting the value of y^2 in eqn. (2)

$9x^2 + 4(90 – x^2) = 405$

$9x^2 + 360 – 4X^2 – 405 = 0$

$5x^2 – 45 = 0$

$5(x^2 – 9) = 0$

$x^2 – 9 = 0$

$x^2 = 9$

x=√9$x = \sqrt{9}$ x=±3$x = \pm 3$

Since, the side of triangle can never be negative

Therefore, when x = 3

Then, $y^2 = 90 - x^2 = 90 - (3)^2 = 90 - 9 = 81$

y=√81$y = \sqrt{81}$ y=±9$y = \pm 9$

Hence, the length of smaller side of right triangle is = 3cm and larger side is = 9cm

Q.4) A pole has to be erected at a point on the boundary of a circular park of diameter meters in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 meters. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

Sol:

let P be the required location on the boundary of circular park such that its distance from the gate B is x metres that is BP = x metres

Then, AP = x + 7

In right triangle ABP, by using Pythagoras theorem,

$AP^2 + BP^2 = AB^2$

$(x + 7)^2 + x^2 = 13^2$

$x^2 + 14x + 49 + x^2 = 169$

$2x^2 + 14x + 49 - 169 = 0$

$2x^2 + 14x - 120 = 0$

$2(x^2 + 7x - 60) = 0$

$x^2 + 12x - 5x - 60 = 0$

$x(x + 12) - 5(x + 12) = 0$

$(x + 12)(x - 5) = 0$

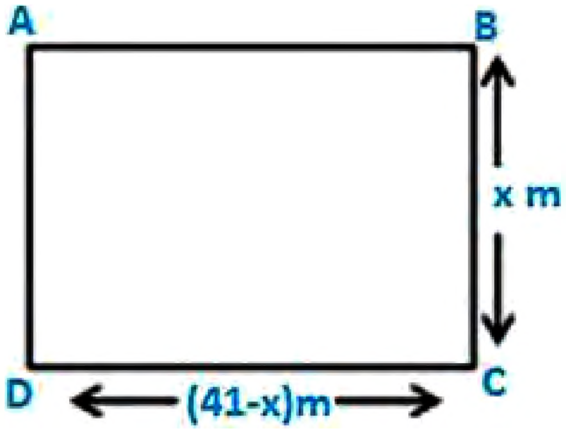
$x = -12$ or $x = 5$

Since, the side of triangle can never be negative

Therefore, P is at a distance of 5 metres from the gate B

Exercise 8.11: Quadratic Equations

Question 1:



The perimeter of the rectangular field is 82m and its area is 400m^2.find the breadth of the rectangle?

Soln:

Let the breadth of the rectangle be (x) m

Given,

Perimeter = 82 m

Area = 400 m^2

Perimeter of a rectangle = 2(length + breadth)

82=2(length + x)

41 = (length + x)

Length = (41-x) m

We know,

Area of the rectangle = length * breadth

400 = (41-x) (x)

$400 = 41x-x^2$

$= x^2-41x+400 = 0$

$= x^2-25x-16x+400 = 0$

= x(x-25)-16(x-25) =0

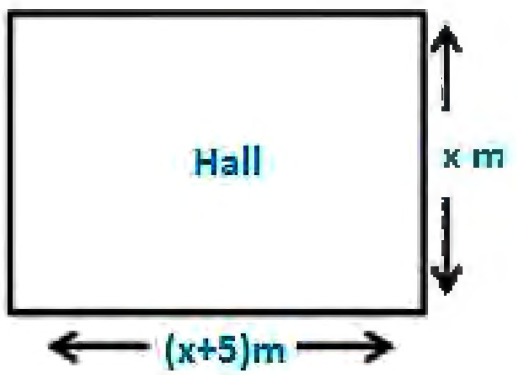
= (x-16)(x-25) =0

Either x-16 =0 therefore x=16

Or, x-25=0 therefore x =25

Hence the breadth of the above mentioned rectangle is either 16 m or 25 m respectively.

Question 2:



The length of the hall is 5 m more than its breadth. If the area of the floor of the hall is 84 m^2 , what is the length and breadth of the hall?

Soln:

Le the breadth of the rectangle be x m

Let the length of the hall is 5 m more than its breadth =(x+5) m

Also given that,

Area of the hall is = 84 m^2

The shape of the hall is rectangular

Area of the rectangular hall = length * breadth

84 = x(x+5)

= $x^2+5x-84=0$

= $x^2+12x-7x-84=0$

= x(x+12)-7(x+12) =0

= (x+12)(x-7) =0

Either x+12 =0 therefore x = -12

Or, x-7 =0 therefore x =7

Since the value of x cannot be negative

So x = 7

= x+5 = 12

The length and breadth of the rectangle is 7 and 12 respectively.

Question 3: Two squares have sides x and (x+4) cm. The sum of their area is 656 cm^2 . Find the sides of the square.

Soln:

Let S_1 and S_2 be the two square

Let x cm be the side square S_1 and (x+4) cm be the side of the square S_2 .

Area of the square $S_1 = x^2\ cm^2$

Area of the square $S_2 =(x+4)^2\ cm^2$

According to the question,

Area of the square S_1 + Area of the square $S_2 = 656\ cm^2$

$= x^2\ cm^2 + (x+4)^2\ cm^2 = 656\ cm^2$

$= x^2+ x^2+16+8x-656 =0$

$=2 x^2+16+8x-656 =0$

$= 2 (x^2+4x-320) =0$

$= x^2+4x-320 =0$

$= x^2+20x-16x-320 =0$

$= x(x+20)-16(x+20) =0$

$= (x+20)(x-16) =0$

Either x+20 = 0 therefore x = -20

Or, x-16 =0 therefore x =16

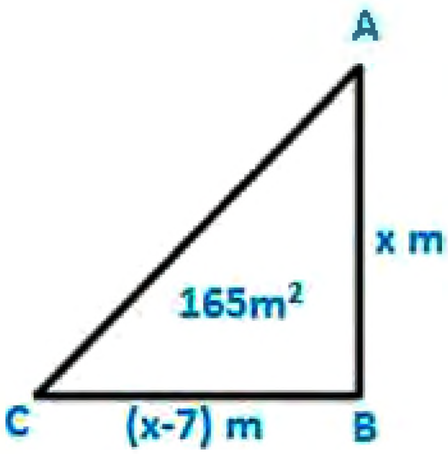
Since the value of x cannot be negative so the value of x = 16

The side of the square S_1= 16 cm

The side of the square S_2 = 20 cm

Question 4: The area of the right-angled triangle is 165 cm^2. Determine the base and altitude if the latter exceeds the former by 7m.

Soln:



Let the altitude of the right angles triangle be denoted by x m

Given that the altitude exceeds the base by 7 m = x-7 m

We know

Area of the triangle = 12×base×altitude$\frac{1}{2} \times base \times altitude$

= 165 = 12×(x−7)×x$\frac{1}{2} \times (x - 7) \times x$

= x(x-7) = 330

= x^2-7x-330 =0

= x^2-22x+15x-330 =0

= x(x-22)+15(x-22) =0

= (x-22)(x+15) =0

Either x-22 =0 therefore x =22

Or, x+15 =0 therefore x =-15

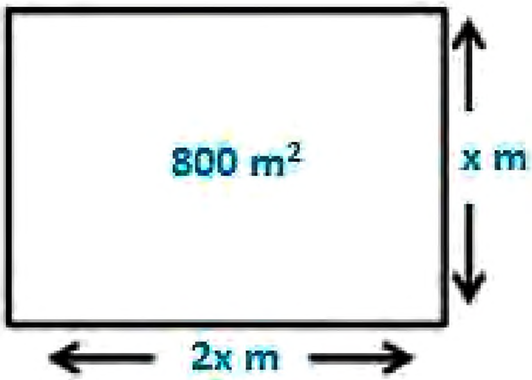
Since the value of x cannot be negative so the value of x = 22

=x-7 = 15

The base and altitude of the right angled triangle are 15 cm and 22 cm respectively.

Question 5: Is it possible to design a rectangular mango grove whose length is twice its breadth and area is 800 m^2 .find its length and breadth.

Soln:



Let the breadth of the rectangular mango grove be x m

Given that length of rectangle is twice of its breadth

Length = 2x

Area of the grove = 800 m^2

We know,

Area of the rectangle = length * breadth

= 800 = x(2x)

= $2x^2$-800 =0

= x^2-400 =0 = x^2=400 = x=√400=20$x = \sqrt{400} = 20$

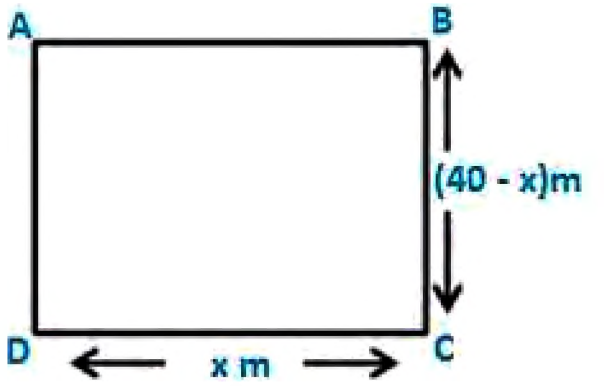
Breadth of the rectangular groove is 20 m

Length of the rectangular groove is 40 m

Yes, it is possible to design a rectangular groove whose length is twice of its breadth.

Question 6: Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so find its length and breadth.

Soln:



In order to prove the given condition let us assume that the length of the rectangular park is denoted by x m

Given that,

Perimeter = 8 cm

Area = 400 cm ^{2}

Perimeter of the rectangle = 2(length +breadth)

80 = 2(x + breadth)

Breadth = (40-x) m

We know,

Area of the rectangle = (length) (breadth)

= 400 = x(40-x)

= $40x-x^2=400$

= $x^2-40x+400=0$

= $x^2-20x-20x+400=0$

= x(x-20)-20(x-20) =0

$= (x-20)(x-20) = 0$

$= (x-20)^2 = 0$

$= x-20 = 0$ therefore $x=20$

Length of the rectangular park is = 20 m

Breadth of the rectangular park =(40-x) = 20 m

Yes, it is possible to design a rectangular Park of perimeter 80 m and area $400m^2$

Question 7: Sum of the area of the square is 640 m^2.if the difference of their perimeter is 64 m, find the sides of the two squares.

Soln:

Let the two squares be S_1 and S_2 respectively. let he sides of the square S_1 be x m and the sides of the square S_2 be y m

Given that the difference of their perimeter is 64 m

We know that the

Perimeter of the square = 4(side)

Perimeter of the square S_1 = 4x m

Perimeter of the square S_2 = 4y m

Now, difference of their perimeter is 64 m

$=4x-4y = 64$

$x-y = 16$

$x = y + 16$

Also, given that the sum of their two areas

= area of the square 1 +area of the square 2

$= 640 = x^2 + y^2$

$= 640 = (y+16)^2 + y^2$

$= 2y^2+32y+256-640 = 0$

$= 2y^2+32y-384 = 0$

$= 2(y^2+16y-192) = 0$

$= y^2+16y-192 = 0$

$= y^2+24y-8y-192 = 0$

$= y(y+24)-8(y+24) = 0$

$= (y+24)(y-8) = 0$

Either $y+24 = 0$ therefore $y = -24$

Or, $y-8 = 0$ therefore $y=8$

Since the value of y cannot be negative so $y = 8$

Side of the square 1 = 8 m

Side of the square 2 = 8+16 = 24 m

The sides of the squares 1 and 2 are 8 and 24 respectively.

Exercise 8.12: Quadratic Equations

Question 1: A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish a work in 12 days, find the time taken by B to finish the piece of work.

Sol:

Let us consider B tales x days to complete the piece of work

B's 1 day work = 1x $\frac{1}{x}$

Now, A takes 10 days less than that of B to finish the same piece of work that is (x-10) days

A's 1 day work = 1x−10 $\frac{1}{x-10}$

Same work in 12 days

(A and B)'s 1 day's work = 112 $\frac{1}{12}$

According to the question

A's 1 day work + B's 1 day work = 1x−10 $\frac{1}{x-10}$ + 1x $\frac{1}{x}$

= 1x+1x−10=112 $\frac{1}{x}+\frac{1}{x-10}=\frac{1}{12}$

= x−10+xx(x−10)=112 $\frac{x-10+x}{x(x-10)} = \frac{1}{12}$

= 12(2x-10) = x(x-10)

= 24x-120 = x^2-10x

= x^2-10x-24x+120 =0

= x^2-34x+120 =0

= x^2-30x-4x+120 =0

= x(x-30)-4(x-30) =0

=(x-30)(x-4) =0

Either x-30 =0 therefore x =30

Or, x-4 =0 therefore x=4

We observe that the value of x cannot be less than 10 so the value of x = 30

Time taken by B to finish the piece of work is 30 days

Question 2: If two pipes function simultaneously, a reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours will the second pipe take to fill the reservoir?

Soln:

Let us assume that the faster pipe takes x hours to fill the reservoir

Portion of reservoir filled by faster pipe in one hour = 1x $\frac{1}{x}$

Now, slower pipe takes 10 hours more than that of faster pipe to fill the reservoir that is (x+10) hours

Portion of reservoir filled by slower pipe = 1x+10 $\frac{1}{x+10}$

Given that, if both the pipes function simultaneously, the same reservoir can be filled in 12 hours

Portion of the reservoir filled by both pipes in one hour = 112 $\frac{1}{12}$

Now ,

Portion of reservoir filled by slower pipe in one hour + Portion of reservoir filled by faster pipe in one hour = 1x $\frac{1}{x}$ + 1x+10 $\frac{1}{x+10}$

And portion of reservoir filled by both pipes = 112 $\frac{1}{12}$

= 1x $\frac{1}{x}$ + 1x+10 $\frac{1}{x+10}$ = 112 $\frac{1}{12}$

= 12(2x+10) = x(x+10)

= x^2-14x-120 =0

= x^2-20x+6x-120 =0

= x(x-20)+6(x-20) =0

=(x-20)(x+6) =0

Either x-20 = 0 therefore x =20

Or, x+6 =0 therefore x =-6

Since the value of time cannot be negative so the value of x is 20 hours

Time taken by the slower pipe to fill the reservoir = x+10 = 30 hours

Question 3: Two water taps together can fill a tank in 9 38 $9\frac{3}{8}$. The tap of larger diameter takes 10hours less than the smaller one to fill the tank separately. Find the time in which each tap can be fill **separately the tank.**

Soln:

Let the time taken by the tap of smaller diameter to fill the tank be x hours

Portion of tank filled by smaller pipe in one hour = 1x $\frac{1}{x}$

Now, larger pipe diameter takes 10 hours less than the smaller diameter pipe in one hour = 1x−10 $\frac{1}{x-10}$

Given that,

Two taps together can fill the tank in 9 38 $9\frac{3}{8}$.

= 758 $\frac{75}{8}$.

Now, portion of the tank filled by both the taps together in one hour

= 1 758 =875 $\frac{1}{\frac{75}{8}} = \frac{8}{75}$

We have ,

Portion of tank filled by smaller pipe in one hour + Portion of tank filled by larger pipe in one hour

= 875 $\frac{8}{75}$

= 1x + 1x−10 =875 $\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

= x−10+xx(x−10) =875 $\frac{x-10+x}{x(x-10)} = \frac{8}{75}$

= 75(2x-10) = 8x(x-10)

= 150x-750 = $8x^2$-80x

= $8x^2$-230x+750 =0

= $4x^2$-115x+375 =0

Here a= 4 , b = -115 , c = 375

x= −b±√b2−4ac2a $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

= x= 115±√(−115)2−4(4)(375)2(4) $x = \frac{115 \pm \sqrt{(-115)^2-4(4)(375)}}{2(4)}$

= x= 115±√(13225−6000)8 $x = \frac{115 \pm \sqrt{(13225-6000)}}{8}$

= x= 115±√858 $x = \frac{115 \pm \sqrt{85}}{8}$

The value of x can either be 8 or 3.75 hours.

The value of x is 8 hours

Question 4: Two pipes running together can fill the tank in 11 19 $11\frac{1}{9}$ minutes. if one pipe takes 5 minutes more than the other to fill the tank separately. Find the time in which each pipe would fill the tank separately.

Sol:

Let us take the time taken by the faster pipe to fill the tank as x minutes

Portion of tank filled by faster pipe in one minute = 1x $\frac{1}{x}$

Now,

Time taken by the slower pipe to fill the same tank is 5 minutes more than that of faster pipe = x+5 minutes

Portion of the tank filled by the slower pipe = 1x+5 $\frac{1}{x+5}$

Given that,

The two pipes together can fill the tank in 11 19 $11\frac{1}{9}$ = 1009 $\frac{100}{9}$

Portion of tank filled by two pipes together in 1 minute = 9100 $\frac{9}{100}$

Portion of tank filled by faster pipe in one minute + Portion of the tank filled by the slower pipe

= 9100 $\frac{9}{100}$ = 1x + 1x+5 $\frac{1}{x} + \frac{1}{x+5}$

= x+5+xx(x+5) = 9100 $\frac{x+5+x}{x(x+5)} = \frac{9}{100}$

= 9x(x+5) = 100(2x+5)

= $9x^2+45x = 200x+500$

= $9x^2-155x-500 = 0$

= $9x^2-180x+25x-500 = 0$

= 9x(x-20)+25(x-20) =0

= (x-20)(9x+25) =0

Either x-20 therefore x =20

Or, 9x+25 =0 therefore $-259\ \frac{-25}{9}$

Since time cannot be negative

So the value of x =20 minutes

The required time taken to fill the tank is 20 minutes

Time taken by the slower pipe is x+5 = 20+5 = 25 minutes

Times taken by the slower and faster pipe are 25 minutes and 20 minutes respectively.

Exercise 8.13: Quadratic Equations

Question 1: Find the roots of the equation (x – 4) (x + 2) = 0

The given equation is (x-4)(x+2)=0

Either x-4 =0 therefore x= 4

Or, x+2=0 therefore x= -2

The roots of the above mentioned quadratic equation are 4 and -2 respectively.

Question 2: Find the roots of the equation (2x+3) (3x-7)=0

The given equation is (2x+3)(3x-7)=0 .

Either 2x+3 =0, therefore x=−32 $x = \frac{-3}{2}$

Or, 3x-7 =0, therefore x=73 $x = \frac{7}{3}$

The roots of the above mentioned quadratic equation are x=−32 $x = \frac{-3}{2}$ and x=73 $x = \frac{7}{3}$ respectively.

Question 3: Find the roots of the quadratic equation $3x^2$-14x-5 = 0

The given equation is $3x^2$-14x-5 = 0

= $3x^2$-14x-5 = 0

= $3x^2$-15x+x-5 = 0

= 3x(x-5)+1(x-5) =0

= (3x+1)(x-5) = 0

Either 3x+1 =0 therefore X=−13 $x = \frac{-1}{3}$

Or, x-5 =0 therefore x=5

The roots of the given quadratic equation are 5 and X=−13 $x = \frac{-1}{3}$ respectively.

Question 4: Find the roots of the equation $9x^2$-3x-2=0.

The given equation is $9x^2$-3x-2 =0.

= $9x^2$-3x-2 =0.

= $9x^2$ -6x+3x-2 =0

= 3x (3x-2)+1(3x-2) =0

= (3x-2)(3x+1) = 0

Either, 3x-2 =0 therefore X=23 $x = \frac{2}{3}$

Or, 3x+1 = 0 therefore X=−13 $x = \frac{-1}{3}$

The roots of the above mentioned quadratic equation are x=23 $x = \frac{2}{3}$ and x=-13 $x = \frac{-1}{3}$ respectively.

Question 5: Find the roots of the quadratic equation 1x−1−1x+5=67 $\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}$.

The given equation is 1x−1−1x+5=67 $\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}$

= 1x−1−1x+5=67 $\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}$

= x+5−x+1(x−1)(x+5)=67 $\frac{x+5-x+1}{(x-1)(x+5)} = \frac{6}{7}$

= 6x2+4x−5=67 $\frac{6}{x^2+4x-5} = \frac{6}{7}$

Cancelling out the like terms on both the sides of the numerator. We get,

= 1x2+4x−5=17 $\frac{1}{x^2+4x-5} = \frac{1}{7}$

= $x^2+4x-5 = 7$

= $x^2+4x-12 = 0$

= $x^2+6x-2x-12 = 0$

= x(x+6)-2(x-6) =0

=(x+6)(x-2) =0

Either x+6 =0

Therefore x= -6

Or, x-2 =0

Therefore x=2

The roots of the above mentioned quadratic equation are 2 and -6 respectively.

Question 6: Find the roots of the equation $6x^2+11x+3=0$.

The given equation is $6x^2+11x+3=0$.

= $6x^2+11x+3=0$.

= $6x^2+9x+2x+3=0$

= 3x(2x+3) +1(2x+3) =0

= (2x+3)(3x+1) =0

Either, 2x+3 =0 therefore x=−32 $x=\frac{-3}{2}$

Or, 3x+1 =0 therefore x=−13 $x=\frac{-1}{3}$

The roots of the above mentioned quadratic equation are x=−32 $x=\frac{-3}{2}$ and x=−13 $x=\frac{-1}{3}$ respectively .

Question 7: Find the roots of the equation $5x^2-3x-2=0$

The given equation is $5x^2-3x-2=0$.

= $5x^2-3x-2=0$.

= $5x^2-5x+2x-2=0$

= 5x(x-1) +2(x-1) =0

= (5x+2) (x-1) =0

Either 5x+2 =0 therefore x=−25 $x=\frac{-2}{5}$

Or, x-1 =0 therefore x= 1

The roots of the above mentioned quadratic equation are 1 and x=−25 $x=\frac{-2}{5}$ respectively.

Question 8: Find the roots of the equation $48x^2-13x-1=0$

The given equation is $48x^2-13x-1=0$.

= $48x^2-13x-1=0$.

= $48x^2-16x+3x-1=0$.

= 16x (3x-1) +1(3x-1) =0

= (16x+1)(3x-1) =0

Either 16x+1 =0 therefore X=−116 $x = \frac{-1}{16}$

Or, 3x-1 =0 therefore X=13 $x = \frac{1}{3}$

The roots of the above mentioned quadratic equation are X=−116 $x = \frac{-1}{16}$

And X=13 $x = \frac{1}{3}$ respectively.

Question 9: Find the roots of the equation $3x^2$=-11x-10

The given equation is $3x^2$=-11x-10

= $3x^2$=-11x-10

= $3x^2$+11x+10 = 0

= $3x^2$+6x+5x+10 =0

= 3x(x+2) +5(x+2) =0

= (3x+2)(x+2) =0

Either 3x+2 =0 therefore X=−23 $x = \frac{-2}{3}$

Or, x+2 =0 therefore x= -2

The roots of the above mentioned quadratic equation are X=−23 $x = \frac{-2}{3}$ and -2 respectively.

Question 10

Find the roots of the equation 25x(x+1) =-4

The given equation is 25x(x+1) =4

$= 25x(x+1) = -4$

$= 25x^2+25x+4 = 0$

$= 25x^2+20x+5x+4 = 0$

$= 5x(5x+4)+1(5x+4) = 0$

$= (5x+4)(5x+1) = 0$

Either 5x+4 = 0 therefore X=−45 $x = \frac{-4}{5}$

Or, 5x+1 =0 therefore X=−15 $x = \frac{-1}{5}$

The roots of the quadratic equation are X=−45 $x = \frac{-4}{5}$ and X=−15 $x = \frac{-1}{5}$ respectively.

Question 12

Find the roots of the quadratic equation 1x−1x−2=3 $\frac{1}{x} - \frac{1}{x-2} = 3$

The given equation is 1x−1x−2=3 $\frac{1}{x} - \frac{1}{x-2} = 3$

= 1x−1x−2=3 $\frac{1}{x} - \frac{1}{x-2} = 3$

= x−2−xx(x−2)=3 $\frac{x-2-x}{x(x-2)} = 3$

= 2x(x−2)=3 $\frac{2}{x(x-2)} = 3$

Cross multiplying both the sides. We get,

$=2 = 3x(x-2)$

$= 2 = 3x^2-6x$

$= 3x^2-6x-2 = 0$

$= 3x^2-3x-3x-2 = 0$

=3x2−(3+√3)x−(3−√3)x+[(√3²)−1²] $3x^2 - (3+\sqrt{3})x - (3-\sqrt{3})x + [(\sqrt{3}^2) - 1^2]$

= $3x^2-(3+\sqrt{3})x-(3-\sqrt{3})x+[(\sqrt{3}^2)-1^2][(\sqrt{3}^2)-1^2]$

$$3x^2-(3+\sqrt{3})x-(3-\sqrt{3})x+[(\sqrt{3}^2)-1^2][(\sqrt{3}^2)-1^2]$$

= $\sqrt{3}^2x^2-\sqrt{3}(\sqrt{3}+1)x-\sqrt{3}(\sqrt{3}-1)x+(\sqrt{3}+1)(\sqrt{3}-1)$

$$\sqrt{3}^2x^2-\sqrt{3}(\sqrt{3}+1)x-\sqrt{3}(\sqrt{3}-1)x+(\sqrt{3}+1)(\sqrt{3}-1)$$

= $\sqrt{3}x(\sqrt{3}+1)x-(\sqrt{3}x-(\sqrt{3}+1))(\sqrt{3}-1)\sqrt{3}x(\sqrt{3}+1)x-(\sqrt{3}x-(\sqrt{3}+1))(\sqrt{3}-1)$

= $(\sqrt{3}x-\sqrt{3}-1)(\sqrt{3}x-\sqrt{3}+1)(\sqrt{3}-1)(\sqrt{3}x-\sqrt{3}-1)(\sqrt{3}x-\sqrt{3}+1)(\sqrt{3}-1)$

Either = $(\sqrt{3}x-\sqrt{3}-1)(\sqrt{3}x-\sqrt{3}-1)$

Therefore x= $\sqrt{3}+1\sqrt{3}\,x=\frac{\sqrt{3}+1}{\sqrt{3}}$

Or, $(\sqrt{3}x-\sqrt{3}+1)(\sqrt{3}-1)(\sqrt{3}x-\sqrt{3}+1)(\sqrt{3}-1)$

Therefore, x= $\sqrt{3}-1\sqrt{3}\,x=\frac{\sqrt{3}-1}{\sqrt{3}}$

The roots of the above mentioned quadratic equation are x= $\sqrt{3}-1\sqrt{3}\,x=\frac{\sqrt{3}-1}{\sqrt{3}}$ and x= $\sqrt{3}+1\sqrt{3}\,x=\frac{\sqrt{3}+1}{\sqrt{3}}$ respectively.

Question 13

Find the roots of the quadratic equation x− 1x =3 $x-\frac{1}{x}=3$

The given equation is x− 1x =3 $x-\frac{1}{x}=3$

= x− 1x =3 $x-\frac{1}{x}=3$

= x2−1x =3 $\frac{x^2-1}{x}=3$

= x^2-1 = 3x

= x^2-1 -3x=0

= $x^2-(32+32)x-1=0x^2-(\frac{3}{2}+\frac{3}{2})x-1=0$

= x2− 3+√32x− 3−√32x−1=0 $x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - 1 = 0$

= x2− 3+√32x− 3−√32x− −44 =0 $x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - \frac{-4}{4} = 0$

= x2− 3+√32x− 3−√32x− 9−134 =0 $x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - \frac{9-13}{4} = 0$

= x2− 3+√32x− 3−√32x− (3)2−(√132)(2)2 =0 $x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x - \frac{(3)^2-(\sqrt{13}^2)}{(2)^2} = 0$

= x2− 3+√32x− 3−√32x+(3+√132)(3−√132)=0

$x^2 - \frac{3+\sqrt{3}}{2}x - \frac{3-\sqrt{3}}{2}x + (\frac{3+\sqrt{13}}{2})(\frac{3-\sqrt{13}}{2}) = 0$

= (x− 3+√132)(x− 3−√132)=0 $(x - \frac{3+\sqrt{13}}{2})(x - \frac{3-\sqrt{13}}{2}) = 0$

Either (x− 3+√132)=0 $(x - \frac{3+\sqrt{13}}{2}) = 0$

Therefore 3+√132 $\frac{3+\sqrt{13}}{2}$

Or, (x− 3−√132)=0 $(x - \frac{3-\sqrt{13}}{2}) = 0$

Therefore 3−√132 $\frac{3-\sqrt{13}}{2}$

The roots of the above mentioned quadratic equation are 3+√132 $\frac{3+\sqrt{13}}{2}$ and 3−√132 $\frac{3-\sqrt{13}}{2}$ respectively.

Question 14

Find the roots of the quadratic equation 1x+4 − 1x−7 = 1130 $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

The given equation is 1x+4 − 1x−7 = 1130 $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

= 1x+4 − 1x−7 = 1130 $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

= x−7−x−4(x+4)(x−7) = 1130 $\frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$

= −11(x+4)(x−7) = 1130 $\frac{-11}{(x+4)(x-7)} = \frac{11}{30}$

Cancelling out the like numbers on both the sides of the equation

= $-1(x+4)(x-7)=130$ $\frac{-1}{(x+4)(x-7)} = \frac{1}{30}$

= x^2-3x-28 =-30

= x^2-3x-2 =0

= x^2-2x-x-2 =0

= x(x-2)-1(x-2) =0

= (x-2)(x-1) =0

Either x-2 = 0

Therefore x= 2

Or, x-1 = 0

Therefore x= 1

The roots of the above mentioned quadratic equation are 1 and 2 respectively.

Question 16

Find the roots of the quadratic equation a^2x^2-3abx+2b^2=0

The given equation is a^2x^2-3abx+2b^2=0

= a^2x^2-3abx+2b^2=0

= a^2x^2-abx-2abx+2b^2=0

= ax(ax-b)-2b(ax-b) =0

= (ax-b)(ax-2b) =0

Either ax-b=0 therefore X=ba $x = \frac{b}{a}$

Or, ax-2b =0 therefore X=2ba $x = \frac{2b}{a}$

The roots of the quadratic equation are X=2ba $x = \frac{2b}{a}$ and X=ba $x = \frac{b}{a}$ respectively.

Question 18

Find the roots of the $4x^2+4bx-(a^2-b^2)$ =0

$-4(a^2-b^2) = -4(a-b)(a+b)$

= -2(a-b) * 2(a+b)

=2(b-a) * 2(b+a)

= $4x^2$+ (2(b-a) + 2(b+a)) – (a-b)(a+b) = 0

= $4x^2$ + 2(b-a)x++ 2(b+a)x+(b-a)(a+b) =0

= 2x(2x+(b-a)) +(a+b)(2x+(b-a)) = 0

= (2x+(b-a))(2x+b+a) = 0

Either, (2x+(b-a)) = 0

Therefore X=a−b2 $x = \frac{a-b}{2}$

Or, (2x+b+a) =0

Therefore X=−a−b2 $x = \frac{-a-b}{2}$

The roots of the above mentioned quadratic equation are X=−a−b2 $x = \frac{-a-b}{2}$ and X=a−b2 $x = \frac{a-b}{2}$ respectively.

Question 19

Find the roots of the equation $ax^2+(4a^2-3b)x$ -12ab =0

The given equation is $ax^2+(4a^2-3b)x$ -12ab =0

= $ax^2+(4a^2-3b)x$ -12ab =0

= ax^2+4a^2x-3bx -12ab =0

= ax(x-4a) – 3b(x-4a) =0

= (x-4a)(ax-4b) = 0

Either x-4a =0

Therefore x= 4a

Or, ax-4b = 0

Therefore X=4ba $x = \frac{4b}{a}$

The roots of the above mentioned quadratic equation are X=4ba $x = \frac{4b}{a}$ and 4a respectively.

Question 22

Find the roots of x+3x+2=3x−72x−3 $\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$

The given equation is x+3x+2=3x−72x−3 $\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$

= =(x+3)(2x-3)=(x+2){3x-7)

$=2x^2-3x+6x-9=3x^2-x-14$

$=2x^2+3x-9=3x^2-x-14$

$=x^2-3x-x-14+9=0$

$= x^2-5x+x-5 = 0$

=x(x-5)+1(x-5)=0

=(x-5) (x+l)-0

Either x-5-0 or x+1=0

x=5 and x=-1

The roots of the above mentioned quadratic equation are 5 and -1 respectively.

Question 23

Find the roots of the equation 2xx−4+2x−5x−3=253 $\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$

The given equation is 2xx−4+2x−5x−3=253 $\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$

= 2x(x−3)+(2x−5)(x−4)(x−4)(x−3)=253 $\frac{2x(x-3)+(2x-5)(x-4)}{(x-4)(x-3)}=\frac{25}{3}$

= 2x²−6x+2x²−5x−8x+20x²−4x−3x+12=253 $\frac{2x^2-6x+2x^2-5x-8x+20}{x^2-4x-3x+12}=\frac{25}{3}$

= 4x²−19x+20x²−7x+12=253 $\frac{4x^2-19x+20}{x^2-7x+12}=\frac{25}{3}$

= $3(4x^2-19x+20) = 25(x^2-7x+12)$

= $12x^2-57x+60 = 25x^2 - 175x+300$

= $13x^2-78x-40x+240=0$

= $13x^2-118x+240=0$

= $13x^2-78x-40x+240=0$

= $13x(x-6)-40(x-6) =0$

= $(x-6)(13x-40) =0$

Either x-6 = 0 therefore x= 6

Or , 13x-40 = 0 therefore x = 4013 $\frac{40}{13}$

The roots of the above mentioned quadratic equation are 6 and 4013 $\frac{40}{13}$ respectively.

Question 24

Find the roots of the quadratic equation x+3x−2−1−xx=174 $\frac{x+3}{x-2}-\frac{1-x}{x}=\frac{17}{4}$

The given equation is x+3x−2−1−xx=174 $\frac{x+3}{x-2}-\frac{1-x}{x}=\frac{17}{4}$

= x(x+3)−(x−2)(1−x)x(x−2)=174 $\frac{x(x+3)-(x-2)(1-x)}{x(x-2)}=\frac{17}{4}$

= x²+3x−x+x²+2−2xx²−2x=174 $\frac{x^2+3x-x+x^2+2-2x}{x^2-2x}=\frac{17}{4}$

= $2x^2$+$2x^2$−2x=174 $\frac{2x^2+2}{x^2-2x} = \frac{17}{4}$

= $4(2x^2+2) = 17(x^2-2x)$

= $8x^2+8 = 17x^2-34x$

= $9x^2-34x-8 = 0$

= $9x^2-36x+2x-8 = 0$

= $9x(x-4)+2(x-4) =0$

= $(9x+2)(x-4) =0$

Either 9x+2 = 0 therefore X=−29 $x = \frac{-2}{9}$

Or, x-4 = 0 therefore x= 4

The roots of the above mentioned quadratic equation are X=−29 $x = \frac{-2}{9}$ and 4 respectively.

Question 26

Find the roots of the quadratic equation 1x−2+2x−1=6x $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$

The equation is 1x−2+2x−1=6x $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$

= (x−1)+2(x−2)(x−2)(x−1)=6x $\frac{(x-1)+2(x-2)}{(x-2)(x-1)} = \frac{6}{x}$

= (x−1)+2x−4(x^2−2x−x+2)=6x $\frac{(x-1)+2x-4}{(x^2-2x-x+2)} = \frac{6}{x}$

= 3x−5(x^2−3x+2)=6x $\frac{3x-5}{(x^2-3x+2)} = \frac{6}{x}$

= $x(3x-5) = 6(x^2-3x+2)$

= $3x^2-5x= 6x^2-18x+12$

= $3x^2-13x+12 =0$

= $3x^2-9x-4x+12 =0$

= 3x(x-3)-4(x-3) =0

= (x-3)(3x-4) = 0

Either x-3 = 0 therefore x= 3

Or, 3x-4 = 0 therefore 43 $\frac{4}{3}$

The roots of the above mentioned quadratic equation are 3 and 43 $\frac{4}{3}$ respectively.

Question 27

Find the roots of the quadratic equation x+1x−1−x−1x+1=56 $\frac{x+1}{x-1} - \frac{x-1}{x+1} = \frac{5}{6}$

The equation is x+1x−1−x−1x+1=56 $\frac{x+1}{x-1} - \frac{x-1}{x+1} = \frac{5}{6}$

= (x+1)2−(x−1)2x2−1=56 $\frac{(x+1)^2-(x-1)^2}{x^2-1} = \frac{5}{6}$

= 4xx2−1=56 $\frac{4x}{x^2-1} = \frac{5}{6}$

= 6(4x) = 5(x^2-1)

= 24x= 5x^2-5

= 5x^2-24x-5 =0

= 5x^2-25x+x-5 =0

= 5x(x-5)+1(x-5) =0

= (5x+1)(x-5) =0

Either x-5 =0

Therefore x= 5

Or, 5x+1 = 0

Therefore X=−15 $x = \frac{-1}{5}$

The roots of the above mentioned quadratic equation are X=−15 $x = \frac{-1}{5}$ and 5 respectively.

Question 28

Find the roots of the quadratic equation x−12x+1+2x+1x−1=52 $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=\frac{5}{2}$

The equation is x−12x+1+2x+1x−1=52 $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=\frac{5}{2}$

= (x−1)2+(2x+1)22x2−2x+x−1=52 $\frac{(x-1)^2+(2x+1)^2}{2x^2-2x+x-1}=\frac{5}{2}$

= x2−2x+1+4x2+4x+12x2−x−1=52 $\frac{x^2-2x+1+4x^2+4x+1}{2x^2-x-1}=\frac{5}{2}$

= 5x2+2x+22x2−x−1=52 $\frac{5x^2+2x+2}{2x^2-x-1}=\frac{5}{2}$

= $2(5x^2+2x+2) = 5(2x^2-x-1)$

= $10x^2+4x+4 = 10x^2-5x-5$

Cancelling out the equal terms on both sides of the equation. We get,

= 4x+5x+4+5=0

= 9x+9=0

= 9x = -9

X = -1

X = -1 is the only root of the given equation.

Question 44

Find the roots of the quadratic equation mnx2+nm=1−2x $\frac{m}{n}x^2+\frac{n}{m}=1-2x$

The given equation is mnx2+nm=1−2x $\frac{m}{n}x^2+\frac{n}{m}=1-2x$

= mnx2+nm=1−2x $\frac{m}{n}x^2+\frac{n}{m}=1-2x$

= m2x2+n2mn=1−2x $\frac{m^2x^2+n^2}{mn}=1-2x$

$= m^2x^2+2mnx+(n^2-mn) = 0$

Now we solve the above quadratic equation using factorization method

Therefore

$= (m^2x^2+mnx+m\sqrt{mnx})+(mnx-m\sqrt{mnx}(n+\sqrt{mn})(n-\sqrt{mn}))=0$
$(m^2x^2+mnx+m\sqrt{mnx})+(mnx-m\sqrt{mnx}(n+\sqrt{mn})(n-\sqrt{mn}))=0$

$= (m^2x^2+mnx+m\sqrt{mnx})+(mx(n-\sqrt{mn})+(n+\sqrt{mn})(n-\sqrt{mn}))=0$
$(m^2x^2+mnx+m\sqrt{mnx})+(mx(n-\sqrt{mn})+(n+\sqrt{mn})(n-\sqrt{mn}))=0$

$= mx(mx+n+\sqrt{mn})+(n-\sqrt{mn})(mx+n+\sqrt{mn})=0$
$mx(mx+n+\sqrt{mn})+(n-\sqrt{mn})(mx+n+\sqrt{mn})=0$

$= (mx+n+\sqrt{mn})(mx+n-\sqrt{mn})=0$ $(mx+n+\sqrt{mn})(mx+n-\sqrt{mn})=0$

Now, one of the products must be equal to zero for the whole product to be zero for the whole product to be zero. Hence, we equate both the products to zero in order to find the value of x .

Therefore,

$(mx+n+\sqrt{mn})=0$ $(mx+n+\sqrt{mn})=0$ $mx=-n-\sqrt{mn}$ $mx=-n-\sqrt{mn}$
x= $-n-\sqrt{mn}m$ $x=\frac{-n-\sqrt{mn}}{m}$

Or

$(mx+n-\sqrt{mn})=0$ $(mx+n-\sqrt{mn})=0$

x= $-n+\sqrt{mn}m$ $x=\frac{-n+\sqrt{mn}}{m}$ x= $-n+\sqrt{mn}m$ $x=\frac{-n+\sqrt{mn}}{m}$

The roots of the above mentioned quadratic equation are x= $-n+\sqrt{mn}m$ $x=\frac{-n+\sqrt{mn}}{m}$ and x= $-n-\sqrt{mn}m$ $x=\frac{-n-\sqrt{mn}}{m}$ respectively.

Question 45

Find the roots of the quadratic equation x−ax−b+x−bx−a=ab+ba $\frac{x-a}{x-b}+\frac{x-b}{x-a}=\frac{a}{b}+\frac{b}{a}$

The given equation is x−ax−b + x−bx−a = ab + ba $\frac{x-a}{x-b}+\frac{x-b}{x-a}=\frac{a}{b}+\frac{b}{a}$

= x−ax−b + x−bx−a = ab + ba $\frac{x-a}{x-b}+\frac{x-b}{x-a}=\frac{a}{b}+\frac{b}{a}$

= $(x-a)^2+(x-b)^2(x-a)(x-b)$ = ab + ba $\frac{(x-a)^2+(x-b)^2}{(x-a)(x-b)}=\frac{a}{b}+\frac{b}{a}$

= $x^2-2ax+a^2+x^2-2bx+b^2x^2+ab-bx-ax = a^2+b^2ab$ $\frac{x^2-2ax+a^2+x^2-2bx+b^2}{x^2+ab-bx-ax}=\frac{a^2+b^2}{ab}$

= $(2x^2-2x(a+b)+a^2+b^2)ab = (a^2+b^2)(x^2-(a+b)x+ab)$

= $(2abx^2-2abx(a+b)+ab(a^2+b^2)) = (a^2+b^2)(x^2-(a^2+b^2)(a+b)x+(a^2+b^2)(ab)$

= $(a^2+b^2-2ab)x-(a+b)(a^2+b^2-2ab)x=0$

= $(a-b)^2x^2-(a+b)(a+b)^2x^2=0$

= $x(a-b)^2(x-(a+b))=0$

= $x(x-(a+b))=0$

Either x= 0

Or, (x-(a+b))=0

Therefore x= a+b

The roots of the above mentioned quadratic equation are 0 and a+b respectively.

Question 46

Find the roots of the quadratic equation 1(x−1)(x−2) + 1(x−2)(x−3) + 1(x−3)(x−4) = 16

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{1}{6}$$

The given equation is 1(x−1)(x−2) + 1(x−2)(x−3) + 1(x−3)(x−4) = 16

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{1}{6}$$

= 1(x−1)(x−2) + 1(x−2)(x−3) + 1(x−3)(x−4) = 16 $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{1}{6}$

$= (x-3)(x-4)+(x-1)(x-4)+(x-1)(x-2)(x-1)(x-2)(x-3)(x-3)(x-4) = 16$

$$\frac{(x-3)(x-4)+(x-1)(x-4)+(x-1)(x-2)}{(x-1)(x-2)(x-3)(x-3)(x-4)} = \frac{1}{6}$$

$= (x-3)(x-4)+(x-1)[(x-4)+(x-2)](x-1)(x-2)(x-3)(x-3)(x-4) = 16$ $\frac{(x-3)(x-4)+(x-1)[(x-4)+(x-2)]}{(x-1)(x-2)(x-3)(x-3)(x-4)} = \frac{1}{6}$

$= (x-3)(x-4)+(x-1)(2x-6)(x-1)(x-2)(x-3)(x-3)(x-4) = 16$ $\frac{(x-3)(x-4)+(x-1)(2x-6)}{(x-1)(x-2)(x-3)(x-3)(x-4)} = \frac{1}{6}$

$= (x-3)(x-4)+(x-1)2(x-3)(x-1)(x-2)(x-3)(x-3)(x-4) = 16$ $\frac{(x-3)(x-4)+(x-1)2(x-3)}{(x-1)(x-2)(x-3)(x-3)(x-4)} = \frac{1}{6}$

$= (x-3)[(x-4)+(2x-2)](x-1)(x-2)(x-3)(x-3)(x-4) = 16$ $\frac{(x-3)[(x-4)+(2x-2)]}{(x-1)(x-2)(x-3)(x-3)(x-4)} = \frac{1}{6}$

$= (x-3)(3x-6)(x-1)(x-2)(x-3)(x-3)(x-4) = 16$ $\frac{(x-3)(3x-6)}{(x-1)(x-2)(x-3)(x-3)(x-4)} = \frac{1}{6}$

$= 3(x-3)(x-2)(x-1)(x-2)(x-3)(x-3)(x-4) = 16$ $\frac{3(x-3)(x-2)}{(x-1)(x-2)(x-3)(x-3)(x-4)} = \frac{1}{6}$

Cancelling out the like terms on both the sides of numerator and denominator. We get,

$= 3(x-1)(x-2)(x-4) = 16$ $\frac{3}{(x-1)(x-2)(x-4)} = \frac{1}{6}$

= (x-1)(x-4) = 18

= x^2-4x-x+4 =18

= x^2-5x-14=0

= x^2-7x+2x-14=0

= x(x-7)+2(x-7)=0

= (x-7)(x+2)=0

Either x-7 = 0

Therefore x=7

Or, x+2 =0

Therefore x= -2

The roots of the above mentioned quadratic equation are 7 and -2 respectively.

Question 49

Find the roots of the quadratic equation ax−a+bx−b=2cx−c $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$

The given equation is ax−a+bx−b=2cx−c $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$

= ax−a+bx−b=2cx−c $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$

= a(x−b)+b(x−a)(x−b)(x−a)=2cx−c $\frac{a(x-b)+b(x-a)}{(x-b)(x-a)} = \frac{2c}{x-c}$

= ax−ab+bx−ab(x2−bx−ax+ab)=2cx−c $\frac{ax-ab+bx-ab}{(x^2-bx-ax+ab)} = \frac{2c}{x-c}$

= (x-c)(ax-2ab+bx) = 2c(x^2-bx-ax+ab)

= (a+b)x^2-2abx-(a+b)cx+2abc= 2cx^2-2c(a+b)x+2abc

Question 50

Find the roots of the Question x^2+2ab=(2a+b)x

The given equation is x^2+2ab=(2a+b)x

= x^2+2ab = (2a+b)x

= x^2-(2a+b)x+2ab = 0

= x^2-2ax-bx+2ab = 0

= x(x-2a)-b(x-2a) =0

= (x-2a)(x-b) =0

Either x-2a = 0

Therefore x= 2a

Or, x-b =0

Therefore x= b

The roots of the above mentioned quadratic equation are 2a and b respectively.

Question 51

Find the roots of the quadratic equation $(a+b)^2x^2$-4abx-$(a-b)^2$ =0

The given equation is $(a+b)^2x^2$-4abx-$(a-b)^2$ =0

= $(a+b)^2x^2$-4abx-$(a-b)^2$ =0

= $(a+b)^2x^2$-($(a+b)^2$ –$(a-b)^2$)x-$(a-b)^2$ =0

= $(a+b)^2x^2$-$(a+b)^2$ x +$(a-b)^2$ x-$(a-b)^2$ =0

= $(a+b)^2$x(x-1) $(a+b)^2$ (x-1)=0

= (x-1) $(a+b)^2$x+$(a+b)^2$) =0

Either x-1 =0

Therefore x= 1

Or, $(a+b)^2$x+$(a+b)^2$) =0

Therefore $-(a-ba+b)^2-(\frac{a-b}{a+b})^2$

The roots of the above mentioned quadratic equation are $-(a-ba+b)^2-(\frac{a--b}{a+b})^2$ and 1 respectively .

Question 52

Find the roots of the quadratic equation a(x^2+1)-x(a^2+1)= 0

The given equation is a(x^2+1)-x(a^2+1)= 0

= a(x^2+1)-x(a^2+1)= 0

= ax^2+a-a^2x-x= 0

= ax(x-a)-1(x-a) =0

= (x-a)(ax-1) =0

Either x-a =0

Therefore x= a

Or, ax-1 =0

Therefore X=1a$x = \frac{1}{a}$

The roots of the above mentioned quadratic equation are (a) and X=1a$x = \frac{1}{a}$ respectively.

Question 54

Find the roots of the quadratic equation x²+(a+1a)x+1=0$x^2 + (a + \frac{1}{a})x + 1 = 0$

The given equation is x²+(a+1a)x+1=0$x^2 + (a + \frac{1}{a})x + 1 = 0$

= x²+(a+1a)x+1=0$x^2 + (a + \frac{1}{a})x + 1 = 0$

= x²+ax+xa+(a×1a)=0$x^2 + ax + \frac{x}{a} + (a \times \frac{1}{a}) = 0$

= x(x+a)+1a(x+a)=0$x(x + a) + \frac{1}{a}(x + a) = 0$

= (x+a)(x+1a)=0$(x + a)(x + \frac{1}{a}) = 0$

Either x+a = 0

Therefore x= -a

Or , (x+1a)=0$(x + \frac{1}{a}) = 0$

Therefore X=1a$x = \frac{1}{a}$

The roots of the above mentioned quadratic equation are X=1a$x = \frac{1}{a}$ and –a respectively.

Question 55

Find the roots of the quadratic equation $abx^2+(b^2-ac)x-bc=0$

The given equation is $abx^2+(b^2-ac)x-bc=0$

$= abx^2+(b^2-ac)x-bc=0$

$= abx^2+b^2x-acx-bc=0$

$= bx(ax+b)-c(ax+b)=0$

$= (ax+b)(bx-c) = 0$

Either, $ax+b = 0$

Therefore X=−ba $x = \frac{-b}{a}$

Or, $bx-c = 0$

Therefore X=cb $x = \frac{c}{b}$

The roots of the above mentioned quadratic equation are X=cb $x = \frac{c}{b}$ and X=−ba $x = \frac{-b}{a}$ respectively.

Question 56

Find the roots of the quadratic equation $a^2b^2x^2+b^2x-a^2x-1=0$

The given equation is $a^2b^2x^2+b^2x-a^2x-1=0$

$= a^2b^2x^2+b^2x-a^2x-1=0$

$= b^2x(a^2x+1)-1(a^2x+1)$

$= (a^2x+1)(b^2x-1)=0$

Either $(a^2x+1)=0$

Therefore X=−1a² $x = \frac{-1}{a^2}$

Or, $(b^2x-1)=0$

Therefore X=1b² $x = \frac{1}{b^2}$

The roots of the above mentioned quadratic equation are X=1b^2 $x = \frac{1}{b^2}$ and X=−1a^2 $x = \frac{-1}{a^2}$ respectively.