

Exercise 7.1: Statistics

Q.1: Calculate the mean for the following distribution:

| | | | | | |
|-----------|----------|----------|-----------|-----------|----------|
| x: | 5 | 6 | 7 | 8 | 9 |
| f: | 4 | 8 | 14 | 11 | 3 |

Sol:

| X | f | fx |
|----------|---------------|------------|
| 5 | 4 | 20 |
| 6 | 8 | 48 |
| 7 | 14 | 98 |
| 8 | 11 | 88 |
| 9 | 3 | 27 |
| | N = 40 | 281 |

$$\text{Mean} = 281/4 = 7.025$$

2. Find the mean of the following data:

| | | | | | | | |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| x: | 19 | 21 | 23 | 25 | 27 | 29 | 31 |
| f: | 13 | 15 | 16 | 18 | 16 | 15 | 13 |

Soln:

| X | f | fx |
|-----------|----------------|-------------------|
| 18 | 13 | 247 |
| 21 | 15 | 315 |
| 23 | 16 | 368 |
| 25 | 18 | 450 |
| 27 | 16 | 432 |
| 29 | 15 | 435 |
| 31 | 13 | 403 |
| | N = 106 | Sum = 2620 |

$$\text{Mean (x)} = 2680/106 = 25$$

3. If the mean of the following data is 20.6. Find the value of p.

| | | | | | |
|-----------|-----------|-----------|-----------|-----------|-----------|
| x: | 10 | 15 | p | 25 | 35 |
| f: | 3 | 10 | 25 | 7 | 5 |

Soln:

| | | |
|--|--|--|
| | | |
|--|--|--|

| X | f | fx |
|----|---------|------------|
| 10 | 3 | 30 |
| 5 | 10 | 150 |
| P | 25 | 25p |
| 25 | 7 | 175 |
| 35 | 5 | 175 |
| | N = 106 | Sum = 2620 |

Given

$$\text{Mean} = 20.6$$

$$(530 + 25p)/50 = 20.6$$

$$25p = 20.6$$

$$P = 20$$

4. If the mean of the following data is 15, find p

| | | | | | |
|-----------|----------|-----------|-----------|-----------|-----------|
| x: | 5 | 10 | 15 | 20 | 25 |
| f: | 6 | p | 6 | 10 | 5 |

Soln:

| X | f | fx |
|----|----------|-----------------|
| 5 | 6 | 30 |
| 10 | P | 10p |
| 15 | 6 | 90 |
| 20 | 10 | 200 |
| 25 | 5 | 125 |
| | N = p+27 | Sum = 10p + 445 |

Given

Mean =15

$$(10p + 445)/(p+27) = 15$$

$$10p + 445 = 15p + 405$$

$$15p - 10p = 445 - 405$$

$$5p = 40$$

$$P = 8$$

5.Find the value of p for the following distribution whose mean is 16.6

| | | | | | | | |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| X: | 8 | 12 | 15 | p | 20 | 25 | 30 |
| F: | 12 | 16 | 20 | 24 | 16 | 8 | 4 |

Soln:

| X | f | fx |
|----|---------|------------------|
| 8 | 12 | 96 |
| 12 | 12 | 192 |
| 15 | 20 | 300 |
| P | 24 | 24p |
| 20 | 16 | 320 |
| 25 | 8 | 200 |
| 30 | 4 | 120 |
| | N = 100 | Sum = 24p + 1228 |

Given

Mean = 16.6

$$(24p+1228)/100 = 16.6$$

$$24p + 1228 = 1660$$

$$24p = 1660 - 1228$$

$$P = 432/24$$

$$P = 18$$

6. Find the missing value of p for the following distribution whose mean is 12.58

| | | | | | | | |
|-----------|----------|----------|-----------|-----------|----------|-----------|-----------|
| x: | 5 | 8 | 10 | 12 | p | 20 | 25 |
| f: | 2 | 5 | 8 | 22 | 7 | 4 | 2 |

Soln:

| x | f | fx |
|----|--------|----------------|
| 5 | 2 | 10 |
| 8 | 5 | 40 |
| 10 | 8 | 80 |
| 12 | 22 | 264 |
| P | 7 | 7p |
| 20 | 4 | 480 |
| 25 | 2 | 50 |
| | N = 50 | Sum = 524 + 7p |

Given mean = 12.58

$$\text{Sum}/N = 12.58$$

$$(524 + 7p)/50 = 12.58$$

$$524 + 7p = 629$$

$$7p = 105$$

$$P = 15$$

7. Find the missing frequency (p) for the following distribution whose mean is 7.68.

| | | | | | | |
|-----------|----------|----------|-----------|----------|-----------|-----------|
| x: | 3 | 5 | 7 | 9 | 11 | 13 |
| f: | 6 | 8 | 15 | p | 8 | 4 |

Soln:

| X | f | fx |
|----|------------|--------------|
| 3 | 6 | 18 |
| 5 | 8 | 40 |
| 7 | 15 | 105 |
| 9 | p | 9p |
| 11 | 8 | 88 |
| 13 | 4 | 52 |
| | N = P + 41 | Sum 9p = 303 |

Given

$$\text{Mean} = 7.68$$

$$(7p+303)/p+41 = 7.68$$

$$9p + 303 = p(7.68) + 314.88$$

$$9p - p(7.68) = 314.88 - 303$$

$$1.32p = 11.88$$

$$P = (11.88)/1.32$$

$$P = 9$$

8.The following table gives the number of boys of a particular age in a class of 40 students. Calculate the mean age of the students.

| Ages (in years): | 15 | 16 | 17 | 18 | 19 | 20 |
|-------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| No of students: | 3 | 8 | 10 | 10 | 5 | 4 |

Soln:

| x | f | fx |
|----------|---------------|------------------|
| 15 | 3 | 45 |
| 16 | 8 | 128 |
| 17 | 10 | 170 |
| 18 | 10 | 180 |
| 19 | 5 | 95 |
| 20 | 4 | 80 |
| | N = 40 | Sum = 698 |

$$\text{Mean age} = \text{sum} / N$$

$$= 698 / 40$$

$$= 17.45 \text{ years}$$

9.Candidates of four schools appear in a mathematics test. The data were as follows:

| Schools | No of candidates | Average score |
|---------|------------------|---------------|
| I | 60 | 75 |
| II | 48 | 80 |
| III | P | 55 |
| IV | 40 | 50 |

If the average score of the candidates of all the four schools is 66, find the number of candidates that appeared from school III.

Soln: Let the number candidates from school III = P

| Schools | No of candidates N_i | Average scores (x_i) |
|---------|------------------------|--------------------------|
| I | 60 | 75 |
| II | 48 | 80 |
| III | P | 55 |
| IV | 40 | 50 |

Given

Average score or all schools = 66

$$N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4 = (N_1 + N_2 + N_3 + N_4) \times \text{Average}$$

$$4500 + 3340 + 55p + 2000 = (60 + 48 + p + 40) \times 66$$

$$10340 + 55p = 66p + 9768$$

$$10340 - 9768 = (66 - 55)p$$

$$P = 572/11$$

10. Five coins were simultaneously tossed 1000 times and at each toss, the number of heads was observed. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss.

| No of heads per toss | No of tosses |
|----------------------|--------------|
| 0 | 38 |
| 1 | 144 |
| 2 | 342 |
| 3 | 287 |
| 4 | 164 |
| 5 | 25 |
| Total | 1000 |

Soln:

| No of heads per toss | No of tosses |
|----------------------|--------------|
| 0 | 38 |
| 1 | 144 |
| 2 | 342 |
| 3 | 287 |
| 4 | 164 |
| 5 | 25 |

| No of heads per toss | No of tosses | fx |
|----------------------|--------------|-----|
| 0 | 38 | 0 |
| 1 | 144 | 144 |

| | | |
|---|-----|-----|
| 2 | 342 | 684 |
| 3 | 287 | 861 |
| 4 | 164 | 656 |
| 5 | 25 | 125 |

Mean number of heads per toss = $2470/1000 = 2.47$

Mean = 2.47

12. The arithmetic mean of the following data is 25. Find the value of k.

| | | | | | |
|--------------------------|----------|-----------|-----------|-----------|-----------|
| X_i: | 5 | 15 | 25 | 35 | 45 |
| f_i: | 3 | k | 3 | 6 | 2 |

Sol:

| X | f | fx |
|----|---------------|-------------------|
| 5 | 3 | 15 |
| 15 | k | 15k |
| 25 | 3 | 75 |
| 35 | 6 | 210 |
| 45 | 2 | 90 |
| | $N = k + 120$ | Sum = $15k + 390$ |

Given mean = 25

Sum/ N = 25

$15k + 390 = 25k + 350$

$25k - 15k = 40$

$10k = 40$

$$k = 4$$

13. If the mean of the following data is 18.75. Find the value of p.

| | | | | | |
|---------|----|----|---|----|----|
| X_i : | 10 | 15 | p | 25 | 30 |
| F_i : | 5 | 10 | 7 | 8 | 2 |

Soln:

| X | f | fx |
|----|---------------|-------------------------|
| 10 | 5 | 50 |
| 15 | 10 | 150 |
| P | 7 | 7p |
| 25 | 8 | 200 |
| 30 | 2 | 60 |
| | $N = k + 120$ | $\text{Sum} = 1p + 460$ |

Given mean = 18.75

$$\text{Sum} / N = 18.75$$

$$7p + 460 = 600$$

$$7p = 140$$

$$P = 20$$

14. Find the value of p. If the mean of the following distribution is 20.

| | | | | | |
|-------|----|----|----|----------|----|
| x : | 15 | 17 | 19 | $20 + p$ | 23 |
|-------|----|----|----|----------|----|

| | | | | | |
|----|---|---|---|----|---|
| f: | 2 | 3 | 4 | 5p | 6 |
|----|---|---|---|----|---|

Soln:

| X | f | fx |
|--------|--------------|------------------------------------|
| 15 | 2 | 30 |
| 17 | 3 | 51 |
| 19 | 4 | 76 |
| 20 + p | 5p | 100p+5p ² |
| 23 | 6 | 138 |
| | N = 5p + 15` | Sum = 295 + 100p + 5p ² |

Given Mean= 2n

Sum/ N = 20

$$(295 + 100p - 5p^2) / (5 + 15) = 20$$

$$295 + 100p + 5p^2 = 100p + 300$$

$$5p^2 - 5 = 0$$

$$5(p^2 - 1) = 0$$

$$p^2 - 1 = 0$$

$$P = (+1, -1)$$

$$\text{If } p + 1 = 0$$

$$P = -1$$

$$\text{Or } p - 1 = 0$$

$$P = 1$$

15. Find the missing frequencies in the following frequency distribution if it is known that the mean of the distribution is 50.

| | | | | | |
|-----------|-----------|-------------------------|-----------|-------------------------|-----------|
| X: | 10 | 30 | 50 | 70 | 90 |
| f: | 17 | f_1 | 32 | f_2 | 19 |

Soln:

| x | f | fx |
|----|---------|------------------------------|
| 10 | 17 | 170 |
| 30 | f_1 | $30f_1$ |
| 50 | 32 | 1600 |
| 70 | f_2 | $70f_2$ |
| 90 | 19 | 1710 |
| | N = 120 | Sum = $30f_1 + 70f_2 + 3480$ |

Given mean

$$\text{Sum} / N = 50$$

$$30f_1 + 70f_2 + 3480 / 120 = 50$$

$$30f_1 + 70f_2 + 3480 = 6000 \text{ --- (1)}$$

Also, sum of f = 120

$$17 + f_1 + 32 + f_2 + 19 = 120$$

$$f_1 + f_2 = 52$$

$$f_1 = 52 - f_2$$

Substituting the value of f_1 in (1)

$$30(52 - f_2) + 70f_2 + 3480 = 6000$$

$$f_2 = 24$$

Hence $f_1 = 52 - 24 = 28$

$f_1 = 28$; $f_2 = 24$

Exercise 7.2: Statistics

1. The number of telephone calls received at an exchange per interval for 250 successive one- minute intervals are given in the following frequency table:

| No. of calls(x): | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|--------------------------|----|----|----|----|----|----|----|
| No. of intervals (f): | 15 | 24 | 29 | 46 | 54 | 43 | 39 |

Compute the mean number of calls per interval.

Soln: Let be assumed mean (A) = 3

| No. of calls x_i | No. of intervals f_i | $u_1 = x_i - A = x_i - 3$ $u_1 = x_i - A = x_i - 3$ | $f_i u_i$ |
|-----------------------|------------------------------|--|-----------|
| 0 | 15 | -3 | -45 |
| | | | |

| | | | |
|---|--------|----|------------|
| 1 | 24 | -2 | -47 |
| 2 | 29 | -1 | -39 |
| 3 | 46 | 0 | 0 |
| 4 | 54 | | 54 |
| 5 | 43 | 2 | 43(2) = 86 |
| 6 | 39 | 3 | 47 |
| | N= 250 | | Sum = 135 |

Mean number of cells = $3 + \frac{135}{250} = \frac{885}{250} = 3.54$

2. Five coins were simultaneously tossed 1000 times, and at each toss the number of heads was observed. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss.

| No of heads per toss (x): | 0 | 1 | 2 | 3 | 4 | 5 |
|---------------------------|----|-----|-----|-----|-----|----|
| No of tosses (f): | 38 | 144 | 342 | 287 | 164 | 25 |

Soln: Let the assumed mean (A) = 2

| No. of heads per toss x_i | No of intervals f_i | $u_i = \frac{A_i - x_i}{h} = \frac{2 - x_i}{1} = 2 - x_i$ | $f_i u_i$ |
|--------------------------------|--------------------------|---|-----------|
| 0 | 38 | -2 | -76 |
| 1 | 144 | -1 | -144 |
| 2 | 342 | 0 | 0 |
| 3 | 287 | 1 | 287 |
| | | | |

| | | | |
|---|---------|---|-----------|
| 4 | 164 | 2 | 328 |
| 5 | 25 | 3 | 75 |
| | N= 1000 | | Sum = 470 |

Mean number of per toss = $2 + 470/1000 = 2 + 0.47 = 2.47$

3. The following table gives the number of branches and number of plants in the garden of a school.

| No of branches (x): | 2 | 3 | 4 | 5 | 6 |
|---------------------|----|----|----|----|----|
| No of plants (f): | 49 | 43 | 57 | 38 | 13 |

Calculate the average number of branches per plant.

Soln:

Let the assumed mean (A) = 4

| No of branches x_i | No of plants f_i | $u_i = x_i - A = x_i - 4$ | $f_i u_i$ |
|-------------------------|--------------------|---------------------------|-----------|
| 2 | 49 | -2 | -98 |
| 3 | 43 | -1 | -43 |
| 4 | 57 | 0 | 0 |
| 5 | 38 | 1 | 38 |
| 6 | 13 | 2 | 26 |
| | N = 200 | | Sum = -77 |

Average number of branches per plant = $4 + (-77/200) = 4 - 77/200 = (800 - 77)/200 = 3.615$

4. The following table gives the number of children of 150 families in a village

| | | | | | | |
|---------------------|----|----|----|----|----|---|
| No of children (x): | 0 | 1 | 2 | 3 | 4 | 5 |
| No of families (f): | 10 | 21 | 55 | 42 | 15 | 7 |

Find the average number of children per family.

Soln: Let the assumed mean (A) = 2

| No of children x_i | No of families f_i | $u_i = x_i - A = x_i - 2$ | $f_i u_i$ |
|-------------------------|-------------------------|---------------------------|-----------|
| 0 | 10 | -2 | -20 |
| 1 | 21 | -1 | -21 |
| 3 | 42 | 1 | 42 |
| 4 | 15 | 2 | 30 |
| 5 | 7 | 5 | 35 |
| | N = 20 | | Sum = 52 |

Average number of children for family = $2 + 52/150 = (300 + 52)/150 = 352/150 = 2.35$
(approx)

5. The marks obtained out of 50, by 102 students in a physics test are given in the frequency table below:

| | | | | | | | | | |
|----------------|----|----|----|----|----|----|----|----|----|
| Marks (x): | 15 | 20 | 22 | 24 | 25 | 30 | 33 | 38 | 45 |
| Frequency (f): | 5 | 8 | 11 | 20 | 23 | 18 | 13 | 3 | 1 |

Find the average number of marks.

Soln: Let the assumed mean (A) = 25

| Marks x_i | Frequency f_i | $u_i = x_i - A = x_i - 25$ | $f_i u_i$ |
|-------------|-----------------|----------------------------|-----------|
| 15 | 5 | -10 | -50 |
| 20 | 8 | -5 | -40 |
| 22 | 8 | -3 | -24 |
| 24 | 20 | -1 | -20 |
| 25 | 23 | 0 | 0 |
| 30 | 18 | 5 | 90 |
| 33 | 13 | 8 | 104 |
| 38 | 3 | 12 | 36 |
| 45 | 3 | 20 | 60 |
| | N = 122 | | Sum = 110 |

Average number of marks = $25 + 110/122$

= $(2550 + 110)/122$

= $2660/122$

= 26.08 (Approx)

6. The number of students absent in a class was recorded every day for 120 days and the information is given in the following

| | | | | | | | |
|----------------------------|--------|--------|----|----|----|----|---|
| No of students absent (x): | 0 6 | 1 7 | 2 | 3 | 4 | 5 | |
| No of days (f): | 1 2 | 4 | 10 | 50 | 34 | 15 | 4 |

Find the mean number of students absent per day.

Soln: Let mean assumed mean (A) = 3

| No of students absent x_i | No of days f_i | $u_i = x_i - A = x_i - 3$ $u_i = x_i - A = x_i - 3$ | $f_i u_i$ |
|--------------------------------|---------------------|--|-----------|
| 3 | 1 | -3 | -3 |
| 1 | 4 | -2 | -8 |
| 2 | 10 | -1 | -10 |
| 3 | 50 | 0 | 0 |
| 4 | 34 | 1 | 34 |
| 5 | 15 | 2 | 30 |
| 6 | 4 | 3 | 12 |
| 7 | 2 | 4 | 8 |
| | N = 120 | | Sum = 63 |

Mean number of students absent per day = $3 + 63/120$

$$= (360 + 63)/120$$

$$= 423/120$$

$$= 3.53$$

7. In the first proof of reading of a book containing 300 pages the following distribution of misprints was obtained:

| No of misprints per page (x): | 0 | 1 | 2 | 3 | 4 | 5 |
|----------------------------------|-----|----|----|---|---|---|
| No of pages (f): | 154 | 96 | 36 | 9 | 5 | 1 |

Find the average number of misprints per page.

Soln: Let the assumed mean (A) = 2

| No of misprints per page x_i | No of days f_i | $u_i = x_i - A = x_i - 3$ | $f_i u_i$ |
|-----------------------------------|---------------------|---------------------------|-----------|
| 0 | 154 | -2 | -308 |
| 1 | 95 | -1 | -95 |
| 2 | 36 | 0 | 0 |
| 3 | 9 | 1 | 9 |
| 4 | 5 | 2 | 10 |
| 5 | 1 | 3 | 3 |
| | N = 300 | | Sum = 381 |

Average number of misprints per day = $2 + (-381/300)$

$$= 2 - 381/300$$

$$= (600-381)/300$$

$$= 219/300$$

$$= 0.73$$

8. The following distribution gives the number of accidents met by 160 workers in a factory during a month.

| No of accidents (x): | 0 | 1 | 2 | 3 | 4 |
|-------------------------|----|----|----|---|---|
| No of workers (f): | 70 | 52 | 34 | 3 | 1 |

Find the average number of accidents per worker.

Soln: Let the assumed mean (A) = 2

| No of accidents | No of workers $f_i f_i$ | $u_i = x_i - A = x_i - 3$ $u_i = x_i - A = x_i - 3$ | $f_i u_i f_i u_i$ |
|-----------------|----------------------------|--|-------------------|
| 0 | 70 | -2 | -140 |
| 1 | 52 | -1 | -52 |
| 2 | 34 | 0 | 0 |
| 3 | 3 | 1 | 3 |
| 4 | 1 | 2 | 2 |
| | N = 100 | | Sum = -187 |

Average no of accidents per day workers

$$\Rightarrow x + (-187/160)$$

$$= 133/160$$

$$= 0.83$$

9. Find the mean from the following frequency distribution of marks at a test in statistics:

| Marks (x): | 5 40 | 10 45 | 15 50 | 20 | 25 | 30 | 35 |
|---------------------|---------|----------|----------|----|----|----|----|
| No of students (f): | 15 9 | 50 8 | 80 6 | 76 | 72 | 45 | 39 |

Soln: Let the assumed mean (A) = 25

| Marks x_i | No of students $f_i f_i$ | $u_i = x_i - A = x_i - 25$ $u_i = x_i - A = x_i - 25$ | $f_i u_i f_i u_i$ |
|-------------|-----------------------------|--|-------------------|
| 5 | 15 | -20 | -300 |
| 10 | 50 | -15 | -750 |
| 15 | 80 | -10 | -800 |
| 20 | 76 | -5 | -380 |
| 25 | 72 | 0 | 0 |

| | | | |
|----|---------|----|-------------|
| 30 | 45 | 5 | 225 |
| 35 | 39 | 10 | 390 |
| 40 | 9 | 15 | 135 |
| 45 | 8 | 20 | 160 |
| 50 | 6 | 25 | 150 |
| | N = 400 | | Sum = -1170 |

$$\text{Mean} = 25 + (-1170)/400 = 22.075$$

Exercise 7.3: Statistics

1. The following table gives the distribution of total household expenditure (in rupees) of manual workers in a city.

| Expenditure (in rupees) (x) | Frequency (f_i) | Expenditure (in rupees) (x_i) | Frequency (f_i) |
|---------------------------------|---------------------|-----------------------------------|---------------------|
| 100 – 150 | 24 | 300 – 350 | 30 |
| 150 – 200 | 40 | 350 – 400 | 22 |
| 200 – 250 | 33 | 400 – 450 | 16 |
| 250 – 300 | 28 | 450 – 500 | 7 |

Find the average expenditure (in rupees) per household

Soln: Let the assumed mean (A) = 275

| Class interval | Mid value (x_i) | $d_i = x_i - 275$ | $u_i = (x_i - 275)/50$ | Frequency f_i | $f_i u_i$ |
|----------------|---------------------|-------------------|------------------------|-----------------|-----------|
|----------------|---------------------|-------------------|------------------------|-----------------|-----------|

| | | | | | |
|-----------|-----|------|----|---------|-----------|
| 100 – 150 | 125 | -150 | -3 | 24 | -12 |
| 150 – 200 | 175 | -100 | -2 | 40 | -80 |
| 200 – 250 | 225 | -50 | -1 | 33 | -33 |
| 250 – 300 | 275 | 0 | 0 | 28 | 0 |
| 300 – 350 | 325 | 50 | 1 | 30 | 30 |
| 350 – 400 | 375 | 100 | 2 | 22 | 44 |
| 400 – 450 | 425 | 150 | 3 | 16 | 48 |
| 450 – 500 | 475 | 200 | 4 | 7 | 28 |
| | | | | N = 200 | Sum = -35 |

We have

$$A = 275, h = 50$$

$$\text{Mean} = A + h * \text{sum}/N = 275 + 50 * -35/200 = 275 - 8.75 = 266.25$$

2. A survey was conducted by a group of students as a part of their environmental awareness program, in which they collected the following data regarding the number of plants in 200 houses in a locality. Find the mean number of plants per house.

| Number of plants: | 0-2 | 2-4 | 4-6 | 6-8 | 8-10 | 10-12 | 12-14 |
|--------------------------|------------|------------|------------|------------|-------------|--------------|--------------|
| Number of houses: | 1 | 2 | 1 | 5 | 6 | 2 | 3 |

Which method did you use for finding the mean, and why?

Soln:

Let us find class marks (x_i) = (upper class limit + lower class limit)/2

Now we may compute x_i and $f_i x_i$ as following.

| Number of plants | Number of house (f_i) | x_i | $F_i x_i$ |
|------------------|---------------------------|-------|-----------|
| 0 – 2 | 1 | 1 | 1 |
| 2 – 4 | 2 | 3 | 6 |
| 4 – 6 | 1 | 5 | 5 |
| 6 – 8 | 5 | 7 | 35 |
| 8 – 10 | 6 | 9 | 54 |
| 10 – 12 | 2 | 11 | 22 |
| 12 – 14 | 3 | 13 | 39 |
| Total | N = 20 | | Sum = 162 |

From the table we may observe that

$$N = 20$$

$$\text{Sum} = 162$$

$$\text{Mean } \bar{x} = \text{Sum}/N$$

$$162/20 = 8.1$$

So mean number of plants per house is 8.1

We have used for the direct method values X_i and f_i are very small

3. Consider the following distribution of daily wages of workers of a factory

| Daily wages (in Rs) | 100-120 | 120-140 | 140-160 180-200 | 160-180 |
|---------------------|---------|---------|--------------------|---------|
| Number of workers: | 12 | 16 6 | 8 10 | |

Find the mean daily wages of the workers of the factory by using an appropriate method.

Soln: Let the assume mean (A) = 150

| Class interval | Mid value x_i | $d_i = x_i - 150$ | $u_i = (x_i - 150)/20$ | Frequency f_i | $F_i u_i$ |
|----------------|-----------------|-------------------|------------------------|-----------------|-----------|
| 100 – 120 | 110 | -40 | -2 | 12 | -24 |
| 120 – 140 | 130 | -20 | -1 | 14 | -14 |
| 140 – 160 | 150 | 0 | 0 | 8 | 0 |
| 160 – 180 | 170 | 20 | 1 | 6 | 6 |
| 180 – 200 | 190 | 40 | 2 | 10 | 20 |
| | | | $N = 50$ | Sum = -12 | |

We have

$$N = 50, h = 20$$

$$\text{Mean} = A + h \times \text{sum}/N$$

$$= 150 + 2 \times (-12)/50$$

$$= 150 - 4.8$$

$$= 145.2$$

4. Thirty women were examined in a hospital by a doctor and the number of heart beats per minute recorded and summarized as follows. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heart

| Beats Per minute: | 65 – 68 68 – 71 71 – 74 74 – 77 77 – 80 80 – 83 83 – 86 | | | | | | |
|-------------------|---|---|---|---|---|---|---|
| Number of women: | 2 | 4 | 3 | 8 | 7 | 4 | 2 |

Soln: we may find marks of each interval (x_i) by using the relation

$$(x_i) = (\text{upper class limit} + \text{lower class limit})/2$$

Class size of this data = 3

Now taking 75.5 as assumed mean (a)

We may calculate d_i , u_i , $f_i u_i$ as following

| Number of heart beats per minute | Number of women (x_i) | x_i | $d_i = x_i - 75.5$ | $u_i = (x_i - 75.5)/h$ | $f_i u_i$ |
|----------------------------------|---------------------------|-------|--------------------|------------------------|-----------|
| 65-68 | 2 | 66.5 | -9 | -3 | -6 |
| 68-71 | 9 | 69.5 | -6 | -2 | -8 |
| 71-74 | 3 | 72.5 | -3 | -1 | -3 |
| 74-77 | 8 | 75.5 | 0 | 0 | 0 |
| 77-80 | 7 | 78.5 | 3 | 1 | 7 |
| 80-83 | 4 | 81.5 | 6 | 2 | 8 |
| 83-86 | 2 | 84.5 | 9 | 3 | 6 |
| | N = 30 | | | | Sum = 4 |

Now we may observe from table that $N = 30$, sum = 4

$$\text{Mean } \bar{x} = 75.5 + (4/3) \times 3$$

$$= 75.5 + 0.4$$

$$= 75.9$$

So mean heart beats per minute for those women are 75.9 beats per minute

5. Find the mean of each of the following frequency distributions: (5 – 14)

| Class interval: | 0-6 | 6-12 | 12-18 | 18-24 | 24-30 |
|-----------------|-----|------|-------|-------|-------|
| Frequency: | 6 | 8 | 10 | 9 | 7 |

Soln:

Let us assume mean be 15

| Class interval | Mid – value | $d_i = x_i - 15$ | $u_i = (x_i - 15)/6$ | f_i | $f_i u_i$ |
|----------------|-------------|------------------|----------------------|-------|-----------|
| 0 – 6 | 3 | -12 | -2 | 6 | -12 |
| 6 – 12 | 9 | -6 | -1 | 8 | -8 |
| 12 – 18 | 15 | 0 | 0 | 10 | 0 |

| | | | | | |
|---------|----|----|---|--------|---------|
| 18 – 24 | 21 | 6 | 1 | 9 | 9 |
| 24 – 30 | 27 | 18 | 2 | 7 | 14 |
| | | | | N = 40 | Sum = 3 |

$$A = 15, h = 6$$

$$\text{Mean} = A + h(\text{sum}/A)$$

$$= 15 + 6(3/40)$$

$$= 15 + 0.45$$

$$= 15.45$$

6.

| | | | | | | |
|------------------------|--------------|--------------|---------------|----------------|----------------|----------------|
| Class interval: | 50-70 | 70-90 | 90-110 | 110-130 | 130-150 | 150-170 |
| Frequency: | 18 | 12 | 13 | 27 | 8 | 22 |

Soln: Let us assumed mean be 100

| Class interval | Mid-value x_i | $d_i = x_i - 100$ | $u_i = (x_i - 100)/20$ | f_i | $f_i u_i$ |
|----------------|-----------------|-------------------|------------------------|-------|-----------|
| 50 – 70 | 60 | -40 | -2 | 18 | -36 |
| 70 – 90 | 80 | -20 | -1 | 12 | -12 |
| 90 – 110 | 100 | 0 | 0 | 13 | 0 |
| 110 – 130 | 120 | 20 | 1 | 27 | 27 |
| 130 – 150 | 140 | 40 | 2 | 8 | 16 |
| 150 – 170 | 160 | 60 | 3 | 22 | 66 |
| | | | | | 61 |

$$A = 100, h = 20$$

$$\text{Mean} = 100 + 20 (61/100)$$

$$= 100 + 12.2$$

$$= 112.2$$

7.

| | | | | | |
|------------------------|------------|-------------|--------------|--------------|--------------|
| Class interval: | 0-8 | 8-16 | 16-24 | 24-32 | 32-40 |
| Frequency: | 6 | 7 | 10 | 8 | 9 |

Soln: Let the assumed mean (A) = 20

| Class interval | Mid- value | $d_i = x_i - 20$ | $u_i = (x_i - 20)/8$ | f_i | $f_i u_i$ |
|----------------|------------|------------------|----------------------|--------|-----------|
| 0-8 | 4 | -16 | -2 | 6 | -12 |
| 8-16 | 12 | -8 | -1 | 7 | -7 |
| 16-24 | 20 | 0 | 0 | 10 | 0 |
| 24-32 | 28 | 8 | 1 | 8 | 8 |
| 32-40 | 36 | 16 | 2 | 9 | 18 |
| | | | | N = 40 | Sum = 7 |

We have A = 20, h = 8

Mean= A + h (sum/N)

= 20 + 8 (7/40)

= 20 + 1.4

= 21.4

8.

| | | | | |
|------------------------|--------------------------------|----------------------|----------------|----------------|
| Class interval: | 0 – 6 24 – 30 | 6 – 12 | 12 – 18 | 18 – 24 |
| Frequency: | 7 12 | 5 6 | 10 | |

Soln: Let the assumed mean be (A) = 15

| Class interval | Mid – value | $d_i = x_i - 15$ | $u_i = (x_i - 15)/6$ | Frequency f_i | $f_i u_i$ |
|----------------|-------------|------------------|----------------------|-----------------|-----------|
| 0 – 6 | 3 | -12 | -2 | -1 | -14 |
| 6 – 12 | 9 | -6 | -1 | 5 | -5 |

| | | | | | |
|---------|----|----|---|--------|---------|
| 12 – 18 | 15 | 0 | 0 | 10 | 0 |
| 18 – 24 | 21 | 6 | 1 | 12 | 12 |
| 24 – 30 | 27 | 12 | 2 | 6 | 12 |
| | | | | N = 40 | Sum = 5 |

We have

$$A = 15, h = 6$$

$$\text{Mean} = A + h(\text{sum}/N)$$

$$= 15 + 6 (5/40)$$

$$= 15 + 0.75$$

$$= 15.75$$

9.

| | | | | |
|------------------------|----------------------------|----------------------------------|----------------|-------------|
| Class interval: | 0 – 10 40 | 10 – 20 40 – 50 | 20 – 30 | 30 – |
| Frequency: | 9 10 | 12 14 | 15 | |

Soln: Let the assumed mean (A) = 25

| Class interval | Mid – value | $d_i = x_i - 25$ | $u_i = (x_i - 25)/10$ | Frequency f_i | $f_i u_i$ |
|----------------|-------------|------------------|-----------------------|-----------------|-----------|
| 0 – 10 | 5 | -20 | -2 | 9 | -18 |
| 10 – 20 | 15 | -10 | -1 | 10 | -12 |
| 20 – 30 | 25 | 0 | 0 | 15 | 0 |
| 30 – 40 | 35 | 10 | 1 | 10 | 10 |
| 40 – 50 | 45 | 20 | 2 | 14 | 28 |
| | | | | N = 60 | Sum = 8 |

We have A = 25, h = 10

$$\text{Mean} = A + h(\text{sum}/N)$$

$$= 25 + 19 (8/60)$$

$$= 25 + (4/3)$$

$$= 26.333$$

10.

| Class interval: | 0-8 | 8-16 | 16-24 | 24-32 | 32-40 |
|-----------------|-----|------|-------|-------|-------|
| Frequency: | 5 | 9 | 10 | 8 | 8 |

Soln: Let the assumed mean (A) = 20

| Class interval | Mid value x_i | $d_i = x_i - 20$ | $u_i = (x_i - 20)/8$ | Frequency f_i | $f_i u_i$ |
|----------------|-----------------|------------------|----------------------|-----------------|-----------|
| 0-8 | 4 | -16 | -2 | 5 | -10 |
| 8-16 | 12 | -8 | -1 | 9 | -9 |
| 16-24 | 20 | 0 | 0 | 10 | 0 |
| 24-32 | 28 | 8 | 1 | 8 | 8 |
| 32-40 | 36 | 16 | 2 | 8 | 16 |
| | | | | N = 40 | Sum = 5 |

We have,

$$A = 20, h = 8$$

$$\text{Mean} = A + h (\text{sum}/N)$$

$$= 20 + 8 (5/40)$$

$$= 20 + 1$$

$$= 21$$

11.

| Class interval: | 0-8 | 8-16 | 16-24 | 24-32 | 32-40 |
|-----------------|-----|------|-------|-------|-------|
| Frequency: | 5 | 6 | 4 | 3 | 2 |

Soln: Let the assumed mean (A) = 20

| Class | Mid value x_i | $d_i = x_i - 20$ | $u_i = (x_i - 20)/8$ | Frequency f_i | $f_i u_i$ |
|-------|-----------------|------------------|----------------------|-----------------|-----------|
|-------|-----------------|------------------|----------------------|-----------------|-----------|

| interval | | | | | |
|----------|----|-----|----|--------|----------|
| 0-8 | 4 | -16 | -2 | -2 | -10 |
| 8-16 | 12 | -8 | -1 | -1 | -6 |
| 16-24 | 20 | 0 | 0 | 0 | 0 |
| 24-32 | 28 | 8 | 1 | 1 | 3 |
| 32-40 | 36 | 16 | 2 | 2 | 4 |
| | | | | N = 20 | Sum = -9 |

We have,

$$A = 20, h = 8$$

$$\text{Mean} = A + h (\text{sum}/N)$$

$$= 20 + 8 (-9/ 20)$$

$$= 20 - (72/20)$$

$$= 20 - 3.6$$

$$= 16.4$$

12.

| Class interval: | 10-30 | 30-50 | 50-70 | 70-90 | 90-110 | 110-130 |
|-----------------|-------|-------|-------|-------|--------|---------|
| Frequency: | 5 | 8 | 12 | 20 | 3 | 2 |

Soln: Let the assumed mean (A) = 60

| Class interval | Mid value x_i | $d_i = x_i - 60$ | $u_i = (x_i - 60)/20$ | Frequency f_i | $f_i u_i$ |
|----------------|-----------------|------------------|-----------------------|-----------------|-----------|
| 10 – 30 | 20 | -40 | -2 | 5 | -10 |
| 30 – 50 | 40 | -20 | -1 | 8 | -8 |
| 50 – 70 | 60 | 0 | 0 | 12 | 0 |
| 70 – 90 | 80 | 20 | 1 | 20 | 20 |
| 90 – 110 | 100 | 40 | 2 | 3 | 6 |
| 110 – 130 | 120 | 60 | 3 | 2 | 6 |
| | | | | | |

| | | | | | |
|--|--|--|--|--------|----------|
| | | | | N = 50 | Sum = 14 |
|--|--|--|--|--------|----------|

We have

$$A = 60, h = 20$$

$$\text{Mean} = A + h (\text{sum}/N)$$

$$= 60 + 20 (14/ 5)$$

$$= 60 + 5.6$$

$$= 65.6$$

13.

| Class interval: | 25-35 | 35-45 | 45-55 | 55-65 | 65-75 |
|-----------------|-------|-------|-------|-------|-------|
| Frequency: | 6 | 10 | 8 | 10 | 4 |

Soln: Let the assumed mean (A) = 50

| Class interval | Mid value x_i | $d_i = x_i - 50$ | $u_i = (x_i - 50)/10$ | Frequency f_i | $f_i u_i$ |
|----------------|-----------------|------------------|-----------------------|-----------------|-----------|
| 25 – 35 | 30 | -20 | -2 | 6 | -12 |
| 35 – 45 | 40 | -10 | -1 | 10 | -10 |
| 45 – 55 | 50 | 0 | 0 | 8 | 0 |
| 55 – 65 | 60 | 10 | 1 | 12 | 12 |
| 65 – 75 | 70 | 20 | 2 | 4 | 8 |
| | | | | N = 40 | Sum = -2 |

We have

$$A = 50, h = 10$$

$$\text{Mean} = A + h (\text{sum}/N)$$

$$= 50 + 10 (-2/ 40)$$

$$= 50 - 0.5$$

$$= 49.5$$

14.

| | | | | | | | |
|------------------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Class interval: | 25-29 | 30-34 | 35-39 | 40-44 | 45-49 | 50-54 | 55-59 |
| Frequency: | 14 | 22 | 16 | 6 | 5 | 3 | 4 |

Soln: Let the assumed mean (A) = 42

| Class interval | Mid value x_i | $d_i = x_i - 42$ | $u_i = (x_i - 42)/5$ | Frequency f_i | $f_i u_i$ |
|----------------|-----------------|------------------|----------------------|-----------------|-----------|
| 25 – 29 | 27 | -15 | -3 | 14 | -42 |
| 30 – 34 | 32 | -10 | -2 | 22 | -44 |
| 35 – 39 | 37 | -5 | -1 | 16 | -16 |
| 40 – 44 | 42 | 0 | 0 | 6 | 0 |
| 45 – 49 | 47 | 5 | 1 | 5 | 5 |
| 50 – 54 | 52 | 10 | 2 | 3 | 6 |
| 55 – 59 | 57 | 15 | 3 | 4 | 12 |
| | | | | N = 70 | Sum = -79 |

We have

$$A = 42, h = 5$$

$$\text{Mean} = A + h (\text{sum}/N)$$

$$= 42 + 5 (-79/70)$$

$$= 42 - 79/14$$

$$= 36.357$$

15. For the following distribution, calculate mean using all suitable methods:

| | | | | |
|----------------------|--------------|--------------|---------------|----------------|
| Size of item: | 1 – 4 | 4 – 9 | 9 – 16 | 16 – 20 |
| Frequency: | 6 | 12 | 26 | 20 |

Soln: By direct method

| Class interval | Mid value x_i | Frequency f_i | $f_i x_i$ |
|----------------|-----------------|-----------------|-----------|
| 1 – 4 | 2.5 | 6 | 15 |
| 4 – 9 | 6.5 | 12 | 18 |
| 9 – 16 | 12.5 | 26 | 325 |
| 16 – 27 | 21.5 | 20 | 430 |
| | | N = 64 | Sum = 848 |

$$\text{Mean} = (\text{sum}/N) + A$$

$$= 848/64$$

$$= 13.25$$

By assuming mean method

Let the assumed mean (A) = 65

| Class interval | Mid value x_i | $u_i = (x_i - A) = \frac{x_i - 65}{65}$ | Frequency f_i | $f_i u_i$ |
|----------------|-----------------|---|-----------------|-----------|
| 1 – 4 | 2.5 | -4 | 6 | -25 |
| 4 – 9 | 6.5 | 0 | 12 | 0 |
| 9 – 16 | 12.5 | 6 | 26 | 196 |
| 16 – 27 | 21.5 | 15 | 20 | 300 |
| | | | N = 64 | Sum = 432 |

$$\text{Mean} = A + \text{sum}/N$$

$$= 6.5 + 6.75$$

$$= 13.25$$

16. The weekly observation on cost of living index in a certain city for the year 2004 – 2005 are given below. Compute the weekly cost of living index.

| Cost of living index | Number of students | Cost of living index | Number of students |
|----------------------|--------------------|----------------------|--------------------|
| 1400 – 1500 | 5 | 1700 – 1800 | 9 |
| 1500 – 1600 | 10 | 1800 – 1900 | 6 |

| | | | |
|-------------|----|-------------|---|
| 1600 – 1700 | 20 | 1900 – 2000 | 2 |
|-------------|----|-------------|---|

Soln: Let the assumed mean (A) = 1650

| Class interval | Mid value x_i | $d_i = x_i - A$ $= x_i - 1650$ | $u_i = \frac{x_i - 1650}{100}$ | Frequency f_i | $f_i u_i$ |
|----------------|-----------------|-----------------------------------|--------------------------------|-----------------|-----------|
| 1400 – 1500 | 1450 | -200 | -2 | 5 | -10 |
| 1500 – 1600 | 1550 | -100 | -1 | 10 | -10 |
| 1600 – 1700 | 1650 | 0 | 0 | 20 | 0 |
| 1700 – 1800 | 1750 | 100 | 1 | 9 | 9 |
| 1800 – 1900 | 1850 | 200 | 2 | 6 | 12 |
| 1900 – 2000 | 1950 | 300 | 3 | 2 | 6 |
| | | | | N = 52 | Sum = 7 |

We have

$$A = 16, h = 100$$

$$\text{Mean} = A + h (\text{sum}/N)$$

$$= 1650 + 100 (7/52)$$

$$= 1650 + (175/13)$$

$$= 21625/13$$

$$= 1663.46$$

17. The following table shows the marks scored by 140 students in an examination of a certain paper:

| Marks: | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|---------------------|------|-------|-------|-------|-------|
| Number of students: | 20 | 24 | 40 | 36 | 20 |

Calculate the average marks by using all the three methods: direct method, assumed mean deviation and shortcut method.

Soln: (i) Direct method:

| Class interval | Mid value x_i | Frequency f_i | $f_i x_i$ |
|----------------|-----------------|-----------------|------------|
| 0 – 10 | 5 | 20 | 100 |
| 10 – 20 | 15 | 24 | 360 |
| 20 – 30 | 25 | 40 | 1000 |
| 30 – 40 | 35 | 36 | 1260 |
| 40 – 50 | 45 | 20 | 900 |
| | | N = 140 | Sum = 3620 |

$$\text{Mean} = \text{sum} / N$$

$$= 3620 / 140$$

$$= 25.857$$

(ii) Assumed mean method:

Let the assumed mean = 25

$$\text{Mean} = A + (\text{sum} / N)$$

| Class interval | Mid value x_i | $u_i = (x_i - A)$ | Frequency f_i | $f_i u_i$ |
|----------------|-----------------|-------------------|-----------------|-----------|
| 0 – 10 | 5 | -20 | 20 | -400 |
| 10 – 20 | 15 | -10 | 24 | -240 |
| 20 – 30 | 25 | 0 | 40 | 0 |
| 30 – 40 | 35 | 10 | 36 | 360 |
| 40 – 50 | 45 | 20 | 20 | 400 |
| | | | N = 140 | Sum = 120 |

$$\text{Mean} = A + (\text{sum} / N)$$

$$= 25 + (120 / 140)$$

$$= 25 + 0.857$$

$$= 25.857$$

(iii) Step deviation method:

Let the assumed mean (A) = 25

| Class interval | Mid value x_i | $d_i = x_i - A$ $= x_i - 25$ | $u_i = \frac{(x_i - 25)}{10}$ | Frequency f_i | $f_i u_i$ |
|----------------|-----------------|---------------------------------|-------------------------------|-----------------|-----------|
| 0 – 10 | 5 | -20 | -2 | 20 | -40 |
| 10 – 20 | 15 | -10 | -1 | 24 | -24 |
| 20 – 30 | 25 | 0 | 0 | 40 | 0 |
| 30 – 40 | 35 | 10 | 1 | 36 | 36 |
| 40 – 50 | 45 | 20 | 2 | 20 | 40 |
| | | | | N = 140 | Sum = 12 |

$$\text{Mean} = A + h(\text{sum} / N)$$

$$= 25 + 10(12 / 140)$$

$$= 25 + 0.857$$

$$= 25.857$$

18. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Compute the miss frequency f_1 and f_2 .

| Class: | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 | 100-120 |
|------------|------|-------|-------|-------|--------|---------|
| Frequency: | 5 | f_1 | 10 | f_2 | 7 | 8 |

Soln:

| Class interval | Mid value x_i | Frequency f_i | $f_i x_i$ |
|----------------|-----------------|-----------------|-----------|
| 0 – 20 | 10 | 5 | 50 |
| 20 – 40 | 30 | f_1 | $30f_1$ |
| 40 – 60 | 50 | 10 | 500 |
| 60 – 80 | 70 | f_2 | $70f_2$ |
| 80 – 100 | 90 | 7 | 630 |
| 100 – 120 | 110 | 8 | 880 |

| | | | |
|--|--|--------|------------------------------|
| | | N = 50 | Sum = $30f_1 + 70f_2 + 2060$ |
|--|--|--------|------------------------------|

Given,

sum of frequency = 50

$$5 + f_1 + 10 + f_2 + 7 + 8 = 50$$

$$f_1 + f_2 = 20$$

$$3f_1 + 3f_2 = 60 \text{ --- (1) [multiply both side by 3]}$$

And mean = 62.8

$$\text{Sum} / N = 62.8$$

$$(30f_1 + 70f_2 + 2060) / 50 = 62.8$$

$$30f_1 + 70f_2 = 3140 - 2060$$

$$30f_1 + 70f_2 = 1080$$

$$3f_1 + 7f_2 = 108 \text{ --- (2) [divide it by 10]}$$

subtract equation (1) from equation (2)

$$3f_1 + 7f_2 - 3f_1 - 3f_2 = 108 - 60$$

$$4f_2 = 48$$

$$f_2 = 12$$

Put value of f_2 in equation (1)

$$3f_1 + 3(12) = 60$$

$$f_1 = 24/3 = 8$$

$$f_1 = 8, f_2 = 12$$

19. The following distribution shows the daily pocket allowance given to the children of a multistory building. The average pocket allowance is Rs 18.00. Find out the missing frequency.

| Class interval: | 11-13 | 13-15 | 15-17 | 17-19 | 19-21 | 21-23 | 23-25 |
|-----------------|-------|-------|-------|-------|-------|-------|-------|
| | | | | | | | |

| | | | | | | | |
|------------|---|---|---|----|---|---|---|
| Frequency: | 7 | 6 | 9 | 13 | – | 5 | 4 |
|------------|---|---|---|----|---|---|---|

Soln: Given mean = 18,

Let the missing frequency be v

| Class interval | Mid value x_i | Frequency f_i | $f_i x_i$ |
|----------------|-----------------|-----------------|-------------------|
| 11 – 13 | 12 | 7 | 84 |
| 13 – 15 | 14 | 6 | 88 |
| 15 – 17 | 16 | 9 | 144 |
| 17 – 19 | 18 | 13 | 234 |
| 19 – 21 | 20 | x | $20x$ |
| 21 – 23 | 22 | 5 | 110 |
| 23 – 25 | 14 | 4 | 56 |
| | | $N = 44 + x$ | Sum = $752 + 20x$ |

Mean = sum/ N

$$18 = \frac{752 + 20x}{44 + x}$$

$$792 + 18x = 752 + 20x$$

$$2x = 40$$

$$x = 20$$

20.If the mean of the following distribution is 27. Find the value of p.

| | | | | | |
|------------|------|-------|-------|-------|-------|
| Class: | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
| Frequency: | 8 | p | 12 | 13 | 10 |

Soln:

| Class interval | Mid value x_i | Frequency f_i | $f_i x_i$ |
|----------------|-----------------|-----------------|-----------|
| 0 – 10 | 5 | 8 | 40 |
| 10 – 20 | 15 | P | 152 |
| 20 – 30 | 25 | 12 | 300 |

| | | | |
|---------|----|--------------|---------------------------|
| 30 – 40 | 35 | 13 | 455 |
| 40 – 50 | 45 | 16 | 450 |
| | | $N = 43 + P$ | $\text{Sum} = 1245 + 15p$ |

Given mean = 27

Mean = sum/ N

$$1245 + 15p \div 43 + p = 27$$

$$1245 + 15p = 1161 + 27p$$

$$12p = 84$$

$$P = 7$$

21. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contain varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

| | | | | | |
|---------------------------|--------------|--------------|--------------|--------------|--------------|
| Number of mangoes: | 50-52 | 53-55 | 56-58 | 59-61 | 62-64 |
| Number of boxes: | 15 | 110 | 135 | 115 | 25 |

Find the mean number of mangoes kept in packing box. Which method of finding the mean did you choose?

Soln:

| Number of mangoes | Number of boxes (f_i) |
|-------------------|---------------------------|
| 50 – 52 | 15 |
| 53 – 55 | 110 |
| 56 – 58 | 135 |
| 59 – 61 | 115 |
| 62 – 64 | 25 |

We may observe that class intervals are not continuous

There is a gap between two class intervals. So we have to add $\frac{1}{2}$ from lower class limit of each interval and class mark (x_i) may be obtained by using the relation

$$x_i = \text{upperlimit} + \text{lowerclasslimit} \times \frac{\text{upperlimit} + \text{lowerclasslimit}}{2}$$

Class size (h) of this data = 3

Now taking 57 as assumed mean (a) we may calculated $d_i, u_i, f_i u_i$ as follows

| Class interval | Frequency f_i | Mid value x_i | $d_i = x_i - A$ $= x_i - 25$ | $u_i = \frac{(x_i - 25)}{10}$ | $f_i u_i$ |
|----------------|-----------------|-----------------|---------------------------------|-------------------------------|-----------|
| 49.5 – 52.5 | 15 | 51 | -6 | -2 | -30 |
| 52.5 – 55.5 | 110 | 54 | -3 | -1 | -110 |
| 55.5 – 58.5 | 135 | 57 | 0 | 0 | 0 |
| 58.5 – 61.5 | 115 | 60 | 3 | 1 | 115 |
| 61.5 – 64.5 | 25 | 63 | 6 | 2 | 50 |
| Total | N = 400 | | | | Sum = 25 |

Now we have N = 400

Sum = 25

Mean = $A + h (\text{sum} / N)$

= $57 + 3 (25/400)$

= $57 + 3/ 16$

= $57 + 0.1875$

= 57.19

Clearly mean number of mangoes kept in packing box is 57.19

22.The table below shows the daily expenditure on food of 25 households in a locality

| Daily expenditure (in Rs): | 100-150 | 150-200 | 200-250 | 250-300 |
|----------------------------|---------|---------|---------|---------|
| | 300-350 | | | |
| Number of households: | 4 | 5 | 12 | |
| | 2 | 2 | | |

Find the mean daily expenditure on food by a suitable method.

Soln: we may calculate class mark (x_i) for each interval by using the relation

$$x_i = \frac{\text{upperlimit} + \text{lowerclasslimit}}{2}$$

Class size = 50

Now, taking 225 as assumed mean (x_i) we may calculate $d_i, u_i, f_i u_i$ as follows

| Daily expenditure | Frequency f_i | Mid value x_i | $d_i = x_i - 225$ | $u_i = \frac{x_i - 225}{50}$ | $f_i u_i$ |
|-------------------|-----------------|-----------------|-------------------|------------------------------|-----------|
| 100 – 150 | 4 | 125 | -100 | -2 | -8 |
| 150 – 200 | 5 | 175 | -50 | -1 | -5 |
| 200 – 250 | 12 | 225 | 0 | 0 | 0 |
| 250 – 300 | 2 | 275 | 50 | 1 | 2 |
| 300 – 350 | 2 | 325 | 100 | 2 | 4 |
| | N = 25 | | | | Sum = -7 |

Now we may observe that

$$N = 25$$

$$\text{Sum} = -7$$

$$\text{Mean } \bar{x} = a + \left(\frac{\text{sum}}{N} \right) \times h$$

$$225 + 50 \left(\frac{-7}{25} \right)$$

$$225 - 14 = 211$$

So, mean daily expenditure on food is Rs 211

23. To find out the concentration of SO_2 in the air (in parts per million i.e ppm) the data was collected for 30 localities in a certain city and is presented below:

| | |
|--|-----------|
| | Frequency |
|--|-----------|

| Concentration of SO ₂ (in ppm) | |
|---|---|
| 0.00 – 0.04 | 4 |
| 0.04 – 0.08 | 9 |
| 0.08 – 0.12 | 9 |
| 0.12 – 0.16 | 2 |
| 0.16 – 0.20 | 4 |
| 0.20 – 0.24 | 2 |

Find the mean concentration of SO₂ in the air

Soln: we may find class marks for each interval by using the relation

$$X = \frac{\text{upperlimit} + \text{lowerclasslimit}}{2} = \frac{\text{upperlimit} + \text{lowerclasslimit}}{2}$$

Class size of this data = 0.04

Now taking 0.04 assumed mean (x_i) we may calculate $d_i, u_i, f_i u_i$ as follows

| Concentration of SO ₂ | Frequency f_i | Class interval x_i | $d_i = x_i - 0.14$ | u_i | $f_i u_i$ |
|----------------------------------|-----------------|----------------------|--------------------|-------|-----------|
| 0.00 – 0.04 | 4 | 0.02 | -0.12 | -3 | -12 |
| 0.04 – 0.08 | 9 | 0.06 | -0.08 | -2 | -18 |
| 0.08 – 0.12 | 9 | 0.10 | -0.04 | -1 | -9 |
| 0.12 – 0.16 | 2 | 0.14 | 0 | 0 | 0 |
| 0.16 – 0.20 | 4 | 0.18 | 0.04 | 1 | 4 |
| 0.20 – 0.24 | 2 | 0.22 | 0.08 | 2 | 4 |
| Total | N = 30 | | | | Sum = -31 |

From the table we may observe that

$$N = 30$$

$$\text{Sum} = -31$$

$$\text{Mean } \bar{x} = a + \left(\frac{\text{sum}}{N} \right) \times h$$

$$= 0.14 + (0.04)(-31/30)$$

$$= 0.099 \text{ ppm}$$

So mean concentration of SO₂ in the air is 0.099 ppm

24. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

| Number of days: | 0-6 | 6-10 | 10-14 | 14-20 | 20-28 | 28-38 | 38-40 |
|---------------------|-----|------|-------|-------|-------|-------|-------|
| Number of students: | 11 | 10 | 7 | 4 | 4 | 3 | 1 |

Soln: We may find class mark of each interval by using the relation

$$X = \frac{\text{upper limit} + \text{lower class limit}}{2} \quad x = \frac{\text{upper limit} + \text{lower class limit}}{2}$$

Now, taking 16 as assumed mean (a) we may

Calculate d_i and f_id_i as follows

| Number of days | Number of students f _i | X _i | d = x _i + 10 | f _i d _i |
|----------------|-----------------------------------|----------------|-------------------------|-------------------------------|
| 0 – 6 | 11 | 3 | -13 | -143 |
| 6 – 10 | 10 | 8 | -8 | -280 |
| 10 – 14 | 7 | 12 | -4 | -28 |
| 14 – 20 | 7 | 16 | 0 | 0 |
| 20 – 28 | 8 | 24 | 8 | 32 |
| 28 – 36 | 3 | 33 | 17 | 51 |
| 30 – 40 | 1 | 39 | 23 | 23 |
| Total | N = 40 | | | Sum = -145 |

Now we may observe that

$$N = 40$$

$$\text{Sum} = -145$$

$$\text{Mean } \bar{x} = a + \left(\frac{\text{sum}}{N} \right) \bar{x} = a + \left(\frac{\text{sum}}{N} \right)$$

$$= 16 + (-145/40)$$

$$= 16 - 3.625$$

$$= 12.38$$

So mean number of days is 12.38 days, for which student was absent

25. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

| Literacy rate (in %): | 45-55 | 55-65 | 65-75 | 75-85 | 85-95 |
|-----------------------|-------|-------|-------|-------|-------|
| Number of cities: | 3 | 10 | 11 | 8 | 3 |

Soln: We may find class marks by using the relation

$$X = \frac{\text{upper limit} + \text{lower class limit}}{2} = \frac{\text{upper limit} + \text{lower class limit}}{2}$$

Class size (h) for this data = 10

Now taking 70 as assumed mean (a) wrong

Calculate d_i , u_i , $f_i u_i$ as follows

| Literacy rate (in %) | Number of cities (f_i) | Mid value x_i | $d_i = x_i - 70$ | $u_i = \frac{d_i}{h}$ | $f_i u_i$ |
|----------------------|----------------------------|-----------------|------------------|-----------------------|-----------|
| 45 – 55 | 3 | 50 | -20 | -2 | -6 |
| 55 – 65 | 10 | 60 | -10 | -1 | -10 |
| 65 – 75 | 11 | 70 | 0 | 0 | 0 |
| 75 – 85 | 8 | 80 | 10 | 1 | 8 |
| 85 – 95 | 3 | 90 | 20 | 2 | 6 |
| Total | N = 35 | | | | Sum = -2 |

Now we may observe that

$$N = 35$$

$$\text{Sum} = -2$$

$$\text{Mean } \bar{x} = a + \left(\frac{\sum N}{N} \right) \times h$$

$$= 70 + (-2/35)$$

$$= 70 - 4/7$$

$$= 70 - 0.57$$

$$= 69.43$$

So, mean literacy rate is 69.43 %

Exercise 7.4: Statistics

1. Following are the lives in hours of 15 pieces of the components of aircraft engine. Find the median: 715, 724, 725, 710, 729, 745, 694, 699, 696, 712, 734, 728, 716, 705, 719

Soln:

Lives in hours of is pieces are = 715, 724, 725, 710, 729, 745, 694, 699, 696, 712, 734, 728, 716, 705, 719

Arrange the above data in ascending order = 694, 696, 699, 705, 710, 712, 715, 716, 719, 721, 725, 728, 729, 734, 745

$N = 15$ (odd)

$$\text{Median} = (N+1)^{\text{th}} \left(\frac{N+1}{2} \right)^{\text{th}} \text{ terms}$$

$$= (15+1)^{\text{th}} \left(\frac{15+1}{2} \right)^{\text{th}} \text{ terms} = 8^{\text{th}} \text{ terms} = 716$$

2. The following is the distribution of height of students of a certain class in a certain city:

| Height (in cm): | 160-162 | 163-165 | 166-168 172-174 | 169-171 |
|-----------------|---------|------------|--------------------|---------|
| No of students: | 15 | 118 127 | 142 18 | |

Find the median height.

Soln:

| Class interval (exclusive) | Class interval (inclusive) | Class interval frequency | Cumulative frequency |
|-------------------------------|-------------------------------|-----------------------------|-------------------------|
| 160 – 162 | 159.5 – 162.5 | 15 | 15 |
| 163 – 165 | 162.5 – 165.5 | 118 | 133 (F) |
| 166 – 168 | 165.5 – 168.5 | 142 (f) | 275 |
| 169 – 171 | 168.5 – 171.5 | 127 | 402 |
| 172 – 174 | 171.5 – 174.5 | 18 | 420 |
| | | N = 420 | |

We have

$$N = 420$$

$$N/2 = 420/2 = 210$$

The cumulative frequency just greater than $N/2$ is 275 then 165.5 – 168.5 is the median class such, that

$$L = 165.5, f = 142, F = 133 \text{ and } h = 168.5 - 165.5 = 3$$

$$\text{Mean} = L + \frac{N/2 - F}{f} \times h$$

$$= 165.5 + \frac{210 - 133}{142} \times 3 = 165.5 + 1.63 = 167.13$$

3. Following is the distribution of I.Q of 100 students. Find the median I.Q.

| I.Q: | 55-64 | 65-74 | 75-84 | 85-94 134 | 95-104 135-144 | 105-114 | 115-124 | 125- |
|-----------------|-------|-------|-------|--------------|-------------------|---------|---------|------|
| No of students: | 1 | 2 | 9 | 22 2 | 33 1 | 22 | 8 | |

Soln:

| Class interval (exclusive) | Class interval (inclusive) | Class interval frequency | Cumulative frequency |
|-------------------------------|-------------------------------|-----------------------------|-------------------------|
| 55 – 64 | 54.5 – 64.5 | 1 | 1 |
| 65 – 74 | 64.5 – 74.5 | 2 | 3 |
| 75 – 84 | 74.5 – 84.5 | 9 | 12 |
| 85 – 94 | 84.5 – 94.5 | 22 | 34 (f) |
| 95 – 104 | 94.5 – 104.5 | 33 (f) | 67 |
| 105 – 114 | 104.5 – 114.5 | 22 | 89 |
| 115 – 124 | 114.5 – 124.5 | 8 | 97 |
| 125 – 134 | 124.5 – 134.5 | 2 | 99 |
| 135 – 144 | 134.5 – 144.5 | 1 | 100 |
| | | N = 100 | |

We have $N = 100$

$$N/2 = 100/2 = 50$$

The cumulative frequency just greater than $N/2$ is 67 then the median class is 94.5-104.5 such that

$$L=94.5, F = 33 \text{ } h = 104.5 - 94.5 = 10$$

$$\text{Mean} = L + \frac{N/2 - F}{f} \times h$$

$$= 94.5 + \frac{50 - 34}{33} \times 10 = 94.5 + 4.88 = 99.35$$

4. Calculate the median from the following data:

| Rent (in Rs): | 15-25 | 25-35 | 35-45 | 45-55 | 55-65 | 65-75 | 75-85 | 85-95 |
|---------------|-------|-------|-------|-------|-------|-------|-------|-------|
| No of houses: | 8 | 10 | 15 | 25 | 40 | 20 | 15 | 7 |

Soln:

| Class interval | Frequency | Cumulative frequency |
|----------------|-----------|----------------------|
| 15 – 25 | 8 | 8 |
| 25 – 35 | 10 | 18 |
| 35 – 45 | 15 | 33(f) |
| 45 – 55 | 25 | 58 |
| 55 – 65 | 40(f) | 28 |
| 65 – 75 | 20 | 38 |
| 75 – 85 | 15 | 183 |
| 85 – 95 | 7 | 140 |
| | N = 140 | |

We have N = 140

$$N/2 = 140/2 = 70$$

The cumulative frequency just greater than N/2 is 98 then median class is 55 – 65 such that

$$L = 55, f = 40, F = 58, h = 65 - 55 = 10$$

$$\text{Mean} = L + \frac{N/2 - F}{f} \times h$$

$$= 55 + \frac{70 - 58}{40} \times 10 = 55 + 3 = 58$$

5. Calculate the median from the following data:

| | | | | | | | | |
|------------------------|-----------|-----------|-----------|-----------|-----------|------------|------------|------------|
| Marks below: | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| No of students: | 15 | 35 | 60 | 84 | 96 | 127 | 198 | 250 |

Soln:

| Marks below | No of students | Class interval | Frequency | Cumulative frequency |
|-------------|----------------|----------------|-----------|----------------------|
| 10 | 15 | 0 – 10 | 15 | 15 |
| 20 | 35 | 10 – 20 | 20 | 35 |
| 30 | 60 | 20 – 30 | 25 | 60 |
| 40 | 84 | 30 – 40 | 24 | 84 |
| 50 | 96 | 40 – 50 | 12 | 96(F) |
| 60 | 127 | 50 – 60 | 31 (f) | 127 |
| 70 | 198 | 60 – 70 | 71 | 198 |
| 80 | 250 | 70 – 80 | 52 | 250 |
| | | | N = 250 | |

We have N = 250

$$N/2 = 250/2 = 125$$

The cumulative frequency just greater than N/2 is 127 then median class is 50 – 60 such that

$$L = 50, f = 31, F = 96, h = 60 - 50 = 10$$

$$\text{Mean} = L + \frac{N - F}{f} \times h$$

$$= 50 + \frac{125 - 96}{31} \times 10 = 50 + 9.35 = 59.35$$

6. Calculate the missing frequency from the following distribution, it being given that the median of the distribution is 24.

| Age in years: | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|----------------|------|-------|-------|-------|-------|
| No of persons: | 5 | 25 | ? | 18 | 7 |

Soln:

| Class interval | Frequency | Cumulative frequency |
|----------------|-----------|----------------------|
| 0 – 10 | 5 | 5 |
| 10 – 20 | 25 | 30 (F) |
| 20 – 30 | x (f) | 30 + x |
| 30 – 40 | 18 | 48 + x |
| 40 – 50 | 7 | 55 + x |
| | N = 170 | |

Given

$$\text{Median} = 24$$

Then, median class = 20 – 30

$$L = 20, h = 30 - 20 = 10, f = x, F = 30$$

$$\text{Median} = L + \frac{N - F}{f} \times h$$

$$24 = 20 + \frac{55 + x - 30}{x} \times 10$$

$$24 - 20 = \frac{55 + x - 30}{x} \times 10$$

$$4x = (55+x-30) \times 10 \left(\frac{55+x}{2} - 30 \right) \times 10$$

$$4x = 275 + 5x - 300$$

$$4x - 5x = -25$$

$$-x = -25$$

$$x = 25$$

Missing frequency = 25

7. The following table gives the frequency distribution of married women by age at marriage.

| Age (in years) | Frequency | Age (in years) | Frequency |
|----------------|-----------|----------------|-----------|
| 15 – 19 | 53 | 40 – 44 | 9 |
| 20 – 24 | 140 | 45 – 49 | 5 |
| 25 – 29 | 98 | 50 – 54 | 3 |
| 30 – 34 | 32 | 55 – 59 | 3 |
| 35 – 39 | 12 | 60 and above | 2 |

Calculate the median and interpret the results

Soln:

| Class interval (exclusive) | Class interval (inclusive) | Frequency | Cumulative frequency |
|-------------------------------|-------------------------------|-----------|-------------------------|
| 15 – 19 | 14.5 – 19.5 | 53 | 53 (F) |
| 20 – 24 | 19.5 – 24.5 | 140 (f) | 193 |
| 25 – 29 | 24.5 – 29.5 | 98 | 291 |
| 30 – 34 | 29.5 – 34.5 | 32 | 323 |
| 35 – 39 | 34.5 – 39.5 | 12 | 335 |
| 40 – 44 | 39.5 – 44.5 | 9 | 344 |
| 45 – 49 | 44.5 – 49.5 | 5 | 349 |
| 50 – 54 | 49.5 – 54.5 | 3 | 352 |

| | | | |
|--------------|----------------|---------|-----|
| 55 – 54 | 54.5 – 59.5 | 3 | 355 |
| 60 and above | 59.5 and above | 2 | 357 |
| | | N = 357 | |

$$N = 357$$

$$N/2 = 357/2 = 178.5$$

The cumulative frequency just greater than $N/2$ is 193,

Then the median class is 19.5 – 24.5 such that $l = 19.5$, $f = 140$, $F = 53$, $h = 25.5 - 19.5 = 5$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\text{Median} = 19.5 + \frac{178.5 - 53}{140} \times 5$$

$$\text{Median} = 23.98$$

Nearly half the women were married between the ages of 15 and 25

8. The following table gives the distribution of the life time of 400 neon lamps:

| Life time: | Number of lamps |
|-------------|-----------------|
| 1500 – 2000 | 14 |
| 2000 – 2500 | 56 |
| 2500 – 3000 | 60 |
| 3000 – 3500 | 86 |
| 3500 – 4000 | 74 |
| 4000 – 4500 | 62 |
| 4500 – 5000 | 48 |

Find the median life.

Soln: We can find cumulative frequencies with their respective class intervals as below

| | | |
|--|--|--|
| | | |
|--|--|--|

| Life time | Number of lamps f_i | Cumulative frequency (cf) |
|-------------|-----------------------|---------------------------|
| 1500 – 2000 | 14 | 14 |
| 2000 – 2500 | 56 | 70 |
| 2500 – 3000 | 60 | 130 |
| 3000 – 3500 | 86 | 216 |
| 3500 – 4000 | 74 | 290 |
| 4000 – 4500 | 62 | 352 |
| 4500 – 5000 | 48 | 400 |
| Total (n) | 400 | |

Now we may observe that cumulative frequency just greater than $n/2$ ($400/2 = 200$) is 216 belongs to class interval 3000 – 3500

Median class = 3000 – 3500

Lower limits (l) of median class = 3000

Frequency (f) of median class = 86

Cumulative frequency (cf) of class preceding median class = 130

Class size (h) = 500

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 3000 + \left(\frac{200 - 130}{86} \right) \times 500$$

$$= 3000 + (35000/86)$$

$$= 3406.98$$

So, median life time of lamps is 3406.98 hours

9. The distribution below gives the weight of 30 students in a class. Find the median weight of students:

| Weight (in kg): | 40-45 | 45-50 | 50-55 | 55-60 | 60-65 | 65-70 | 70-75 |
|-----------------|-------|-------|-------|-------|-------|-------|-------|
| No of students: | 2 | 3 | 8 | 6 | 6 | 3 | 2 |

Soln: We may find cumulative frequency with their respective class intervals as below

| Weight (in kg) | Number of students f_i | Cumulative frequency (cf) |
|----------------|--------------------------|---------------------------|
| 40 – 45 | 2 | 2 |
| 45 – 50 | 3 | 5 |
| 50 – 55 | 8 | 13 |
| 55 – 60 | 6 | 19 |
| 60 – 65 | 6 | 25 |
| 65 – 70 | 3 | 28 |
| 70 – 75 | 2 | 30 |

Cumulative frequency just greater than $n/2$ (i.e. $30/2 = 15$) is 19, belonging to class interval 55 – 60

Median class = 55 – 60

Lower limit (l) of median class = 55

Frequency (f) of median class = 6

Cumulative frequency (cf) = 13

Class size (h) = 5

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 55 + \left(\frac{15 - 13}{6} \right) \times 5$$

$$= 55 + 10/6$$

$$= 56.666$$

So, median weight is 56.67 kg

10. Find the missing frequencies and the median for the following distribution if the mean is 1.46

| | | | | | | |
|----------------------------------|-----------|----------|----------|-----------|-----------|--------------|
| No of accidents: | 0 | 1 | 2 | 3 | | Total |
| | 4 | 5 | | | | |
| Frequencies (no of days): | 46 | ? | ? | 25 | 10 | 200 |
| | 5 | | | | | |

Soln:

| No of accidents (x) | No of days (f) | fx |
|---------------------|----------------|--------------------|
| 0 | 46 | 0 |
| 1 | x | x |
| 2 | y | 2y |
| 3 | 25 | 75 |
| 4 | 10 | 40 |
| 5 | 5 | 25 |
| | N = 200 | Sum = x + 2y + 140 |

Given

$$N = 200$$

$$46 + x + y + 25 + 10 + 5 = 200$$

$$x + y = 200 - 46 - 25 - 10 - 5$$

$$x + y = 114 \text{ --- (1)}$$

$$\text{And, Mean} = 1.46$$

$$\text{Sum} / N = 1.46$$

$$(x + 2y + 140) / 200 = 1.46$$

$$x + 2y = 292 - 140$$

$$x + 2y = 152 \text{ --- (2)}$$

Subtract equation (1) from equation (2)

$$x + 2y - x - y = 152 - 114$$

$$y = 38$$

Putting the value of y in equation (1), we have $x = 114 - 38 = 76$

| No of accidents | No of days | Cumulative frequency |
|-----------------|------------|----------------------|
| 0 | 46 | 46 |
| 1 | 76 | 122 |
| 2 | 38 | 160 |
| 3 | 25 | 185 |
| 4 | 10 | 195 |
| 5 | 5 | 200 |
| | N = 200 | |

We have,

$$N = 200$$

$$N/2 = 200/2 = 100$$

The cumulative frequency just more than $N/2$ is 122 then the median is 1

11. An incomplete distribution is given below:

| Variable: | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|------------|-------|-------|-------|-------|-------|-------|-------|
| Frequency: | 12 | 30 | ? | 65 | ? | 25 | 18 |

You are given that the median value is 46 and the total number of items is 230.

(i) Using the median formula fill up the missing frequencies.

(ii) Calculate the AM of the completed distribution.

Soln:

(i)

| Class interval | Frequency | Cumulative frequency |
|----------------|-----------|----------------------|
| 10 – 20 | 12 | 12 |
| | | |

| | | |
|---------|---------|-------------|
| 20 – 30 | 30 | 42 |
| 30 – 40 | x | 42+ x (F) |
| 40 – 50 | 65 (f) | 107 + x |
| 50 – 60 | Y | 107 + x + y |
| 60 – 70 | 25 | 132 + x + y |
| 70 – 80 | 18 | 150 + x + y |
| | N = 150 | |

Given

Median = 46

Then, median class = 40 – 50

L = 40, h = 50 – 40 = 10, f = 65, F = 42 + x

$$\text{Median} = L + \frac{\frac{N}{2} - F}{f} \times h$$

$$46 = 40 + \frac{115 - (42 + x)}{65} \times 10$$

$$46 - 40 = \frac{115 - 42 - x}{65} \times 10$$

$$6 (65 / 10) = 73 - x$$

$$39 = 73 - x$$

$$x = 73 - 39 = 34$$

Given

N = 230

$$12 + 30 + 34 + 65 + y + 25 + 18 = 230$$

$$184 + y = 230$$

$$Y = 230 - 184$$

$$Y = 46$$

(ii)

| | | | |
|--|--|--|--|
| | | | |
|--|--|--|--|

| Class interval | Mid value x | Frequency f | Fx |
|----------------|-------------|-------------|---------------------|
| 10 – 20 | 15 | 12 | 180 |
| 20 – 30 | 25 | 30 | 750 |
| 30 – 40 | 35 | 34 | 1190 |
| 40 – 50 | 45 | 65 | 2925 |
| 50 – 60 | 55 | 46 | 2530 |
| 60 – 70 | 65 | 25 | 1625 |
| 70 – 80 | 75 | 18 | 1350 |
| | | N = 230 | $\Sigma fx = 10550$ |

$$\text{Mean} = \frac{\Sigma fx}{N}$$

$$= 10550 / 230 = 45.87$$

12.If the median of the following frequency distribution is 28.5 find the missing frequencies:

| Class interval: | 0-10 60 | 10-20 | 20-30 | 30-40 | 40-50 | 50- 60 | Total |
|-----------------|------------|-------|-------|-------|-------|-----------|-------|
| Frequency: | 5 | f_1 | 20 | 15 | f_2 | | 60 |

Soln:

| Class interval | Frequency | Cumulative frequency |
|----------------|-----------|----------------------|
| 0 – 10 | 5 | 5 |
| 10 – 20 | f_1 | $5 + f_1$ (F) |
| 20 – 30 | 20 (f) | $25 + f_1$ |
| 30 – 40 | 15 | $40 + f_1$ |
| 40 – 50 | f_2 | $40 + f_1 + f_2$ |
| | N = 60 | |

Given

$$\text{Median} = 28.5$$

$$\text{Then, median class} = 20 - 30$$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$28.5 = 20 + \frac{30 - (5 + f_1)}{20} \times 10$$

$$28.5 - 20 = \frac{30 - 5 - f_1}{20} \times 10$$

$$8.5 = \frac{25 - f_1}{2}$$

$$17 = 25 - f_1$$

$$f_1 = 25 - 17 = 8$$

Given

$$\text{Sum of frequencies} = 60$$

$$5 + f_1 + 20 + 15 + f_2 + 5 = 60$$

$$5 + 8 + 20 + 15 + f_2 + 5 = 60$$

$$f_2 = 7$$

$$f_1 = 8 \text{ and } f_2 = 7$$

13. The median of the following data is 525. Find the missing frequency, if it is given that there are 100 observations in the data.

| Class interval | Frequency | Class interval | Frequency |
|----------------|-----------|----------------|-----------|
| 0 – 100 | 2 | 500 – 600 | 20 |
| 100 – 200 | 5 | 600 – 700 | f_2 |
| 200 – 300 | f_1 | 700 – 800 | 9 |
| 300 – 400 | 12 | 800 – 900 | 7 |
| | | | |

| | | | |
|-----------|----|------------|---|
| 400 – 500 | 17 | 900 – 1000 | 4 |
|-----------|----|------------|---|

Soln:

| Class interval | Frequency | Cumulative frequency |
|----------------|-----------|----------------------|
| 0 – 100 | 2 | 2 |
| 100 – 200 | 5 | 7 |
| 200 – 300 | f_1 | $7 + f_1$ |
| 300 – 400 | 12 | $19 + f_1$ |
| 400 – 500 | 17 | $36 + f_1$ (F) |
| 500 – 600 | 20 (f) | $56 + f_1$ |
| 600 – 700 | f_2 | $56 + f_1 + f_2$ |
| 700 – 800 | 9 | $65 + f_1 + f_2$ |
| 800 – 900 | 7 | $72 + f_1 + f_2$ |
| 900 – 1000 | 4 | $76 + f_1 + f_2$ |
| | N = 100 | |

Given

Median = 525

Then, median class = 500 – 600

$L = 500$, $f = 20$, $F = 36 + f_1$, $h = 600 - 500 = 100$

$$\text{Median} = L + \frac{N - F}{f} \times h$$

$$525 = 500 + \frac{50 - (36 + f_1)}{20} \times 100$$

$$525 = 500 + \frac{50 - 36 - f_1}{20} \times 100$$

$$25 = (14 - f_1) \times 5$$

$$5 = 14 - f_1$$

$$f_1 = 14 - 5 = 9$$

Given

Sum of frequencies = 100

$$2 + 5 + f_1 + 12 + 17 + 20 + f_2 + 9 + 7 + 4 = 100$$

$$2 + 5 + 9 + 12 + 17 + 20 + f_2 + 9 + 7 + 4 = 100$$

$$85 + f_2 = 100$$

$$f_2 = 100 - 85 = 15$$

$$f_1 = 9 \text{ and } f_2 = 15$$

14.If the median of the following data is 32.5, find the missing frequencies.

| Class interval: | 0-10 60-70 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | Total |
|-----------------|---------------|--------|-------|-------|-------|-------|-------|
| Frequency: | f_1 3 | 5 2 | 9 | 12 | f_2 | | 40 |

Soln:

| Class interval | Frequency | Cumulative frequency |
|----------------|-----------|----------------------|
| 0 – 10 | f_1 | f_1 |
| 10 – 20 | 5 | $5 + f_1$ |
| 20 – 30 | 9 | $14 + f_1$ |
| 30 – 40 | 12 (f) | $26 + f_1$ |
| 40 – 50 | f_2 | $26 + f_1 + f_2$ |
| 50 – 60 | 3 | $29 + f_1 + f_2$ |
| 60 – 70 | 2 | $31 + f_1 + f_2$ |
| | N = 40 | |

Given

$$\text{Median} = 32.5$$

$$\text{The median class} = 90 - 40$$

$$L = 30, h = 40 - 30 = 10, f = 12, F = 14 + f_1$$

$$\text{Median} = L + \frac{\frac{N}{2} - F}{f} \times h$$

$$32.5 = 30 + \frac{20 - (14 + f_1)}{12} \times 10$$

$$32.5 - 30 = \frac{20 - (14 + f_1)}{12} \times 10$$

$$2.5 (12) = (6 - f_1) * 10$$

$$30 = (6 - f_1) * 10$$

$$3 = 6 - f_1$$

$$f_1 = 6 - 3 = 3$$

Given

$$\text{Sum of frequencies} = 40$$

$$f_1 + 5 + 9 + 12 + f_2 + 3 + 2 = 40$$

$$3 + 5 + 9 + 12 + f_2 + 3 + 2 = 40$$

$$34 + f_2 = 40$$

$$f_2 = 40 - 34 = 6$$

$$f_1 = 3 \text{ and } f_2 = 6$$

15. Compute the median for each of the following data

| (i) | | (ii) | |
|--------------|----------------|---------------|----------------|
| Marks | No of students | Marks | No of students |
| Less than 10 | 0 | More than 80 | 150 |
| Less than 30 | 10 | More than 90 | 141 |
| Less than 50 | 25 | More than 100 | 124 |

| | | | |
|----------------------|------------|----------------------|------------|
| Less than 70 | 43 | More than 110 | 105 |
| Less than 90 | 65 | More than 120 | 60 |
| Less than 110 | 87 | More than 130 | 27 |
| Less than 130 | 96 | More than 140 | 12 |
| Less than 150 | 100 | More than 150 | 0 |

Soln:(i)

| Marks | No of students | Class interval | Frequency | Cumulative frequency |
|---------------|----------------|----------------|-----------|----------------------|
| Less than 10 | 0 | 0 – 10 | 0 | 0 |
| Less than 30 | 10 | 10 – 30 | 10 | 10 |
| Less than 50 | 25 | 30 – 50 | 15 | 25 |
| Less than 70 | 43 | 50 – 70 | 18 | 43 (F) |
| Less than 90 | 65 | 70 – 90 | 22 (f) | 65 |
| Less than 110 | 87 | 90 – 110 | 22 | 87 |
| Less than 130 | 96 | 110 – 130 | 9 | 96 |
| Less than 150 | 100 | 130 – 150 | 4 | 100 |
| | | | N = 100 | |

We have

$$N = 100$$

$$N/2 = 100/2 = 50$$

The cumulative frequency just greater than $N/2$ is 65 then median class is 70 – 90 such that

$$L = 70, h = 90 - 70 = 20, f = 22, F = 43$$

$$\text{Median} = L + \frac{N/2 - F}{f} \times h$$

$$= 70 + \frac{50 - 43}{22} \times 20$$

$$= 70 + \frac{7 \times 20}{22}$$

$$= 70 + 6.36$$

$$= 76.36$$

(ii)

| Marks | No of students | Class interval | Frequency | Cumulative frequency |
|---------------|----------------|----------------|-----------|----------------------|
| More than 80 | 150 | 80 – 90 | 9 | 9 |
| More than 90 | 141 | 90 – 100 | 17 | 26 |
| More than 100 | 124 | 100 – 110 | 19 | 45 (F) |
| More than 110 | 105 | 110 – 120 | 45 (f) | 90 |
| More than 120 | 60 | 120 – 130 | 33 | 123 |
| More than 130 | 27 | 130 – 140 | 15 | 138 |
| More than 140 | 12 | 140 – 150 | 12 | 150 |
| More than 150 | 0 | 150 – 160 | 0 | 150 |
| | | | N = 150 | |

We have

$$N = 150$$

$$N/2 = 150/2 = 75$$

The cumulative frequency just more than $N/2$ is 90 then the median class is 110 – 120 such that

$$L = 70, h = 120 - 110 = 10, f = 45, F = 45$$

$$\text{Median} = L + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 110 + \frac{75 - 45}{45} \times 10$$

$$= 110 + \frac{30 \times 10}{45}$$

$$= 110 + 6.67$$

$$= 116.67$$

16.A survey regarding the height (in cm) of 51 girls of class X of a school was conducted and the following data was obtained:

| Height in cm | number of girls |
|---------------|-----------------|
| Less than 140 | 4 |
| Less than 145 | 11 |
| Less than 150 | 29 |
| Less than 155 | 40 |
| Less than 160 | 46 |
| Less than 165 | 51 |

Find the median height.

Soln: To calculate the median height, we need to find the class intervals and their corresponding frequencies.

The given distribution being of the less than type, 140, 145, 150, 155, 160, 165 give the upper limits of the corresponding class intervals. So, the classes should be below 140, 140-145, 145-150, 150-155, 155-160, 160-165. Observe that from the given distribution, we find that there are 4 girls with height less than 140, i.e. the frequency of class interval below 140 is 4. Now, there are 11 girls with heights less than 145 and 4 girls with height less than 140. Therefore, the number of girls with height in the interval 140 – 145 is $11 - 4 = 7$. Similarly, the frequency of 145 – 150 is $29 - 11 = 18$, for 150 – 155, it is $40 - 29 = 11$, and so on. So, our frequency distribution table with given cumulative frequencies becomes:

| Class interval | Frequency | Cumulative frequency |
|----------------|-----------|----------------------|
| Below 140 | 4 | 4 |
| 140 – 145 | 7 | 11 |
| 145 – 150 | 18 | 29 |
| 150 – 155 | 11 | 40 |
| 155 – 160 | 6 | 46 |
| 160 – 165 | 5 | 51 |

Now $n = 51$. So, $n/2 = 51/2 = 25.5$ this observation lies in the class 145 – 150

Then,

L (the lower limit) = 145

cf (the cumulative frequency of the class preceding 145 – 150) = 11

f (the frequency of the median class 145 – 150) = 18

h (the class size) = 5

Using the formula, median = $L + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$, we have

Median = Missing close brace

Missing close brace

= $145 + 72.5/18 = 149.03$

So, the median height of the girls is 149.03 cm

This means that the height of about 50% of the girls is less than this height, and 50% are taller than this height.

17. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are only given to persons having age 18 years onwards but less than 60 years.

| Ages in years | Number of policy holders |
|---------------|--------------------------|
| Below 20 | 2 |
| Below 25 | 6 |
| Below 30 | 24 |
| Below 35 | 45 |
| Below 40 | 78 |
| Below 45 | 89 |
| Below 50 | 92 |
| Below 55 | 98 |
| Below 60 | 100 |

Soln: Here class width is not same. There is no need to adjust the frequencies according to class interval. Now given frequencies table is less type represented with upper class limits. As policies were given only to persons having age 18 years onwards but less than 60 years we can define class intervals with their respective cumulative frequency as below.

| Age (in years) | Number of policy holders f_i | Cumulative frequency (cf) |
|----------------|--------------------------------|---------------------------|
| 18 – 20 | 2 | 2 |
| 20 – 25 | $6 - 2 = 4$ | 6 |
| 25 – 30 | $24 - 6 = 18$ | 24 |
| 30 – 35 | $45 - 24 = 21$ | 45 |
| 35 – 40 | $78 - 45 = 33$ | 78 |
| 40 – 45 | $89 - 78 = 11$ | 89 |
| 45 – 50 | $92 - 89 = 3$ | 92 |
| 50 – 55 | $98 - 92 = 6$ | 98 |
| 55 – 60 | $100 - 98 = 2$ | 100 |
| Total | | |

Now from table we may observe that $n = 100$

Cumulative frequency (cf) just greater than $n/2$ (i.e. $100/2 = 50$) is 78 belonging to interval 35 – 40

So median class = 35 – 40

Lower limit (l) of median class = 35

Class size (h) = 5

Frequency (f) of median class = 33

Cumulative frequency (cf) of class preceding median class = 45

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 35 + \left(\frac{50 - 45}{33} \right) \times 5$$

$$= 35 + \frac{25}{33}$$

$$= 35.76$$

So median age is 35.76 years

18. The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

| Length (in mm) | No of leaves |
|----------------|--------------|
| 118 – 126 | 3 |
| 127 – 135 | 5 |
| 136 – 144 | 9 |
| 145 – 153 | 12 |
| 154 – 162 | 5 |
| 163 – 171 | 4 |
| 172 – 180 | 2 |

Find the mean length of life

Soln: The given data is not having continuous class intervals. We can observe the difference between two class intervals is 1. So we have to add and subtract

$1/2 = 0.5$ to upper class limits and lower class limits

Now continuous class intervals with respective cumulative frequencies can be represented as below:

| Length (in mm) | Number of leaves f_i | Cumulative frequency (cf) |
|----------------|------------------------|---------------------------|
| 117.5 – 126.5 | 3 | 3 |
| 126.5 – 135.5 | 5 | 8 |
| 135.5 – 144.5 | 9 | 17 |
| 144.5 – 153.5 | 12 | 29 |
| 153.5 – 162.5 | 5 | 34 |
| 162.5 – 171.5 | 4 | 38 |

| | | |
|---------------|---|----|
| 171.5 – 180.5 | 2 | 40 |
|---------------|---|----|

From the table we may observe that cumulative frequency just greater than $n/2$ (i.e. $40/2 = 20$) is 29, belongs to class interval 144.5 – 153.5

Median class = 144.5 – 153.5

Lower limit (l) = 144.5

Class size (h) = 9

Frequency (f) of median class = 12

Cumulative frequency (cf) of class preceding median class = 17

$$\begin{aligned}\text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 144.5 + \left(\frac{20 - 17}{12} \right) \times 9 \\ &= 144.5 + 9/4 = 146.75\end{aligned}$$

So median length of leaves is 146.75mm

19. An incomplete distribution is given as follows:

| Variable: | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 |
|------------|------|-------|-------|-------|-------|-------|-------|
| Frequency: | 10 | 20 | ? | 40 | ? | 25 | 15 |

You are given that the median value is 35 and sum of all the frequencies are 170. Using the median formula, fill up the missing frequencies

Soln:

| Class interval | Frequency | Cumulative frequency |
|----------------|-----------|----------------------|
| 0 – 10 | 10 | 10 |
| 10 – 20 | 20 | 30 |
| 20 – 30 | f_1 | $30 + f_1$ (F) |
| 30 – 40 | 40 (F) | $70 + f_1$ |
| 40 – 50 | | |

| | | |
|---------|-----------|-------------------|
| | f_2 | $70 + f_1 + f_2$ |
| 50 – 60 | 25 | $95 + f_1 + f_2$ |
| 60 – 70 | 15 | $110 + f_1 + f_2$ |
| | $N = 170$ | |

Given

Median = 35

Then median class = 30 – 40

$L = 30$, $h = 40 - 30 = 10$, $f = 40$, $F = 30 + f_1$

$$\text{Median} = L + \frac{\frac{N}{2} - F}{f} \times h$$

$$35 = 30 + \frac{85 - (30 + f_1)}{40} \times 10$$

$$35 - 30 = \frac{85 - 30 - f_1}{40} \times 10$$

$$5 = \frac{55 - f_1}{4}$$

$$20 = 55 - f_1$$

$$f_1 = 55 - 20 = 35$$

Given

Sum of frequencies = 170

$$10 + 20 + f_1 + 40 + f_2 + 25 + 15 = 170$$

$$10 + 20 + 35 + 40 + f_2 + 25 + 15 = 170$$

$$f_2 = 25$$

$$f_1 = 35 \text{ and } f_2 = 25$$

Exercise 7.5: Statistics

1. Find the mode of the following data:

(i) 3, 5, 7, 4, 5, 3, 5, 6, 8, 9, 5, 3, 5, 3, 6, 9, 7, 4

(ii) 3, 3, 7, 4, 5, 3, 5, 6, 8, 9, 5, 3, 5, 3, 6, 9, 7, 4

(iii) 15, 8, 26, 25, 24, 15, 18, 20, 24, 15, 19, 15

Soln:

(i)

| Value (x) | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---------------|---|---|---|---|---|---|---|
| Frequency (f) | 4 | 2 | 5 | 2 | 2 | 1 | 2 |

Mode = 5 because it occurs the maximum number of times.

(ii)

| Value (x) | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------|---|---|---|---|---|---|---|
| | | | | | | | |

| | | | | | | | |
|----------------------|----------|----------|----------|----------|----------|----------|----------|
| Frequency (f) | 5 | 2 | 4 | 2 | 2 | 1 | 2 |
|----------------------|----------|----------|----------|----------|----------|----------|----------|

Mode = 3 because it occurs maximum number of times.

(iii)

| | | | | | | | | |
|----------------------|----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Value (x) | 8 | 15 | 18 | 19 | 20 | 24 | 25 | 26 |
| Frequency (f) | 1 | 4 | 1 | 1 | 1 | 2 | 1 | 1 |

Mode = 15 because it occurs maximum number of times.

2. The shirt size worn by a group of 200 persons, who bought the shirt from a store, are as follows:

| | | | | | | | |
|---------------------------|-----------|-----------|-----------|------------------------|-----------|-----------|-----------|
| Shirt size: | 37 | 38 | 39 | 40 44 | 41 | 42 | 43 |
| Number of persons: | 15 | 25 | 39 | 41 12 | 36 | 17 | 15 |

Find the model shirt size worn by the group.

Soln:

| | | | | | | | | |
|--------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Shirt size | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 |
| Number of persons | 15 | 25 | 39 | 41 | 36 | 17 | 15 | 12 |

Model shirt size = 40 because it occurs maximum number of times.

3. Find the mode of the following distribution.

(i)

| | | | | | | | | |
|------------------------|-------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Class interval: | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
| Frequency: | 5 | 8 | 7 | 12 | 28 | 20 | 10 | 10 |

(ii)

| | | | | | | |
|------------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Class interval: | 10-15 | 15-20 | 20-25 | 25-30 | 30-35 | 35-40 |
| Frequency: | 30 | 45 | 75 | 35 | 25 | 15 |

(iii)

| | | | | | | |
|------------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Class interval: | 25-30 | 30-35 | 35-40 | 40-45 | 45-50 | 50-60 |
| Frequency: | 25 | 34 | 50 | 42 | 29 | 15 |

Soln:

(i)

| | | | | | | | | |
|-----------------------|---------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Class interval | 0 – 10 | 10–20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
| Frequency | 5 | 8 | 7 | 12 | 28 | 20 | 10 | 10 |

Here the maximum frequency is 28 then the corresponding class 40 – 52 is the modal class

$$l = 40, h = 50 - 40 = 10, f = 28, f_1 = 12, f_2 = 20$$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 40 + \frac{28 - 12}{2 \times 28 - 12 - 20} \times 10$$

$$= 40 + 160/24$$

$$= 40 + 6.67$$

$$= 46.67$$

(ii)

| | | | | | | |
|-----------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Class interval | 10-15 | 15–20 | 20-25 | 25-30 | 30-35 | 35-40 |
| Frequency | 30 | 45 | 75 | 35 | 25 | 15 |

Here the maximum frequency is 75, then the corresponding class 20 – 25 is the modal class

$$l = 20, h = 25 - 20 = 5, f = 75, f_1 = 45, f_2 = 35$$

$$\begin{aligned} \text{Mode} &= l + \frac{f - f_1}{2f - f_1 - f_2} \times h \\ &= 20 + \frac{75 - 45}{2 \times 75 - 45 - 35} \times 5 \\ &= 20 + 150/70 \\ &= 20 + 2.14 \\ &= 22.14 \end{aligned}$$

(iii)

| Class interval | 25-30 | 30-35 | 35-40 | 40-45 | 45-50 | 50-60 |
|----------------|-------|-------|-------|-------|-------|-------|
| Frequency | 25 | 34 | 50 | 42 | 38 | 14 |

Here the maximum frequency is 50 then the corresponding class 35 – 40 is the modal class

$$l = 35, h = 40 - 35 = 5, f = 50, f_1 = 34, f_2 = 42$$

$$\begin{aligned} \text{Mode} &= l + \frac{f - f_1}{2f - f_1 - f_2} \times h \\ &= 35 + \frac{50 - 34}{2 \times 50 - 34 - 42} \times 5 \\ &= 35 + 80/24 \\ &= 35 + 3.33 \\ &= 38.33 \end{aligned}$$

4. Compare the modal ages of two groups of students appearing for an entrance test:

| Age (in years): | 16–18 | 18–20 | 20–22 | 22–24 | 24–26 |
|-----------------|-------|-------|-------|-------|-------|
| Group A: | 50 | 78 | 46 | 28 | 23 |
| Group B: | 54 | 89 | 40 | 25 | 17 |

Soln:

| Age in years | 16-18 | 18-20 | 20-22 | 22-24 | 24-26 |
|--------------|-------|-------|-------|-------|-------|
| Group A | 50 | 78 | 46 | 28 | 23 |
| Group B | 54 | 89 | 40 | 25 | 17 |

For Group A

Here the maximum frequency is 78, then the corresponding class 18 – 20 is modal class

$$l = 18, h = 20 - 18 = 2, f = 78, f_1 = 50, f_2 = 46$$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$
$$= 18 + \frac{78 - 50}{2 \times 78 - 50 - 46} \times 2$$

$$= 18 + 56/60$$

$$= 18 + 0.93$$

$$= 18.93 \text{ years}$$

For group B

Here the maximum frequency is 89, then the corresponding class 18 – 20 is the modal class

$$l = 18, h = 20 - 18 = 2, f = 89, f_1 = 54, f_2 = 40$$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$
$$= 18 + \frac{89 - 54}{2 \times 89 - 54 - 40} \times 2$$

$$= 18 + 70/84$$

$$= 18 + 0.83$$

$$= 18.83 \text{ years}$$

Hence the modal age for the Group A is higher than that for Group B

5.The marks in science of 80 students of class X are given below. Find the mode of the marks obtained by the students in science.

| | | | | | | | | | |
|------------|------------|-----------------|---------|-------|-------|-------|-------|-------|-----|
| Marks: | 0-10 90 | 10-20 90-100 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | 80- |
| Frequency: | 3 4 | 5 1 | 16 1 | 12 | 13 | 20 | 5 | | |

Soln:

| | | | | | | | | | | |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|
| Marks | 0 – 10 | 10– 20 | 20– 30 | 30– 40 | 40– 50 | 50– 60 | 60– 70 | 70– 80 | 80– 90 | 90– 100 |
| Frequency | 3 | 5 | 16 | 12 | 13 | 20 | 5 | 4 | 1 | 1 |

Here the maximum frequency is 20, then the corresponding class 50 – 60 is the modal class

$$l = 50, h = 60 - 50 = 10, f = 20, f_1 = 13, f_2 = 5$$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 50 + \frac{20 - 13}{2 \times 20 - 13 - 5} \times 10$$

$$= 50 + 70/22$$

$$= 50 + 3.18$$

$$= 53.18$$

6.The following is the distribution of height of students of a certain class in a city:

| | | | | |
|-----------------|--------------------|-----------|---------|---------|
| Height (in cm): | 160-162 172-174 | 163-165 | 166-168 | 169-171 |
| No of students: | 15 127 | 118 18 | 142 | |

Find the average height of maximum number of students.

Soln:

| | | | | | |
|--|--|--|--|--|--|
| | | | | | |
|--|--|--|--|--|--|

| | | | | | |
|---------------------|-------------|-------------|-------------|-------------|-------------|
| Heights(exclusive) | 160-162 | 163-165 | 166-168 | 169-171 | 172-174 |
| Heights (inclusive) | 159.5-162.5 | 162.5-165.5 | 165.5-168.5 | 168.5-171.5 | 171.5-174.5 |
| No of students | 15 | 118 | 142 | 127 | 18 |

Here the maximum frequency is 142, then the corresponding class 165.5 – 168.5 is the modal class

$$l = 165.5, h = 168.5 - 165.5 = 3, f = 142, f_1 = 118, f_2 = 127$$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 165.5 + \frac{142 - 118}{2 \times 142 - 118 - 127} \times 3$$

$$= 165.5 + 72/39$$

$$= 165.5 + 1.85$$

$$= 167.35 \text{ cm}$$

7. The following table shows the ages of the patients admitted in a hospital during a year:

| | | | | | | |
|-------------------------|-------------|--------------|--------------|--------------|--------------|--------------|
| Ages (in years): | 5-15 | 15-25 | 25-35 | 35-45 | 45-55 | 55-65 |
| No of students: | 6 | 11 | 21 | 23 | 14 | 5 |

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Soln: We may compute class marks (x_i) as per the relation

$$x_i = \frac{\text{upperclasslimit} + \text{lowerclasslimit}}{2}$$

Now taking 30 as assumed mean (a) we may calculate d_i and $f_i d_i$ as follows.

| Age (in years) | Number of patients f_i | Class marks x_i | $d_i = x_i - 275$ | $f_i d_i$ |
|----------------|--------------------------|-------------------|-------------------|-----------|
| 5 – 15 | 6 | 10 | -20 | -120 |

| | | | | |
|---------|----|----|-----|------|
| 15 – 25 | 11 | 20 | -10 | -110 |
| 25 – 35 | 21 | 30 | 0 | 0 |
| 35 – 45 | 23 | 40 | 10 | 230 |
| 45 – 55 | 14 | 50 | 20 | 280 |
| 55 – 65 | 5 | 60 | 30 | 150 |
| Total | 80 | | | 430 |

From the table we may observe that

$$\sum f_i \sum f_i = 80$$

$$\sum f_i d_i \sum f_i d_i = 430$$

$$\text{Mean } \bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} \bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$30 + 430 / 80$$

$$30 + 5.375$$

$$35.375$$

$$\approx 35.38$$

Clearly, mean of this data is 35.38. It represents that on an average the age of a patients admitted to hospital was 35.38 years.

As we may observe that maximum class frequency is 23 belonging to class interval 35 – 45

So, modal class = 35 – 45

Lower limit (l) of modal class = 35

Frequency (f) of modal class = 23

Class size (h) = 10

Frequency (f_1) of class preceding the modal class = 21

Frequency (f_2) of class succeeding the modal class = 14

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 35 + \frac{23-21}{2 \times 23-21-14} \times 10$$

$$= 35 + \frac{2}{46-35} \times 10$$

$$= 35 + 1.81$$

$$= 36.8$$

Clearly mode is 36.8. It represents that maximum number of patients admitted in hospital were of 36.8 years.

8. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

| Lifetimes (in hours): | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 | 100-120 |
|-----------------------|------|-------|-------|-------|--------|---------|
| No of components: | 10 | 35 | 52 | 61 | 38 | 29 |

Determine the modal lifetimes of the components.

Soln: From the data given as above we may observe that maximum class frequency is 61 belonging to class interval 60 – 80

So, modal class limit (l) of modal class = 60

Frequency (f) of modal class = 61

Frequency (f_1) of class preceding the modal class = 52

Frequency (f_2) of class succeeding the modal class = 38

Class size (h) = 20

$$\text{Mode} = l + \frac{f-f_1}{2f-f_1-f_2} \times h$$

$$= 60 + \frac{61-52}{2 \times 61-52-38} \times 20$$

$$= 60 + \frac{9}{122-90} \times 20$$

$$= 60 + \frac{9 \times 20}{32}$$

$$= 60 + 90/16$$

$$= 60 + 5.625$$

$$= 65.625$$

So, modal lifetime of electrical components is 65.625 hours

9.The following table gives the daily income of 50 workers of a factory:

| Daily income | 100 – 120 | 120 – 140 | 140 – 160 | 160 – 180 | 180 – 200 |
|-------------------|-----------|-----------|-----------|-----------|-----------|
| Number of workers | 12 | 14 | 8 | 6 | 10 |

Find the mean, mode and median of the above data.

Soln:

| Class interval | Mid value (x) | Frequency (f) | fx | Cumulative frequency |
|----------------|---------------|---------------|--------------------|----------------------|
| 100 – 120 | 110 | 12 | 1320 | 12 |
| 120 – 140 | 130 | 14 | 1820 | 26 |
| 140 – 160 | 150 | 8 | 1200 | 34 |
| 160 – 180 | 170 | 6 | 1000 | 40 |
| 180 – 200 | 190 | 10 | 1900 | 50 |
| | | N = 50 | $\Sigma fx = 7260$ | |

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{7260}{50} = 145.2$$

We have,

$$N = 50$$

$$\text{Then, } N/2 = 50/2 = 25$$

The cumulative frequency just greater than $N/2$ is 26, then the median class is 120 – 140 such that

$$l = 120, h = 140 - 120 = 20, f = 14, F = 12$$

$$\begin{aligned} \text{Median} &= l + \frac{N/2 - F}{f} \times h \\ &= 120 + \frac{25 - 12}{14} \times 20 \\ &= 120 + 260/14 \\ &= 120 + 18.57 \\ &= 138.57 \end{aligned}$$

Here the maximum frequency is 14, then the corresponding class 120 – 140 is the modal class

$$l = 120, h = 140 - 120 = 20, f = 14, f_1 = 12, f_2 = 8$$

$$\begin{aligned} \text{Mode} &= l + \frac{f - f_1}{2f - f_1 - f_2} \times h \\ &= 120 + \frac{14 - 12}{2 \times 14 - 12 - 8} \times 20 \\ &= 120 + 40/8 \\ &= 120 + 5 \\ &= 125 \end{aligned}$$

10. The following distribution gives the state-wise teachers-students ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures:

| Number of students per teacher | Number of states/ U.T | Number of students per teacher | Number of states/ U.T |
|--------------------------------|-----------------------|--------------------------------|-----------------------|
| 15 – 20 | 3 | 35 – 40 | 3 |
| 20 – 25 | 8 | 40 – 45 | 0 |
| 25 – 30 | 9 | 45 – 50 | 0 |
| | | | |

| | | | |
|---------|----|---------|---|
| 30 – 35 | 10 | 50 – 55 | 2 |
|---------|----|---------|---|

Soln: WE may observe from the given data that maximum class frequency is 10 belonging to class interval 30 – 35.

So, modal class = 30 – 35

Class size (h) = 5

Lower limit (l) of modal class = 30

Frequency (f) of modal class = 10

Frequency (f_1) of class preceding modal class = 9

Frequency (f_2) of class succeeding modal class = 3

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 30 + \frac{10 - 9}{2 \times 10 - 9 - 3} \times 5$$

$$= 30 + \frac{1}{20 - 12} \times 5$$

$$= 30 + 5/8$$

$$= 30.625$$

$$\text{Mode} = 30.6$$

It represents that most of states/ U.T have a teacher- student ratio as 30.6

Now we may find class marks by using the relation

$$\text{Class mark} = \frac{\text{upperclasslimit} + \text{lowerclasslimit}}{2}$$

Now taking 32.5 as assumed mean (a) we may calculate d_i , u_i , and $f_i u_i$ as following

| Number of students per teacher | Number of states/ U.T (f_i) | x_i | $d_i = x_i - 32.5$ | U_i | $f_i u_i$ |
|--------------------------------|---------------------------------|-------|--------------------|-------|-----------|
| 15 – 20 | 3 | 17.5 | -15 | -3 | -9 |
| 20 – 25 | 8 | 22.5 | -10 | -2 | -16 |
| 25 – 30 | 9 | 27.5 | -5 | -1 | -9 |

| | | | | | |
|---------|----|------|----|---|-----|
| 30 – 35 | 10 | 32.5 | 0 | 0 | 0 |
| 35 – 40 | 3 | 37.5 | 5 | 1 | 3 |
| 40 – 45 | 0 | 42.5 | 10 | 2 | 0 |
| 45 – 50 | 0 | 47.5 | 15 | 3 | 0 |
| 50 – 55 | 2 | 52.5 | 20 | 4 | 8 |
| Total | 35 | | | | -23 |

Now Mean $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} \times h$

$$= 32.5 + \frac{-23}{35} \times 5$$

$$= 32.5 - 23/7$$

$$= 32.5 - 3.28$$

$$= 29.22$$

So mean of data is 29.2

It represents that on an average teacher-student ratio was 29.2

11. Find the mean, median and mode of the following data:

| | | | | | | | |
|-------------------|---------------|-----------------|------------------|------------------|------------------|------------------|------------------|
| Classes: | 0 – 50 | 50 – 100 | 100 – 150 | 150 – 200 | 200 – 250 | 250 – 300 | 300 – 350 |
| Frequency: | 2 | 3 | 5 | 6 | 5 | 3 | 1 |

Soln:

| Class interval | Mid value (x) | Frequency (f) | fx | Cumulative frequency |
|----------------|---------------|---------------|-----|----------------------|
| 0 – 50 | 35 | 2 | 50 | 2 |
| 50 – 100 | 75 | 3 | 225 | 5 |
| 100 – 150 | 125 | 5 | 625 | 10 |

| | | | | |
|-----------|-----|--------|--------------------|----|
| 150 – 200 | 175 | 6 | 1050 | 16 |
| 200 – 250 | 225 | 5 | 1127 | 21 |
| 250 – 300 | 275 | 3 | 825 | 24 |
| 300 – 350 | 325 | 1 | 325 | 25 |
| | | N = 25 | $\Sigma fx = 4225$ | |

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{4225}{25} = 169$$

We have,

$$N = 25$$

$$\text{Then, } N/2 = 25/2 = 12.5$$

The cumulative frequency just greater than $N/2$ is 16, then the median class is 150 – 200 such that

$$l = 150, h = 200 - 150 = 50, f = 6, F = 10$$

$$\begin{aligned} \text{Median} &= l + \frac{\frac{N}{2} - F}{f} \times h \\ &= 150 + \frac{12.5 - 10}{6} \times 50 \\ &= 150 + 125/6 \\ &= 150 + 20.83 \\ &= 170.83 \end{aligned}$$

Here the maximum frequency is 6, then the corresponding class 150 – 200 is the modal class

$$l = 150, h = 200 - 150 = 50, f = 6, f_1 = 5, f_2 = 5$$

$$\begin{aligned} \text{Mode} &= l + \frac{f - f_1}{2f - f_1 - f_2} \times h \\ &= 150 + \frac{6 - 5}{2 \times 6 - 5 - 5} \times 50 \\ &= 150 + 50/2 \end{aligned}$$

$$= 150 + 25$$

$$= 175$$

12. A student noted the number of cars that pass through a spot on a road for 100 periods each of 3 minutes and summarized it in the table given below. Find the mode of the data.

Soln: From the given data we may observe that maximum class frequency is 20 belonging to 40 – 50 class intervals.

So, modal class = 40 – 50

Lower limit (l) of modal class = 40

Frequency (f) of modal class = 20

Frequency (f_1) of class preceding modal class = 12

Frequency (f_2) of class succeeding modal class = 11

Class size = 10

$$\begin{aligned} \text{Mode} &= l + \frac{f - f_1}{2f - f_1 - f_2} \times h \\ &= 40 + \frac{20 - 12}{2(20) - 12 - 11} \times 10 \\ &= 40 + \frac{8}{40 - 23} \\ &= 40 + \frac{80}{17} \\ &= 40 + 4.7 \\ &= 44.7 \end{aligned}$$

So mode of this data is 44.7 cars

13. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

| | | | | | | |
|-----------------------------|----------------------|---------------------------|----------------|----------------|----------------|---------------------|
| Monthly consumption: | 65-85 185 | 85-105 185-205 | 105-125 | 125-145 | 145-165 | 165- 185 |
| No of consumers: | 4 | 5 4 | 13 | 20 | 14 | 8 |

Soln:

| Class interval | Mid value x_i | Frequency f_i | Fx | Cumulative frequency |
|----------------|-----------------|-----------------|------|---------------------------------|
| 65 – 85 | 75 | 4 | 300 | 4 |
| 85 – 105 | 95 | 5 | 475 | 9 |
| 105 – 125 | 115 | 13 | 1495 | 22 |
| 125 – 145 | 135 | 20 | 2700 | 42 |
| 145 – 165 | 155 | 14 | 2170 | 56 |
| 165 – 185 | 175 | 8 | 1400 | 64 |
| 185 – 205 | 195 | 4 | 780 | 68 |
| | | $N = 68$ | | $\Sigma fx = \Sigma f x = 9320$ |

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{9320}{68} = 137.05$$

We have, $N = 68$

$$N/2 = 68/2 = 34$$

The cumulative frequency just greater than $N/2$ is 42 then the median class is 125 – 145 such that

$$l = 125, h = 145 - 125 = 20, f = 20, F = 22$$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 125 + \frac{34-22}{20} \times 20$$

$$= 125 + 12$$

$$= 137$$

Here the maximum frequency is 20, then the corresponding class 125 – 145 is the modal class

$$l = 125, h = 145 - 125 = 20, f = 20, f_1 = 13, f_2 = 14$$

$$\text{Mode} = l + \frac{f-f_1}{2f-f_1-f_2} \times h$$

$$= 125 + \frac{20-13}{2(20)-13-14} \times 20$$

$$= 125 + 140/13$$

$$= 135.77$$

14.100 surnames were randomly picked up from a local telephone directly and the frequency distribution of the number of letter English alphabets in the surnames was obtained as follows:

| Number of letters: | 1-4 | 4-7 | 7-10 | 10-13 | 13-16 | 16-19 |
|--------------------|-----|-----|------|-------|-------|-------|
| Number surnames: | 6 | 30 | 40 | 16 | 4 | 4 |

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames. Also, find the modal size of the surnames.

Soln:

| Class interval | Mid value (x) | Frequency (f) | fx | Cumulative frequency |
|----------------|---------------|---------------|-----|----------------------|
| 1 – 4 | 2.5 | 6 | 15 | 6 |
| 4 – 7 | 5.5 | 30 | 165 | 36 |
| 7 – 10 | 8.5 | 40 | 340 | 76 |
| 10 – 13 | 11.5 | 16 | 184 | 92 |
| 13 – 16 | 14.5 | 4 | 58 | 96 |
| | | | | |

| | | | | |
|---------|------|---------|-------------------|-----|
| 16 – 19 | 17.5 | 4 | 70 | 100 |
| | | N = 100 | $\Sigma fx = 832$ | |

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{832}{100} = 8.32$$

We have,

$$N = 100$$

$$N/2 = 100/2 = 50$$

The cumulative frequency just greater than $N/2$ is 76, then the median class is 7 – 10 such that

$$l = 7, h = 10 - 7 = 3, f = 40, F = 36$$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 7 + \frac{50 - 36}{40} \times 3$$

$$= 7 + 52/40$$

$$= 7 + 1.05$$

$$= 8.05$$

Here the maximum frequency is 40, then the corresponding class 7 – 10 is the modal class

$$l = 7, h = 10 - 7 = 3, f = 40, f_1 = 30, f_2 = 16$$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 7 + \frac{40 - 30}{2 \times 40 - 30 - 16} \times 3$$

$$= 7 + 30/34$$

$$= 7 + 0.88$$

$$= 7.88$$

15. Find the mean, median and mode of the following data:

| | | | | | | | |
|--|--|--|--|--|--|--|--|
| | | | | | | | |
|--|--|--|--|--|--|--|--|

| Class | 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 | 100 – 120 | 120 – 140 |
|-----------|--------|---------|---------|---------|----------|-----------|-----------|
| Frequency | 6 | 8 | 10 | 12 | 6 | 5 | 3 |

Soln:

| Class interval | Mid value (x) | Frequency (f) | fx | Cumulative frequency |
|----------------|---------------|---------------|--------------------|----------------------|
| 0 – 20 | 10 | 6 | 60 | 6 |
| 20 – 40 | 30 | 8 | 240 | 17 |
| 40 – 60 | 50 | 10 | 500 | 24 |
| 60 – 80 | 70 | 12 | 840 | 36 |
| 80 – 100 | 90 | 6 | 540 | 42 |
| 100 – 120 | 110 | 5 | 550 | 47 |
| 120 – 140 | 130 | 3 | 390 | 50 |
| | | N = 50 | $\Sigma fx = 3120$ | |

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{3120}{50} = 62.4$$

We have,

$$N = 50$$

$$\text{Then, } N/2 = 50/2 = 25$$

The cumulative frequency just greater than $N/2$ is 36, then the median class is 60 – 80 such that

$$l = 60, h = 80 - 60 = 20, f = 12, F = 24$$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 60 + \frac{25 - 24}{12} \times 20$$

$$= 60 + 20/12$$

$$= 60 + 1.67$$

$$= 61.67$$

Here the maximum frequency is 12, then the corresponding class 60 – 80 is the modal class

$$l = 60, h = 80 - 60 = 20, f = 12, f_1 = 10, f_2 = 6$$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 60 + \frac{12 - 10}{2 \times 12 - 10 - 6} \times 20$$

$$= 60 + 40/8$$

$$= 65$$

16. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure:

| Expenditure | Frequency | Expenditure | Frequency |
|-------------|-----------|-------------|-----------|
| 1000-1500 | 24 | 3000-3500 | 30 |
| 1500-2000 | 40 | 3500-4000 | 22 |
| 2000-2500 | 33 | 4000-4500 | 16 |
| 2500-3000 | 28 | 4500-5000 | 7 |

Soln: We may observe from the given data that maximum class frequency is 40 belonging to 1500 -200 intervals

So, modal class = 1500 -2000

Lower limit (l) of modal class = 1500

Frequency (f) of modal class = 40

Frequency (f₁) of class preceding modal class = 24

Frequency (f₂) of class succeeding modal class = 33

Class size (h) = 500

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 1500 + \frac{40 - 24}{2 \times 40 - 24 - 33} \times 500$$

$$= 1500 + \frac{16}{80 - 57} \times 500$$

$$= 1500 + 347.826$$

$$= 1847.826 \approx 1847.83$$

So modal monthly expenditure was Rs. 1847.83

Now we may find class mark as

$$\text{Class mark} = \frac{\text{upperclasslimit} + \text{lowerclasslimit}}{2}$$

Class size (h) of given data = 500

Now taking 2750 as assumed mean (a) we may calculate d_i u_i as follows:

| Expenditure (in Rs) | Number of families f_i | x_i | $d_i = x_i - 2750$ | U_i | $f_i u_i$ |
|------------------------|-----------------------------|-------|--------------------|-------|-----------|
| 1000-1500 | 24 | 1250 | -1500 | -3 | -72 |
| 1500-2000 | 40 | 1750 | -1000 | -2 | -80 |
| 2000-2500 | 33 | 2250 | -500 | -1 | -33 |
| 2500-3000 | 28 | 2750 | 0 | 0 | 0 |
| 3000-3500 | 30 | 3250 | 500 | 1 | 30 |
| 3500-4000 | 22 | 3750 | 1000 | 2 | 44 |
| 4000-4500 | 16 | 4250 | 1500 | 3 | 48 |
| 4500-5000 | 7 | 4750 | 2000 | 4 | 28 |
| Total | 200 | | | | -35 |

Now from table may observe that

$$\sum f_i = 200$$

$$\sum f_i d_i / \sum f_i = -35$$

$$\text{Mean } \bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} \times h = a + \frac{\sum f_i d_i}{\sum f_i} \times h$$

$$\bar{x} = 2750 + \frac{-35}{200} \times 500 = 2750 - 87.5$$

$$\bar{x} = 2750 - 87.5$$

$$\bar{x} = 2662.5$$

So mean monthly expenditure was Rs. 2662.5

17. The given distribution shows the number of runs scored by some top batsmen of the world in one day international cricket matches.

| Runs scored | No of batsmen | Runs scored | No of batsmen |
|-------------|---------------|---------------|---------------|
| 3000 – 4000 | 4 | 7000 – 8000 | 6 |
| 4000 – 5000 | 18 | 8000 – 9000 | 3 |
| 5000 – 6000 | 9 | 9000 – 10000 | 1 |
| 6000 – 7000 | 7 | 10000 – 11000 | 1 |

Find the mode of the data.

Soln: From the given data we may observe that maximum class frequency is 18 belonging to class interval 4000 – 5000.

So, modal class = 4000 – 5000

Lower limit (l) of modal class = 4000

Frequency (f) of modal class = 18

Frequency (f₁) of class preceding modal class = 4

Frequency (f₂) of class succeeding modal class = 9

Class size (h) = 1000

$$\text{Now mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h = 4000 + \frac{18 - 4}{2 \times 18 - 4 - 9} \times 1000$$

$$= 4000 + \frac{18 - 4}{2(18) - 4 - 9} \times 1000$$

$$= 4000 + (14000 / 23)$$

$$= 4000 + 608.695$$

$$= 4608.695$$

So mode of given data is 4608.7 runs

Exercise 7.6: Statistics

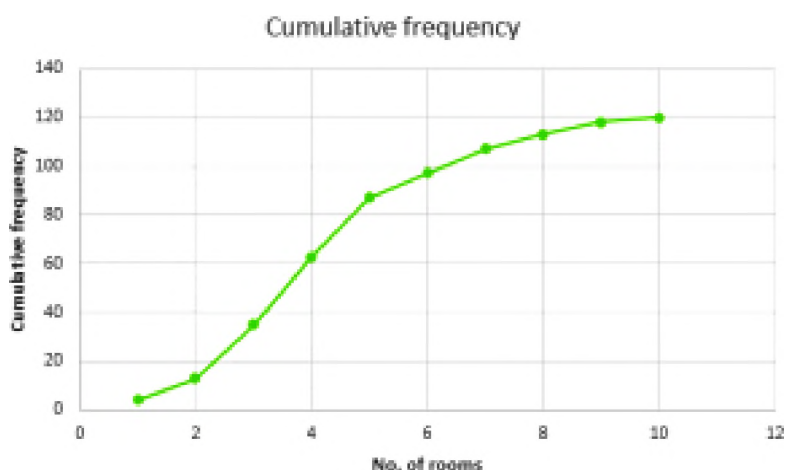
Q.1 Draw an ogive by less than the method for the following data:

| No. of rooms | No. of houses |
|--------------|---------------|
| 1 | 4 |
| 2 | 9 |
| 3 | 22 |
| 4 | 28 |
| 5 | 24 |
| 6 | 12 |
| 7 | 8 |
| 8 | 6 |
| 9 | 5 |
| 10 | 2 |

Soln:

| No. of rooms | No. of houses | Cumulative Frequency |
|--------------------------|---------------|----------------------|
| Less than or equal to 1 | 4 | 4 |
| Less than or equal to 2 | 9 | 13 |
| Less than or equal to 3 | 22 | 35 |
| Less than or equal to 4 | 28 | 63 |
| Less than or equal to 5 | 24 | 87 |
| Less than or equal to 6 | 12 | 99 |
| Less than or equal to 7 | 8 | 107 |
| Less than or equal to 8 | 6 | 113 |
| Less than or equal to 9 | 5 | 118 |
| Less than or equal to 10 | 2 | 120 |

We need to plot the points (1,4) , (2,13) , (3,35) , (4,63) , (5,87) , (6,99) , (7,107) , (8,113) , (9,118) , (10,120), by taking upper class limit over the x-axis and cumulative frequency over the y-axis.



Q.2): The marks scored by 750 students in an examination are given in the form of a frequency distribution table:

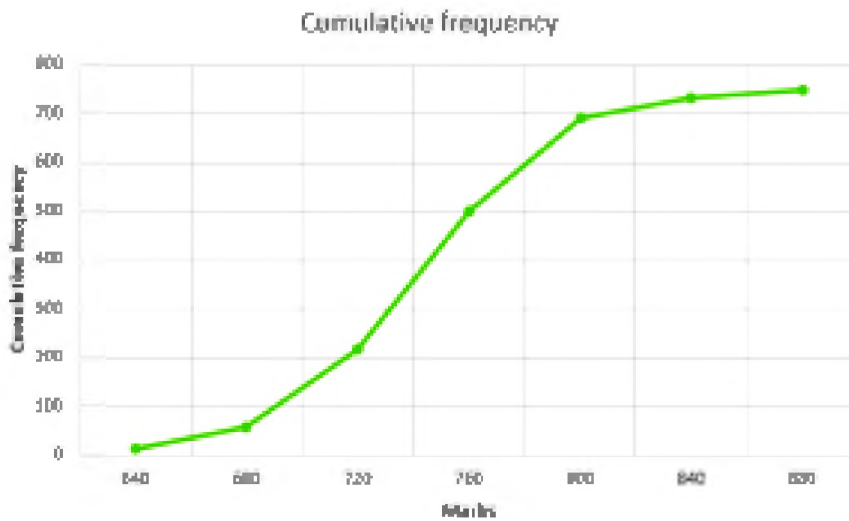
| Marks | No. of Students |
|---------|-----------------|
| 600-640 | |

| | |
|----------------|------------|
| 640-680 | 16 |
| 680-720 | 45 |
| 720-760 | 156 |
| 760-800 | 284 |
| 800-840 | 172 |
| 840-880 | 59 |
| | 18 |

Soln:

| Marks | No. of Students | Marks less than | Cumulative Frequency |
|---------|-----------------|-----------------|----------------------|
| 600-640 | 16 | 640 | 16 |
| 640-680 | 45 | 680 | 61 |
| 680-720 | 156 | 720 | 217 |
| 720-760 | 284 | 760 | 501 |
| 760-800 | 172 | 800 | 693 |
| 800-840 | 59 | 840 | 732 |
| 840-880 | 18 | 880 | 750 |

Plot the points (640-16), (680, 61), (720,217), (760,501), (800,673), (840,732), (880,750) by taking upper class limit over the x-axis and cumulative frequency over the y-axis.



Q.3) Draw an ogive to represent the following frequency distribution:

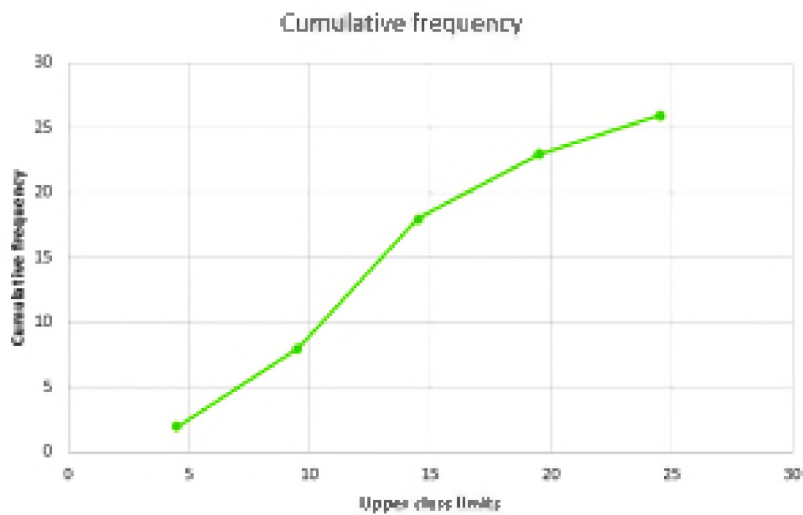
| Class-interval | 0-4 | 5-9 | 10-14 | 15-19 | 20-24 |
|-----------------|-----|-----|-------|-------|-------|
| No. of students | 2 | 6 | 10 | 5 | 3 |

Soln:

The given frequency distribution is not continuous, so we will first make it continuous and then prepare the cumulative frequency:

| Class-interval | No. of Students | Less than | Cumulative frequency |
|----------------|-----------------|-----------|----------------------|
| 0.5-4.5 | 2 | 4.5 | 2 |
| 4.5-9.5 | 6 | 9.5 | 8 |
| 9.5-14.5 | 10 | 14.5 | 18 |
| 14.5-19.5 | 5 | 19.5 | 23 |
| 19.5-24.5 | 3 | 24.5 | 26 |

Plot the points (4.5, 2), (9.5, 8), (14.5, 18), (19.5, 23), (24.5, 26) by taking the upper class limit over the x-axis and cumulative frequency over the y-axis.



Q.4) The monthly profits (in Rs) of 100 shops are distributed as follows:

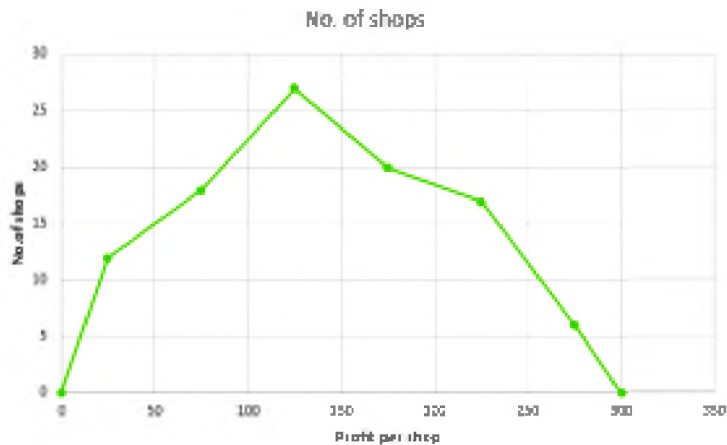
| Profits per shop: | 0-50 | 50-100 | 100-150 | 150-200 | 200-250 | 250-300 |
|-------------------|------|--------|---------|---------|---------|---------|
| No of shops: | 12 | 18 | 27 | 20 | 17 | 6 |

Draw the frequency polygon for it

Soln:

We have

| Profit per shop | Mid-value | No. of shops |
|-----------------|-----------|--------------|
| Less than 0 | 0 | 0 |
| 0-60 | 30 | 12 |
| 60-120 | 90 | 18 |
| 120-180 | 150 | 27 |
| 180-240 | 210 | 20 |
| 240-300 | 270 | 17 |
| 300-360 | 330 | 6 |
| Above 300 | 360 | 0 |



Q.5) The following distribution gives the daily income of 50 workers of a factory:

| Daily income (in Rs): | 100-120 | 120-140 | 140-160 | 160-180 | 180-200 |
|-----------------------|---------|---------|---------|---------|---------|
| No of workers: | 12 | 14 | 8 | 6 | 10 |

Convert the above distribution to a 'less than' type cumulative frequency distribution and draw its ogive.

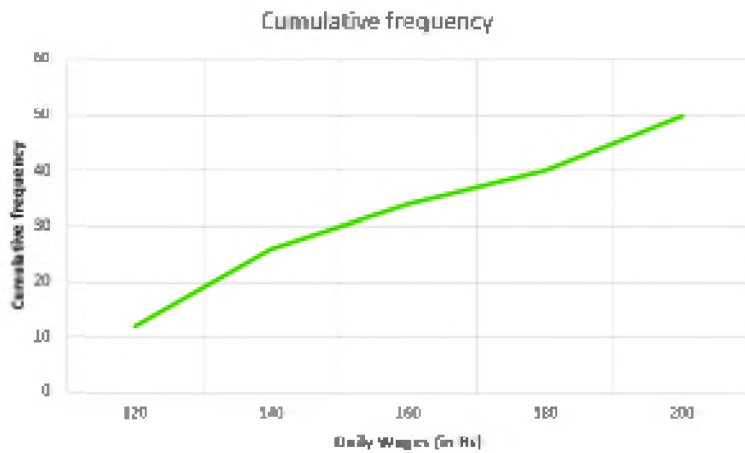
Soln:

We first prepare the cumulative frequency table by less than method as given below

| Daily income | Cumulative frequency |
|--------------|----------------------|
| <120 | 12 |
| <140 | 26 |
| <160 | 34 |
| <180 | 40 |
| <200 | 50 |

Now we mark on x-axis upper class limit, y-axis cumulative frequencies.

Thus we plot the point (120,12)(140,26)(160,34)(180,40)(200,50).



Q.6) The following table gives production yield per hectare of wheat of 100 farms of a village:

| Production yield: | 50-55 | 55-60 | 60-65 | 65-70 | 70-75 | 75-80 in kg per hectare |
|-------------------|-------|-------|-------|-------|-------|-------------------------|
| No of farms: | 2 | 8 | 12 | 24 | 38 | 16 |

Draw 'less than' ogive and 'more than' ogive

Soln:

Less than method:

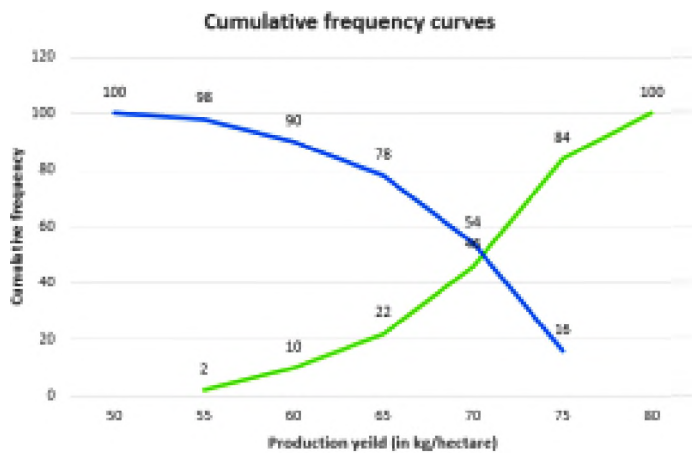
Cumulative frequency table by less than method

| Production yield | Number of farms | Production yield more than | Cumulative frequency |
|------------------|-----------------|----------------------------|----------------------|
| 50-55 | 2 | 55 | 2 |
| 55-60 | 8 | 60 | 10 |
| 60-65 | 12 | 65 | 22 |
| 65-70 | 24 | 70 | 46 |
| 70-80 | 38 | 75 | 84 |

| | | | |
|-------|----|----|-----|
| 75-80 | 16 | 80 | 100 |
|-------|----|----|-----|

Now we mark on x-axis upper class limit, y-axis cumulative frequencies.

We plot the point (50,100) (55, 98) (60, 90) (65, 78) (70, 54) (75, 16)



Q.7) During the medical check-up of 35 students of a class, their weight recorded as follows:

| Weight (in kg) | No of students |
|----------------|----------------|
| Less than 38 | 0 |
| Less than 40 | 3 |
| Less than 42 | 5 |
| Less than 44 | 9 |
| Less than 46 | 14 |
| Less than 48 | 28 |
| Less than 50 | 32 |
| Less than 52 | 35 |

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula.

Soln: Less than method

It is given that

On x-axis upper class limits. Y-axis cumulative frequency

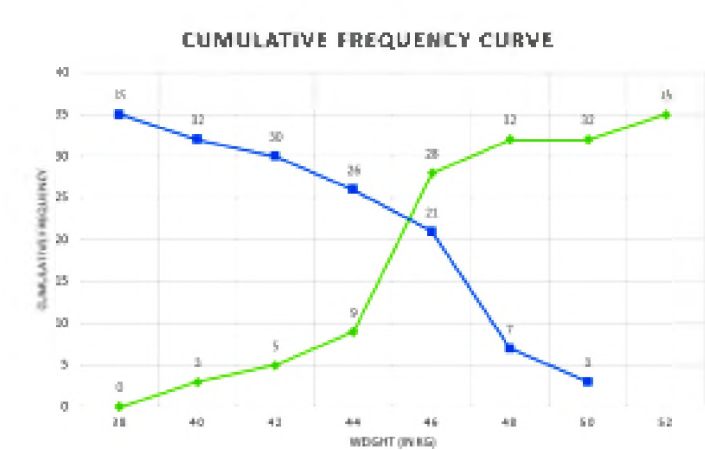
We plot the points (38,0) (40,3)(42,5)(44,9)(46,4)(48,28)(50,32)(52,35)

More than method: cumulative frequency

| Weight | No. of students | Weight more than | Cumulative frequency |
|--------|-----------------|------------------|----------------------|
| 38-40 | 3 | 38 | 34 |
| 40-42 | 2 | 40 | 32 |
| 42-44 | 4 | 42 | 30 |
| 44-46 | 5 | 44 | 26 |
| 46-48 | 14 | 46 | 21 |
| 48-50 | 4 | 48 | 7 |
| 50-52 | 3 | 50 | 3 |

X-axis lower class limits on y-axis cf

We plot the points (38,35)(40,32)(42,30)(44,26)(46,26)(48,7)(50,3)



We find the two types of curves intersect at a point P. From point P perpendicular PM is draw on x-axis

The verification,

We have

| Weight (in kg) | No. of students | Cumulative frequency |
|----------------|-----------------|----------------------|
| | | |

| | | |
|-------|----|----|
| 36-38 | 0 | 0 |
| 38-40 | 3 | 3 |
| 40-42 | 2 | 5 |
| 42-44 | 4 | 9 |
| 44-46 | 5 | 28 |
| 46-48 | 14 | 32 |
| 48-50 | 4 | 32 |
| 50-0 | 3 | 35 |

Now, $N = 35$

$$N_2 = 17.5 \frac{N}{2} = 17.5$$

The cumulative frequency just greater than $N_2 \frac{N}{2}$ is 28 and the corresponding class is 46 – 48

Thus 46 – 48 is the median class such that

$$L = 46, f = 14, C_1 = 14, h = 2$$

$$\text{Median} = L + \frac{\frac{N}{2} - C_1}{f} \times h$$

$$= 46 + \frac{17.5 - 14}{14} \times 2$$

$$= 46 + 7/14$$

$$46.5$$

Median = 46.5 kg

Hence verified

Q.9) The following table shows the height of trees:

| Height | No. of trees |
|--------------|--------------|
| Less than 7 | 26 |
| Less than 14 | 57 |

| | |
|---------------------|------------|
| Less than 21 | 92 |
| Less than 28 | 134 |
| Less than 35 | 216 |
| Less than 42 | 287 |
| Less than 49 | 341 |
| Less than 56 | 360 |

Draw 'less than 'ogive and 'more than 'ogive

Soln:

By less than method

| Height | No. of trees |
|--------------|--------------|
| Less than 7 | 26 |
| Less than 14 | 57 |
| Less than 21 | 92 |
| Less than 28 | 134 |
| Less than 35 | 216 |
| Less than 42 | 287 |
| Less than 49 | 341 |
| Less than 56 | 360 |

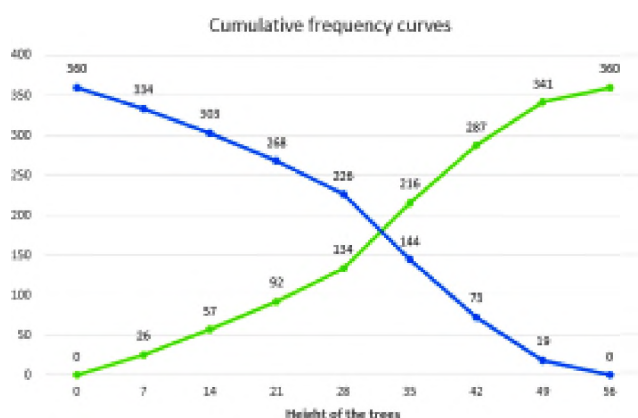
Plot the points (7,26) , (14,57) , (21,92) , (28,134) , (35,216) , (42,287) , (49,341) , (56,360) by taking upper class limit over the x-axis and cumulative frequency over the y-axis.

By more than method:

| Height | Frequency | Height more than | C.F. |
|--------|-----------|------------------|------|
| 0-7 | 26 | 0 | 360 |
| 7-14 | 31 | 5 | 334 |
| 14-21 | 35 | 10 | 303 |
| 21-28 | 42 | 15 | 268 |

| | | | |
|-------|----|----|-----|
| 28-35 | 82 | 20 | 226 |
| 35-42 | 71 | 25 | 144 |
| 24-49 | 54 | 30 | 73 |
| 49-56 | 19 | 35 | 19 |

Take lower class limit over the x-axis and CF over the y-axis and plot (0,360) , (7,334) , (14,303) , (21,268) (28,226) , (35,144) , (42,73) , (49,19).



Q.10) The annual profits earned by 30 shops of a shopping complex in a locality give rise to the following distribution:

| Profit (In lakhs In Rs) | Number of shops (frequency) |
|--------------------------|-----------------------------|
| More than or equal to 5 | 30 |
| More than or equal to 10 | 28 |
| More than or equal to 15 | 16 |
| More than or equal to 20 | 14 |
| More than or equal to 25 | 10 |
| More than or equal to 30 | 7 |
| More than or equal to 35 | 3 |

Draw both ogive for the above data and hence obtain the median.

Soln:**More than method**

| Profit (In lakhs in Rs) | Number of shops (frequency) |
|--------------------------|-----------------------------|
| More than or equal to 5 | 30 |
| More than or equal to 10 | 28 |
| More than or equal to 15 | 16 |
| More than or equal to 20 | 14 |
| More than or equal to 25 | 10 |
| More than or equal to 30 | 7 |
| More than or equal to 35 | 3 |

Now, we mark on x-axis lower class limits, y-axis cumulative frequency

Thus, we plot the points (5,30)(10,28)(15,16)(20,14)(25,10)(30,7) and (35,3)

Less than method

| Profit in lakhs | No of shops | Profits less than | C.F |
|-----------------|-------------|-------------------|-----|
| 0-10 | 2 | 10 | 2 |
| 10-15 | 12 | 15 | 14 |
| 15-20 | 2 | 20 | 16 |
| 20-25 | 4 | 25 | 20 |
| 25-30 | 3 | 30 | 23 |
| 30-35 | 4 | 35 | 27 |
| 35-40 | 3 | 40 | 30 |

Now we mark the upper class limits along x-axis and cumulative frequency along y-axis.

Thus we plot the points (10,2)(15,14)(20,16)(25,20)(30,23)(35,27)(40,30)

We find that the two types of curves intersect of P from point L it is drawn on x-axis

The value of a profit corresponding to M is 17.5. Hence median is 17.5 lakh

